Consumption and hedging in oil-importing developing countries*

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Abstract

We present a dynamic model of an oil-importing developing country that faces multiple crude oil shocks. Developing countries has two particular characteristics: their technologies are more intense and less efficient in energy usage, and their economies are mainly driven by their natural resources. The exports from these natural resources can sometimes be correlated with the crude oil shocks. We study the consumption and hedging strategies of the country and compare its decisions with those of more developed countries. The country takes long/short positions in the existent crude oil futures contracts. We find that both, inefficiencies in energy usage and shocks to the crude oil price, generate a negative income effect because they lower the productivity of capital. There is also a positive substitution effect that makes today's consumption relatively cheaper than tomorrow's consumption. The optimal consumption of depends on the magnitudes of these effects and the risk-aversion degree of the country. Shocks to other crude oil factors, such as the convenience yield, are also studied. We find that the persistence of the shocks magnifies the income and substitution effects on consumption, thus affecting also the hedging strategy of the country. The demand for futures contracts is decomposed in a myopic demand, a pure hedging term and the productive hedging demands. These hedging demands arise to hedge against changes in the productivity of capital due to changes in the crude oil spot price in the future. We calibrate the model for Chile and study up to what extent the country's copper exports can be used to hedge the crude oil risk.

Keywords: Crude oil prices, convenience yields, risk management, emerging markets, government policy, two-sector economies

JEL Classification: G11, Q43, Q48, D92, O41, C60

Contents

1 Introduction

The recent steeply rise in crude oil prices is comparable to the hike observed in prices in the 70's and early 80's. The fact that nine out of the previous ten US recessions were preceded by an increase in oil prices has brought back the interest of researchers and policymakers in understanding the effect of energy shocks in the economy.[1] But the impact of oil-price shocks in oil-importing developing economies is different than the one in more developed countries. In general, oil-importing developing countries have higher energy-intensive manufacturing as a fraction of their GDP and use energy less efficiently (see International Energy Agency (2004)). Also, many of these economies are less diversified than developed economies and rely on the export of a few primary commodities that flow from their natural resources.[2] These exports are sometimes called the *natural exports*. Interestingly, changes in the natural exports due to variations in the domestic commodity prices are sometimes correlated with crude oil shocks.[3] This correlation added to the higher energy usage makes some developing economies extremely vulnerable to crude oil shocks. Clearly, changes in crude oil prices affect the consumption pattern of these economies. Surprisingly, there has not been enough attention to the risk-management policy that the countries can implement to confront these fluctuations. Nowadays crude oil futures are the most actively traded contracts and can significantly reduce the exposure of an economy to crude oil risk.

In this paper we study the consumption and hedging strategies of an oil-importing developing country that faces exogenous multiple crude oil shocks. To capture the relation between oil and the developing economy, we consider that the country has two productive sectors. First, it has a technology that produces capital and combines oil and capital as inputs. There are some particular parameters in this technology that regulate the efficiency

[1]See the reviews of Jones, Leiby, and Paik (2004) and Kilian (2007) for the current state of this literature.

[2]For example, using the data from Table 1 of **?** (**?**) we find that between 1991 and 1999, copper accounted for 85% of the exports of Zambia and 41% of the exports of Chile. In the same period, gold corresponded to 34%, 18% and 17% of Burundi, South Africa and Ghana exports, respectively.

[3]Using monthly average prices from Sep-1995 to Aug-2007 we find a correlation between oil and copper returns of 28.9% and between oil and gold returns of 16.4%.

of oil usage. The country chooses how much capital to consume and how much oil to import at the prevailing market prices. The other productive technology is the natural resource sector that generates the natural exports of the country. We assume that changes in these exports can be correlated with the oil price shocks. Under this setting, we consider that a more developed country has less natural exports relative to its capital. Other types of exports are included in the capital's production technology.

Recent financial studies have developed multi-factor Gaussian models that correctly captures the dynamics of crude oil prices (see for example Schwartz (1997) and Casassus and Collin-Dufresne (2005)). We consider a generalization of these models. A multi-factor model is important because the risk management techniques involve trading in oil futures contracts that can be subject to numerous sources of uncertainty.[4] The optimal hedging strategy imply long/short positions in the existing crude oil futures contracts. There are at least as many futures contracts available as crude oil risk factors, so that the developing country can fully hedge the oil risk if it's optimal to do so. These financial instruments enhances the investment opportunity set of the country. We assume that the country only chooses to hedge against the crude oil risk factors. The country has a comparative advantage over other commodity producers and decides not to hedge against its own commodity price risk. Studying the hedging policies of some emerging countries in our sample shows that our assumption is quiet reasonable. For example Codelco, Chile's public copper mining company, has only 9% of its future production hedged for the period between 2006 and 2012.

