Do Inflation-Linked Bonds Still Diversify?

M. Brière[1,2,3] and O. Signori[1,*]

October 30, 2007

Abstract

This paper examines the dynamics of conditional volatilities and correlations of three asset classes: inflation-linked (IL) bonds, nominal bonds and equities in the United States and Europe for the period 1997-2007. Using a DCC-MVGARCH model, we highlight the significant change that has taken place in the dynamics of correlations and volatilities since 2003. Inflation-linked bonds have become much more volatile and, at the same time, much more highly correlated with nominal bonds. Monthly portfolio optimization since 1997, using our estimates of conditional correlations and volatilities, clearly demonstrates the decreasing weight of inflation-linked bonds in an optimal allocation. This weighting should now be partially reallocated to equities in a US portfolio, while in Europe, the decreased weight of IL bonds is redistributed, with about one-third going to equities and two-thirds to nominal bonds.

Keywords: inflation-linked bonds, optimal allocation, portfolio choice, conditional volatility, conditional correlation

JEL Classification: G11, G12

[1] Crédit Agricole Asset Management, Paris
90 bd Pasteur, 75015 Paris

[2] Centre Emile Bernheim
Solvay Business School
Université Libre de Bruxelles

[3] Paris Jourdan Sciences Economiques

Comments could be sent to ombretta.signori@caam.com *or* marie.briere@caam.com.

1. Introduction

The first inflation-linked (IL) bonds were issued in 1780 in the United States, but they fell off the radar until twenty-odd years ago, when governments of developed countries began to issue them again, starting with the United Kingdom in 1981. Since then, they have found favor both with governments and with markets, which, besides seeing them as a hedge against inflation, have embraced them as a means of diversifying their portfolios. Currently, 13 developed countries, including the United States, the United Kingdom, France, Germany, Italy, Japan and Canada, have issued substantial amounts of debt in IL bonds. And the list keeps growing. Indexed bond issues now account for more than one-third of new issuance in the U.S. and about 10% in the eurozone.

IL bonds differ from conventional–or nominal–bonds in two important ways: (1) the nominal value of their coupons is the sum of a real coupon that is constant and fixed in advance, and observed inflation (in general, a consumer price index); (2) the principal is indexed to observed inflation, but it is also generally guaranteed in case of deflation. IL bonds serve a dual purpose. First, they hedge against inflation, unlike nominal bonds (Campbell and Viceira (2002)) or equities (Campbell and Shiller (1996)). Second, many studies have pointed out IL bonds' usefulness for portfolio diversification in a mean-variance framework (Lamm (1998), Roll (2004), Kothari and Shanken (2004), Mamun and Visaltanachoti (2006a)).

Unfortunately, this analysis is extremely sensitive to assumptions regarding expected returns, volatility and correlation. Roll (2004) and Mamun and Visaltanachoti (2006b) have shown that inflation expectation hypotheses strongly influence the attractiveness of IL bonds relative to other assets.

Yet even though the return aspect of IL bonds has been widely studied, their risk component has received much less attention. There is very little research into the dynamics of volatility and correlations of IL bonds with other asset classes, or the influence of these factors on the optimal allocation for indexed bonds. To the best of our knowledge, only Hunter and Simon (2005) have modeled conditional volatilities and correlations through a bivariate GARCH model in order to calculate conditional Sharpe ratios. Unfortunately, their study ends with 2001. Now, with more than ten years' data available, we can study changes in correlations and volatilities.

This article contributes to the existing body of research in two ways. First, we propose an estimate for conditional correlations and volatilities between IL bonds, nominal bonds and equities, by means of a DCC-MVGARCH model (Engle (2002)). This has been used successfully to model correlation dynamics between exchange rates (van Dijk et al. (2006)), equities (Kearney and Potì (2003)), and equities and bonds (Cappiello et al. (2003)), but it does not appear to have been applied to IL bonds. In addition, we examine dynamic portfolio optimization, taking conditional volatilities and correlations into account. The results allow us to precisely study the diversifying power of IL bonds and to show how it has changed over time.

We focus on the dynamics of volatilities and conditional correlations between IL bonds, nominal bonds and equities, in the United States and Europe, over 1997-2007. We demonstrate that these dynamics have completely changed in recent years. The volatility of IL bonds, formerly weaker than that of nominal bonds, has increased sharply, reaching levels that are now equal to or slightly higher than those of nominal bonds. At the same time, indexed bonds have become very highly correlated with nominal bonds, thus losing much of their ability to diversify.

This paper is organized as follows: Section 2 presents the data. Section 3 shows that correlations are unstable and presents estimated conditional volatilities and correlations using a DCC-MVGARCH model. Section 4 presents the results of a dynamic portfolio allocation in a mean-variance framework, taking into account conditional correlations and volatilities. Finally, Section 5 concludes.

2. Data

The dataset is composed of daily returns for equities, nominal bonds and IL bonds, in the U.S. and the eurozone. For equities, we use the S&P500 index for the U.S. and the DJ Euro Stoxx for the eurozone. For IL bonds, we use Barclays Global Inflation Total Return indices in US and France. To qualify for inclusion in an IL bond index, a security must meet five criteria: the type of market and bond, the type of coupon, the maturity, the issue date and issue size[1]. For the purposes of the analysis we rely on the French "linker" market as representative of the European market. This is because France has the largest IL bond market[2] in terms of outstanding amounts, number of securities, liquidity and length of sample period. Using it as a proxy also avoids the problem of mixing bonds with different credit ratings[3]. The French index includes bonds linked to French and European inflation rates since October 2001. For nominal bonds, we use the respective Barclays Breakeven Comparator Bond indices for the U.S. and France. These indices are composed of nominal securities, maturity matched with linkers. We thus overcome the problem of dealing with returns that are "contaminated" by differences in duration.

