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Volatility Regimes and Cross-market Correlation Dynamics in the Determination of the Optimal International Equity Portfolio

Ming-Yuan Leon Li
Associate Professor
Department of Accountancy and Graduate Institute of Finance and Banking
National Cheng Kung University, Taiwan
No.1, Ta-Hsueh Road, Tainan 701, Taiwan
TEL: 886-6-2757575 Ext. 53421, FAX: 886-2744104
E-mail: lmyleon@mail.ncku.edu.tw

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ABSTRACT

A Markov-switching technique is used to examine co-movement dynamics between domestic and global stock markets and to develop a state-varying approach to the design of an international portfolio. A bivariate Markov switching ARCH (SWARCH) model is designed to evaluate four possible combinations of the volatility states which can characterize both the domestic and world markets, thus allowing the generation of state-varying portfolio loadings on the basis of 4-state correlations. The following conclusions are drawn. First, the domestic and global markets are more strongly correlated when both simultaneously find themselves in the same state of volatility. Conversely, this correlation is weaker when a different volatility state characterizes each market. Furthermore, the circumstances in which both the domestic and global stock markets simultaneously experience high levels of volatility prove strongest cross-market correlations, and result in least effective at reducing the risk stemming from international portfolio diversification. Next, this study determines state-varying portfolio loadings which prove effective at the task of asset allocation, particularly under conditions of market volatility. Finally, filtering structural changes out from the variance-switching process leads to a considerable decrease of the time variation involved in generating state-varying portfolio loadings, which, in turn, leads to lower transaction costs when compared to the performance of the conventional GARCH model.

JEL classification: G11, G15

Keywords: International diversification; volatility; cross-market correlation; Markov-switching model; GARCH

I. Introduction

A state-varying approach forms the backbone of this analysis of the dynamics underlying interactions among global stock markets. The purpose of the study is to determine the optimal allocation of international equity to a portfolio consisting of differentially weighted stock assets on the basis of their state-varying characteristics. More specifically, four configurations of the volatility states of domestic and world market returns are determined by the bivariate Markov-switching ARCH (hereafter, SWARCH) model established by this study. Portfolio loading under state-varying conditions can then be determined on the basis of the measures of the correlations among the four states. Data from the stock markets of G7 countries are employed to demonstrate the feasibility of the proposed model. Finally, a comparative analysis is performed between portfolio weights calculated using either the bivariate SWARCH or the bivariate GARCH model, in order to identify differences between state-varying and time-varying characteristics.[1]

The benefits of investing in international equity are widely recognized, especially with regards to increased efficiency and decreased risk. Research into the international diversification of investment portfolios often emphasizes that the potential of such behavior for bringing about a reduction of systematic risk below the

[1] Modeling of the volatility of stock returns is commonly done using an ARCH (auto-regressive conditional heteroskedasticity) model originally proposed by Engle (1982) and subsequently extended by Bollerslev's (1986) GARCH (generalized ARCH) model.

level incurred by investing in domestic securities alone results from structural and cyclical differences among various economies.[2] While risk reduction is a well-known phenomenon in this context, its extent may be attenuated by the strength of the correlation existing between domestic and global markets. In fact, such a cross-market correlation and the potential for risk reduction are negatively correlated; that is, the more strongly are correlated different markets, the smaller is the potential reduction in risk. In terms of portfolio diversification, an additional difficulty is ascertaining the optimal weights to be given to the different stock assets under consideration. Market correlation strength also exerts a considerable influence on the determination of such portfolio loadings. Consequently, this study considers the accurate measurement of the strength of the correlation between domestic and international markets to be a key issue in international stock allocation.

A number of studies have contributed to the literature on the diversification of international investments. For example, Byers and Peel (1993) examine stock market interdependence and returns on investment resulting from international diversification on the basis of data from the national stock markets of Japan, the Netherlands, the U.K., the U.S., and West Germany. Chang (2001) provides evidence that there exist

[2] International diversification offers benefits which stem from different circumstances. For example, individual national stock markets vary considerably in terms of returns and risks, a fact which becomes clear when one considers the persistence of barriers to international diversification, such as market segmentation, insufficient liquidity, exchange rate controls, and inaccessibility of information. As a result of such circumstances, cross-market correlations have been relatively weak.

long-run benefits for investors in Taiwan who include in their portfolios shareholdings from the equity markets of the country's major trading partners, including Hong Kong, Japan, South Korea, Thailand, and the U.S. Investing heavily in an emerging market such as Thailand seems to imply increased risk, a concern that is addressed by Fifield *et al.* (2002) in their discussion of the costs and benefits of investing in emerging stock market equities.

Market integration has been another focal point of the research, some of which investigates the effects of recent developments in Europe. Kempa and Nelles (2001), for instance, analyze international correlations and excess returns in European stock markets both before and after the EMU's coming into effect. The countries of the industrialized world in general are frequently targeted by such research. Tahai *et al.* (2004) employ a vector error correction model to investigate financial co-integration among G7 equity markets. Heimonen (2002) evaluates stock market integration between Finland, Germany, Japan, the U.K., and the U.S., from the point of view of the international investor.

Finally, the phenomenon of market co-movements has also been analyzed, in particular by Syriopoulos (2004), who shows that long-run co-movements exist among various Central European stock markets. It is suggested that such behavior implies that the potential to diversify risk and attain superior portfolio returns by investing in different Central European markets may be limited for international investors.

As opposed to these several studies, this paper emphasizes that international stock markets are much more closely correlated when markets are struggling through a period of crisis. Along similar lines of reasoning, country portfolio returns have been shown to correlate much more strongly during turbulent times in securities markets (e.g., King and Wadhwani, 1990; Erb *et al.* 1994; Lin *et al.* 1994; Longin and Solnik, 1995; Karolyi and Stulz, 1996; Boyer *et al*. 1999; Longin and Solnik, 2001; Jacquier and Marcus, 2001; Ang and Bekaert, 2002; Forbes and Rigobon, 2002; Bae *et al.* 2003 and Das and Uppal, 2004). Another case in point is the fact that declines of most major stock indices occurred concurrently during the crash of October 1987 (Roll, 1988). At such times of chaos and instability, the need is more acutely felt for the benefits derived from diversification-related risk reduction. However, as previously noted, the stronger correlations observed among markets suggest that these benefits are largely lost at such times.

Upon closer scrutiny of studies such as these, it is possible to discern a common, time-based factor from which stem global stock price fluctuations. For instance, Longin and Solnik (1995) identify an intensification of cross-market correlations at times of crisis when markets are highly volatile. This implies that inevitable changes in volatility over the long run adversely impact the advantages of international diversification. This contention is supported by a study of the linkages between the U.S. and Latin American stock markets, which was conducted using co-integration models highlighting structural shifts in long-run dynamics (Fernandez-Serrano and Sosvilla-Rivero, 2003). The primary conclusion drawn from this analysis is that gains derived from international diversification are limited in the case of long-term investments.

This study attempts to extend existing research and to improve on some of its perceived shortcomings. On the one hand, many studies consider time to be the primary explanatory variable in the analysis of non-constant correlations; consequently, researchers proceed either by dividing the entire sampling period into several sub-periods or by employing a time-varying approach. In contrast to such an approach, a state-varying framework here forms the basis of our investigation of cross-market correlation dynamics.

On the other hand, it is generally accepted that market volatility is a crucial element of the study of stock market behavior. In times of crises, financial or political upheavals are associated with increased market volatility, as revealed by the examination of realized stock returns. In terms of the overseas extension of portfolio diversification, distinct high/low volatility (HV/LV) regimes have been shown to exist in international stock markets (Hamilton and Susmel, 1994; Ramchand and Susmel, 1998; and Li and Lin, 2003). Unfortunately, the accurate characterization of various volatility regimes is impossible in the context of frequently encountered research based on simplified settings with constant parameters.

Based on such observations, therefore, we develop the bivariate SWARCH model to determine whether a HV/LV regime exists at certain dates, and to measure the size of the co-movements occurring among domestic and world markets in various states of volatility. The model also serves the purpose of devising a strategy for the development of a state-varying international portfolio.

Investigating much the same relationships as those under scrutiny in the present study, such as Ramchand and Susmel (1998) observed that the correlation between the U.S. domestic stock market and other national markets was strongest when the former was experiencing high volatility; Ang and Bekaert (2002) used a regime switching model to analyze the effect of time-varying correlation on the benefit of international diversification and demonstrated that the existence of high correlation and high volatility bear market regime does not negate the benefits of international diversification; Das and Uppal (2004) characterized the returns on international equity market by jumps occurring at the same time and studied the effect of the systematic risk induced by the jumps. They found that while systematic risk affects the allocation of wealth between the riskless and risky assets, it has a small effect on the composition of the portfolio of only-risky assets, and reduces marginally the gains to a US investor from international diversification.

While based on a similar framework consisting of state variables, the analysis presented in this study differs from theirs as follows. First, this study adopts the CAPM perspective to investigate two independent components of the risks at play in international diversification: (1) the world market risk factor (systematic risk), and (2) the domestic market risk factor (nonsystematic risk). This bifurcation leads to a further distinction with the study of Ramchand and Susmel. Whereas they utilized a dual-state specification (HV/LV states) in their analysis of cross-market correlations, we consider both the market-wide (systematic risk) and the idiosyncratic (nonsystematic risk) components to be subject to distinct volatility state switching processes. Therefore, different combinations of volatility states are analyzed on the basis of four-way correlations among domestic and world markets.

In sum, a novel approach is presented to research into issues related to volatility regimes and market correlations with the aim of designing a strategy enabling the development of a dynamic investment portfolio. Its novelty results from the fact that the determination of the optimal allocation ratio of stock assets relies heavily on being able to accurately measure domestic-global market correlations, an aspect addressed in this study.

Finally, the following are examples of issues addressed in this study:

- Is the magnitude of market correlations consistent across various combinations of volatility regimes? If no such consistency exists, what are the relationships among various market volatility regimes and correlations?

- If cross-market correlations are state dependent, can a state-varying framework help investors design a more effective strategy for investing in international stock?

- Is the state-varying framework more or less valid under conditions of highly variable states?

- Is the loading of a portfolio based on state-varying considerations associated with the higher magnitude of variations in time?

- Are the benefits of the reduction in risk stemming from the international diversification of an investment portfolio consistent among various market volatility states? If not, what are the relationships among such benefits and market volatility states?

To our knowledge, few studies have addressed such significant issues regarding the determination of international equity asset allocation.

The remainder of this paper is organized as follows. Section 2 outlines the models used in this study: (1) a conventional bivariate GARCH model for the development of a time-varying framework, and (2) a bivariate SWARCH model in the context of the state-varying approach. Subsequently, Section 3 presents the empirical results and provides economic and financial explanations. Finally, Section 4 draws conclusions.

2. Model Specifications

2.1 Bivariate GARCH Model: Optimal Asset Allocation via a Time-Varying System

Tools facilitating the design of effective strategies for the dynamic allocation of stock assets have long been sought, a fact attested to by the considerable interest generated by recent relevant research. In the existing research, it is invariably the case that frameworks are elaborated to enable the determination of dynamic portfolio loadings, the estimation of which is based on time-varying variance-covariance matrix derived from ARCH or GARCH models. Furthermore, in academic circles, it is generally acknowledged that correlations among domestic and global market returns, along with corresponding return variances, are the key factors in building the optimal international equity portfolio. Consequently, the possibility of designing a more effective portfolio of international stock assets based on consideration of the dynamic relationships existing between volatility regimes and cross-market correlations is investigated herein by the bivariate GARCH model relying on time-varying characteristics as a benchmark. The following is an outline of the specifications of the model and its potential limitations.

Given that r_t^y and r_t^x stand for the return rates in global and domestic stock markets, respectively, the bivariate GARCH model used in this study can be specified as follows. First:

$$r_t^y = \theta_0^y + \sum_{i=1}^{i=p} \theta_i^y r_{t-i}^y + e_t^y \qquad (1)$$

$$r_t^x = \theta_0^x + \sum_{i=1}^{i=p} \theta_i^x r_{t-i}^x + e_t^x \qquad (2)$$

where e_t^y and e_t^x are residuals at time *t*. These residuals are depicted by the following equation:

$$e_t \mid \psi_{t-1} = \begin{bmatrix} e_t^y \\ e_t^x \end{bmatrix} \mid \psi_{t-1} \sim BN(0, \quad H_t) \qquad (3)$$

where Ψ_{t-1} refers to the information available at time *t-1*; BN denotes the bivariate normal distribution; and H_t is a time-varying 2x2 positive definite conditional variance-covariance matrix. This matrix is depicted as:

$$H_t = \begin{bmatrix} h_t^y & h_t^{y,x} \\ h_t^{y,x} & h_t^x \end{bmatrix} \qquad (4)$$

and its specific elements are specified using the following equations:

$$h_t^y = \alpha_0^y + \sum_{j=1}^{q} \alpha_{t-j}^y (e_{t-j}^y)^2 + \sum_{l=1}^{m} \beta_{t-l}^y h_{t-l}^y \quad (5)$$

$$h_t^x = \alpha_0^x + \sum_{j=1}^{q} \alpha_{t-j}^x (e_{t-j}^x)^2 + \sum_{l=1}^{m} \beta_{t-l}^x h_{t-l}^x \quad (6)$$

$$h_t^{y,x} = \rho \times (h_t^y \cdot h_t^x)^{1/2} \quad (7)$$

where h_t^y and h_t^x are the conditional variances of global and domestic market returns, respectively. Finally, $h_t^{y,x}$ is a measure of conditional covariance, and ρ is the correlation coefficient between global and domestic market returns.

The bivariate GARCH model is plagued by questionable assumptions predicated on notions of continuation and stability. The first problem lies in the fact that, in the context of the ARCH/GARCH family of models, the variance revealed on one date is held to be a function of that observed on the previous date. However, in much research on the estimation of stock return series, it is argued that these models do not account suitably well for structural change and that the presence of unidentified structural breaks is behind what is usually deemed to be a high level of persistence inherent in the models (see Diebold, 1986, and Lamoureux and Lastrapes, 1990). Nelson (1991) and Engle and Mustafa (1992) further showed that the unexpected occurrence of events like the Crash of 1987 cannot be explained on the basis of ARCH/GARCH models.[3]

[3] For additional details on this, see Bollerslev and Engle (1986), Schwert and Seguin (1990), Bollerslev *et al.* (1992), Hamilton and Susmel (1994), and Li and Lin (2003).

Another difficulty with the GARCH model is its assumption of the stability of domestic-global market return correlations. In terms of the conventional modeling framework, the variances (h_t^y and h_t^x) and covariance ($h_t^{y,x}$) are considered to be time-varying, whereas the correlation coefficient (the ρ parameter) is treated as a constant, as designed in Bollerslev (1990), Baillie and Bollerslev (1990), and Chan *et al.* (1991), among many others. However, developing an international portfolio relies heavily on the accurate measurement of these variances and correlations, both of which, in reality, may vary. Moreover, the weight attributed to each domestic/global asset in a given portfolio is a function of these key parameters combined.

Two portfolio establishment strategies are here considered in order to examine the potential usefulness of a state-varying system in devising more effective international stock portfolios. The first strategy rests on the existence of a minimum variance, the second on a given variance. The first portfolio, characterized by two stock assets and a minimum variance, is designed on the basis of the following equations[4]:

$$w_t^x = [h_t^y - \rho \cdot (h_t^y \cdot h_t^x)^{1/2}]/[h_t^x + h_t^y - 2 \cdot w_t^x \cdot w_t^y \cdot \rho \cdot (h_t^y \cdot h_t^x)^{1/2}], \quad 0 \le w_t^x \le 1 \qquad (8)$$

[4] Ramchand and Susmel (1998), following in the footsteps of French and Poterba (1991) and Tesar and Werner (1992), develop minimum variance portfolios based on an exponential utility function. This study also examines portfolios formed on the basis of specific targets, and the empirical results are quite similar. Therefore, in an effort to economize space, we present only the empirical results from our observations of portfolios with a minimum variance.

$$w_t^y = 1 - w_t^x \qquad (9)$$

where w_t^x and w_t^y represent the weights attributed to domestic and world assets, respectively. The conditional variances of domestic and world stock returns are designated as h_t^x and h_t^y, respectively. Finally, ρ is the correlation coefficient between domestic and world stock returns.

On the other hand, the two-asset portfolio with a given variance is developed according to the following equations:

$$w_t^x + w_t^y = 1, \quad 0 \le w_t^x \le 1 \quad \textit{and} \quad 0 \le w_t^y \le 1 \qquad (10)$$

$$\bar{h}_{p,t} = (w_t^x)^2 \cdot h_t^x + (w_t^y)^2 \cdot h_t^y + 2 \cdot \rho \cdot w_t^x \cdot w_t^y \cdot (h_t^x \cdot h_t^x)^{1/2} \qquad (11)$$

where $\bar{h}_{p,t}$ denotes the given variance.

For reasons of feasibility, the average conditional variance of domestic and world markets serves as a benchmark in this case, it is calculated according to $\bar{h}_{p,t} = (1/2) \cdot (h_t^x + h_t^y)$, and it is the basis of the determination of portfolio returns at a given level of risk.[5]

[5] This study also examines the results obtained by varying the settings of this given variance; for example, if $\bar{h}_{p,t} = (1/3) \cdot h_t^x + (2/3) \cdot h_t^y$. However, the conclusion remains the same.

2.2 Bivariate SWARCH Model: Optimal Asset Allocation via a State-Varying System

The volatility of stock returns have been shown to be substantially higher than average at certain times on the basis of the careful examination of the rates of realized stock returns. Furthermore, a highly persistent volatility is associated with lower predictive accuracy in many studies employing GARCH models. For instance, two influential such studies attribute the high level of persistence in GARCH models to structural changes emerging in the process of estimating stock return volatility (Diebold, 1986; Lamoureux and Lastrapes, 1990).