We use an asymptotic expansion technique to find approximate analytical expressions for the country's consumption and contract holdings. This technique expands the solution of our problem around the closed-form solution of a particular case (see Kogan and Uppal (2003)). Indeed, as the input share of oil in the economy and the natural exports of the country goes to zero, the solution converges to the portfolio selection model of Merton (1969) and Merton (1971).

[4]For example, futures prices in a 3-factor model depend on three sources of uncertainty that can be interpreted as the level, slope and curvature of the futures curve.

We find that consumption increases with the natural exports of the country, because an increase in the exports increases the country's wealth. We also find that the relative risk-aversion degree plays a crucial role in the country's decisions. In terms of oil usage, less efficient countries consume a lower fraction of their wealth if they are mainly worried about smoothing consumption (i.e. they have a risk-aversion degree greater than 1). For lower risk-aversion degrees less efficient countries consume more than developed countries because of the well-known substitution effect. Indeed, the consumption good is more scarce in the future for less efficient countries, implying that today's consumption good is relatively cheaper than tomorrow's consumption. The crude oil price has a negative effect for oil-importing economies, because it implies a decrease in productivity of capital. They affect the current state of the economy, but also the state in the future, specially if these shocks are persistent. Highly risk-averse countries decrease their consumption if a price shock occurs, but countries less interested in consumption smoothing may increase today's consumption due to the substitution effect of crude oil prices. Shocks to other variables related to the crude oil dynamics, such as the convenience yield, alter consumption through their effect on the expected change in the crude oil price. If the futures price is a good predictor of the expected spot price, then a positive shock to the convenience yield has a positive effect for oil-importing economies because it decreases the expected oil price. This causes an increase in today's consumption if the country has a risk-aversion degree greater than 1.

With respect to the hedging strategy of the developing country we find that the positions in the futures contracts can be decomposed in three components. First, we obtain the standard myopic demand related to the risk-return trade-off of the financial instruments. Second, we find that the country takes positions in contracts for pure hedging purposes in order to minimize the variance of the country's wealth. The natural exports and their correlation with the crude oil shocks have a fundamental role in determining the size of this component. A higher correlation implies short positions in the futures that can potentially offset the other demands. Finally, we get hedging demands with respect to each one of the crude oil risk factors. As expected, these demands are very sensitive to the risk-aversion of the country, because the positions depend on the effect of each oil risk factor in consumption. Therefore, the persistence of the crude oil shocks have a significant impact in the magnitud of the positions. We calibrate our model for a simple case. We consider Chile as the benchmark developing economy and study its decisions in the case that the crude oil price is driven by a one-factor model. We find that a positive correlation between the Chilean natural exports and the crude oil price reduces considerably the positions in the crude oil futures contracts. The natural exports can potentially work as a natural hedge against crude oil risk. If we concentrate only on the hedging characteristics of the futures contacts and assume a high risk aversion degree for Chile, we obtain that the country hedges between -30% and 50% of the annual crude oil imports depending on the natural exports and their correlation with the crude oil shocks.

An extensive literature studies the link between oil prices and economic activity. Rasche and Tatom (1977), Darby (1982), Hamilton (1983), Hamilton (1988) and Mork (1989) report evidence supporting the hypothesis that oil prices have a significant effect on output. More recently, Hamilton (2003) propose a non-linear specifications for an oil shock considering the smaller effect of price shocks on real economic activity detected since the mid-1980s. The mechanism by which oil affects the economy remains to be unclear, specially because on average oil accounts only for a small part of the total marginal cost of production. Kim and Loungani (1992) explicitly include energy as an input in a RBC model and find that oil price shocks should account only for a minor part of the output volatility. Rotemberg and Woodford (1996) consider the effects of imperfect competition and find that a model involving implicit collusion in the product market can significantly increase the effect of an energy price shocks on output. Finn (2000) proposes an explanation based on the relation between the capital utilization rate and energy prices. Finally, Bernanke, Gertler, and Watson (1997), Bernanke, Gertler, and Watson (2004), Hamilton and Herrera (2004) and Leduc and Sill (2004) have contributed to the ongoing debate about the role of the monetary policy that responded to oil-price shocks in causing US recessions.