The market value of U.S. IL bonds has grown enormously, from $168 billion in 2002 to $446 billion in August 2007. Market capitalization of French linkers rose from €31.5 billion in 2002 to €129 billion in August 2007 (Figures 1 and 2 in Appendix 1).

The data cover the period from October 1, 1998 to August 31, 2007 for the eurozone, and from March 3, 1997 to August 31, 2007 for the U.S. Stripping market holidays out of the sample, we obtain a total of 2,294 and 2,666 observations, respectively. All indices include coupon or dividend returns. Figures 3 and 4 in Appendix 1 show the cumulative daily returns of the six asset classes, and Table 1 below displays their summary statistics. All returns have been tested for stationarity, with positive results (not reported here).

---

[1] See "Barclays Inflation-Linked Bond Indices, Index Products", *Barclays Capital Research*, 2006.

[2] It represents more than 50% of the eurozone market.

[3] Italy and Greece have lower ratings than France.

Table 1: Descriptive statistics, U.S. and Eurozone
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
		<u>US (1997-2007)</u>	
Average	0.03%	0.03%	0.04%
Ann. Average	6.55%	6.55%	9.10%
Median	0.03%	0.02%	0.05%
Min	-1.81%	-1.43%	-6.87%
Max	1.61%	1.24%	5.73%
Std Dev	0.40%	0.28%	1.13%
Ann. Std. Dev.	6.30%	4.43%	18.00%
Skewness	-0.27	-0.15	-0.01
Kurtosis	1.45	2.34	3.05
		<u>Eurozone*(1998-2007)</u>	
Average	0.02%	0.02%	0.04%
Ann. Average	4.65%	5.09%	9.37%
Median	0.02%	0.01%	0.08%
Min	-1.84%	-1.29%	-6.38%
Max	1.33%	1.10%	6.35%
Std Dev	0.32%	0.26%	1.29%
Ann. Std. Dev.	5.06%	4.06%	20.42%
Skewness	-0.41	-0.29	-0.04
Kurtosis	1.93	2.26	2.82

* *French data for IL bonds and nominal bonds*

During our sample period, the average return on linkers was higher than that on nominal bonds in Europe (5.09% versus 4.65%), and equal to nominal bonds in the U.S. (6.55%). Theoretically, nominal bond returns should include compensation for the risk of unexpected future inflation: the inflation risk premium (Sarte (1998), Shen (1998), McCulloch and Kochin (2000)). Nominal bonds are thus expected to provide slightly higher returns and risks than IL bonds with similar maturities. In that context, the impressive performance of linkers compared to nominal bonds is surprising. It may be due in part to the sample period, which began with low supply and very high demand from institutional investors (insurance companies and pension funds) seeking protection from inflation. Over the entire study period, IL bonds exhibit lower volatility than nominal bonds in both the U.S. and Europe: 4.4% versus 6.3% in the U.S. and 4.1% versus 5.1% in France, consistent with the theoretical framework and previous findings (Hunter and Simon (2002)). In addition, equities provide a risk premium over Treasuries of around 2.55% in the U.S. and 4.72% in Europe during the study period. The reason for the sharp difference between the two is that average nominal bonds returns were much lower in the eurozone than in the U.S.

3. Modeling time-varying correlation

3.1 Instability of correlations

Tables 2 and 3 present the unconditional correlation matrices calculated in the U.S. and the eurozone over the entire sample period. One oft-cited source of diversification provided by IL bonds is the fact that they exhibit negative correlation with other assets classes (especially equities) and moderate correlation with nominal bonds. In our sample, IL bonds display negative correlations with equities (-0.14 in the U.S., -0.24 in the eurozone), and very high correlations with nominal bonds (0.75 in the U.S., 0.70 in the eurozone).

Table 2: Correlation matrix, U.S., 1997-2007
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
Nominal Bonds		0.75	-0.17
IL Bonds			-0.14
Equities			

Table 3: Correlation matrix, Eurozone, 1998-2007
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
Nominal Bonds		0.70	-0.25
IL Bonds			-0.24
Equities			

As mentioned previously, the linkers market is relatively new, and many studies arguing for their strong diversifying power (Lamm (1998), Roll (2004)) refer to their early history. It is thus reasonable to wonder if, after 10 years of growth and change in the market, we will reach a different conclusion.

As a preliminary step, we split the full sample into two equal-length sub-periods: from the inception date to 2002, and from 2003 to 2007. The choice of dates may be debatable, but volume and turnover in this market clearly doubled after 2003, with growth in issuance, liquidity, and the number of market participants. Shen (2006) finds, for example, a decreasing liquidity premium after 2003 in U.S. TIPS, and Hordahl and Tristani (2007) cite high liquidity premiums before that date. In Europe, 2003 coincides with the issuance of Italian and Greek IL bonds. Tables 4 to 7 present the correlation matrices for the two sub-samples.

Table 4: Correlation matrix, U.S., 1997-2002
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
Nominal Bonds		0.67	-0.17
IL Bonds			-0.15
Equities			

Table 5: Correlation matrix, Eurozone, 1998-2002
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
Nominal Bonds		0.53	-0.20
IL Bonds			-0.21
Equities			

Table 6: Correlation matrix, U.S., 2003-2007
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
Nominal Bonds		0.88	-0.17
IL Bonds			-0.16
Equities			

Table 7: Correlation matrix, Eurozone, 2003-2007
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
Nominal Bonds		0.93	-0.34
IL Bonds			-0.31
Equities			

We notice a relevant change in the nominal / IL bonds correlation since 2003. In the U.S., the correlation increases from 0.67 to 0.88. In France, the increase is even larger from 0.53 to 0.93. Correlations of nominal and IL bonds with equities increased less significantly, and remained completely stable in the U.S.