The partitioning of the period covered by the sample into distinct phases on the basis of dummy variables allows not only the control of volatility levels but also the conceptualization of these structural changes. However, defining such dummy variables rests on the subjective selection of cutoff dates. The resultant arbitrariness prevents the accurate prediction of the timing of the structural changes.

This study contends that a model making it possible to identify the timing of structural changes on the basis of the available data alone would better account for the behavior of stock returns. To this end, a bivariate volatility-switching model is developed in order to accurately identify discrete jumps in stock return volatility. Such a model identifies the HV/LV regimes of domestic and global market returns at each distinct point in time. A state-varying framework is then developed for devising asset allocation strategies.

Given that r_t^y and r_t^x represent the return rates in global and domestic stock markets, respectively, the bivariate SWARCH model used in this study can be specified according to the following equations:

$$r_t^y = \theta_0^y + \sum_{i=1}^{i=p} \theta_i^y r_{t-i}^y + e_t^y \tag{12}$$

$$r_t^x = \theta_0^x + \sum_{i=1}^{i=p} \theta_i^x r_{t-i}^x + e_t^x \tag{13}$$

$$e_t \mid \psi_{t-1} = \begin{bmatrix} e_t^y \\ e_t^x \end{bmatrix} \mid \psi_{t-1} \sim BN(0, \quad H_t) \tag{14}$$

$$H_t = \begin{bmatrix} h_t^y & h_t^{y,x} \\ h_t^{y,x} & h_t^x \end{bmatrix} \tag{15}$$

These variables are generally defined as those of the GARCH model described in the previous section were, with one crucial difference, which lies in the fact that the variance-covariance matrix (H_t) of the SWARCH model is characterized as both time-varying and state-dependent. Therefore, the following equations determine the conditional variance settings of global (Eq. 16) and domestic (Eq. 17) market returns:

$$\frac{h_t^y}{g_{s_t^y}^y} = \alpha_0^y + \sum_{j=1}^{q} \alpha_{t-j}^y \frac{(e_{t-j}^y)^2}{g_{s_{t-j}^y}^y} \tag{16}$$

$$\frac{h_t^x}{g_{s_t^x}^x} = \alpha_0^x + \sum_{j=1}^{q} \alpha_{t-j}^x \frac{(e_{t-j}^x)^2}{g_{s_{t-j}^x}^x} \qquad (17)$$

where s_t^y and s_t^x are unobservable state variables, for which the possible values are 1, 2, 3,… n, and which are employed to indicate the volatility regime at time t characterizing global and domestic market returns, respectively.

This model, thus developed on the basis of two dimensions, is intended as an extension of Hamilton and Susmel's (1994) one-dimensional SWARCH model. It should be noted, however, that the bivariate framework used herein involves extremely intensive computations. Therefore, in an effort to keep time requirements within reasonable bounds, two volatility regimes only are examined: The HV and LV states for both global market returns ($s_t^y = \{1, 2\}$) and domestic market returns ($s_t^x = \{1, 2\}$).

Certain manipulations are required while developing such a model. For instance, without reducing generalization, g_1^y and g_1^x, the scale coefficients for regime I, are normalized to unity; whereas $g_2^y>1$ and $g_2^x>1$ in the case of regime II. More specifically, the dynamics underlying the conditional variances involved in regime I are accounted for by the conventional ARCH (q) process. On the other hand, the conditional variances at play in regime II are defined as g_2^y and g_2^x times those of regime I in the equations for global and individual market returns, respectively. In the special case in which $g_1^y = g_2^y = 1$ and $g_1^x = g_2^x = 1$, the two residual terms, $e^y{}_t$ and $e^x{}_t$, remain consistent with the conventional ARCH (q) process.

Based on the assumption that two distinct volatility regimes characterize each market return component, both global and domestic, covariance is modeled according to the possible occurrence of four different volatility states. Such covariance can be formulated as follows:

$$h_t^{y,x} = \rho_{s_t^y, s_t^x} \times (h_t^y \cdot h_t^x)^{1/2} \tag{18}$$

Furthermore, the possibility of different combinations of volatility regimes brings about a general system consisting of 4-state correlations: (1) $\rho_{1,1}$, in which case both global and individual market returns are in a state of low volatility (s_t^y=1 and s_t^x=1; World=LV and Indl.=LV); (2) $\rho_{2,1}$, where volatility is high in the case of global market returns but low for individual market returns (s_t^y=2 and s_t^x =1; World=HV and Indl.=LV); (3) $\rho_{1,2}$, where volatility is low in the case of global market returns and high for individual market returns (s_t^y=1 and s_t^x=2; World=LV and Indl.=HV); and (4) $\rho_{2,2}$, in which case both global and individual market returns are in a state of high volatility (s_t^y =2 and s_t^x =2; World=HV and Indl.=HV).

Based on this system thus described, the construction of the latent variable s_t on the basis of the separate latent processes s_t^y and s_t^x can be visually depicted as follows:

s_t=1: if s_t^y=1 and s_t^x=1 -or- World=LV and Indl.=LV

s_t=2: if s_t^y=2 and s_t^x=1 -or- World=HV and Indl.=LV

s_t=3: if s_t^y=1 and s_t^x=2 -or- World=LV and Indl.=HV

s_t=4: if s_t^y=2 and s_t^x=2 -or- World=HV and Indl.=HV

As was assumed in the case of the univariate analysis, s_t is an unobservable state variable; however, in the present case, the state variable s_t is associated with possible outcomes of 1, 2, 3 and 4. Furthermore, this state variable is held to follow a first-order 4-state Markov chain model formulated as follows:

$$p(s_t = j \mid s_{t-1} = i) = p_{ij} \qquad (19)$$

The transition probability matrix of this model can be depicted as follows:

$$P = \begin{bmatrix} p_{11} & p_{21} & p_{31} & p_{41} \\ p_{12} & p_{22} & p_{32} & p_{42} \\ p_{13} & p_{23} & p_{33} & p_{43} \\ p_{14} & p_{24} & p_{34} & p_{44} \end{bmatrix} \qquad (20)$$

The inclusion of multiple volatility states in the present model requires further explanation. While it is true that, theoretically, the number of states under consideration in a study such as this one can be extended to infinity, in actuality, their number is limited. In the case of the present model, which is set to accommodate two regimes only, a 2×2 transition probability matrix, with only 2 unknown probability parameters, appears to be all that is required. However, the complexity inherent in volatility switching models increases considerably when they are applied to the estimation of multivariate systems. Consequently, accounting for the regime switching process in the 4-state model employed in this study necessitates the adoption of a 4×4 transition probability matrix and the estimation of 12 unknown probability parameters.

Moreover, in this study, the world market index is considered to serve as a synthetic holding of global assets, and the idiosyncratic risk pertaining to domestic market assets is understood to decrease to an arbitrarily low level. In line with such reasoning, variance of global and domestic market returns is here seen as being subject to the distinct processes of the switching of volatility states characterizing each of these types of market component. Thus, the volatility state variable for global market returns (s_t^y) is expected to be independent of that for individual market returns (s_τ^x) in the case of all t and τ. According to this hypothesis, the transition probability matrix for the variable s_t of this 4-state model can be depicted as:

$$P = \begin{bmatrix} p_{11}^y \cdot p_{11}^x & p_{21}^y \cdot p_{11}^x & p_{11}^y \cdot p_{21}^x & p_{21}^y \cdot p_{21}^x \\ p_{12}^y \cdot p_{11}^x & p_{22}^y \cdot p_{11}^x & p_{12}^y \cdot p_{21}^x & p_{22}^y \cdot p_{21}^x \\ p_{11}^y \cdot p_{12}^x & p_{21}^y \cdot p_{12}^x & p_{11}^y \cdot p_{22}^x & p_{21}^y \cdot p_{22}^x \\ p_{12}^y \cdot p_{12}^x & p_{22}^y \cdot p_{12}^x & p_{12}^y \cdot p_{22}^x & p_{22}^y \cdot p_{22}^x \end{bmatrix} \tag{21}$$

where (p_{11}^y, p_{22}^y) and (p_{11}^x, p_{22}^x) are the transition probabilities for s_t^y and s_t^x, respectively, and are derived on the basis of the following equations:

$$p(s_t^y = 1 \mid s_{t-1}^y = 1) = p_{11}^y, \quad p(s_t^y = 2 \mid s_{t-1}^y = 2) = p_{22}^y \tag{22}$$

$$p(s_t^x = 1 \mid s_{t-1}^x = 1) = p_{11}^x, \quad p(s_t^x = 2 \mid s_{t-1}^x = 2) = p_{22}^x \tag{23}$$

A comparison of the two ostensibly identical 4-state switching models depicted in Eqs. 20 and 21 reveal significantly different in the numbers of population parameters required to estimate. In the case of the general model presented in Eq. 20, s_t follows a 4-state Markov chain, the transition matrix of which is restricted by the condition that each column must sum to unity, hence the need for 12 probability parameters. On the other hand, the estimation of only 4 (2+2) probability parameters is required in the restricted 4-state model presented in Eq. 21.

Of course, the specifications underlying the elaboration of transition probability matrix implied by Eq. 20 are very general in that they encompass various interactions among the variance states of market returns at both the global and domestic levels (see Hamilton and Lin, 1996). On the other hand, the necessity of accommodating a very specific hypothesis about underlying variance states undeniably limits to a considerable extent the generalizability of the preferred model presented in Eqs. 21-23. Despite such limitations, the latter restricted model was decided upon in order to ensure the manageability of the 4-state system studied herein and to facilitate the convergence of the maximum likelihood procedure.[6] This choice is further justified by the very act of postulating the independence of the systematical and non-systematical risk components examined in this study.

As has already been suggested, the bivariate SWARCH model developed in this study bears a strong resemblance to those presented in previous studies (see Ramchand and Susmel, 1998; Edwards and Susmel, 2003). However, whereas such prior attempts rest on the assumption that the correlations examined are a function only of the state of a single return series and employ a setting restricted to dual correlations, we go one step further to define a setting of 4-state correlations.

With regard to the observed data, $r_t = (r_t^y\ , r_t^x)$ is taken to represent a 2×1 vector

[6] This study has also developed a bivariate SWARCH model according to a 4X4 transition probability matrix. Although such an unrestricted model is very general and encompasses different interactions among the variance states of both global and domestic market returns, it is intractable and its likelihood function fails to converge in some cases. Moreover, some elements of the 4X4 switching probability matrix considerably approach zero.

consisting of the return rates of global and domestic markets, and the log-likelihood function can be formulated as follows:

$$L(\Omega)=\sum_{t=1}^{T}\log f(r_t \mid r_{t-1},r_{t-2},\cdots,r_{t-q};\Omega) \tag{24}$$

, where Ω is a vector of population parameters comprising the unknown values of the following elements: p^y_{11}, p^y_{22}, p^x_{11}, p^x_{22}, θ^y_0, $\theta^y_1,\ldots,\theta^y_p$, θ^x_0, $\theta^x_1,\ldots,\theta^x_p$, α^y_0, $\alpha^y_1,\ldots,\alpha^y_q$, α^x_0, $\alpha^x_1,\ldots,\alpha^x_q$, g^y_1, g^y_2, g^x_1 and g^x_2. The numerical maximization of the log-likelihood with respect to Ω follows from these restrictions[7]: $g_1^y=1$, $g_1^x=1$, $p^y_{11}+p^y_{12}=p^y_{21}+p^y_{22}=1$, $p^x_{11}+p^x_{12}=p^x_{21}+p^x_{22}=1$, $0<p^y_{11}$ and $p^y_{22}<1$, $0<p^x_{11}$ and $p^x_{22}<1$.

Furthermore, although the state variables (s_t^y and s_t^x) which have been incorporated into the present model are unobservable, the specific probability of the volatility regime existing at each point in time can be calculated on the basis of both the observed data and estimates of the maximum likelihood ($\hat{\Omega}$).

The optimization of a portfolio is determined on the basis of such regime probabilities. More specifically, the optimal portfolio loadings at any specific point in time are identified as the average loadings in different states of volatility, which are weighted according to their respective filtering probabilities.[8]

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[7] See Hamilton and Lin (1996) for a relevant detailed discussion.
[8] Although the state variables (s^y_t and s^x_t) cannot be observed at time t, the probability of the existence of a specific regime at any given time can be estimated based on the data itself. For example, when the

As a final point on the design of the model used herein, it must be noted that the discrete state variable is given two possible outcome values to reflect HV/LV regimes, and the *q*-order obtained on the basis of prior-period error squares is used in determining the conditional variance settings (see Eqs. 16 and 17). This implies that the possible existence of $2^{(q+1)}$ volatility states for every univariate stock return gained on each date must be taken into account, which, in turn, suggests the need to consider the possibility of facing $(2^{q+1})^2$ variance-state combinations at each point in time in the case of a bivariate SWARCH model. As a result, the value of *q* is set at 1 in the following empirical analysis, thus bringing about the necessity of considering 16 possible variance-sate combinations on each date.

In sum, the bivariate SWARCH model defined in this section holds certain key advantages over the conventional GARCH model presented earlier. First, discrete adjustments made to the volatility switching system in the latter model resolve difficulties stemming from the unrealistic assumption of the existence of a single, constant correlation posited in the context of the former model. In other words, the single measurement of correlation carried out with the GARCH model corresponds to four such measurements in the case of the SWARCH model (compare Eqs. 7 and 18).

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information set used to carry out this estimation includes signals dated up to time *t*, the regime probability is a filtering probability. It is also possible to use the information set encompassing the overall sample period to provide an estimation at time *t* of the regime probability, which is then called a smoothing probability. Finally, a predicting probability refers to the regime probability based on an ex ante estimation, in which case the information set includes signals dated up to period *t-1*.

Second, the bivariate SWARCH model is set in such a way that the measurement of variance is sensitive to changes according to both time and state. Specifically, the variance dynamics at play in the context of an LV regime (regime I) are accounted for by the conventional ARCH (q) process incorporated into the SWARCH model (see Eqs. 16 and 17).

Finally, the use of a discrete jump process in the bivariate SWARCH model allows the examination of structural changes occurring as a result of variations in scale from g^y_1 $(=1)$ to $g^y_2(>1)$ in the case of global market returns, and from g^x_1 $(=1)$ to $g^x_2(>1)$ for domestic market returns. Thus being able to account for events reflecting structural change effectively corrects the problem of a highly persistent volatility which characterizes the conventional ARCH/GARCH models.

3. Empirical Results

3.1 Data

The overall data represent two levels of observation, the global and the national. In the case of the latter category, the domestic market component of the model is expressed in terms of the stock price indices of each individual G7 country, which were collected from the Data Stream database. These indices, which account for a minimum of 80% of the stock market capitalization of each country, are valued in U.S. dollar. On the other hand, the following two indices are seen as proxies for assessing the global market component of the model: (1) the world index, which is provided by Morgan Stanley Capital International (MSCI hereafter), and (2) the equally weighted world stock index (EWW hereafter), which averages various market indices by attributing an equal weight to each one of them. The data thus collated was used to create 14 (7X2) international portfolios which form the core of the analysis which follows this section.

With regard to the specifics of the data, each entry corresponds to a recorded observation, and each two consecutive such observations are separated by a week (Wednesday to Wednesday). Collectively, these observations cover the period from January 1980 to May 2007. The entire sample consists of 1,426 observations. The descriptive statistics and correlation matrices performed on the data are presented in

Table 1.

3.2 Parameter Estimates of the Bivariate GARCH and SWARCH Models

Model settings must be determined before being able to proceed to the analysis. To this end, the order of the auto-regression analysis of the returns on global and domestic markets was set at unity (p=1) in the case of both the GARCH and SWARCH bivariate models (See Eqs. 1 and 2 (12 and 13) for the GARCH (SWARCH)). Furthermore, a conventional setting of GARCH (1, 1), with values of both q and m equaling one (See Eqs. 5 and 6), was incorporated into the former model in order to account for the variance dynamics of stock returns.[9] On the other hand, the setting of q=1 in the case of the latter model indicates that the number of orders in ARCH was set at unity (See Eqs. 16 and 17).[10] It must finally be noted that OPTIMUM, a GAUSS application package, was applied, in conjunction with the BFGS algebra functions built into GAUSS, to the task of deriving the negative minimum likelihood function[11].

The parameter estimates of the bivariate GARCH model are listed in Table 2.

9 As is well known, the GARCH (1, 1) model with q=1 and l=1 is the most commonly used setting for depicting stock return variance dynamics.

10 With such a simple structure consisting of a single lag ARCH component, 18 parameters must be estimated. However, a more general structure involving a higher-order ARCH term considerably increases the number of parameters requiring estimation. In the case of a SWARCH system set at q=2, for example, it is necessary to take into consideration 8 (=2^3) possible states of any univariate stock return and 64 (=8^2) possible states of a bivariate structure. Nevertheless, the higher-order ARCH parameter estimates in the SWARCH model do not appear to differ significantly from zero in most cases. Therefore, for the sake of expediency, the results in cases where model settings include higher lag orders are not reported herein.

11 It is possible to effectively derive the maximum value of the non-linear likelihood functions by methods of Boyden, Fletcher, Goldfarb, and Shanno (BFGS) algebra. See Luenberger (1984) for details.

First, the sum of the two GARCH parameter estimates, namely α^y_1 and β^y_1 for world market returns and α^x_1 and β^x_1 for domestic market returns, approximates unity in most cases. For instance, in the case of MSCI-U.S., the sum of α^y_1 and β^y_1 is 0.9362 (=0.0706 + 0.8656), while that of α^x_1 and β^x_1, reflecting domestic U.S. market returns, is 0.9473 (=0.0649 + 0.8824).