Several papers study the connection between the economic performance of developing countries and the price of the commodities that these countries export (Deaton and Miller (1995), Hoffmaister, Roldos, and Wickham (1998), Deaton (1999), Kose and Riezman (2001) among others). Only few papers deal with the management of oil price risk in developing countries. Daniel (2001) and Devlin and Titman (2004) study the effectiveness of oil stabilization funds compared to managing risk with financial instruments when the country is a net exporter of oil. Both papers find that in theory the usage of derivatives dominates the stabilization fund approach, but in practice governments have favored the latter alternative. The authorities fear the political cost of ending up worse off and also lack of know how to implement these financial strategies. Devlin and Titman (2004) also argues that stabilization funds solution is even less efficient if oil price shocks are persistent. Claessens and Varangis (1991) studies a historical simulation of different hedging strategies of a state oil-importing company for the period 1986-1990. They show that the the company would have benefited substantially with the usage of futures contracts even if is were subject to basis risk.

The paper is organized as follows. Section 2 presents the model and provides an analytical solution to the Hamilton-Jacobi-Bellman equation, discussing the resulting hedging and consumption strategies. Section 4 presents the empirical estimation and analyzes the economic implications for a one-factor crude oil pricing model. Finally, Section 5 concludes.

2 The Model

2.1 Production Technologies in the Developing Country

We assume that the emerging economy has two productive sectors: a capital sector and a natural resource sector that exports the production of local commodity (i.e. the natural exports).

The capital sector $K(t)$ has a Cobb-Douglas production technology that uses capital and crude oil as inputs. We consider the following dynamics for the developing country's capital stock:

$$dK(t) = \big(\alpha\, K(t)^{1-\eta}(\omega\, Q(t))^{\eta} - S(t)Q(t) + X(t) - C(t)\big)\, dt, \tag{1}$$

where $K(t)$ is the stock of capital, α is the total factor productivity, η denotes the oil share of input in the production of capital, $Q(t)$ is the demand for crude oil, $S(t)$ is the price of a barrel of crude oil, $X(t)$ are the natural exports and $C(t)$ is consumption. The parameter ω regulates the efficiency of oil usage. It is higher for countries with more efficient technologies, because oil is a more productive input. The country chooses how many barrels of oil to demand and how much capital to consume at any given time t. The demand for oil is relatively small compared to the global aggregate demand, thus the country is assumed to be a crude oil price taker. Other types of exports from alternative sources are included in the capital's production technology.

Rather than assuming a process for the natural resource stock, we directly model the exports from this sector. We consider the natural exports, $X(t)$, to follow a geometric Brownian motion:

$$dX(t) = -\phi\, X(t)dt + \sigma_X X(t) d\widehat{Z}(t), \tag{2}$$

where ϕ and σ_X are the 'depreciation rate' and the volatility of the export changes, respectively. The natural exports decreases over time because the natural resource is assumed to be exhaustible. Another interpretation for a decrease in the natural exports is that the economy develops over time, meaning that more developed countries have lower exports to capital ratios. Finally, $\widehat{Z}(t)$ is a standard Brownian motion, that can be correlated with the crude oil shocks described in the next section.

The emerging country maximizes the expected utility of consumption given by

$$U(t, C) = e^{-\beta t} \frac{C^{1-\gamma}}{1-\gamma} \quad \text{for} \quad \gamma > 0, \gamma \neq 1 \tag{3}$$

The effect of crude oil in the developing country is twofold. First, it has a direct impact in the economy's marginal productivity of capital, since the crude oil is as input to the economy. The higher the price of the oil, the lower the country's output. The second effect, is through a possible correlation between the crude oil shocks and the natural exports of the country. If they are positively correlated, then an increase in the oil price can generate an increase in the exports. In this case, the two oil effects have opposite directions, implying that the exports can potentially act as a natural hedge against crude oil shocks. In a dynamic economy like ours, crude oil shocks can also have a substantial effect in the economy's productivity in subsequent periods.

2.2 General Gaussian Crude Oil Price Process

For the crude oil price process we extend the approach of Casassus and Collin-Dufresne (2005) (CCD) to multiple sources of uncertainty. We introduce a canonical representation of an n-factor Gaussian model for crude oil (log) prices similar to the standard affine models from the term structure literature.[5] The model is in the $A_0(n)$ family using the terminology of Dai and Singleton (2000).