Volatilities also appear very different in the two sub-periods, as can be seen in Tables 8 and 9. Equity index volatility for the second period is roughly 0.6 times that for the first sub-period (which included the equity market crash), whereas nominal bond volatility decreases slightly in the second period in both areas. Again, a dramatic change occurred for IL bonds. Their volatility increased considerably–the volatility ratio between the second and first period is 1.79 in the U.S. and 1.55 in France)–to levels comparable to nominal bonds (or even slightly higher in Europe).

Table 8: Volatilities and volatility ratio, U.S., 1997-2002 and 2003-2007
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
1997 - 2002	6.53%	3.15%	21.29%
2003 - 2007	6.00%	5.64%	12.71%
Ratio	0.92	1.79	0.60

Table 9: Volatilities and volatility ratio, Eurozone, 1998-2002 and 2003-2007
Daily returns in local currency

	Nominal Bonds	IL Bonds	Equities
1997 - 2002	5.75%	3.08%	24.69%
2003 - 2007	4.31%	4.79%	15.49%
Ratio	0.75	1.55	0.63

This preliminary analysis clarifies the problem of considering a covariance matrix calculated on the whole period. This may partly conceal the real-world situation, since correlations and volatilities are very unstable. The χ^2 test by Engle and Sheppard (2001) allows us to test econometrically the null hypothesis of constant correlations against the alternative of dynamic conditional correlations. The results, reported in Tables 10 and 11 in Appendix 2, show strong rejection of the null hypothesis. In that context, the DCC-MVGARCH model will be of great interest, because it can cope with the varying correlations and volatilities over time.

3.2. Dynamic Conditional Correlation modeling

We apply the simplest form of the DCC-MVGARCH model first presented by Engle and Sheppard (2001) and Engle (2002). The original approach of this model uses a two-stage procedure to maximize the log likelihood[4] for the specification of the dynamic conditional correlation framework. This new class of model has the advantage of a parsimonious[5], straightforward and less time-consuming estimation, even for a large number of series, as in Bollerslev's (1990) constant correlation model, while allowing correlation to vary over time.

Consider the daily returns r_t of k asset classes and assume that the returns are conditionally normal[6] with a mean of zero and covariance matrix H_t:

$$r_t | I_{t-1} \sim N(0, H_t)\,.$$

H_t represents the conditional covariance matrix and can be decomposed as follows:

[4] The two-step estimator is shown to be consistent and asymptotically normal and a consistent estimator of standard errors is provided. Another strength of the DCC model is its great flexibility, since it can be easily extended to include other exogenous variables in the correlation model or to adopt other parameterizations.

[5] For example, in a BEKK GARCH specification, to guarantee the stability of the estimate and to limit the estimation time, the maximum number of series usually accepted is five.

$$H_t = D_t R_t D_t$$

where R_t is the conditional correlation matrix and $D_t = \sqrt{h_{it}}$ is a kxk diagonal matrix whose elements are the conditional standard deviations of the returns on each asset class, typically thought of as univariate GARCH(p,q) models.

In other words, the elements of D_t are generated from a process

$$h_{it} = \omega_i + \sum_{p=1}^{P_i} \alpha_{ip} r_{it-p}^2 + \sum_{q=1}^{Q_i} \beta_{iq} h_{it-q} \quad \text{for i=1,2, ..k.}$$

and $\varepsilon_t = D_t^{-1} r_t$

The standardized residuals resulting from the univariate GARCH process are used in the model proposed by Engle to model the dynamic correlation, through a process called DCC(M,N) with M the number of lags of the innovation terms (α_m^*) and N the number of lags of the autoregressive terms (β_n^*) of this type:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}$$

$$Q_t = \left(1 - \sum_{m=1}^{M} \alpha_m^* - \sum_{n=1}^{N} \beta_n^*\right)\overline{Q} + \sum_{m=1}^{M} \alpha_m^* \left(\varepsilon_{t-m} \varepsilon_{t-m}^{'}\right) + \sum_{n=1}^{N} \beta_n^* Q_{t-n}$$

Q_t^* is a diagonal matrix containing the standard deviation of the diagonal of Q_t, which guarantees that R_t is the correlation matrix as long as the positive definiteness of Q_t.

$\overline{Q}$ is the unconditional covariance matrix of the standardized residuals resulting from the first stage estimation and α_m^* and β_n^* are scalars.

The log likelihood for this estimator can be written:

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log\left|H_t\right| + r_t^{'} H_t^{-1} r_t\right)$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log\left|D_t R_t D_t\right| + r_t^{'} D_t^{-1} R_t^{-1} D_t^{-1} r_t\right)$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log\left|D_t\right| + \log\left|R_t\right| + \varepsilon_t^{'} R_t^{-1} \varepsilon_t\right)$$

[6] In case of absence of normality the results are still valid and have a QMLE interpretation.

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log\left|D_t\right| + r_t^{'}D_t^{-1}D_t^{-1}r_t - \varepsilon_t^{'}\varepsilon_t + \log\left|R_t\right| + \varepsilon_t^{'}R_t^{-1}\varepsilon\right)$$

This log likelihood function can be considered as the sum of a volatility component $L_v(\theta)$ and a correlation component $L_C(\theta,\phi)$:

$$L_v(\theta) = n\log(2\pi) + 2\log\left|D_t\right| + r_t^{'}D_t^{-1}D_t^{-1}r_t$$

$$L_C(\theta,\phi) = -\varepsilon_t^{'}\varepsilon_t + \log\left|R_t\right| + \varepsilon_t^{'}R_t^{-1}\varepsilon\ .$$

$$L(\theta,\phi) = L_V(\theta) + L_C(\theta,\phi)$$

The DCC model maximizes the log likelihood in two steps. The first stage is to find $\hat{\theta} = \arg\max\{L_V(\theta)\}$, i.e. the coefficients of a GARCH (p,q) process applied to each asset class. In the second stage the correlation coefficients are estimated conditionally to the parameters estimated in the first stage likelihood: $\max_{\iota}\left\{L_C\left(\hat{\theta},\phi\right)\right\}$.