This finding provides some support for the notion that GARCH models are handicapped by the inability to account for structural changes during the estimation period and thus suffers from a high persistence problem in variance settings. In an effort to correct such weaknesses, this study holds that structural changes, which are related to market volatility, are responsible for such high persistence of variance in GARCH models.

This claim is premised on the fact that a distinguishing characteristic of the SWARCH model is its ability to account for structural changes in the dynamics of stock return variances on the basis of a discrete state variable incorporated into the model design. The parameter estimates calculated for world (g_2^y) and domestic (g_2^x) market returns in the case of our bivariate SWARCH model illustrate the validity of this contention (see Table 3). The first point that must be made is that, in all cases, these estimates exceed unity by a highly significant margin. Again taking the case of MSCI-U.S. as an example, the estimated value of g_2^y is 2.0875 with a standard deviation of 0.1472, and the estimated value of g_2^x is 2.4436 with a standard deviation of 0.1827. These estimates reflect differences in market volatility between regimes I and II. Specifically, the level of volatility inherent in the world (domestic) market returns of MSCI-U.S. under regime II is held to be 2.0875 (2.4436) times higher than that of regime I.

Furthermore, the 99% confidence level of the estimations of g_2^y and g_2^x do not overlap with those of g_1^y and g_1^x, that is, the values at unity. Consequently, in terms of the SWARCH model designed in this study, s_t^y=2 (the state variable for global markets) and s_t^x=2 (the state variable for domestic markets) are confidently held to stand for the HV state, whereas s_t^y=1 and s_t^x=1 represent the LV state.

This study further incorporates into its model a 4-state correlation system based on the examination of various combinations of variance states. Corresponding correlation estimates are shown to diverge significantly among various state combinations in all cases. More specifically, with three exceptions only (MSCI-Canada, EWW-Canada and EWW-Japan), rejection of the null hypothesis postulating the existence of identical correlations is warranted on the basis of the LR statistical analysis[12] at a 1% level of significance[13] (see the last row of Table 3).

[12] To test this null hypothesis, this bivariate SWARCH model is first estimated on the basis of 4-state correlations and $L(H_A)$, representing the log likelihood function. The model is then estimated assuming the existence of a single constant correlation ($\rho_{1,1}=\rho_{2,1}=\rho_{1,2}=\rho_{2,2}=\rho$), which allows for the subsequent derivation of the log likelihood function of the restricted model, $L(H_0)$. Finally, this function is used to carry out a likelihood ratio test, LR=-2[$L(H_0)$- $L(H_A)$]. In terms of the null hypothesis, this test displays a χ^2 distribution with 3 (=4-1) degrees of freedom.

Furthermore, in terms of values calculated by averaging all 14 cases under examination, correlation estimates of $\rho_{2,2}$ = 0.7993, under HV states in the case of both global and domestic markets ("HV-HV" hereafter), and $\rho_{1,1}$ = 0.7905, given LV states in both types of market ("LV-LV" hereafter), were obtained. These values exceed those calculated for the other two possible state combinations ($\rho_{2,1}$ = 0.6164, under World=HV and Indl.=LV, "HV-LV" hereafter; and $\rho_{1,2}$ = 0.5790, under World=LV and Indl.=HV, "LV-HV" hereafter).

Let us recall that the GARCH model is premised on the existence of a single constant correlation which is represented by ρ. Therefore, in terms of model comparisons, the corresponding average correlation estimate for the GARCH model, ρ = 0.7009, is both lower than the values of $\rho_{2,2}$ and $\rho_{1,1}$, and higher than those of $\rho_{2,1}$ and $\rho_{1,2}$, all of which were calculated according to the SWARCH model. Clearly then, the GARCH model setting involving a single correlation measure underestimates the actual magnitude of co-movements between global and domestic markets in the case of two of the possible state combinations: "HV-HV" and "LV-LV", and overestimates it for the other two combinations: "HV-LV" and "LV-HV".[14]

This conclusion requires some elucidation. The "HV-HV" combination reflects

[13] The LR statistic of the three exceptions (MSCI-Canada, EWW-Canada and EWW-Japan) is calculated to have a 10% level of significance.

[14] Undeniably, these relationships among correlation estimates are not perfectly consistent across all cases. For instance, in the case of MSCI-U.K., both $\rho_{1,1}$ (0.7933) and $\rho_{2,2}$ (0.6661) are lower than $\rho_{2,1}$ (0.8057).

circumstances in which both global and domestic stock markets concurrently experience extremely volatile stock price movements. This study posits that risk-averse investors are expected to adjust their behavior under such conditions according to a "stock-to-bond" asset reallocation process, which involves a general redirection of capital flows from stock markets to non-stock markets, such as bond markets, at such chaotic times. The perceptions of heightened risk which inevitably accompany highly volatile market conditions encourage investors to employ an across-market-hedging strategy, based on which they begin to simultaneously short sell their stock assets and purchase other assets, such as bonds. Under "HV-HV" market conditions; this strategy causes stock prices on both the domestic and global markets to move in similar directions, thus increasing the magnitude of co-movements between these different markets. Experimentally, this contention is validated by the finding that the maximum correlation estimate corresponds to the "HV-HV" state combination in 10 of 14 cases (see Table 3).

Both the "LV-HV" and "HV-LV" combinations, on the other hand, give rise to an inter-market "stock-to-stock" asset reallocation process. In the case of the "LV-HV" state, in which only the domestic market experiences a high level of volatility, a flight of capital is triggered from the domestic stock market to the global stock market. Under these conditions, risk-averse investors tend to employ a within-market-hedging strategy, which leads them to short sell their domestic stock market assets, and to purchase global stock market assets in their place. Consequently, stock prices on both the domestic and global markets move in opposite directions, thus reducing the magnitude of co-movements between these different markets. A similar line of reasoning can be applied to the “HV-LV” combination. In this case, the domestic stock market presents relatively less risk; accordingly, capital flows in the opposite direction towards the domestic stock market. Furthermore, given that both the “stock-to-stock” asset reallocation process and the underlying strategy are the same here as in the preceding case, a reduction in the magnitude of cross-market correlations occurs.

To sum up, two asset reallocation processes have been described which are held to explain variations in the strength of cross-market correlations existing under different states of market volatility. Specifically, the “stock-to-bond” process is primarily associated with the “HV-HV” state, and it considerably increases the magnitude of inter-market co-movements. By contrast, in both the “LV-HV” and “HV-LV” states, the strength of cross-market correlations is significantly reduced as a result of the “stock-to-stock” reallocation process.

There remains a state combination which has not yet been addressed. In the case of the “LV-LV” combination, both the domestic and global stock markets are held to be relatively stable, thus presenting minimal risk to wary investors. Under such conditions, the aforementioned asset reallocation processes are deemed inapplicable because unnecessary. As a result, in terms of the analyses conducted in this study, the value of the correlation estimate of the "LV-LV' state is lower than that of the "HV-HV" state and higher than that of both the "HV-LV" and "LV-HV" states.

Finally, it must be noted that it is impossible to compare the SWARCH and GARCH models by conventional LR statistical analyses because the models are not strictly nested[15]. As a result, the AIC and Schwarz value statistics were applied to the evaluation of relative model performance.[16] The comparison of Tables 2 and 3 reveals that the bivariate SWARCH model outperforms the bivariate GARCH model in all cases in terms of these two measures of statistical effectiveness.

3.3 Asset Allocation Effectiveness: In-sample Tests

It is well known that the effectiveness of an international stock portfolio relies on the prior careful consideration of a market specific factor, the level of variance within both the domestic and global markets, and an inter-market factor, the strength with which these different markets are correlated. If the weight attributed to each asset

[15] The Markov-switching mechanism and the GARCH model are widely considered impossible to combine, with the exception of Gray (1996), who established a system which permits such a combination on the basis of specific assumptions. See Hamilton and Susmel (1994) for a more in-depth discussion of this issue.

[16] See Schwarz (1978) for the Schwarz value and Akaike (1976) for AIC.

included in the portfolio is a function of both these factors, then obtaining accurate correlation/variance measurements is obviously crucial. This study investigates the feasibility of generating an international portfolio on the basis of state-varying correlations. This possibility is put to the test in the context of different strategies for the development of stock portfolios, one of which defines variance in relation to its minimum level, whereas the other attributes to the variance a given value.

In the case of the first strategy, that is the design of a minimum variance portfolio consisting of both domestic and world stock assets (See Eqs. 8 and 9), the in-sample evaluation of the effectiveness of asset allocation strategies is detailed in Table 4. These results indicate differences between portfolios designed according to either of the models being studied; that is, the return mean for a SWARCH-based portfolio is higher than that for a GARCH-based portfolio. However, although this difference is prevalent, having been observed in 10 of 14 cases, it is statistically insignificant.[17] Furthermore, the variance of the SWARCH-based portfolio is significantly lower (at a 1% level) than that of the GARCH-based portfolio in all cases[18].

The annual return/risk (R/R) ratio of the GARCH-based portfolio is here used as a benchmark against which is measured the degree to which the R/R ratio achieved by

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[17] The case of MSCI-Canada is exceptional in that the return mean for the GARCH-based portfolio exceeds that for the SWARCH-based portfolio at a 5% level of significance.

[18] Notably, this investigation devises a statistical test for examining whether the variances of the international portfolio varies significantly between using the bivariate GARCH and SWARCH models. Please see the appendix for the detail.

the SWARCH model can be deemed a success. As shown in panel (c) of Table 4, the relative improvement of the R/R ratio in the case of the portfolio designed on the basis of the SWARCH model suggests that the portfolio development strategy involved is more effective than its GARCH-based counterpart (again with the exception of MSCI-Canada).

The obvious conclusion is that the modeling of market correlations and corresponding variances on the basis of variations in volatility states is both statistically significant and strategically effective. Moreover, reductions in risk, rather than increases in return mean, are to thank for the benefits stemming from such improved effectiveness.

The second strategy under investigation proposes investing in a variety of domestic and world assets to build a given variance portfolio (See Eqs. 10 and 11). Table 5 presents the effectiveness of in-sample asset allocation based on this strategy. As was true of the first strategy, the empirical results here indicate that the SWARCH-based portfolio outperforms the GARCH-based portfolio in most cases. In particular, the portfolio designed on the basis of the SWARCH model shows a relative improvement of the R/R ratio in 10 of 14 cases.

3.4 Comparative Analysis across Various Volatility Regimes

The findings thus far demonstrate that the bivariate SWARCH model considerably outperforms the GARCH model on the basis of general measures encompassing the entire period concerned in this study. On the other hand, the related question of how effectively the SWARCH model allocates stock assets during distinct periods marked by differences in volatility remains unresolved.

In weekly increments, Fig. 1 depicts global and domestic market returns in the illustrative case of MSCI-U.S., which manifest increased volatility at certain times. Such findings warrant the use of the SWARCH model, which facilitates the investigation of variance dynamics by methods of a jump process. In this way, the unsettled issue mentioned above is addressed by the specific design of the SWARCH model used in this study.

The specific volatility state existing at each point in time is here identified on the basis of the estimation of the filtering probability of a particular state combination and a maximum value criterion. For instance, if, at time t, the estimated filtering probability of the "HV-HV" state combination (that is, $s_t = 4$, or $s_t^y = 2$ and $s_t^x = 2$) exceeds that of the three alternative states, then an "HV-HV" state is said to exist at this point in time. SWARCH-based estimates of the filtering probabilities of specific volatility state combinations for the illustrative case of MSCI-U.S. are shown in Fig. 2. On a more general point, the market as a whole is here considered to be in a specific state of volatility if the estimated filtering probability at any given time is relatively high. Table 6 lists the observation percentage of return rates at which were observed the different volatility state combinations. Appearing in 9 of 14 cases, the "LV-LV" state is set as the maximum value.[19]

With the average standard errors calculated for both domestic and world assets serving as benchmarks, the margin by which risk is reduced in the context of various market volatility state combinations was assessed. The portfolio on which were based the observations consists of equally weighted domestic and global assets. The results, presented in Table 7, reveal that the "HV-HV" state combination represents the minimum value in 10 of 14 cases. Furthermore, attempts at determining the extent to which reductions in risk stemming from the international diversification of an investment portfolio are beneficial are likely to overestimate the advantages perceived in the case of an "HV-HV" state combination and underestimate them in the other cases.

Attention must be brought to the fact that the "HV-HV" state combination corresponds to both the least effective conditions under which risk is reduced by international portfolio diversification, and the context in which the strongest market return correlations exist (in terms of the value of $\rho_{2,2}$; see Table 3).[20] This observation

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[19] This finding is consistent with the notion that the LV state is more persistent than the HV state. This point is illustrated in Table 3, where the values of p^y_{11}/p^x_{11} generally exceed those of p^y_{22}/p^x_{22}.

[20] The finding according to which the "HV-HV" state is associated with the smallest reduction in the level of risk is quite consistent, with the exception of MSCI-Italy, for which the value of $\rho_{2,2}$ is second to last.

suggests that setting a model with a single, constant correlation value is likely to underestimate the actual strength of cross-market correlations and to overestimate the effects of risk reduction under conditions of an "HV-HV" state combination.

The "HV-HV" state combination is thus shown to frequently be the context in which the cross-market correlation is strongest. This phenomenon is supported by factual evidence. On the one hand, volatility in global markets consistently coincides with global financial or economic crises. On the other hand, a recession has been shown to precede high levels of volatility in domestic markets (Chen, Roll and Ross, 1986; Schwert, 1990; Chen, 1991; Hamilton and Lin, 1996). At times when both global crises and domestic recessions coincide, this study contends that the influence of shared factors related to the world market dominate that of the idiosyncratic factors characterizing domestic markets. Consequently, cross-market correlations rise dramatically under such circumstances.

Estimates of risk reduction further serve as a basis for comparing the relative performance of the GARCH and SWARCH models evaluated in this study. Underlying this comparison is the variance of a GARCH-based portfolio, providing a benchmark against which can be measured the extent to which a SWARCH-based portfolio is successful at reducing investment risk. The resultant differences (expressed in percentages), implicit in Table 8, make it possible for the effectiveness of both models at reducing risk to be contrasted in the context of various volatility state combinations. Table 8 also suggests a further comparison on the basis of the type of portfolio considered; that is, panel (a) lists effectiveness ratings in the case of a minimum variance portfolio, whereas panel (b) does the same for a given variance portfolio.

In terms of risk reduction effectiveness, a general impression of the performance of domestic stock market assets under conditions of low variance can be got from the integration of the “HV-LV” and “LV-LV” state combinations listed in both panels of Table 8. Therefore, by averaging all 14 cases in panel (a), the SWARCH model is shown to be 13.48% more effective than the GARCH model at reducing risk in the “HV-LV” state, and 7.21% more effective in the “LV-LV” state. In contrast, the integration of the “HV-HV” and “LV-HV” state combinations reflects how domestic markets react to conditions of high variance. Again, the SWARCH model proves effective at reducing risk both in the “HV-HV” (6.79%) and the “LV-HV” (5.10%) states.

Based on these observations, the largest reductions in investment risk result from the global market being in a state of high volatility, regardless of whether the level of the concurrent volatility of domestic markets is high or low. This conclusion suggests that a state-varying framework is the most effective methods of diversifying an investment portfolio in the context of a highly volatile global market.[21]

This study further investigates the optimization of the loading of a portfolio in terms of time variation. To this end, the standard error of the portfolio weights determined on the basis of the SWARCH model is divided by that for the GARCH model to calculate a relative measure of time variation in loading optimization (see Table 9). In evaluating the results, a value lower than unity suggests that portfolio loadings determined on the basis of the SWARCH model are less volatile than those derived from the GARCH model, whereas a higher such volatility level results from a value higher than unity.

Panel (a) of Table 9 presents the results of the analysis of a minimum variance portfolio consisting of both domestic and global assets. The examination of the period under investigation as a whole reveals that the measures of relative time variation exceed unity in 13 of 14 cases. However, when different combinations of volatility states are taken into account, SWARCH-based portfolio loadings are stabilized to a considerable extent, particularly in the "HV-HV" state, where the relative time variation falls below unity in 10 of 14 cases.[22]

Fig. 3 depicts the optimal loading of a minimum variance portfolio with global stock market assets in the illustrative case of MSCI-U.K. According to this figure, the

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[21] This conclusion remains valid when a similar analysis of the data for a given variance portfolio is performed (see panel (b) of Table 8).

[22] Furthermore, our analyses show that the relative time variation falls below unity in 5 of 14 cases in the "LV-LV" state, 6 of 14 in the "HV-LV" state, and 3 of 14 in the "LV-HV" state.

GARCH model frequently produces a corner solution; that is, it accounts for 100% of global asset loadings. This observation explains why GARCH-generated portfolio loadings tend to be more stable than those produced by the SWARCH model. However, SWARCH-based portfolio loadings stabilize, in the case of the "HV-HV" state in particular, after structural changes in the volatility of returns have been filtered out. In contrast, the conventional GARCH model with a time-varying parameter fails to account for structural changes in return variance dynamics. Therefore, it generates hedge ratio estimates with greater time variation, particularly in times marked by the "HV-HV" state.