We assume that the spot crude oil (log) price, $u(t) = \log S(t)$, follows the standard no-arbitrage dynamics under the equivalent martingale measure $\mathbb{Q}$:

$$du(t) = \left(r - \delta(t) - \frac{1}{2}\sigma_u^2\right) dt + \sigma_u \left(\sqrt{1 - \varsigma^\top \varsigma}\, dZ_u^{\mathbb{Q}}(t) + \varsigma^\top dZ_v^{\mathbb{Q}}(t)\right) \tag{4}$$

[5] See Duffie and Kan (1996), Duffie, Pan, and Singleton (2000) and Dai and Singleton (2000).

where r is the interest rate, $\delta(t)$ is the convenience yield, σ_u is the volatility of oil returns and $Z_u^{\mathbb{Q}}(t)$ and $Z_v^{\mathbb{Q}}(t)^\top = \left\{Z_{v_1}^{\mathbb{Q}}(t), \ldots, Z_{v_{n-1}}^{\mathbb{Q}}(t)\right\}$ are n independent standard Brownian motions. The $n \times 1$ vector ς, defines the instantaneous correlation structure of the (log) price with other factors affecting the oil price dynamics.

The proposed Gaussian model considers time-varying expected crude oil returns. Its flexibility to fit the data is given by a stochastic specification for the convenience yield $\delta(t)$.[6] Empirical studies (Schwartz (1997), CCD among others) suggest that the variability of crude oil returns are mostly explained by changes in the convenience yield, rather than by changes in interest rates. For this reason and to keep the model simple, we assume a constant interest rate. We generalize the model in CCD and assume that the convenience yield is a linear function of the (log) price and $n-1$ other factors represented by $v(t)^\top = \{v_1(t), v_2(t), \ldots, v_{n-1}(t)\}$:

$$\delta(t) = \psi_0 + \psi_u u(t) + \psi_v^\top v(t) \tag{5}$$

The vector $v(t)$ follows a Gaussian diffusion process under the equivalent martingale measure $\mathbb{Q}$:

$$dv(t) = -\kappa_v v(t)dt + dZ_v^{\mathbb{Q}}(t), \tag{6}$$

where κ_v is an $n \times n$ upper triangular matrix.[7]

The parameter ψ_u in equation (5) plays a crucial role in the dynamics of the oil price. This relation between convenience yields and oil prices allows the model to generate both, contango and backwardation in the futures curve. Indeed, if ψ_u is positive, the expected change in oil prices (under the $\mathbb{Q}$ measure) is lower for high prices because the convenience yield is high, implying higher degrees of backwardation. The opposite effect occurs for low

[6]The convenience yield is defined as the implied benefit associated with holding the underlying physical good, in this case, a barrel of oil.

[7] From an empirical point of view, it is worth noting the parameters r and ψ_0 cannot be separately identified. If we replace equation (5) in (4) we can see that the constant in the expected oil return is $r - \psi_0$. As we will see later, we estimate the model only with futures prices data, and since the convenience yield is an unobservable variable, it is impossible to identify ψ_0 from the estimate of $r - \psi_0$. To circumvent this empirical issue we assume a value for r and estimate ψ_0 from the data. We prefer this overidentified representation to isolate the convenience yield effect from the interest rates.

oil prices.

We have chosen a slightly different canonical representation than in CCD where the (log) spot price, $u(t)$, is a function of the latent factors. We want to explicitly have the oil price as a factor in the crude oil dynamics in order to understand the direct effect of this variable in the consumption and hedging strategies. Under the CCD representation, the hedging strategy will be in terms of n latent factors, rather than in the spot price $u(t)$ and $n-1$ latent factors.

To simplify the notation we define $Y(t)$ as the stacked vector of the n crude oil factors, $Y(t)^\top = \{u(t), v_1(t), \ldots, v_{n-1}(t)\}$. Using equations (4)-(6) we obtain the dynamics of $Y(t)$:

$$dY(t) = (\kappa_0 - \kappa_Y Y(t))dt + \sigma_Y dZ_Y^{\mathbb{Q}}(t), \tag{7}$$

where $Z_Y^{\mathbb{Q}}(t)^\top = \left\{Z_u^{\mathbb{Q}}(t), Z_{v_1}^{\mathbb{Q}}(t), \ldots, Z_{v_{n-1}}^{\mathbb{Q}}(t)\right\}$ and the $n \times 1$ vector κ_0, and the $n \times n$ matrices κ_Y and σ_Y, collect the parameters from the dynamics of the crude oil factors $u(t)$ and $v(t)$.