3.3. Estimation results

A preliminary analysis (not reported there) has been conducted to choose the best univariate GARCH process for our assets. Tables 12 and 13 in Appendix 3 display the estimates of GARCH parameters for each series. Looking at the significance of parameters and information criteria, the model selected is a GARCH (1,1) for the six asset classes. All series present a high degree of persistence (long memory of the process), although the parameter β is more pronounced for bonds than for equities. By contrast, equities demonstrate an innovation parameter (α) which is higher than that for bonds, meaning that a volatility shock lasts longer in equity markets than in bond markets.

Figures 5 to 8 plot the estimated conditional volatilities. The greater volatility of equities is conspicuous with respect to the two bond indices. The recent period has been characterized by a relatively low-volatility environment for equities and decreasing volatility for bonds. Volatility peaks are detectable, coinciding with crises (Schwert (1989), Bekaert and Wu (2000), Caporale et al. (2000)).

Figure 5: Conditional volatility of equities, U.S.

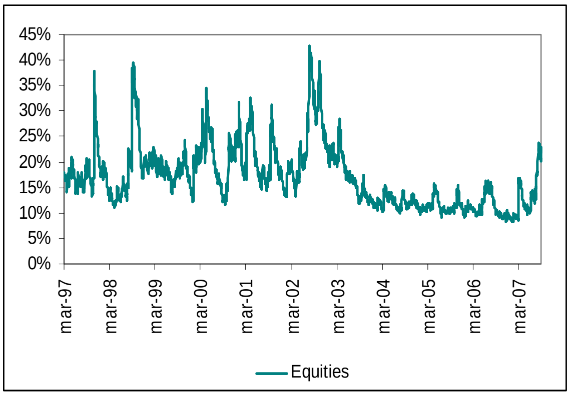


Figure 6: Conditional volatility of equities, Eurozone

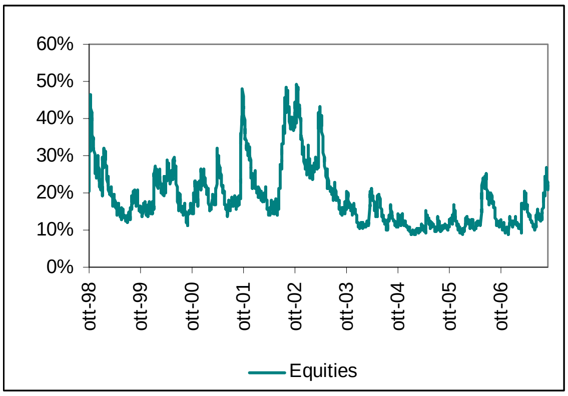


Figure 7: Conditional volatilities of nominal and IL bonds, U.S.

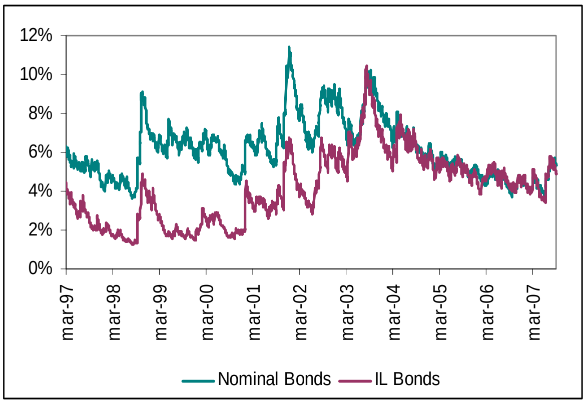
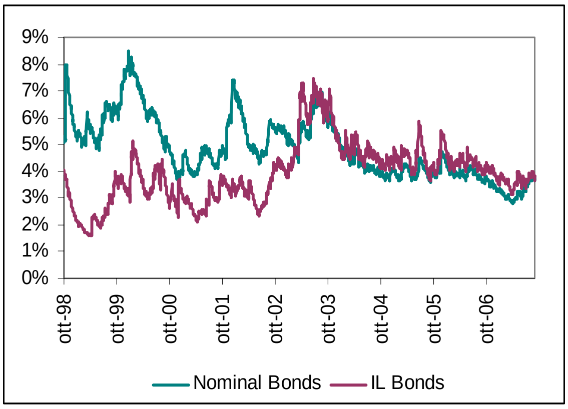


Figure 8: Conditional volatilities of nominal and IL bonds, Eurozone



Until 2003, consistent with theory, IL bonds were less volatile than nominal bonds. But curiously, since 2004, IL bond volatility increased sharply, reaching levels almost identical to those of nominal bonds, and even slightly higher in Europe.

This phenomenon, which is scarcely documented in the academic literature, can probably be explained by the greater stability of inflation expectations (Kahn et al. (2002), Ahmed et al. (2004), Bernanke (2006), Aglietta et al. (2007)). This stability has led to more stable inflation breakevens[7], and therefore to real interest rates moving almost in parallel with nominal rates. Accordingly, the volatility of IL bonds returns is bound to be identical to that of nominal bonds,

[7] Inflation breakevens are expressed as the difference between the nominal rate (quoted on nominal bonds) and the real rate (quoted on IL bonds) of the same maturity. Breakeven measures the market's inflation expectation, plus premiums linked to inflation risk and to the difference in liquidity between the nominal bond market and the IL bond market.

since the returns of both bond classes are dominated by their common component (Hordahl and Tristani (2007)).