Because the strategy for the development of an investment portfolio based on the setting of a minimum level of variance repeatedly produces a corner solution, the alternative strategy of fixing the variance at a given level is here considered. The optimal portfolio loading corresponding to this alternative strategy for the illustrate case of MSCI-U.K. is depicted in Fig. 4, which shows a sizable decrease in the frequency of GARCH-produced corner solutions. Furthermore, the portfolio loading determined on the basis of the SWARCH model appears to be much more stable than that generated by the GARCH model. This finding of the greater stability of the SWARCH model in the case of a given variance portfolio is further validated by the findings listed in panel (b) of Table 9. In the case of data covering the period under observation as a whole, the relative time variation is below unity in 10 of 14 cases.[23]

There are practical implications to the findings reported in this section. The higher levels of time variation associated with certain portfolio loading strategies imply that the holders of such portfolios must frequently reposition their investments, thus incurring higher transaction costs. Such strategies are here considered a direct consequence of the failure of the GARCH model to account for structural changes in return variance dynamics. Furthermore, investors opting for a GARCH-inspired strategy for the diversification of their investment portfolios must cope with a relatively ineffective asset allocation in addition to the extra transaction costs mentioned above.

On the other hand, it was demonstrated that filtering out the structural dynamics underlying volatility switching by the SWARCH model considerably decreases time variation. This finding leads to the conclusion that the determination of portfolio loadings based on the consideration of variations in volatility states corrects the shortcomings inherent in the GARCH model, thus implying a more effective allocation of assets and lower transaction costs.

3.5 Asset Allocation Effectiveness: Out-of-Sample Test

The relative effectiveness of two models in the design of strategies for the

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[23] Furthermore, the relative time variation is lower than unity in 5 of 14 cases in the “LV-LV” state, in 12 of 14 in the “HV-LV” state, in 6 of 14 in the “LV-HV” state, and in 10 of 14 in the “HV-HV” state.

international diversification of investment portfolios has so far been evaluated on the basis of historical data. However, the primary concern of investors lies in determining the likelihood of the future success of alternative portfolio development models. In an attempt to resolve this apparent contradiction, this study further evaluates the effectiveness of asset allocation strategies by out-of-sample tests using the rolling-estimation technique.[24]

According to the requirements of this technique, the final 200 weekly observations of the sample (representing approximately four-year's worth of data) were omitted from the initial sample, and formed the sequential inputs of the rolling estimation. The following is a more detailed description of how this technique is carried out.

To begin with, at time t, 1,226 (equal to 1,426 minus 200) historical data are incorporated into the estimation of the model parameters[25], which is carried out on the basis of $\{r_{t-i}^{y}, r_{t-i}^{x}\}_{i=1}^{1,226}$. The ex ante regime probabilities are then estimated, to be used in the determination of the portfolio weights in the case of each volatility state. At time t+1, a multi-step procedure based on the state-varying SWARCH model is

[24] The out-of-sample tests conducted in this study resemble those reported by West, Edison and Cho (1993).

[25] How far back it is necessary to go when using historical data in model estimation presents some difficulties in the field of portfolio management, and involves a trade-off between the quantity and the freshness of the information submitted to analyses. This study contends that estimations based on samples of limited size are less accurate because there is less chance of encountering data reflecting the rarer, more extreme market movements that are associated with the greatest losses. Consequently, the nonlinear SWARCH model used here requires the inclusion of much data and is thus more likely to account for such rare and extreme behavior.

employed to determine the optimal portfolio loadings. First, the estimation of the regime probabilities at time *t*+1 is carried out on the basis of population parameters which were estimated at time *t*, $\hat{\Omega}_t$. The one-step-ahead forecasts thus derived are then used to determine the loading of a SWARCH-based portfolio at time *t*+1.[26]

With the addition of each subsequent observation, the same multi-step procedure is repeated. Furthermore, following each such addition, the sample is rolled; that is, the deliberate deletion of the oldest observation coincides with the addition of the most recent one. This technique thus fixes the sample size at 1,226 observations.

In the case of a minimum variance portfolio consisting of both domestic and world stock assets, the out-of-sample evaluation of the effectiveness of asset allocation strategies is detailed in Table 10. The corresponding comparative analysis of the two models was performed on six selected cases.[27] Contrary to expectations, however, the SWARCH model underperforms the GARCH model in most cases. More specifically, the variance of the SWARCH-based portfolio is higher than that of its GARCH-based counterpart in 5 of 6 cases, and it is significantly so in two instances. In line with such findings, the R/R ratio of the portfolio designed on the basis of the SWARCH model actually decreases in relation to the GARCH

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[26] To facilitate the process of convergence, this study uses the estimate for a given individual period as the initial value in the non-linear estimation of the period immediately following it.

[27] As is well known, estimations carried on the basis of the non-linear SWARCH model are time-consuming. In order to economize time, the out-of-sample test presented in this study is performed on six cases only.

benchmark.

Fig. 5 depicts regime probability forecasts derived on the basis of the SWARCH model in the illustrative case of MSCI-Japan.[28] Thus, the likelihood of the different volatility state combinations occurring is predicted. Fig. 6 also consists of results in the case of MSCI-Japan. Specifically, GARCH-derived estimates of the rolling loadings of a portfolio with global assets are presented in Fig. 6(a), whereas those based on the SWARCH model are presented in Fig. 6(b). As was observed in the case of Fig. 3 above, Fig. 6 suggests that the GARCH model seems to frequently produce a corner solution during the process of conducting rolling estimations. This observation partially explains the relatively poor performance of the SWARCH model in the context of the out-of-sample testing of a minimum variance portfolio.

Turning now to the diversification of a given variance portfolio, Table 11 describes the effectiveness of out-of-sample asset allocation. Close examination of these results shows them to be the diametrical opposite of those for the minimum variance portfolio. First, a comparative analysis reveals the SWARCH model to be more effective than the GARCH model in most cases. In particular, the mean return for a SWARCH-based portfolio is higher than that for a GARCH-based portfolio in all selected cases.[29] Furthermore, the variance of the SWARCH-based portfolio is lower

[28] In the case of a SWARCH-based portfolio, the relative improvement of the R/R ratio for MSCI-Japan is the smallest among the six cases under investigation here.

[29] Note that only one of them is significant at the 5% level.

than that of the GARCH-based in 5 of 6 cases, and it is significantly so in three instances. Further evidence of the superior performance of the SWARCH model is provided by the finding that the R/R ratio of the portfolio designed on the basis of the SWARCH model shows improvement in relation to the GARCH benchmark in all six cases.

Again using MSCI-Japan as an illustrative case, Fig. 7(a) shows estimates of the rolling loadings of a given variance portfolio with global assets derived by the GARCH model; those based on the SWARCH model are presented in Fig. 7(b). According to the results depicted in Fig. 7, it would appear that using a given level of variance as a target on which to base portfolio design decision-making effectively reduces the likelihood of producing corner solutions, which in turn corresponds to the increased relative effectiveness of asset allocation strategies derived on the basis of the SWARCH model.

4. Conclusions and Future Research Directions

On the basis of a Markov-switching technique, this study analyzes the dynamics underlying the cross-market correlations existing between domestic and global stock markets. Various combinations of high/low volatility states characterizing both types of markets are examined with the aim, first, of investigating how variations in such correlations correspond to changes in combined volatility states, and, second, of identifying the most effective strategy for determining optimal portfolio loadings.

The following conclusions are drawn on the basis of the analyses carried out in this study.

First, the domestic and global markets are more strongly correlated when both simultaneously find themselves in the same state of volatility (i.e., “HV-HV” and “LV-LV”). Conversely, this correlation is weaker when a different volatility state characterizes each market (i.e., “LV-HV” and “HV-LV”). Furthermore, the circumstances in which both the domestic and global stock markets are simultaneously being disrupted by high levels of volatility (i.e., “HV-HV”) , on the one hand, prove least effective at reducing the risk stemming from international portfolio diversification, and, on the other, result in the strongest cross-market correlations.

Second, the determination of optimal portfolio loadings according to a state-varying framework proves a highly effective strategy for the allocation of assets, and this due to reductions in risk, rather than increases in mean returns, thus produced. This framework is fleshed out in the form of a SWARCH model, the overall superior performance of which is demonstrated by conducting in-sample tests, although out-of-sample testing shows the relative performance of the SWARCH model to be less promising, especially where corner solutions are concerned. On a related point, filtering structural changes in the volatility of returns out from the variance-switching process leads to a considerable decrease of the time variation involved in generating state-varying portfolio loadings. These reductions in time variation result in lower transaction costs when they fall below the level of the time variation involved in generating conventional time-varying loadings,.

In light of certain limitations of this study, the following are suggestions for future research.

First, the observations are made exclusively on data from the stock markets of G7 countries, thus severely limiting their generalizability. Future studies could examine a range of stock markets, especially emerging stock markets. Specifically, this suitability of the Markov-switching system rests on its sensitivity to rare and extreme price fluctuations to which emerging stock markets are much more prone than mature stock markets are. Therefore, it would be useful to conduct a comparative analysis of mature and emerging stock markets.

Second, two alternative strategies for the determination of the loading of an internationally diversified stock portfolio (i.e. state-varying loading implied by the SWARCH model and time-varying loading obtained from the GARCH model) are evaluated both independently and comparatively. However, it would be useful to carry out comparisons of these approaches with other models based on different portfolio loading designs. Finally, two variance targets are incorporated into the models studied, but other such targets could be used to reexamine the extent to which state-varying portfolio designs are effective at asset allocation.

Appendix

To examine the variances of international portfolio varies significantly between using the bivariate GARCH and SWARCH models; this investigation devises a statistical test as below. First, this study calculates the returns of international portfolio involving domestic and world market assets for each period, as follows:

$$r_t^{GARCH} = w_t^{y,GARCH} \cdot r_t^y + w_t^{x,GARCH} \cdot r_t^x, \qquad t = 1,\ldots,T \tag{1}$$

$$r_t^{SWARCH} = w_t^{y,SWARCH} \cdot r_t^y + w_t^{x,SWARCH} \cdot r_t^x, \qquad t = 1,\ldots,T \tag{2}$$

,where r_t^y and r_t^x represent return rates of global and domestic markets, respectively. Additionally, $w_t^{y,GARCH}$ and $w_t^{x,GARCH}$ ($w_t^{y,SWARCH}$ and $w_t^{x,SWARCH}$) are the optimal portfolio loadings established by the bivariate GARCH (SWARCH) model, and r_t^{GARCH} (r_t^{SWARCH}) are the corresponding return rates of international portfolio. Next, the sample variances of the two return rates: $VAR(r^{GARCH})$ for r_t^{GARCH} and $VAR(r^{SWARCH})$ for r_t^{SWARCH}, are calculated, respectively.

The test mainly examines whether the difference of the two variances, $VAR(r^{GARCH}) - VAR(r^{SWARCH})$ significantly exceeds zero. Owing to the mean value of daily stock returns being very close to zero, this study simplifies the difference between the two variances as follows:

$$E[(r^{GARCH})^2] - E[(r^{SWARCH})^2] \qquad (3)$$

Furthermore, the above equation is rewritten as follows:

$$\begin{aligned} &E[(r^{GARCH})^2] - E[(r^{SWARCH})^2] \\ &= E[(r^{GARCH})^2 - (r^{SWARCH})^2] \\ &= E[(r^{GARCH} - r^{SWARCH})(r^{GARCH} + r^{SWARCH})] \end{aligned} \qquad (4)$$

Clearly, to test if the difference of two variances is significantly greater than 0 equals test if the correlation coefficient between two return rates: $(r^{GARCH} - r^{SWARCH})$ and $(r^{GARCH} + r^{SWARCH})$ is significantly exceeding 0. To achieve this, this study calculates the following two return rates: r_t^{G-SW} and r_t^{G+SW}, as follows:

$$r_t^{G-SW} = r_t^{GARCH} - r_t^{SWARCH}, \quad t = 1,\ldots,T \qquad (5)$$

$$r_t^{G+SW} = r_t^{GARCH} + r_t^{SWARCH}, \quad t = 1,\ldots,T \qquad (6)$$

To test the significance of the correlation coefficient between r_t^{G-SW} and r_t^{G+SW}, this investigation establishes a *d* statistic, as follows:

$$d = \sqrt{T} \times \frac{(1/T) \times \sum_{t=1}^{T} (r_t^{G-SW} \times r_t^{G+SW})}{SD(r_t^{G-SW} \times r_t^{G+SW})} \tag{7}$$

, where the $(1/T)\sum_{t=1}^{T}(r_t^{G-SW} \times r_t^{G+SW})$ and $SD(r_t^{G-SW} \times r_t^{G-SW})$ are the sample mean and standard error estimate of the cross return rates, $(r_t^{G-SW} \times r_t^{G+SW})$, respectively. Furthermore, under the central limit theorem, the *d* statistic exhibits a standard normal distribution.

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Table(s)

Table 1 Summary of Main Descriptive Statistics and Correlation Matrix of Various Weekly Stock Index Returns

(a) The Descriptive Statistics for Various Stock Markets

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	MSCI-WORLD	EWW
Mean	0.1519	0.1905	0.1763	0.1947	0.1505	0.1763	0.1841	0.1755	0.1749
S.D.	2.4260	2.9417	2.9198	3.3330	3.0305	2.5149	2.1582	1.9317	2.0108
Kurtosis	2.6868	3.4016	2.3822	1.7629	1.1887	3.1246	3.9213	3.7147	2.8881
Skewness	-0.4082	-0.5817	-0.4636	-0.3203	0.1143	-0.3095	-0.4797	-0.5118	-0.6049
Maximum	11.4506	12.8290	13.9773	12.4910	14.6184	11.5638	10.3441	9.2676	9.3684
Minimum	-14.1583	-18.2848	-14.4390	-18.9359	-12.1452	-17.7245	-16.4158	-14.5267	-12.3932

(b) The Correlation Matrix

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	MSCI-WORLD	EWW
Canada	1.000	0.442	0.462	0.327	0.304	0.497	0.690	0.693	0.698
France		1.000	0.694	0.512	0.370	0.586	0.477	0.665	0.808
Germany			1.000	0.513	0.377	0.557	0.508	0.695	0.812
Italy				1.000	0.281	0.470	0.327	0.504	0.701
Japan					1.000	0.356	0.312	0.654	0.601
U.K.						1.000	0.515	0.706	0.769
U.S.							1.000	0.857	0.714
MSCI-WORLD								1.000	0.920
EWW									1.000

Notes:

1. Collected from the Data Stream database, the data represent weekly observations (taken on Wednesdays) and cover the period Jan. 1980-May 2007. The entire sample consists of 1,426 observations.
2. Data for the domestic market component of the model are the stock price indices of individual G7 countries, which account for a minimum of 80% of the stock market capitalization of each country. All market price indices are valued in U.S. dollars and were obtained from the Data Stream database.
3. This study employs two proxies for assessing the global component of the model: (1) the world index, which is provided by Morgan Stanley Capital International (MSCI), and (2) the equally weighted world (EWW) stock index, which averages various market indices by attributing an equal weight to each one of them. The data thus collated was used to create 14 (7X2) international portfolios on which were performed a series of analyses.

Table 2 Parameter Estimates of the Bivariate GARCH Model with One Single Correlation

(a) MSCI World Index -G7 Stock Markets

	MSCI-Canada	MSCI-France	MSCI-Germany	MSCI-Italy	MSCI-Japan	MSCI-U.K.	MSCI-U.S.
World Market Eq.							
θ^y_0	0.2294 (0.0435)***	0.2365 (0.0440)***	0.2018 (0.0446)***	0.2108 (0.0449)***	0.2630 (0.0458)***	0.2075 (0.0444)***	0.2171 (0.0456)***
θ^y_1	-0.0100 (0.0140)	-0.0214 (0.0246)	-0.0062 (0.0413)	-0.0082 (0.0240)	-0.0466 (0.0256)*	0.0134 (0.0158)	0.0145 (0.0265)
α^y_0	0.2055 (0.0469)***	0.1592 (0.0358)***	0.2520 (0.0520)***	0.1923 (0.0451)***	0.2429 (0.0525)***	0.1391 (0.0342)***	0.2276 (0.0437)***
α^y_1	0.0940 (0.0138)***	0.0861 (0.0119)***	0.0906 (0.0132)***	0.1013 (0.0141)***	0.1141 (0.0148)***	0.0908 (0.0125)***	0.0706 (0.0101)***
β^y_1	0.8518 (0.0203)***	0.8729 (0.0155)***	0.8405 (0.0203)***	0.8496 (0.0185)***	0.8237 (0.0217)***	0.8752 (0.0165)***	0.8656 (0.0168)***
Individual Market Eq.							
θ^x_0	0.2138 (0.0567)***	0.3097 (0.0665)***	0.2334 (0.0647)***	0.1991 (0.0760)***	0.2470 (0.0732)***	0.2274 (0.0590)***	0.2197 (0.0496)***
θ^x_1	0.0751 (0.0235)***	-0.0618 (0.0252)***	0.0100 (0.0226)	0.0239 (0.0307)	0.0291 (0.0229)	-0.0253 (0.0223)	-0.0574 (0.0245)***
α^x_0	1.2060 (0.2175)***	0.5840 (0.1159)***	0.4261 (0.0915)***	0.2994 (0.0773)***	0.6514 (0.1940)***	0.3432 (0.0866)***	0.2276 (0.0536)***
α^x_1	0.1431 (0.0222)***	0.1278 (0.0177)***	0.1001 (0.0145)***	0.0863 (0.0134)***	0.0936 (0.0186)***	0.0937 (0.0129)***	0.0649 (0.0096)***
β^x_1	0.6505 (0.0475)***	0.8070 (0.0235)***	0.8491 (0.0199)***	0.8904 (0.0155)***	0.8336 (0.0354)***	0.8576 (0.0193)***	0.8824 (0.0182)***
Correlation							
ρ	0.6833 (0.0142)***	0.6523 (0.0153)***	0.6637 (0.0149)***	0.5133 (0.0200)***	0.6600 (0.0151)***	0.7142 (0.0130)***	0.8413 (0.0079)***
Log-lik.	-5652.1071	-5938.0385	-5904.8489	-6318.9724	-6001.3949	-5652.7313	-5021.5015
AIC value	-5663.1071	-5949.0385	-5915.8489	-6329.9724	-6012.3949	-5663.7313	-5032.5015
Schwarz value	-5692.0516	-5977.9830	-5944.7934	-6358.9169	-6041.3394	-5692.6758	-5061.4460