We assume that there are m crude oil futures contract traded at any time t that matures in $\tau_i > t$ periods more for $i = 1, \ldots, m$. It is well known (e.g., Duffie (2001)) that when interests rates are constant, the futures price $F_i(t)$ with maturity τ_i at time t is:

$$F_i(t) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{u(t+\tau_i)}\right] = e^{B_{0,i}(t) + B_{Y,i}(t)^\top Y(t)}, \tag{8}$$

where $B_{0,i}(t)$ and $B_{Y,i}(t) \equiv \left\{B_{u,i}(t), B_{v_1,i}(t), \ldots, B_{v_{n-1},i}(t)\right\}$ are the solution to the following system of ordinary differential equations:

$$\begin{aligned} \frac{dB_{0,i}(t)}{dt} &= -\frac{1}{2}B_{Y,i}(t)^\top \sigma_Y \sigma_Y^\top B_{Y,i}(t) - \kappa_0^\top B_{Y,i}(t) & (9) \\ \frac{dB_{Y,i}(t)}{dt} &= \kappa_Y^\top B_{Y,i}(t) & (10) \end{aligned}$$

with boundary conditions $B_{0,i}(t+\tau_i) = 0$, $B_{u,i}(t+\tau_i) = 1$ and $B_{v_1,i}(t+\tau_i) = \ldots = B_{v_{n-1},i}(t+$

$\tau_i) = 0$. These conditions ensure that at maturity $F_i(t+\tau_i) = S(t+\tau_i)$.

To complete the model we assume a constant risk-premia specification:

$$dZ_Y^{\mathbb{Q}}(t) = dZ_Y(t) + \lambda_Y dt \tag{11}$$

where $Z_Y(t)$ is a $n \times 1$ vector of Brownian motions on a standard filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\lambda_Y^\top = \left\{\lambda_u, \lambda_{v_1}, \ldots, \lambda_{v_{n-1}}\right\}$ is the risk-premia vector.

Under the physical measure $\mathbb{P}$, the processes for the futures prices maturing τ_i periods from now are given by[8]

$$dF_i(t) = F_i(t)B_{Y,i}(t)^\top \sigma_Y \lambda_Y dt + F_i(t)B_{Y,i}(t)^\top \sigma_Y dZ_Y(t). \tag{12}$$

The processes under $\mathbb{P}$ are relevant for risk management decisions (rather than those under the risk-neutral measure $\mathbb{Q}$), because the implementation of these strategies implies holding futures contracts over time. The country takes positions in futures contracts and demands a compensation (here $B_{Y,i}(t)^\top \sigma_Y \lambda_Y$) for bearing the risk embedded in those contracts.

Finally, we define the process $dF(t)$ as the $m \times 1$ vector of stacked processes $dF_i(t)$:

$$dF(t) = I_F(t)\sigma_F(t)\lambda_Y dt + I_F(t)\sigma_F(t)dZ_Y(t) \tag{13}$$

where $I_F(t)$ is a matrix with the futures prices $F_i(t)$ in the diagonal, and $\sigma_F(t)$ stacks the n row vectors $B_{Y,i}(t)^\top \sigma_Y$.

[8]Note that in the Gaussian model with constant risk premia, the futures returns $dF_i(t)/F_i(t)$ are not affected by the level of $Y(t)$. These state variables enter only through the futures price $F_i(t)$.

3 Optimal Controls in the Developing Country

In this section we study the problem that faces the developing economy. At any given time, the country chooses: (i) how much crude oil to demand for its production technology, (ii) how much capital to consume, and (iii) the positions in the futures contracts in the economy.

We allow the country to take long/short positions in the m available crude oil futures contracts to optimally hedge against the crude oil uncertainty. The primary purpose of the futures contacts is hedging, but they also enhance the investment opportunity set of the developing country. This creates additional incentives to take positions in these financial instruments. The n-factor specification for crude oil dynamics implies that the economy may want to hedge not only against the crude oil price shocks, but also against changes in the convenience yield factors.[9] Without loss of generality, we consider that at every period of time there are as many futures contracts available as risk factors (i.e., $m = n$). Indeed, for an n-factor model for crude oil prices, n futures contracts with different maturities are enough to span the whole futures curve.

Let us define the $n \times 1$ vector $p(t)$ as the *number of crude oil futures contracts* held by the developing country at time t for each one of the n available contracts. A positive (negative) element i of $p(t)$ means that the country takes a long (short) position in the futures contract maturing in τ_i periods from time t. We restrict $p(t)$ to be in the set of admissible strategies that lead to strictly positive capital process ($K(t) > 0$ $a.s.$). We only consider non-negative consumption and crude oil demand strategies.