The second stage of DCC_MVGARCH modeling is estimation of the dynamic conditional correlation. The parameter estimates of the selected specification, a DCC (1,1) [8], are presented in Tables 14 and 15 in Appendix 3. Figures 9 and 10 depict the estimated dynamic correlation between the six asset classes involved.

Figure 9: Conditional correlations between nominal bonds, IL bonds and equities, U.S.

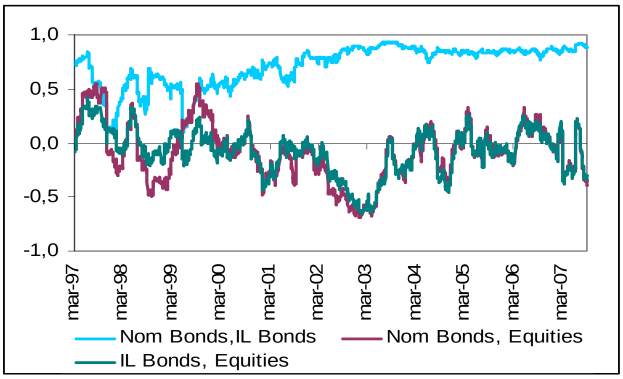
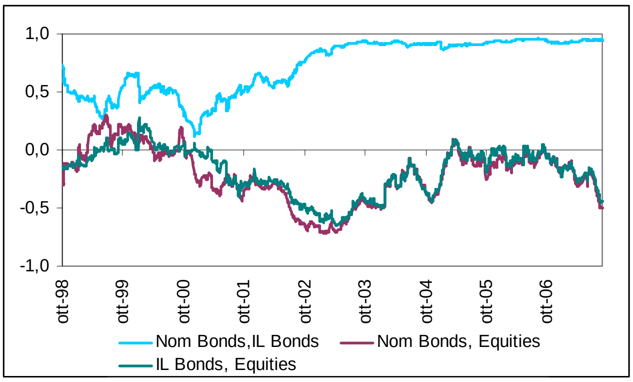


Figure 10: Conditional correlations between nominal bonds, IL bonds and equities, Eurozone



The equity / nominal bond correlation declined significantly beginning in 1998, becoming almost always negative since that date. It is noteworthy that there is a strong analogy between the equity / nominal bond correlation and the equity / IL bonds correlation, especially since 2003. In theory, the correlation of IL bonds with equities should be weaker than that of nominal bonds with equities (Kothari and Shanken (2004)). In fact, nominal bonds and equities are both negatively impacted by unexpected inflation, while indexed bonds are positively impacted, given the negative relationship between inflation and real interest rates.

The most striking fact concerns the change of behavior of the correlation between indexed bonds and nominal bonds. Fluctuating around 60% before 2003, and relatively volatile, the correlation approaches 90% and is extremely stable after that date. Earlier studies (Lucas and

[8] Alternative specifications (DCC(2.1), DCC(1,2), DCC(2,2), etc.) have been tested, but parameters were not significant and the log likelihood did not improve.

Quek (1998), Lamm (1998), Rudolph-Shabinsky and Trainer (1999)) showed that the correlation between the two markets should depend on whether nominal interest rate movements are more reflective of changes in real interest rates or changes in inflation expectations. Thus, when inflation expectations are moving the market more, IL bonds and nominal bonds tend to be less correlated, and when real rates vary, a higher correlation may be expected. The greater stability of inflation expectations could explain why real rates, while fluctuating in parallel with nominal rates, are much more highly correlated with inflation expectations. In addition, aspects of liquidity that are linked to the novelty of IL bonds may explain why their correlation was abnormally low at the beginning of the data series, when the indexed market was subject to significant supply / demand factors.

4. Consequences for Asset allocation

Our objective is to design optimal portfolios with monthly rebalancing, and to measure how their composition changes through time because of the movements in volatility and the correlation process observed in the previous section. The estimated conditional correlations and volatilities allow us to compute daily a conditional covariance matrix that can be used for portfolio optimization, with a standard mean-variance approach.

One difficulty with standard mean-variance optimization is that the appeal of each asset class depends heavily on its expected returns. Many portfolio optimization studies demonstrate that optimal portfolio weights can be highly sensitive to small changes in expected returns. One possibility would be to use average historical returns as a measure of expected returns for the future. As we have seen, the lack of liquidity and the supply/demand mismatch at the beginning of the sample probably made IL bonds returns appear more attractive than they actually were. We can probably not infer from their surprisingly high historical returns that this will remain true.

Since our purpose is to monitor how the optimal asset allocation evolved due to changing correlations and volatilities through time, we want to be fully insensitive to the return hypothesis. Therefore, we make the assumption that each asset's excess return over the risk-free rate is exactly proportionally to risk, i.e. Sharpe ratios are equal for the three asset classes.

This assumption that risk premiums are completely proportional for equities, nominal and IL bonds is a strong hypothesis. It has not been the case historically, but it has the advantage of making portfolio composition dependent only on the risk profile of the assets. Figures 11 and 12 present the patterns of optimal weights in the two portfolios calculated monthly (at month-end) from the daily conditional variance-covariance matrix estimated from the DCC_MVGARCH. We imposed a no-short-selling constraint for all assets.