(b) EWW World Index- G7 Stock Markets

	EWW-Canada	EWW-France	EWW-Germany	EWW-Italy	EWW-Japan	EWW-U.K.	EWW-U.S.
World Market Eq.							
θ^y_0	0.2367 (0.0490)***	0.2581 (0.0475)***	0.2102 (0.0502)***	0.2184 (0.0468)***	0.2651 (0.0475)***	0.2107 (0.0494)***	0.2346 (0.0497)***
θ^y_1	0.0097 (0.0154)	-0.0184 (0.0152)	0.0218 (0.0150)	0.0088 (0.0122)	-0.0073 (0.0134)	0.0469 (0.0209)**	0.0186 (0.0203)
α^y_0	0.3838 (0.0984)***	0.3858 (0.0771)***	0.7670 (0.1767)***	0.5418 (0.1043)***	0.7030 (0.1707)***	0.2816 (0.0547)**	0.4200 (0.0898)***
α^y_1	0.0948 (0.0178)***	0.0717 (0.0117)***	0.0809 (0.0157)***	0.1153 (0.0183)***	0.1359 (0.0244)***	0.0886 (0.0130)***	0.0837 (0.0144)***
β^y_1	0.8089 (0.0372)***	0.8292 (0.0265)***	0.7182 (0.0543)***	0.7481 (0.0364)***	0.6874 (0.0582)***	0.8427 (0.0217)***	0.8087 (0.0312)***
Individual Market Eq.							
θ^x_0	0.2144 (0.0608)***	0.3151 (0.0662)***	0.2344 (0.0690)***	0.1972 (0.0729)***	0.2363 (0.0705)***	0.2113 (0.0610)***	0.2291 (0.0491)***
θ^x_1	0.0587 (0.0219)***	-0.0763 (0.0199)***	0.0181 (0.0195)	0.0245 (0.0203)	0.0376 (0.0230)	-0.0112 (0.0275)	-0.0909 (0.0230)***
α^x_0	1.2171 (0.2268)***	0.7318 (0.1396)***	0.7150 (0.1529)***	0.3265 (0.0743)***	0.7281 (0.2064)***	0.4395 (0.1242)***	0.2519 (0.0662)***
α^x_1	0.1393 (0.0228)***	0.1118 (0.0159)***	0.0909 (0.0142)***	0.0854 (0.0122)***	0.1048 (0.0203)***	0.0983 (0.0138)***	0.0931 (0.0155)***

$\beta^x{}_1$	0.6517 (0.0504)***	0.8004 (0.0262)***	0.8179 (0.0276)***	0.8883 (0.0146)***	0.8145 (0.0374)***	0.8369 (0.0259)***	0.8506 (0.0254)***
Correlation							
ρ	0.6914 (0.0139)***	0.8024 (0.0095)***	0.7926 (0.0100)***	0.7205 (0.0128)***	0.6069 (0.0168)***	0.7801 (0.0104)***	0.6904 (0.0140)***
Log-lik.	-5714.8788	-5680.9695	-5701.7015	-6093.0025	-6157.6765	-5572.3490	-5502.8377
AIC value	-5725.8788	-5691.9695	-5712.7015	-6104.0025	-6168.6765	-5583.3490	-5513.8377
Schwarz value	-5754.8233	-5720.9140	-5741.6460	-6132.9470	-6197.6210	-5612.2935	-5542.7822

Notes:

1. Collected from the Data Stream database, the data represent weekly observations (taken on Wednesdays) and cover the period Jan. 1980-May 2007. The entire sample consists of 1,426 observations. All weekly returns are stock indices valued in U.S. dollars.

2. Data for the domestic market component of the model are the stock price indices of individual G7 countries, which account for a minimum of 80% of the stock market capitalization of each country. All market price indices are valued in U.S. dollars and were obtained from the Data Stream database.

3. This study employs two proxies for assessing the global component of the model: (1) the world index, which is provided by Morgan Stanley Capital International (MSCI), and (2) the equally weighted world (EWW) stock index, which averages various market indices by attributing an equal weight to each one of them. The data thus collated was used to create 14 (7X2) international portfolios on which were performed a series of analyses.

4. Please refer to Eqs 1 to 7 for the specifications of the bivariate GARCH model used in this study. The key limitation of this model is here considered to be its assumption of one constant correlation.

5. The LR statistical analysis cannot be applied to the SWARCH-GARCH model comparison proposed here because the two models are not strictly nested. Instead, the AIC and Schwarz value are used as two statistical criteria, which are specified as follows:

(1) AIC = Log-likelihood function value - N. (N is the number of model parameters.)

(2) Schwarz value = Log-likelihood function value - (N/2) x ln(T). (N is the number of population parameters in the mode and T is the number of samples.)

6. The values in parentheses are the standard errors of the estimates. The ***, ** and * denote the 1%, 2.5% and 5% levels of significance, respectively.

7. The sum of the GARCH parameter estimates for domestic and global markets is approximately equal to unity in most cases, which supports the contention that variance in the GARCH model is far too persistent. This study posits that this problematically high level of persistence is caused by structural changes in volatility during the estimation period.

Table 3 Parameter Estimates of the Bivariate SWARCH Model with State-varying Correlations

(a) MSCI World Index -G7 Stock Markets

	MSCI-Canada	MSCI-France	MSCI-Germany	MSCI-Italy	MSCI-Japan	MSCI-U.K.	MSCI-U.S.
World Market Eq.							
p^y_{11}	0.9856 (0.0053)***	0.9822 (0.0060)***	0.9905 (0.0043)***	0.9825 (0.0057)***	0.9804 (0.0063)***	0.9908 (0.0045)***	0.9848 (0.0053)***
p^y_{22}	0.9643 (0.0140)***	0.9507 (0.0184)***	0.9834 (0.0062)***	0.9716 (0.0115)***	0.9724 (0.0106)***	0.9854 (0.0072)***	0.9828 (0.0057)***
θ^y_0	0.2106 (0.0441)***	0.2250 (0.0475)***	0.2166 (0.0437)***	0.2452 (0.0454)***	0.2380 (0.0433)***	0.2103 (0.0443)***	0.2374 (0.0436)***
θ^y_1	-0.0097 (0.0169)	-0.0176 (0.0298)	-0.0089 (0.0039)	-0.0133 (0.0108)	-0.0478 (0.0252)*	-0.0054 (0.0178)	-0.0004 (0.0183)
α^y_0	2.0808 (0.1326)***	2.1850 (0.1225)***	1.9625 (0.1193)***	1.8504 (0.1126)***	1.9683 (0.1109)***	2.0425 (0.1530)***	2.1811 (0.1262)***
α^y_1	0.0582 (0.0268)**	0.0324 (0.0237)	0.0683 (0.0232)***	0.0408 (0.0250)*	0.0933 (0.0264)***	0.0572 (0.0223)***	0.0551 (0.0175)***
g^y_2	3.0975 (0.2875)***	3.1460 (0.2887)***	2.7738 (0.2466)***	3.6264 (0.0285)***	2.8331 (0.2409)***	2.6289 (0.2333)***	2.0875 (0.1472)***
Individual Market Eq.							
p^x_{11}	0.9864 (0.0054)***	0.9675 (0.0110)***	0.9891(0.0055)***	0.9731 (0.0096)***	0.9871 (0.0050)***	0.9858 (0.0074)***	0.9705 (0.0091)***
p^x_{22}	0.9619 (0.0148)***	0.9241 (0.0238)***	0.9754 (0.0091)***	0.9822 (0.0068)***	0.9329 (0.0203)***	0.9879 (0.0063)***	0.9668 (0.0094)***
θ^x_0	0.1893 (0.0544)***	0.2790 (0.0701)***	0.2390 (0.0680)***	0.2145 (0.0805)***	0.1481 (0.0731)***	0.1941 (0.0565)***	0.2401 (0.0474)***
θ^x_1	0.0673 (0.0219)***	-0.0622 (0.0297)**	0.0126 (0.0182)	0.0136 (0.0285)	0.0432 (0.0245)*	-0.0505 (0.0162)***	-0.0770 (0.0192)***
α^x_0	3.4797 (0.1926)***	4.6742 (0.3360)***	4.6673 (0.2746)***	4.3561 (0.5295)***	6.4515 (0.3135)***	3.3470 (0.2640)***	2.1870 (0.1371)***
α^x_1	0.0477 (0.0225)**	0.0745 (0.0267)***	0.0517 (0.0188)***	0.0080 (0.0231)	0.0786 (0.0275)***	0.0758 (0.0234)***	0.0853 (0.0208)***
g^x_2	2.9862 (0.2749)***	3.1235 (0.3103)***	2.9275 (0.2358)***	3.3792 (0.4153)***	3.0960 (0.2992)***	2.3111 (0.2092)***	2.4436 (0.1827)
Correlations							
$\rho_{1,1}$	0.7135 (0.0235)***	0.7905 (0.0190)***	0.7260 (0.0289)***	0.7363 (0.0392)***	0.7666 (0.0168)***	0.7933 (0.0198)***	0.9079 (0.0099)***
$\rho_{2,1}$	0.6306 (0.0632)***	0.5539 (0.0619)***	0.5071 (0.0086)***	0.4526 (0.2376)***	0.4911 (0.0067)***	0.8057 (0.0283)***	0.5254 (0.0503)***
$\rho_{1,2}$	0.6204 (0.0460)***	0.2929 (0.0691)***	0.4308 (0.0650)***	0.3408 (0.0430)***	0.3958 (0.0766)***	0.6524 (0.0419)***	0.8835 (0.0148)***
$\rho_{2,2}$	0.7648 (0.0395)***	0.8488 (0.0309)***	0.8433 (0.0193)***	0.6553 (0.0453)***	0.8986 (0.0182)***	0.6661 (0.0331)***	0.9102 (0.0107)***
Log-lik.	-5588.1415	-5841.6290	-5836.1382	-6253.7062	-5949.4139	-5589.9625	-4939.6985
AIC value	-5606.1415	-5859.6290	-5854.1382	-6271.7062	-5967.4139	-5607.9625	-4957.6985
Schwarz value	-5653.5052	-5906.9927	-5901.5019	-6319.0699	-6014.7776	-5655.3262	-5005.0622
LR test for constant correlation	5.6104	74.9768***	43.7948***	47.6564***	103.9620***	125.5376***	163.6060***

(b) EWW World Index -G7 Stock Markets

	EWW-Canada	EWW-France	EWW-Germany	EWW-Italy	EWW-Japan	EWW-U.K.	EWW-U.S.
World Market Eq.							
p^y_{11}	0.9871 (0.0048)***	0.9925 (0.0032)***	0.9908 (0.0044)***	0.9952 (0.0024)***	0.9832 (0.0073)***	0.9902 (0.0060)***	0.9855 (0.0061)***
p^y_{22}	0.9641 (0.0137)***	0.9897 (0.0052)***	0.9727 (0.0076)***	0.9860 (0.0068)***	0.9564 (0.0187)***	0.9775 (0.0124)***	0.9751 (0.0111)***

θ^y_0	0.2211 (0.0504)***	0.2456 (0.0474)***	0.2423 (0.0459)***	0.2401 (0.0425)***	0.2382 (0.0472)***	0.2144 (0.0481)***	0.2220 (0.0482)***
θ^y_1	-0.0099 (0.0193)	-0.0255 (0.0204)	0.0072 (0.0182)	-0.0030 (0.0342)	-0.0116 (0.0211)	0.0292 (0.0238)	0.0120 (0.0168)
α^y_0	2.4647 (0.1404)***	2.5199 (0.1302)***	2.6320 (0.1374)***	2.5238 (0.1190)***	2.3707 (0.1888)***	2.4601 (0.1589)***	2.4314 (0.1474)***
α^y_1	0.0622 (0.0225)***	0.0500 (0.0161)***	0.0837 (0.0199)***	0.0767 (0.0198)***	0.0938 (0.0280)***	0.0570 (0.0222)***	0.0684 (0.0219)***
g^y_2	2.6523 (0.2490)***	2.2429 (0.1611)***	1.9635 (0.1465)***	2.9609 (0.2228)***	2.8872 (0.3157)***	2.5058 (0.2107)***	2.2610 (0.1993)***
Individual Market Eq.							
p^x_{11}	0.9919 (0.0038)***	0.9554 (0.0142)***	0.9811 (0.0064)***	0.9995 (0.0009)***	0.9547 (0.0147)***	0.9693 (0.0109)***	0.9749 (0.0100)***
p^x_{22}	0.9766 (0.0118)***	0.9611 (0.0133)***	0.9631 (0.0121)***	0.9990 (0.0008)***	0.9303 (0.0291)***	0.9769 (0.0093)***	0.9855 (0.0071)***
θ^x_0	0.1781 (0.0593)***	0.3105 (0.0644)***	0.2948 (0.0634)***	0.2596 (0.0661)***	0.1395 (0.0726)***	0.1932 (0.0571)***	0.2239 (0.0470)***
θ^x_1	0.0433 (0.0231)*	-0.0946 (0.0216)***	-0.0060 (0.0144)	0.0119 (0.0238)	0.0446 (0.0260)*	-0.0324 (0.0235)	-0.0880 (0.0221)***
α^x_0	3.5470 (0.1995)***	4.1555 (0.2595)***	4.9475 (0.2698)***	3.3589 (0.2530)***	4.7705 (0.4269)***	3.1981 (0.2440)***	1.5658 (0.1250)***
α^x_1	0.0603 (0.0264)**	0.0939 (0.0222)***	0.0718 (0.0198)***	0.0564 (0.0175)***	0.0212 (0.0503)	0.0789 (0.0234)***	0.1113 (0.0283)***
g^x_2	2.9858 (0.2842)***	2.4804 (0.1769)***	2.5662 (0.1922)***	3.4395 (0.2709)***	3.1289 (0.3467)***	2.3413 (0.1924)***	3.3749 (0.2983)***
Correlations							
$\rho_{1,1}$	0.7133 (0.0193)***	0.9008 (0.0129)***	0.8475 (0.0140)***	0.8961 (0.0124)***	0.6874 (0.0297)***	0.8462 (0.0163)***	0.7415 (0.0280)***
$\rho_{2,1}$	0.6379 (0.0436)***	0.8255 (0.0291)***	0.7639 (0.0303)***	0.6380 (0.0827)***	0.5398 (0.0616)***	0.8607 (0.03080***	0.3967 (0.1127)***
$\rho_{1,2}$	0.6784 (0.0452)***	0.6212 (0.0322)***	0.5703 (0.0416)***	0.6240 (0.0186)***	0.6148 (0.0437)***	0.7114 (0.0240)***	0.6693 (0.0316)***
$\rho_{2,2}$	0.7503 (0.0336)***	0.9078 (0.0104)***	0.9399 (0.0082)***	0.8742 (0.0122)***	0.6094 (0.0580)***	0.7804 (0.0242)***	0.7420 (0.0242)***
Log-lik.	-5649.3655	-5567.0396	-5611.3108	-6017.0687	-6117.5955	-5512.0446	-5465.6024
AIC value	-5667.3655	-5585.0396	-5629.3108	-6035.0687	-6135.5955	-5530.0446	-5483.6024
Schwarz value	-5714.7292	-5632.4033	-5676.6745	-6082.4324	-6182.9592	-5577.4083	-5530.9661
LR test for constant correlation	5.1524	124.6438***	94.3840***	77.9736***	4.681	28.6032***	11.9832***

Notes:

1. Please refer to Eqs. 9 to 20 for the specifications of the bivariate SWARCH model used in this study. The key feature of this model is the fact that it is based on conducting 4-state correlation measurements.
2. To test the null hypothesis of identical correlations, this bivariate SWARCH model is first estimated on the basis of 4-state correlations and $L(H_A)$, representing the log likelihood function. The model is then estimated assuming the existence of a single constant correlation ($\rho_{1,1}=\rho_{2,1}=\rho_{1,2}=\rho_{2,2}=\rho$), which allows for the subsequent derivation of the log likelihood function of the restricted model, $L(H_0)$. Finally, this function is used to carry out a likelihood ratio test, $LR=-2[L(H_0)- L(H_A)]$. In terms of the null hypothesis, this test displays a χ^2 distribution with 3 (= 4 - 1) degrees of freedom. Quite interestingly, the results of this LR statistical analysis make it possible to reject the null hypothesis at a 1% level of significance in most cases except those of MSCI-Canada, EWW-Canada and EWW-Japan, which are significant at the 10% level.
3. The values in parentheses are the standard errors of the estimates. The ***, ** and * denote the 1%, 2.5% and 5% levels of significance, respectively.
4. In terms of values calculated by averaging all 14 cases under examination, the calculated correlation estimates of $\rho_{1,1}$ = 0.7799, under LV states in the case of both global domestic markets, and $\rho_{2,2}$ = 0.6979, given HV states in both types of markets, far exceed the estimates of the other two possible state combinations ($\rho_{2,1}$ = 0.58414, World=HV and Indl.=LV; and $\rho_{1,2}$ = 0.52491, World=LV and Indl.=HV). The average correlation estimate for the GARCH model, ρ = 0.6582, is both lower than the values of $\rho_{1,1}$ and $\rho_{2,2}$, and higher than those of $\rho_{2,1}$ and $\rho_{1,2}$. The obvious conclusion is that the GARCH model setting of a single correlation measure underestimates the actual magnitude of co-movements between global and domestic markets in the case of two of the possible state combinations (“LV-LV” and “HV-HV”), and overestimates it for the other two combinations (“LV-HV” and “HV-LV”).
5. Other notations are consistent with Table 2.