The optimal consumption-demand-hedging strategy of the developing country is the solution to the following problem:

$$J(K(0), X(0), Y(0), 0) \equiv \sup_{\{C,Q,p\}\in\Psi} \mathbb{E}_0 \left[\int_0^\infty U(s, C(s))ds \right] \tag{14}$$

[9] Eventually, by taking long/short positions in a portfolio of futures contracts, the country can hedge against changes in the slope and curvature of the futures curve.

subject to:

$$
\begin{aligned}
dK(t) &= \left(\alpha\, K(t)^{1-\eta}(\omega\, Q(t))^{\eta} - S(t)Q(t) + X(t) - C(t)\right) dt + p(t)^{\top} dF(t) &\quad (15)\\
dX(t) &= -\phi\, X(t)dt + \sigma_X X(t)(\rho_Y^{\top} dZ_Y(t) + \sqrt{1-\rho_Y^{\top}\rho_Y}\, dZ_X(t)) &\quad (16)\\
dY(t) &= (\sigma_Y \lambda_Y + \kappa_0 - \kappa_Y Y(t))dt + \sigma_Y dZ_Y(t) &\quad (17)
\end{aligned}
$$

where $J(K(t), X(t), Y(t), t)$ is the value function associated to the country's problem and $dF(t)$ are the changes in the futures prices as defined in equation (13). The futures contracts are marked to market, which implies an instantaneous flow $p(t)^{\top} dF(t)$ to the capital stock. Also, the country's natural exports can be correlated with both, the oil price and convenience yield shocks. To see this we rewrite the Brownian motion of the natural exports, $\widehat{Z}(t)$, as a linear combination of independent Brownian motions (compare equation (16) to (2)). We define $Z_X(t) \equiv \frac{\widehat{Z}(t)-\rho_Y^{\top} Z_Y(t)}{\sqrt{1-\rho_Y^{\top}\rho_Y}}$ as a Brownian motion that captures the unhedgeable risk of the country's natural exports (i.e. $Z_X(t)$ is independent from the vector $Z_Y(t)$) and $\rho_Y^{\top} = \left\{\rho_u, \rho_{v_1}, \ldots, \rho_{v_{n-1}}\right\}$ as the correlation vector. Here, ρ_u stands for the correlation between the exports and the crude oil shocks, and ρ_v defines the correlation between the exports and each one of the latent factors. In the rest of the paper we drop the time argument from the variables to simplify the notation.

Let us define the 'current' value function $J(K, X, Y)$ of the country's problem, such that $J(K, X, Y, t) = e^{-\beta t} J(K, X, Y)$. The function $J(K, X, Y)$ satisfies the standard Hamilton-Jacobi-Bellman (HJB) equation:

$$
\begin{aligned}
0 \;=\; \max_{\{C,Q,p\}} \Bigg\{ & \frac{C^{1-\gamma}}{1-\gamma} - \beta\, J + (\alpha\, K^{1-\eta}(\omega\, Q)^{\eta} - S\, Q + X - C + p^{\top} I_F \sigma_F \lambda_Y) J_K - \phi\, X\, J_X \\
& + (\sigma_Y \lambda_Y + \kappa_0 - \kappa_Y Y)^{\top} J_Y + \frac{1}{2} p^{\top} I_F \sigma_F \sigma_F^{\top} I_F p J_{KK} + \frac{1}{2}\sigma_X^2 X^2 J_{XX} + \frac{1}{2}\mathrm{Tr}[\sigma_Y \sigma_Y^{\top} J_{YY}] \\
& + \sigma_X X p^{\top} I_F \sigma_F \rho_Y J_{KX} + p^{\top} I_F \sigma_F \sigma_Y^{\top} J_{KY} + \sigma_X X \rho_Y^{\top} \sigma_Y^{\top} J_{XY} \Bigg\}. \qquad (18)
\end{aligned}
$$

where J_i is the partial derivative of J with respect to the state variable i and J_{ij} are the second order derivatives.

The next proposition presents the optimal decisions for the country's problem in equations (14) to (17).