Figure 11: Evolution of the weights of the optimal asset allocation in the US

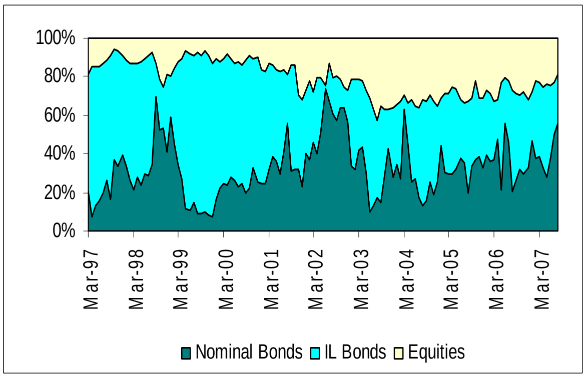
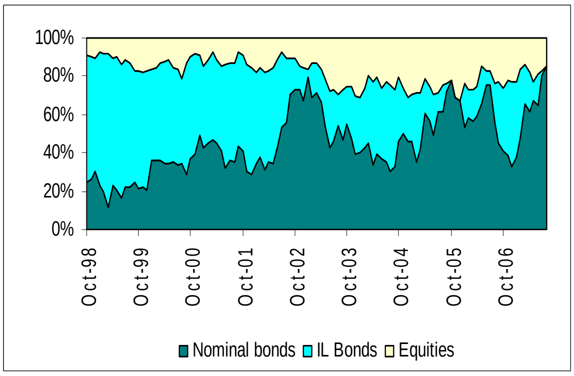


Figure 12: Evolution of the weights of the optimal asset allocation in the Eurozone



The lesser diversifying power of IL bonds is striking in both regions, and has important asset allocation implications. The optimal weight of IL bonds to include in the portfolio, while volatile[9], decreased sharply over the review period, and the decline is even more pronounced in Europe. Tables 16 to 19 examine the change in average optimal weights of the three asset classes (as well as their minimum, maximum and standard deviation) for the two sub-periods split in 2003.

[9] Due to the volatility of conditional correlations and volatilities.

Table 16: Optimal asset allocation weights U.S., 1997-2002

	Nominal Bonds	IL Bonds	Equities
Average	32%	53%	15%
Median	29%	59%	13%
Min	7%	2%	5%
Max	73%	84%	32%
Std Dev	16%	20%	6%

Table 17: Optimal asset allocation weights, Eurozone, 1998-2007

	Nominal Bonds	IL Bonds	Equities
Average	36%	51%	13%
Median	35%	51%	13%
Min	12%	12%	7%
Max	73%	80%	21%
Std Dev	14%	14%	3%

Table 18: Optimal asset allocation weights U.S., 2003-2007

	Nominal Bonds	IL Bonds	Equities
Average	30%	39%	31%
Median	32%	39%	30%
Min	10%	7%	19%
Max	63%	59%	43%
Std Dev	11%	10%	5%

Table 19: Optimal asset allocation weights, Eurozone, 2003-2007

	Nominal Bonds	IL Bonds	Equities
Average	54%	23%	23%
Median	54%	22%	24%
Min	31%	0%	13%
Max	85%	45%	33%
Std Dev	14%	12%	5%

On average the optimal weight of IL bonds decreases from 53% to 39% in U.S. portfolios and from 51% to 23% in Europe. It is always optimal to hold some TIPS in the U.S. portfolio (a minimum optimal weight greater than zero) but this is no longer the case in Europe. The reduction of the maximum weight is also larger in Europe (from 80% to 45%), even though the volatility of weights in the eurozone does not diminish whereas it is halved in the U.S.

Concerning the U.S. asset allocation, the decrease in the weights assigned to linkers moves exclusively in the direction of increasing equities (their weight doubles from 15% to 31%). In the eurozone, however, the share of linkers diminishes, with roughly 60% of the residual going into nominal bonds (the weight increases from 36% to 54%) and 40% into equities (the weighting rises from 13% to 23%). This difference in reallocation between the two regions is linked to the contrasting picture of nominal and IL bonds volatilities in the sample period. In the U.S., the combination of a rise in IL bonds volatility and a fall in nominal volatility leads to similar volatilities for both asset classes in the second period. In Europe, the phenomenon is so pronounced that nominal bonds ultimately appear less volatile than IL bonds, thus attracting a substantial part of the reallocated weights. These results confirm that it is important for investors to be aware of a dynamic variance-covariance structure when composing their portfolios.

5. Conclusion

We have proposed a method for estimating conditional correlations and volatilities between IL bonds, nominal bonds and equities, in the United States and Europe for the period 1997 – 2007, using a DCC-MVGARCH model (Engle (2002)).

The results have enabled us to show that the dynamics of correlations and volatilities have changed radically in recent years. The volatility of IL bonds, which used to be weaker than that of nominal bonds, in keeping with the theory, increased sharply to levels that are now identical to nominal bonds (or even higher in Europe). Indexed bonds have become strongly correlated with nominal bonds, thus losing a large measure of both their diversifying power and their attractiveness for an overall portfolio.

These results shed new light on the subject, in contrast to the earlier literature, which highlighted only the strong diversifying power of IL bonds during their first years in existence. Several factors have contributed to explaining this change in the behavior of linkers, which we mention in passing even though the purpose of this paper is not to examine them in detail. It is possible that a lack of liquidity and a supply / demand imbalance, amid strongly intensifying demand from institutional investors for IL bonds in recent years, made the early price history unrepresentative, and misrepresented the decorrelating power of IL bonds in relation to nominal bonds. Another factor that also played a definite role was the fairly strong decline in inflation expectations and their volatility, in a context of credible inflation-fighting monetary policies and in a more favorable economic environment for low, stable inflation.