Table 4 In-sample Asset Allocation Effectiveness of International Diversification with a Minimum Variance Portfolio of Domestic and World Assets

(a) Return Mean

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
MSCI WORLD			
Canada	0.1759	0.1604	1.7373*
France	0.1685	0.1774	-0.9015
Germany	0.1605	0.1591	0.1411
Italy	0.1589	0.1682	-1.1579
Japan	0.1627	0.1586	0.4412
U.K.	0.1572	0.1667	-1.3622
U.S.	0.1656	0.1730	-0.7340
EWW WORLD			
Canada	0.1639	0.1619	0.2209
France	0.1714	0.1816	-1.3152
Germany	0.1682	0.1683	-0.0143
Italy	0.1707	0.1716	-0.3367
Japan	0.1640	0.1718	-0.8209
U.K.	0.1651	0.1694	-0.6910
U.S.	0.1475	0.1569	-1.1011

(b) Return Variance

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
MSCI WORLD			
Canada	3.5525	3.1963	6.8612***
France	3.8200	3.3374	4.4507***
Germany	3.7466	3.4071	3.8310***
Italy	3.6771	3.3796	6.6110***
Japan	3.6181	3.3743	4.7607***
U.K.	3.6735	3.4329	6.6308***
U.S.	3.6949	3.5091	5.0932***
EWW WORLD			
Canada	3.7418	3.3809	7.5922***
France	4.0655	3.8929	2.9951***
Germany	4.0868	3.9102	3.3885***
Italy	4.0197	3.9700	3.7496***
Japan	3.8776	3.4794	6.2138***
U.K.	3.9684	3.7025	5.5099***
U.S.	3.6563	3.3546	7.1799***

(c) Annual Return/Risk (R/R) Ratio

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
MSCI WORLD			
Canada	0.6599	0.6344	-3.86%
France	0.6096	0.6866	12.63%
Germany	0.5863	0.6095	3.96%
Italy	0.5859	0.6470	10.43%
Japan	0.6048	0.6105	0.94%
U.K.	0.5800	0.6362	9.69%
U.S.	0.6092	0.6530	7.19%
EWW WORLD			
Canada	0.5991	0.6226	3.92%
France	0.6011	0.6508	8.27%
Germany	0.5883	0.6018	2.29%
Italy	0.6020	0.6090	1.16%
Japan	0.5889	0.6513	10.60%
U.K.	0.5860	0.6225	6.23%
U.S.	0.5455	0.6057	11.04%

Notes:
1. The ***, ** and * denote the 1%, 2.5% and 5% levels of significance, respectively.
2. See the appendix for details of the statistical test of whether the variances of portfolio returns vary significantly between the bivariate GARCH and SWARCH models.
3. Results show that modeling domestic-global cross-market correlations and corresponding variances as a state-varying phenomenon is both statistically significant and strategically effective. Moreover, reductions in risk, rather than increases in mean returns, are responsible for the benefits stemming such improved effectiveness.

Table 5 In-sample Asset Allocation Effectiveness of International Diversification with a Given Variance Portfolio of Domestic and World Assets

(a) Return Mean

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
MSCI WORLD			
Canada	0.1428	0.1615	-1.8084*
France	0.1829	0.1815	0.1155
Germany	0.1701	0.1891	-1.2705
Italy	0.1809	0.1793	0.1278
Japan	0.1572	0.1631	-0.4556
U.K.	0.1710	0.1667	0.5554
U.S.	0.1733	0.1736	-0.0292
EWW WORLD			
Canada	0.1511	0.1558	-0.4685
France	0.1805	0.1796	0.6390
Germany	0.1744	0.1810	-0.8139
Italy	0.1832	0.1843	-0.3649
Japan	0.1673	0.1561	0.8780
U.K.	0.1750	0.1669	1.3292
U.S.	0.2200	0.1912	1.9464*

(b) Return Variance

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
MSCI WORLD			
Canada	4.7409	4.6042	2.3144**
France	5.9286	5.6457	3.2433***
Germany	5.9508	5.6461	3.1335***
Italy	6.7184	6.4862	3.9689***
Japan	6.2040	5.8621	5.3391***
U.K.	4.8611	4.8370	0.6808
U.S.	4.1866	4.0719	2.6627***
EWW WORLD			
Canada	4.9628	4.7912	3.0712***
France	6.1573	6.0071	2.6248***
Germany	6.1220	5.9304	3.5705***
Italy	6.9692	6.9930	-1.0797
Japan	6.3152	6.0613	2.6899***
U.K.	5.0334	5.0378	-0.0410
U.S.	4.3065	4.1913	1.6806*

(c) Annual Return/Risk (R/R) Ratio

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
MSCI WORLD			
Canada	0.4637	0.5322	14.76%
France	0.5312	0.5401	1.69%
Germany	0.4931	0.5627	14.13%
Italy	0.4935	0.4978	0.87%
Japan	0.4463	0.4763	6.74%
U.K.	0.5484	0.5360	-2.27%
U.S.	0.5989	0.6083	1.57%
EWW WORLD			
Canada	0.4796	0.5033	4.94%
France	0.5144	0.5182	0.74%
Germany	0.4984	0.5256	5.45%
Italy	0.4907	0.4928	0.43%
Japan	0.4707	0.4483	-4.76%
U.K.	0.5516	0.5258	-4.67%
U.S.	0.7496	0.6604	-11.90%

Notes:
1. On the basis of relative improvements of the R/R ratio, this table indicates that, in terms of asset allocation, the effectiveness of the SWARCH-based portfolio outperforms that of the GARCH-based portfolio in 10 of 14 cases.
2. Other notations are consistent with Table 4.

Table 6 Observation Percentages of Various Market States

	Whole Period	World=LV and Indl.=LV	World=HV and Indl=LV	World=LV and Indl=HV	World=HV and Indl=HV
MSCI-WORLD					
MSCI-Canada	100%	60.86%*	15.11%	13.00%	11.03%
MSCI-France	100%	63.39%*	13.14%	13.91%	9.56%
MSCI-Germany	100%	46.03%*	19.33%	13.63%	21.01%
MSCI-Italy	100%	26.77%	11.04%	38.23%*	23.96%
MSCI-Japan	100%	61.28%*	25.72%	3.30%	9.70%
MSCI-U.K.	100%	33.66%*	11.67%	29.73%	24.94%
MSCI-U.S.	100%	29.80%	19.47%	18.90%	31.83%*
EWW-WORLD					
EWW-Canada	100%	61.21%*	15.39%	12.86%	10.54%
EWW-France	100%	36.05%*	10.19%	29.80%	23.96%
EWW-Germany	100%	55.87%*	12.16%	16.16%	15.81%
EWW-Italy	100%	12.37%	0.56%	61.49%*	25.58%
EWW-Japan	100%	53.90%*	11.38%	24.46%	10.26%
EWW-U.K.	100%	33.73%	8.85%	37.60%*	19.82%
EWW-U.S.	100%	29.73%	5.06%	37.81%*	27.40%
Average	100%	43.19%*	12.79%	25.06%	18.96%

Notes:

1. To identify the specific volatility state existing at each point in time, the estimated filtering probability of a specific state (see Fig. 2 as an example) and a maximum value criterion are here used. In other words, at time t, if the estimated value of the filtering probability of the "HV-HV" state (s_t=4 or s_t^y =2 and s_t^x =2) exceeds that of the three alternative states, then an "HV-HV" state is said to exist at this point in time. More generally, this work defines the market as being in a specific state if the corresponding estimated state probability is relatively high.

2. This table lists observation percentages of various volatility states. The * denotes the maximum value among the four state combinations, which is the "LV and LV" state in 9 of 14 cases. This finding is consistent with the notion that a low volatility state is more persistent than a high volatility state. This claim is further validated by the fact that Table 3 indicates that the p^y_{11} (p^x_{11}) estimates generally exceed the p^y_{22} (p^x_{22}) estimates.

Table 7 Risk Reduction Effectiveness of International Diversification with Equal Weighting of Domestic and World Assets in Various Market States

	Whole Period	World=LV and Indl.=LV	World=HV and Indl.=LV	World=LV and Indl.=HV	World=HV and Indl.=HV
MSCI-WORLD					
MSCI-Canada	7.89%	8.23%	9.70%	7.53%	5.08%*
MSCI-France	8.35%	8.02%	8.15%	14.16%	4.11%*
MSCI-Germany	7.56%	8.48%	8.63%	12.29%	3.91%*
MSCI-Italy	12.29%	10.17%	12.03%	14.13%	9.74%*
MSCI-Japan	8.58%	7.74%	11.05%	7.10%	4.35%*
MSCI-U.K.	7.51%	6.90%	4.87%*	8.95%	7.21%
MSCI-U.S.	3.63%	3.91%	6.21%	3.65%	2.32%*
EWW-WORLD					
EWW-Canada	7.79%	7.65%	9.26%	7.51%	5.31%*
EWW-France	4.75%	4.30%	3.58%	7.81%	2.55%*
EWW-Germany	4.61%	5.29%	4.24%	7.52%	1.97%*
EWW-Italy	7.28%	3.12%*	7.78%	8.33%	3.89%
EWW-Japan	10.06%	10.28%	11.49%	8.81%*	9.37%
EWW-U.K.	5.84%	5.39%	3.54%*	6.40%	5.60%
EWW-U.S.	7.44%	9.55%	11.18%	8.51%	5.49%*

Notes:
1. This table uses the average values of the standard errors of domestic and world assets as the benchmark against which is calculated the percentage with which risk is reduced in the case of a portfolio placing an equal weight on both domestic and global assets in various market volatility states.
2. The * denotes the minimum value in each row. Interestingly, the minimum values occurred in the "HV-HV" state in 10 of 14 cases. Additionally, if the values of the risk reduction percentage for the whole sample period are used to identify the benefits of reductions in risk stemming from the development of an international portfolio, overestimation occurs in the "HV-HV" state and underestimation results in other, alternative situations.
3. It must be noted that, while this Table shows the "HV-HV" state combination corresponding to the least effective conditions under which risk is reduced by international portfolio diversification, this state combination has already been shown to be the context in which the strongest cross-market return correlations exist (see $\rho_{2,2}$ in Table 3). This finding suggests that setting a model with a single, constant correlation value is likely to underestimate the actual strength of cross-market correlations and to overestimate the effects of risk reduction under conditions of an "HV-HV" state.
4. Other notations are consistent with Table 6.

Table 8 Risk Reduction Effectiveness of SWARCH Model against GARCH Model in Various Volatility State Combinations

(a) Minimum Variance Portfolio of Domestic and World Assets

	Whole Period	World=LV and Indl.=LV	World=HV and Indl.=LV	World=LV and Indl.=HV	World=HV and Indl.=HV
MSCI-WORLD					
MSCI-Canada	10.03%	7.93%	21.4%	5.17%	8.91%
MSCI-France	12.63%	9.08%	21.6%	17.33%	9.33%
MSCI-Germany	9.06%	5.95%	17.28%	3.5%	5.66%
MSCI-Italy	8.09%	10.23%	15.7%	5.11%	6.93%
MSCI-Japan	6.74%	5.57%	9.81%	3.15%	10.29%
MSCI-U.K.	6.55%	9.25%	12.48%	4.51%	4.21%
MSCI-U.S.	5.03%	4.44%	4.66%	2.74%	5.98%
EWW-WORLD					
EWW-Canada	9.64%	9.03%	13.43%	5.74%	10.59%
EWW-France	4.24%	0.39%	4.84%	4.38%	5.58%
EWW-Germany	4.32%	4.97%	7.54%	0.00%	3.30%
EWW-Italy	1.24%	1.33%	25.16%	0.47%	1.56%
EWW-Japan	10.27%	9.77%	15.17%	7.26%	10.04%
EWW-U.K.	6.70%	8.46%	13.25%	3.23%	6.57%
EWW-U.S.	8.25%	14.50%	6.33%	8.80%	6.09%
Average	7.34%	7.21%	13.48%	5.10%	6.79%

(b) Given Variance Portfolio of Domestic and World Assets

	Whole Period	World=LV and Indl.=LV	World=HV and Indl.=LV	World=LV and Indl.=HV	World=HV and Indl.=HV
MSCI-WORLD					
MSCI-Canada	2.88%	1.14%	0.69%	5.62%	4.31%
MSCI-France	4.77%	6.25%	12.83%	-8.91%	5.76%
MSCI-Germany	5.12%	6.26%	3.82%	0.55%	6.28%
MSCI-Italy	3.46%	8.29%	18.08%	-3.17%	3.86%
MSCI-Japan	5.51%	7.47%	9.13%	5.4%	-0.24%
MSCI-U.K.	0.50%	1.45%	1.92%	0.63%	-0.74%
MSCI-U.S.	2.74%	1.38%	1.37%	6.01%	2.67%
EWW-WORLD					
EWW-Canada	3.46%	0.70%	7.38%	6.01%	3.97%
EWW-France	2.44%	4.28%	14.00%	-2.67%	2.16%
EWW-Germany	3.13%	2.61%	5.91%	0.49%	3.72%
EWW-Italy	-0.34%	2.85%	-21.10%	-1.74%	2.11%
EWW-Japan	4.02%	6.66%	8.45%	2.32%	-3.38%
EWW-U.K.	-0.09%	2.15%	2.27%	-0.89%	-1.24%
EWW-U.S.	2.68%	5.73%	5.89%	5.58%	-0.65%

Average	2.88%	4.09%	5.05%	1.09%	2.04%

Notes:
1. This study uses the variance of a GARCH-based portfolio as the benchmark against which is calculated the percentage with which risk is reduced by the SWARCH model. In this Table, Panel (a) and (b), respectively, present the results for a minimum variance portfolio and a given variance portfolio consisting of domestic and global assets.
2. The integration of the first two states presented in Table 8 ("LV-LV" and "HV-LV") presents the general results for cases where the domestic market is in a state of low variance, whereas the last two states ("LV-HV" and "HV-HV") can be integrated to yield the results for cases where the domestic market is in a state of high variance.
3. Based on the average value of all 14 cases as a criterion, Panel (a) shows that, in the case of the domestic market in a low state of variance, the SWARCH model reduces risk by an average of 13.48% when the global market is experiencing high levels of variance, and 7.21% when it faces low levels of variance. When the domestic market is in a state of high variance, the corresponding figures are 6.79% (World=HV) and 5.10% (World=LV). These findings are consistent with the notion that the SWARCH model is more valuable than a non-state varying framework when the global market finds itself in a high variance regime. The same conclusion can be drawn in the case of a given variance portfolio (See Panel (b) of Table 8).

Table 9 Relative Time Variation of Optimal Loading of World Market Returns in Various Volatility State Combinations

(a) Minimum Variance Portfolio of Domestic and World Assets

	Whole Period	World=LV and Indl.=LV	World=HV and Indl.=LV	World=LV and Indl.=HV	World=HV and Indl.=HV
MSCI-WORLD					
MSCI-Canada	1.04	1.02	1.00	0.66*	0.61*
MSCI-France	1.37	1.17	1.16	2.65	1.90
MSCI-Germany	1.53	0.85*	1.03	11.22	1.89
MSCI-Italy	1.23	1.21	1.59	1.16	0.77*
MSCI-Japan	1.30	1.92	0.81*	12.82	0.60*
MSCI-U.K.	1.40	1.49	1.21	1.99	0.75*
MSCI-U.S.	1.14	0.83*	0.73*	1.93	0.84*
EWW-WORLD					
EWW-Canada	1.13	0.925*	0.89*	0.87*	0.57*
EWW-France	7.01	19.55	3.00	22.58	47.83
EWW-Germany	2.61	1.39	1.87	11.99	11.03
EWW-Italy	0.75*	0.91*	0.80*	0.53*	0.69*
EWW-Japan	1.26	1.20	0.95*	3.14	0.92*
EWW-U.K.	1.28	1.20	1.41	2.62	0.86*
EWW-U.S.	1.14	0.83*	0.73*	1.93	0.84*

(b) Given Variance Portfolio of Domestic and World Assets

	Whole Period	World=LV and Indl.=LV	World=HV and Indl.=LV	World=LV and Indl.=HV	World=HV and Indl.=HV
MSCI-WORLD					
MSCI-Canada	0.51*	0.82*	0.22*	0.36*	0.22*
MSCI-France	1.08	1.27	0.47*	1.32	1.89

MSCI-Germany	1.59	1.29	0.80*	2.03	1.89
MSCI-Italy	0.90*	1.29	0.54*	0.55*	0.71*
MSCI-Japan	0.88*	1.43	0.62*	1.33	0.57*
MSCI-U.K.	0.53*	0.89*	0.24*	0.66*	0.36*
MSCI-U.S.	0.98*	1.51	0.36*	2.05	0.51*
EWW-WORLD					
EWW-Canada	0.58*	0.70*	0.23*	0.45*	0.19*
EWW-France	1.90	1.05	1.65	1.00	2.16
EWW-Germany	2.11	1.41	2.52	1.18	3.75
EWW-Italy	0.49*	0.75*	0.22*	0.24*	0.49*
EWW-Japan	0.80*	1.21	0.38*	1.08	0.72*
EWW-U.K.	0.43*	0.74*	0.19*	0.75*	0.38*
EWW-U.S.	0.98*	1.51	0.36*	2.05	0.51*

Notes:

1. This table presents the time variation figures of the optimal weighting of portfolios designed according to various alternatives. A fraction, in which the standard error of the GARCH-based portfolio loadings is the denominator and that of the SWARCH model is the numerator, is used to calculate the relative time variation of the optimal loadings determined by means of the SWARCH model. Briefly stated, if the values listed here are lower than unity, the corresponding portfolio loadings derived from the SWARCH model are less volatile than GARCH-derived loadings, and they are more volatile when the values are higher than unity. The * denotes a value that is below unity.