PROPOSITION 1: *The optimal consumption for the developing country is* $C^* = J_K^{-1/\gamma}$ *and the optimal demand for crude oil is given by*

$$Q^* = \frac{K}{\omega}\left(\frac{\alpha\,\eta\,\omega}{S}\right)^{\frac{1}{1-\eta}}. \tag{19}$$

The hedging strategy is determined by the $n \times 1$ *vector of contract holdings*

$$p^* = (I_F)^{-1}\left(\sigma_Y \sigma_F^{-1}\right)^\top \left(\sigma_Y^{-1}\lambda_Y \frac{-J_K}{J_{KK}} + \sigma_X\,\sigma_Y^{-1}\rho_Y \frac{-X\,J_{KX}}{J_{KK}} + \frac{-J_{KY}}{J_{KK}}\right). \tag{20}$$

The optimal consumption of capital, C^*, is derived from the standard envelope condition. Let us define μ as the expected change in the capital stock before consumption:

$$\mu = \frac{\alpha\,K^{1-\eta}(\omega\,Q)^\eta - S\,Q + X}{K}. \tag{21}$$

This variable measures the average productivity of capital in equation (1). The optimal demand for crude oil, Q^*, simply maximizes μ, because of the absence of adjustment costs in the capital technology. Equation (19) shows that Q^* is increasing in ω, because oil is more productive for higher ω. The optimal crude oil demand is decreasing in the price of the crude oil S. It turns out that Q^* equates the marginal benefit and the marginal cost of an extra barrel of oil, thus it is independent from the exports and other variables in the economy. If we replace Q^* in equation (21) we obtain the optimal average productivity of capital μ^*. It is straightforward to show that the productivity of capital is decreasing in the price of the crude oil S, i.e. $\frac{\partial \mu^*}{\partial S} < 0$. This last point is central in what follows, because a higher crude oil price in the future will undoubtedly imply a decrease in the future productivity of capital.

The optimal hedging strategy in Proposition 1 is obtained with the standard first order conditions of the HJB equation in (18) with respect to p. First, we note that what matters for the hedging strategy is the product of quantities and prices (i.e. $(p^*)^\top I_F$) which is in units of the numeraire good. Also, we find that the holdings in equation (20) are amplified by $\left(\sigma_Y \sigma_F^{-1}\right)^\top$. If the futures returns volatilities are low, the country will take a larger position in the futures contracts to have the same hedging effect. This is the only place where the futures returns volatilities matter.[10]

The optimal holdings p^* in (20) result as the summation of three components. The first term is the standard myopic demand present in the classical Merton model and captures the risk-return trade-off of the positions in futures contracts. It's proportional to the Sharpe ratio of each risk factor, i.e. $\sigma_Y^{-1}\lambda_Y$. Its main purpose is to take advantage of the enhanced investment opportunity set rather than hedging against changes in oil prices. If there are no risk premia embedded in the futures contracts (i.e. $\lambda_Y = 0$), there are no incentives for bearing crude oil risk and the myopic demand fade away. This demand is also present in standard static models of portfolio selection.

The second component in equation (20) is a pure hedging term that minimizes the variance of the natural exports. This type of hedging is sometimes called *statistical hedging*, because the coefficients $\sigma_X \, \sigma_Y^{-1} \rho_Y$ are the β's of n regressions where each crude oil factor is regressed on the natural exports. This demand is also myopic in the sense that it appears even in a static version of this model. The risk-averse country is worried about the variance of the natural exports, because it affects the volatility of consumption. These positions are affected by the correlations between the exports and the crude oil shocks, ρ_Y, because these affect the hedging capacity of the futures contracts against shocks in the natural export. If $\rho_Y^\top \rho_Y = 1$, then the natural exports can be fully hedged with the futures contracts. If the country has no natural exports, this type of demand disappears. To see this, note that

[10] From equation (12) we find that the maturities of the futures contracts enter only through the volatility of the futures returns. This implies that the maturities are only relevant in our analysis to determine the amplifying factor $\left(\sigma_Y \sigma_F^{-1}\right)^\top$ in the hedging strategy.

without exports the value function $J(t)$ is independent of $X(t)$ implying that $J_{KX} = 0$. The exports are a *natural hedge* against crude oil shocks as long as this term decreases the absolute holdings of futures contracts.