Portfolio optimization carried out monthly since 1997, using our dynamic conditional correlation and volatility estimates, clearly shows the decreasing weight that would have been allocated to IL bonds in an optimal allocation.. This weighting should now be partially reallocated to equities in US portfolio, while in Europe the decrease of IL bonds' weight moves to about one-third to equities and two-thirds to nominal bonds. It is clear that portfolio managers must today take these changes into account.

We are aware that the scope of our work is limited by the fact that we do not address the question of expected returns. We used the restrictive hypothesis that the return on each asset was proportional to its risk. In an economic environment of resurgent inflationary fears, the expected return on IL bonds may become much higher than that on nominal bonds (Roll (2004), Mamun and Visaltanachoti (2006b)), and their attractiveness in an optimal allocation would, once again, make good sense. One interesting direction for future research would be to determine the extent to which an increase in unexpected inflation could make up for the fact that IL bonds have become less attractive because of the higher volatilities and correlations we have just demonstrated.

References

Ahmed, S., Levin A. and Wilson (2004), “Recent US Macroeconomic Stability: Good Policies, Good Practices or Good Luck,” Review of Economics and Statistics, 86(3), 824-832.

Aglietta M., Berrebi M. and Cohen-Benamran A. (2007), “La mutation des taux d’intérêt américains face à la globalisation”, Expertises 5-2007, Groupama Asset Management.

Bekaert, G. and Wu, G. (2000), “Asymmetric volatility and risk in equity markets” Review of Financial Studies, 13,1, 1-42.

Bernanke, B.S. (2006), “Reflections on the Yield Curve and Monetary Policy,” speech to the Economic Club of New York, 20 March.

Billio, M. and Caporin, M. (2004), “A generalized Dynamic Conditional Correlation model for portfolio risk evaluation,” GRETA Working Paper, 04-05.

Billio, M. and Caporin, M. (2005), “Multivariate Markov Switching Dynamic conditional Correlation GARCH representations for contagion analysis,” GRETA Working Paper, 05-02.

Campbell, J.Y. and Shiller, R.J. (1996), “A Scorecard for Indexed Government Debt,” NBER Working Paper, 5587.

Campbell, J.Y and Viceira, L. (2002), “Strategic Asset Allocation: Portfolio Choice for Long Term Investors,” Clarendon Lectures in Economics, New York, Oxford University Press.

Caporale, G.M., Pittis, N. and Spagnolo, N. (2000), “Volatility Transmission and Financial Crises,” Econometric Society, available at http://www.econometricsociety.org.

Cappiello, L., Engle, R. F. and Sheppard, K. (2003), “Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns”, ECB Working Paper, 204.

Engle, R. F. (2002) “Dynamic Conditional Correlation – A simple class of multivariate GARCH models,” Journal of Business and Economics Statistics, 20(3), p.339-350.

Engle, R. F. and Sheppard, K. (2001), “Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH,” University of California, San Diego, Department of Economics Discussion Paper, 2001-15.

Hordahl, P. and Tristani, O. (2007), “Inflation risk premia in the term structure of interest rates,” ECB Working Paper, 734.

Hunter, D. and Simon, D. (2005), “Are TIPS the “Real” Deal? A Conditional Assessment of Their Role in a Nominal Portfolio,” Journal of Banking and Finance, 29, p. 347-368.

Kahn, J.A., McConnell, M.M. and Perez-Quiros, G. (2002), “On the Cause of the Increased Stability of the US Economy,” Federal Reserve Bank of New York, Economic Policy Review, 8(1), 183-203.

Kearney, C. and Potì, V. (2003), “DCC-GARCH modelling of Market and Firm-Level Correlation Dynamics in the Dow Jones Eurostoxx50 Index”, School of Business Studies, Trinity College Dublin.

Kothari, S.P. and Shanken, J. (2004), “Asset Allocation with Inflation Protected Bonds”, Financial Analyst Journal, January-February, p. 54-70.

Lamm, R. M. (1998), “Asset Allocation Implications of Inflation Protection Securities,” Journal of Portfolio Management, 24, p. 93-100.

Mamun, A. and Visaltanachoti, N. (2006a), “Diversifying Benefits of Treasury Inflation Protected Securities: an Empirical Puzzle,” available at SSRN, http://ssrn.com/abstract_id=663669.

Mamun, A. and Visaltanachoti, N. (2006b), “Inflation Expectation and Asset Allocation in the Presence of an Indexed Bond,” available at SSRN: http://ssrn.com/abstract=907147.

McCulloch, J.H. and Kochin, L.A. (2000), “The Inflation Premium Implicit in the US Real and Nominal Term Structures of Interest Rates,” Ohio State University Economics Dept Working Paper, 98-12, revised in September 2000.

Roll, R. (2004), “Empirical TIPS,” Financial Analyst Journal, 60(1), January-February, p. 31-53.

Sarte, P.G. (1998), “Fisher’s Equation and the Inflation Risk Premium in a Simple Endowment Economy,” Federal Reserve Bank of Richmond Economic Quarterly, 84(4), Fall 1998.

Schwert, G.W. (1989), “Business Cycles, Financial Crises and Stock Volatility,” NBER Working Paper, 2957, May.

Shenk P. (1998), “How Important is the Inflation Risk Premium,” Federal Reserve Bank of Kansas City, Economic review, fourth quarter 1998.