2. Panel (a) shows the results for a minimum variance portfolio consisting of domestic and global assets. First, considering the period under examination as a whole, the relative time variation of the SWARCH-based portfolio loadings exceeds unity in 13 of 14 cases. However, taking into account the existence of various volatility states, the SWARCH-based portfolio loadings stabilize considerably, particularly in the case of the “HV-HV” state, where the value of the relative time variation is below unity in 10 of 14 cases. Moreover, the relative time variation of SWARCH-based portfolio loadings is below unity in 5 of 14 cases in the “LV-LV” state, 6 of 14 for “HV-LV”, and 3 of 14 for “LV-HV”.

3. Panel (b) lists the same figures corresponding to a given variance portfolio consisting of domestic and global assets. The value of the relative time variation is below unity in 10 of 14 cases over the entire observation period. Moreover, the relative time variation of SWARCH-based portfolio loadings is lower than unity in 5 of 14 cases in the “LV-LV” state, 12 of 14 for “HV-LV”, 6 of 14 for “LV-HV”, and 10 of 14 for “HV-HV”.

Table 10 Out-of-Sample Asset Allocation Effectiveness of International Diversification with a Minimum Variance Portfolio of Domestic and World Assets

(a) Return Mean

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
MSCI WORLD			
MSCI-Japan	0.2928	0.2804	0.7842
MSCI-U.K.	0.2871	0.2879	-0.0528
MSCI-U.S.	0.2699	0.2652	0.8802
EWW WORLD			
EWW-Japan	0.3431	0.3455	-0.1779
EWW-U.K.	0.3234	0.3140	0.8366
EWW-U.S.	0.2705	0.2740	-0.4244

(b) Return Variance

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
MSCI WORLD			
MSCI-Japan	1.9137	2.0784	-1.9526*
MSCI-U.K.	1.6786	1.6898	-0.4813
MSCI-U.S.	1.6670	1.6585	1.0273
EWW WORLD			
EWW-Japan	2.6954	2.8709	-2.2239**
EWW-U.K.	2.6638	2.6971	-1.2240
EWW-U.S.	1.8106	1.8260	-0.5809

(c) Annual Return/Risk (R/R) Ratio

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
MSCI WORLD			
MSCI-Japan	1.4967	1.3751	-8.12%
MSCI-U.K.	1.5667	1.5663	-0.03%
MSCI-U.S.	1.4784	1.4563	-1.50%
EWW WORLD			
EWW-Japan	1.4777	1.4417	-2.43%
EWW-U.K.	1.4013	1.3521	-3.51%
EWW-U.S.	1.4213	1.4339	0.88%

Notes:

1. In order to carry out rolling estimations in the context of out-of-sample testing, the last 200 weekly observations of the sample (or nearly four years' worth of data) are withdrawn to make up the inputs in this process.
2. Following the addition of each subsequent observation, the sample is rolled; that is, the deliberate deletion of the oldest observation coincides with the addition of the most recent one. This technique thus fixes the sample size at 1,226 (= 1,426 - 200) observations. In this case, as in that of the in-sample test, the SWARCH and GARCH models are compared in terms of their respective effectiveness in allocating assets.
3. With the SWARCH model, the out-of-sample predicted portfolio loadings at time t+1 are obtained according to a two-step procedure. First, the estimated population parameters at time t are employed to forecast the regime probabilities at time t+1. Second, the one-step-ahead forecasts thus derived are used to determine the loadings of the corresponding state-varying portfolio at time t+1.
4. To facilitate the process of convergence, the estimate for each individual period calculated in the non-linear estimation process is used as the initial value in the calculation of the estimate for the period immediately following it.
5. The ***, ** and * denote the 1%, 2.5% and 5% levels of significance, respectively. See the appendix for details of the statistical test of whether the variances of portfolio returns vary significantly between the bivariate GARCH and SWARCH models.
6. In the case of the minimum variance portfolio tested here, the SWARCH model underperforms the GARCH model in most cases. More specifically, the variance of a SWARCH-based portfolio exceeds that for the GARCH model in 5 of 6 cases, and it does so significantly in two instances. Moreover, the annual R/R ratio calculated for the SWARCH-based portfolio is lower than that obtained in the case of the GARCH model in 5 of 6 cases.

Table 11 Out-of-sample Asset Allocation Effectiveness of International Diversification with a Given Variance Portfolio of Domestic and World Assets

(a) Return Mean

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
MSCI WORLD			
MSCI-Japan	0.3176	0.3291	-0.7411
MSCI-U.K.	0.2908	0.2930	-0.1585
MSCI-U.S.	0.2253	0.2488	-1.9252*
EWW WORLD			
EWW-Japan	0.3379	0.3590	-1.1162
EWW-U.K.	0.3037	0.3071	-0.2978
EWW-U.S.	0.3057	0.3510	-1.4169

(b) Return Variance

	GARCH Model	SWARCH Model	Statistical Difference (GARCH-SWARCH)
MSCI WORLD			
MSCI-Japan	3.8164	3.4714	3.6839***
MSCI-U.K.	2.0415	1.9246	3.7503***
MSCI-U.S.	1.6979	1.6946	0.4943
EWW WORLD			
EWW-Japan	4.3995	3.9329	3.2135***
EWW-U.K.	2.6556	2.6280	0.9436
EWW-U.S.	2.2032	2.1747	-0.0163

(c) Annual Return/Risk (R/R) Ratio

	GARCH Model	SWARCH Model	R/R Ratio Improvement % of SWARCH against GARCH
MSCI WORLD			
MSCI-Japan	1.1495	1.2491	8.67%
MSCI-U.K.	1.4392	1.4936	3.78%
MSCI-U.S.	1.2225	1.3512	10.53%
EWW WORLD			
EWW-Japan	1.1392	1.2799	12.36%
EWW-U.K.	1.3178	1.3396	1.65%
EWW-U.S.	1.4562	1.6831	15.58%

Notes:

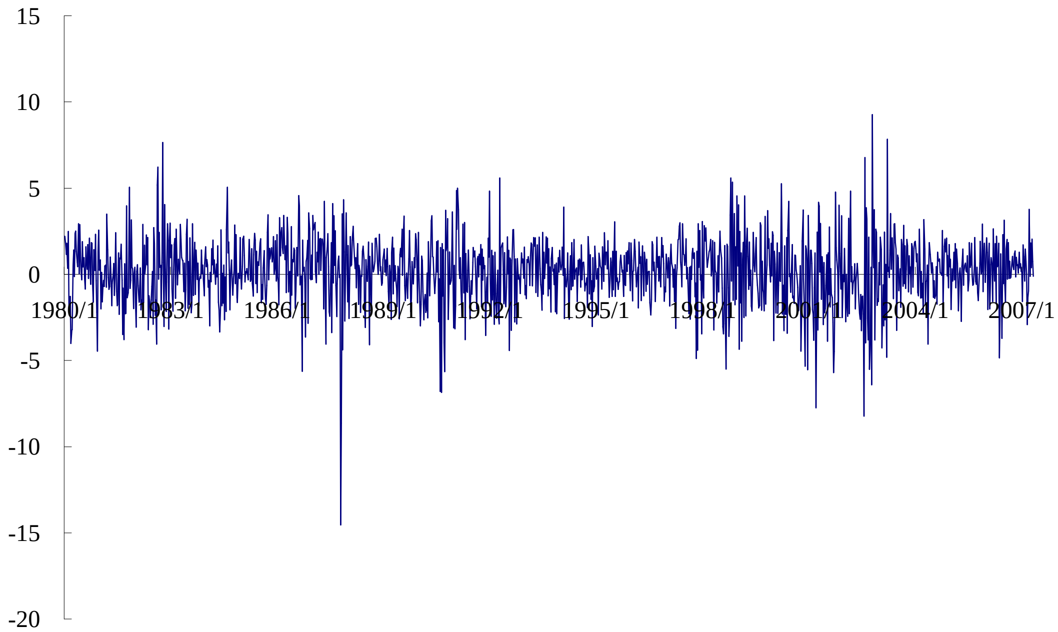
1. The ***, ** and * denote the 1%, 2.5% and 5% levels of significance, respectively. See the appendix for details of the statistical test of whether the variances of portfolio returns vary significantly between the bivariate GARCH and SWARCH models.

2. In the case of the given variance portfolio tested here, the SWARCH model outperforms the GARCH model in most cases. More specifically, the mean return of a SWARCH-based portfolio exceeds that derived from the GARCH model in all six selected cases. The return variance of a portfolio designed on the basis of the SWARCH model is lower than that of a GARCH-based portfolio in 5 of 6 cases, and it is significantly so in three instances. Furthermore, the annual R/R ratio of a SWARCH-based portfolio is higher than that calculated in the case of the GARCH model in all six selected cases.

3. Other notations are consistent with Table 10.

Figure(s)

(a) MSCI World Index



(b) U.S. Stock Market Index

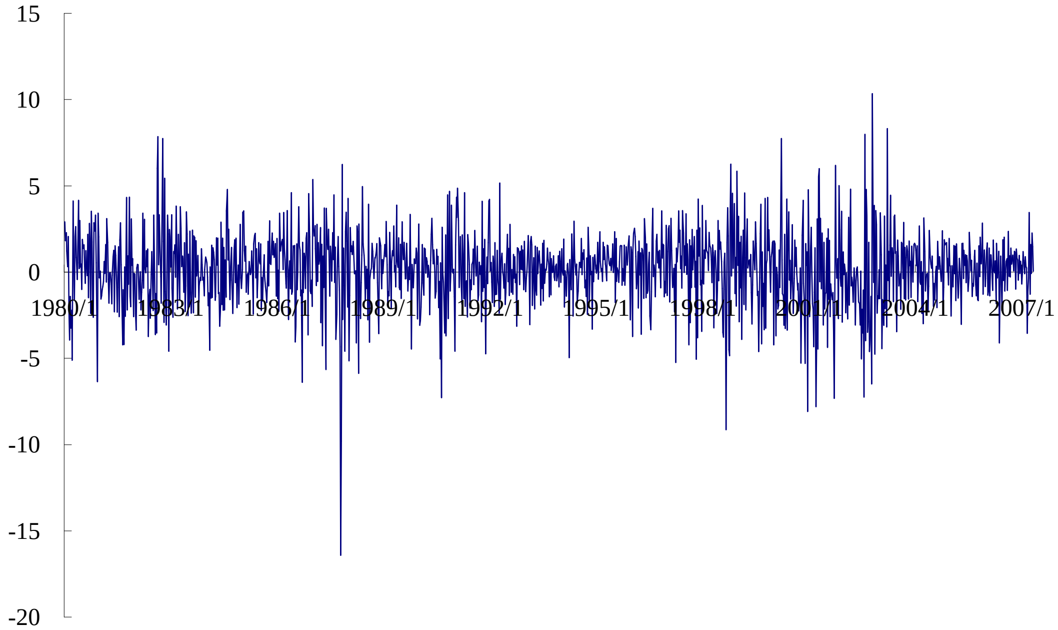


Table 1 Global and Domestic Market Returns: The Case of MSCI-U.S.

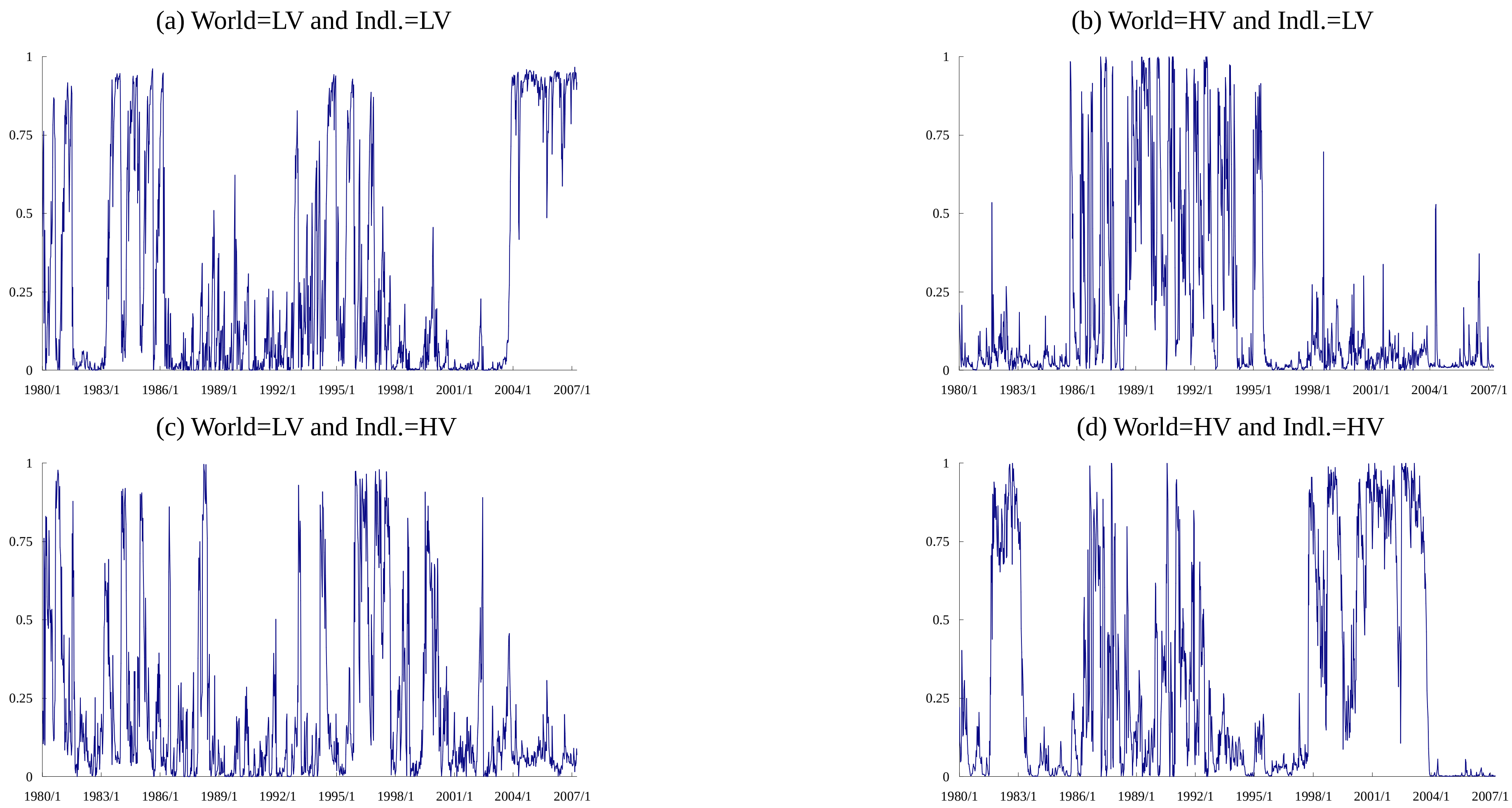
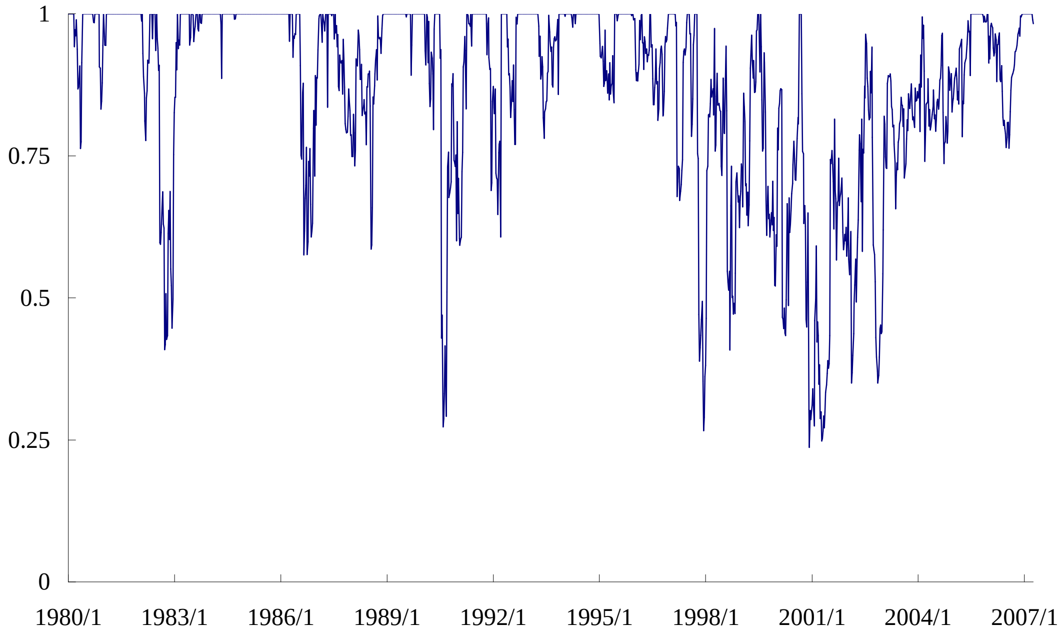


Figure 2 SWARCH-based Estimates of the Filtering Probabilities of Specific Volatility State Combinations for the Illustrative Case of MSCI-U.S.