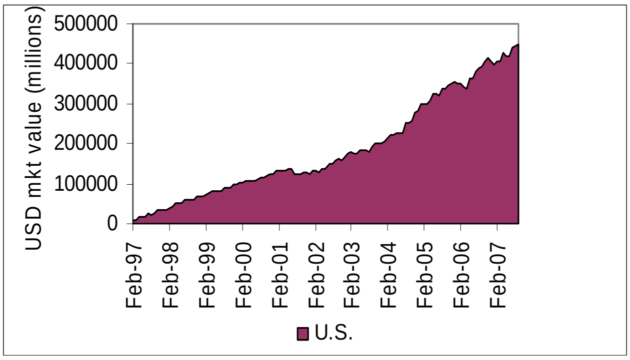
Shen, P. (2006), “Liquidity Risk Premiums and Breakeven Inflation Rates,” Federal Reserve Bank of Kansas City, Economic review, Second Quarter 2006.

Van Dijk, D., Munandar, H. and Hafner, C.M. (2006), “The Euro introduction and Non-Euro Currencies,” Medium Econometrische Toepassingen, 14(1).

Appendix

Appendix 1

Figure 1: IL bonds market capitalization, U.S., 1997-2007



Source : Barclays.

Figure 2: IL bonds market capitalization, Eurozone by country, 1998-2007

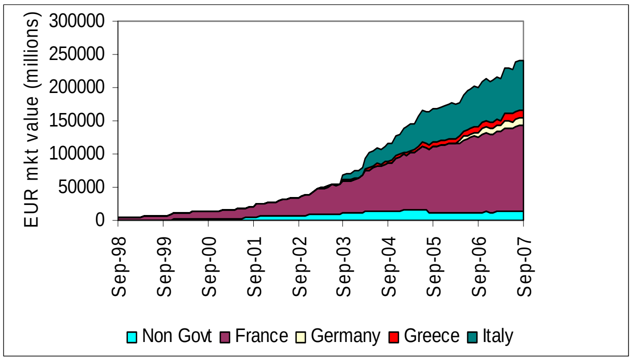


Figure 3: Cumulative daily returns, U.S., 1997 – 2007

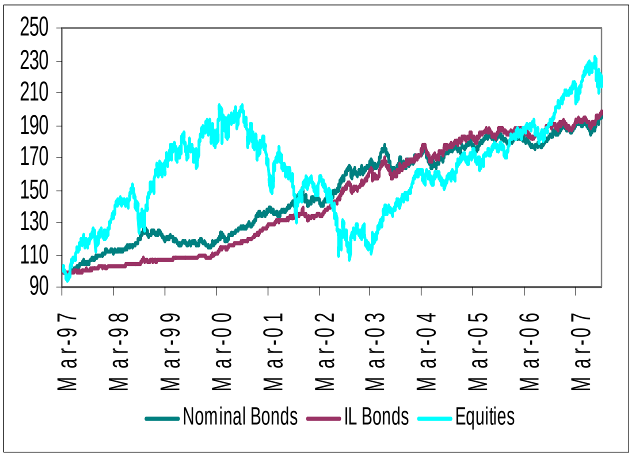
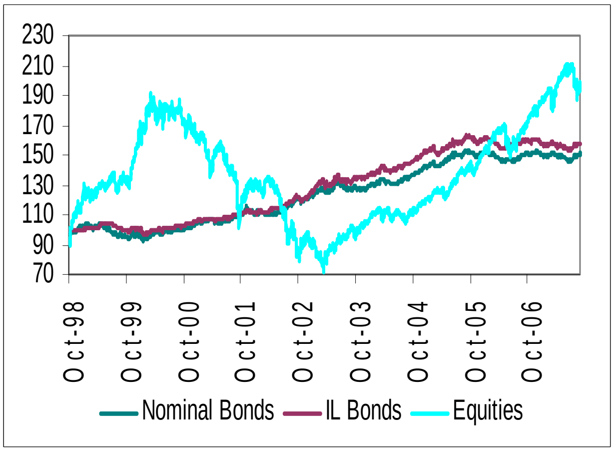


Figure 4: Cumulative daily returns, Eurozone, 1998 - 2007



Appendix 2

Table 10: Constant conditional correlation test*
U.S., 1997-2007

χ_2 **value**	**P- value**
36.48	1.1981e-008

**Engle and Sheppard (2001)*

Ho: $R_t = R \; \forall t \in T$

Table 11: Constant conditional correlation test*
Eurozone, 1998-2007

χ_2 **value**	**P- value**
60.624	6.85e-014

Appendix 3

Table 12: Univariate GARCH parameters estimates, U.S., 1997-2007

	ω	***α***	***β***
Nominal Bonds	1.84E-08	0.02	0.97
	(1.12E-08)	(0.004)	(0.003)
IL Bonds	2.61E-08	0.033	0.963
	(8.94E-09)	(0.009)	(0.010)
Equities	1.32E-06	0.079	0.912
	(4.99E-07)	(0.012)	(0.013)

α represents the ARCH term, β the GARCH term, ω the constant of the variance equation. Standard errors in parenthesis.

Table 13: Univariate GARCH parameters estimates, Eurozone, 1998-2007

	ω	***α***	***β***
Nominal Bonds	4.07E-08	0.03	0.97
	(8.22E-09)	(0.007)	((0.007)
IL Bonds	1.39E-08	0.044	0.956
	(1.83E-08)	(0.008)	(0.009)
Equities	1.08E-06	0.068	0.925
	(5.70E-07)	(0.014)	(0.015)

α represents the ARCH term, β the GARCH term, ω the constant of the variance equation. Standard errors in parenthesis.

Table 14: DCC (1,1) parameters estimates, US, 1997-2007

Parameters	**Estimates**	**St.Dev.**	**z-stat**
α	0.016	0.0028552	5.584
β	0.984	0.0029852	329.551

Table 15: DCC (1,1) parameters estimates, Eurozone, 1997-2007

Parameters	**Estimates**	**St.Dev.**	**z-stat**
α	0.026	0.0059	4.454
β	0.970	0.0072	134.634