(a) Optimal Loading on Global Assets by GARCH Model



(b) Optimal Loading on Global Assets by SWARCH Model

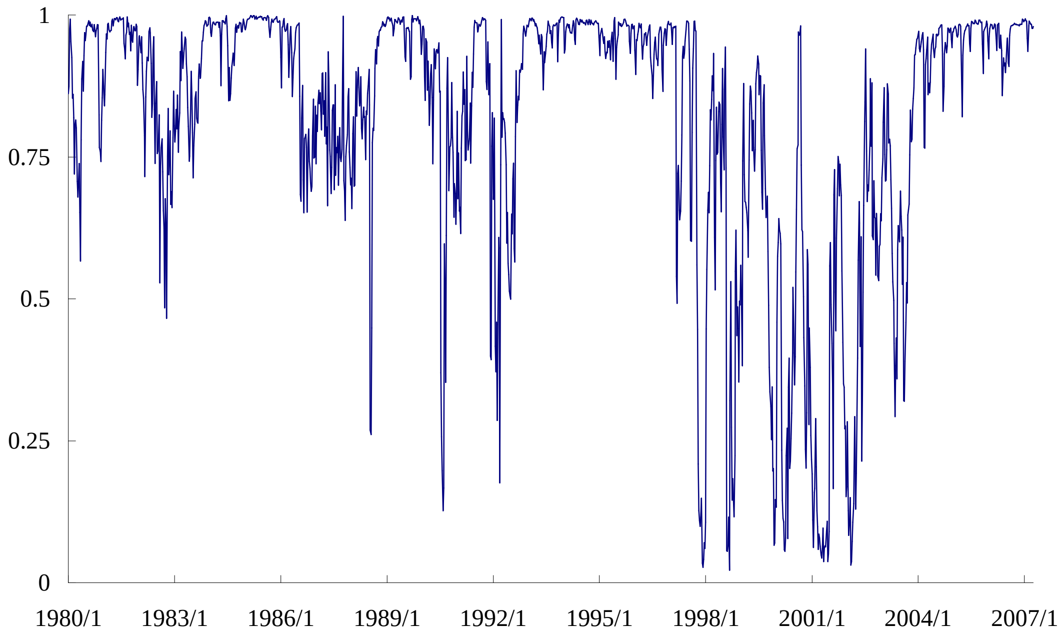
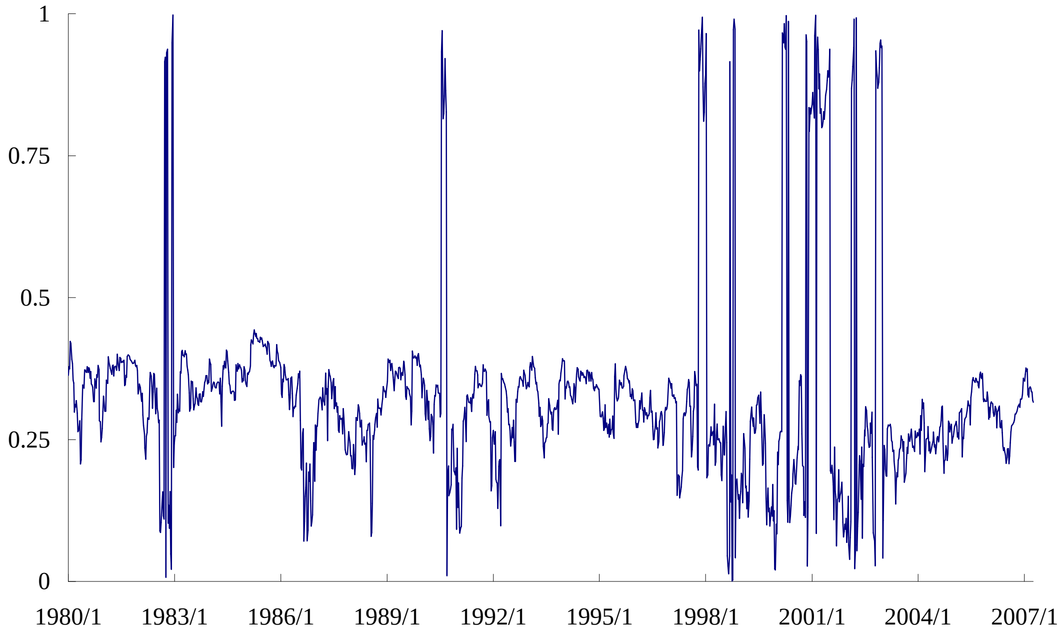


Figure 3 The Optimal Loading of a Minimum Variance Portfolio with Global Stock Market Assets in the Illustrative Case of MSCI-U.K.

(a) Optimal Loading on Global Assets by GARCH Model



(b) Optimal Loading on Global Assets by SWARCH Model

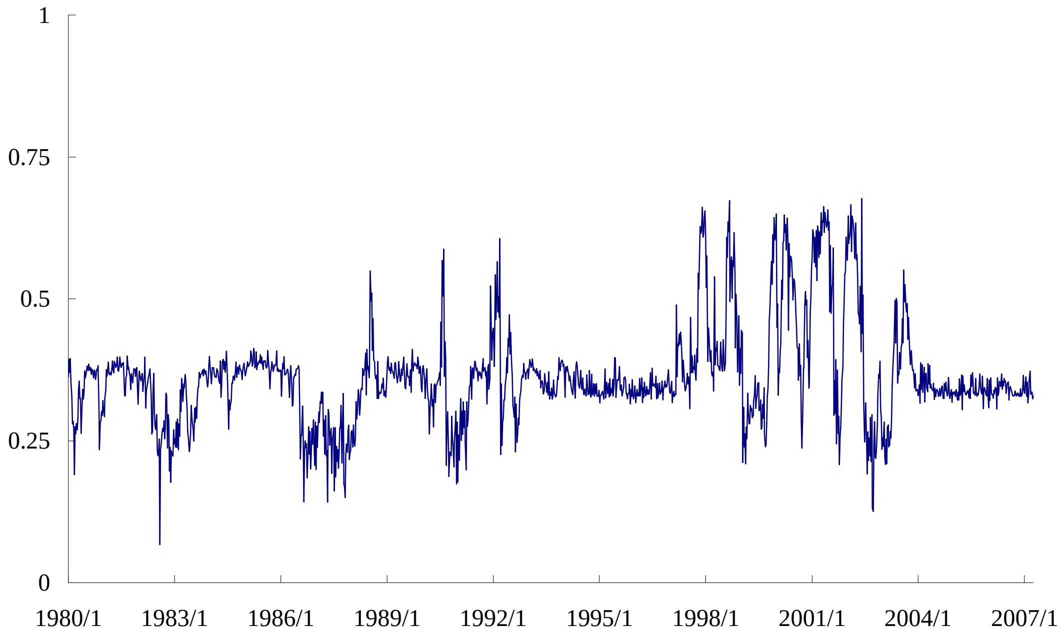


Figure 4 The Optimal Loading of a Given Variance Portfolio with Global Stock Market Assets in the Illustrative Case of MSCI-U.K.

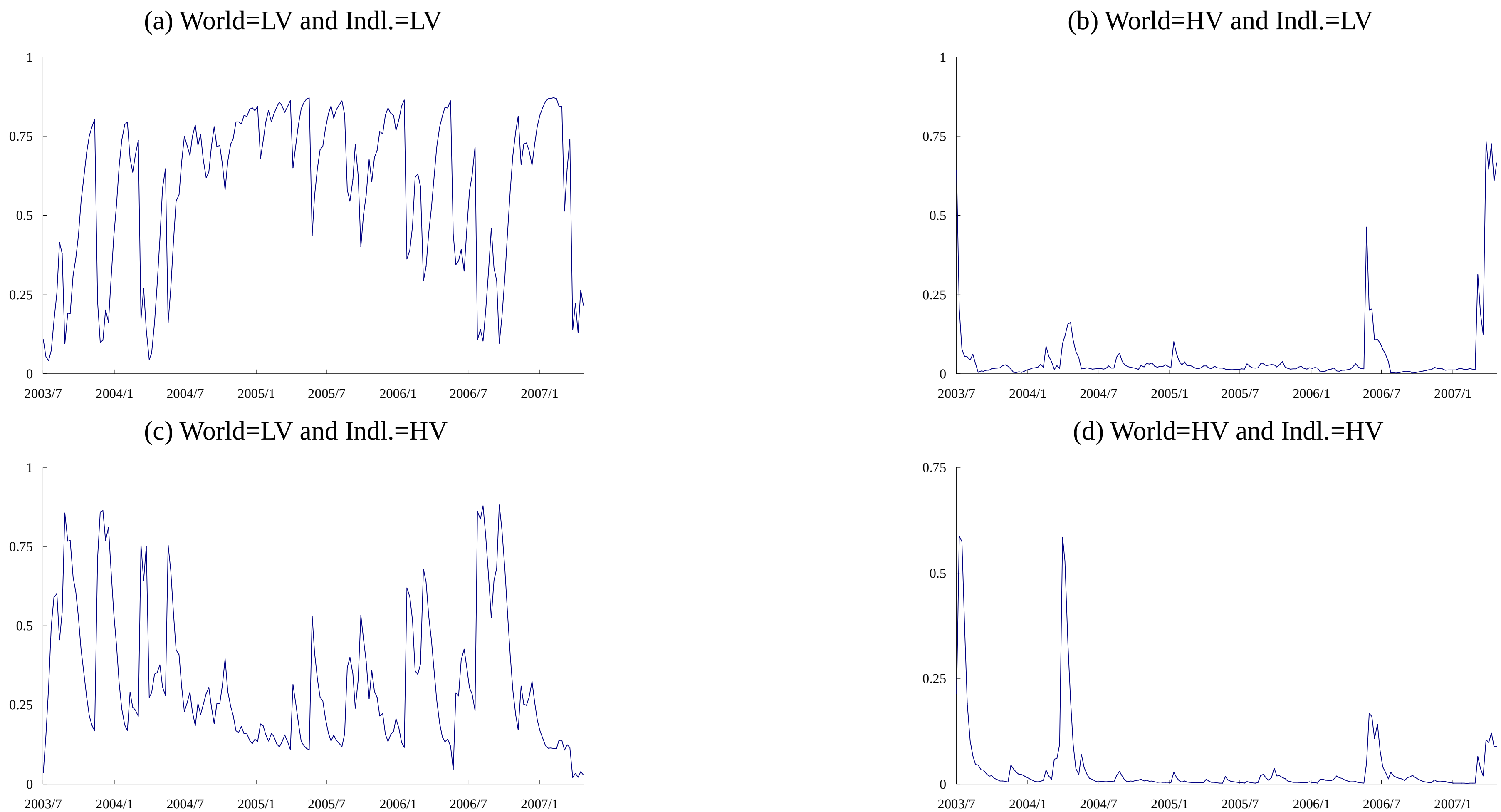
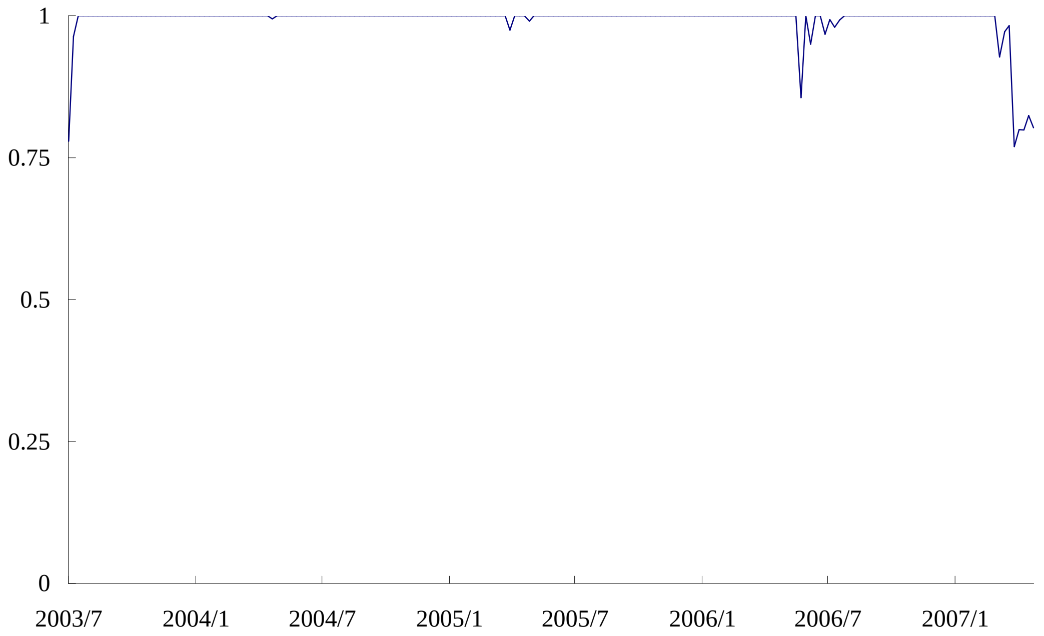


Figure 5 Rolling Regime Probability Forecasts Derived on the Basis of the SWARCH Model in the Illustrative Case of MSCI-Japan

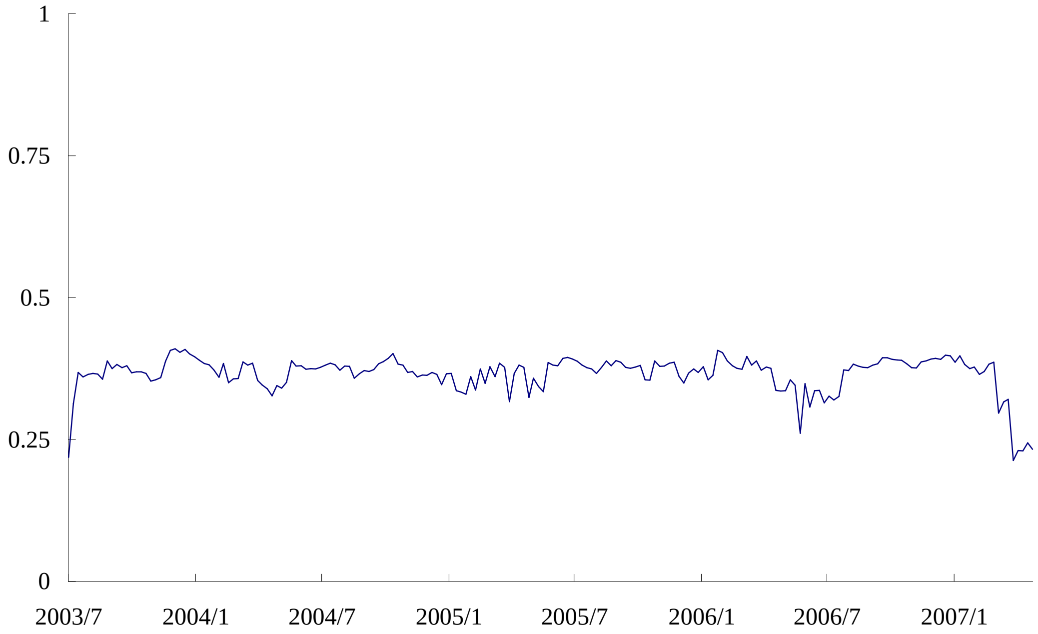
(a) Portfolio Loading on Global Assets by GARCH model



(b) Portfolio Loading on Global Assets by SWARCH model

Figure 6 Estimates of the Rolling Loadings of a Minimum Variance Portfolio with Global Assets: The Case of MSCI-Japan

(a) Portfolio Loading on Global Assets by GARCH model



(b) Portfolio Loading on Global Assets by SWARCH model

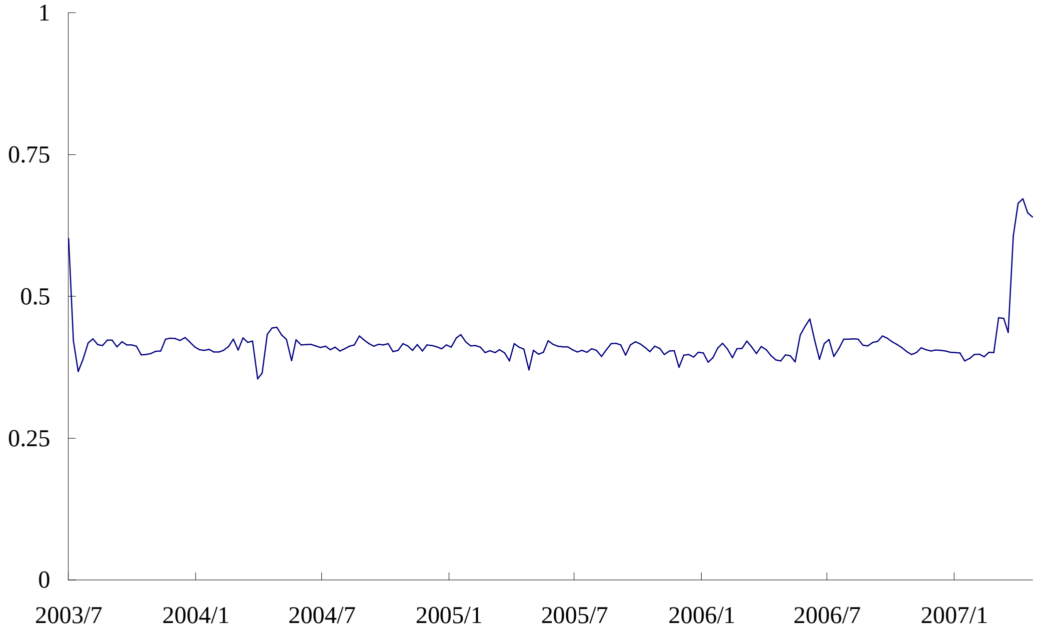


Figure 7 Estimates of the Rolling Loadings of a Given Variance Portfolio with Global Assets: The Case of MSCI-Japan