The last term includes the productive hedging demands due to changes in each crude oil factor in $Y(t)$. The interpretation of this term is similar to the hedging demands in Merton (1973) and Breeden (1979), with the exception that here they hedge against future changes in the productivity of capital rather than against changes in the investment opportunity set.[11] These demands arise because the country worries about changes in the crude oil price since it affects the productivity of capital. For this reason we label these terms as *productive hedging demands*. Recall that the crude oil factors $Y(t)$ can be decomposed in the (log) spot price, $u(t)$, and other latent factors, $v(t)$, associated to the convenience yield. A shock to the spot price can have a disparate effect in the economy depending on whether it is a permanent or a temporal shock. The country is more concerned about crude oil shocks if they persist in the economy for a longer period of time. If this is the case, the productive hedging demand with respect to $u(t)$ is more significant. The latent factors $v(t)$ influence crude oil prices in the future through the convenience yield, thus affecting the future productivity of capital. In the case that oil is useless for the economy (i.e. $\eta = 0$ in equation (1)), the crude oil shocks have no effect in future production, thus these hedging demands disappear.

To the best of our knowledge, the problem that the developing country faces has no closed-form solution. In the next section, we present an approximated solution that is asymptotically exact. This will help us to get a better economic intuition about the decisions that the country takes.

[11] The investment opportunities in our model are given by the positions in the futures contracts, but the futures return are independent of the state variables $Y(t)$.

3.1 An Approximated Solution

In this section we present the steps to obtain closed-form approximations for the consumption and hedging strategies of the developing country in Proposition 1. First, we use the homogeneity of the problem to reduce the number of state variables and then, we apply a dual-asymptotic expansion to get an approximated solution around the standard Merton problem. The approximations deliver various economic insights that are helpful to understand the decisions of the developing country.

We note that consumption is homogeneous of degree one in $K(t)$ and $X(t)$ and that the CRRA utility function is homogeneous of degree $(1-\gamma)$. These two properties imply that the value function $J(t)$ is also homogeneous of degree $(1-\gamma)$. We can use this feature to reduce the state space from $n+2$ to $n+1$ variables. We write the current value function as

$$J(K, X, Y) = A_1^{-\gamma} \frac{\left(K\, e^{h(x,Y)}\right)^{1-\gamma}}{1-\gamma} \tag{22}$$

where the new state variable x is the exports to capital ratio, i.e., $x = K^{-1}X$. Here, we have defined the constant

$$A_1 = \frac{1}{\gamma}\left(\beta - \alpha(1-\gamma) - \frac{1-\gamma}{\gamma}\frac{\lambda_Y^\top \lambda_Y}{2}\right) > 0. \tag{23}$$

Assume that two countries are identical, except that one doubles the other in natural exports and capital stock. The homogeneity feature implies that the bigger country will consume twice the consumption, demand twice the number of crude oil barrels, and take twice the positions in futures contracts than the smaller country. For this reason we discuss the results in terms of the consumption-wealth ratio and the market value of the hedging positions to capital ratio. These variables are homogeneous of degree zero in $K(t)$ and $X(t)$, meaning that the natural exports to capital ratio, $x(t)$, and crude oil factors, $Y(t)$, are enough to characterize the economy.

We replace equation (22) and the optimal controls $\{C^*, Q^*, p^*\}$ from Proposition 1 in equation (18) to get a highly non-linear ODE for $h(x, Y)$ that we write as:[12]

$$0 = ode(x, Y) \tag{24}$$

It is hard to solve this equation numerically because it's a second-order equation in $n + 1$ state variables with complicated boundary conditions. However, it is possible to obtain an approximation by doing an asymptotic expansion of the solution. This approximation method has become increasingly popular in finance lately (see for example, Kogan (2001) and Janecek and Shreve (2004)). This technique is exact in the limit and its main advantage is that provides informative explicit expressions for the optimal consumption and hedging strategies.

The idea behind the asymptotic expansion technique is to do a Taylor expansion of the solution of equation (24) around a particular set of parameters under which this ODE has an exact solution. In our case, we need to restrict two parameters so we perform a dual expansion to achieve the well-known solution of the infinite-horizon model of Merton (1969). A further change of variables is done before continuing. We express the ODE in equation (24) in terms of a new variable $z(t)$ rather than in terms of $x(t)$. Let us define the variable $z(t)$ such that $x(t) = x_0 z(t)$ where x_0 is the initial natural exports to capital ratio and $z(0) = 1$. We note that the problem simplifies considerably if the oil is useless in the economy (i.e. $\eta = 0$) *and* the country has no natural exports or in our new setting, the initial export-to-capital ratio, x_0, is zero.[13] Indeed, under this scenario the production technology $K(t)$ has constant returns to scale, because: (i) $Q^*(t)$ and $X(t)$ are zero, and (ii) the investment opportunity set given by the futures returns is independent of the state variables. The value function is independent of $X(t)$ and $Y(t)$ and the problem reduces to the Merton solution. In this case $h(x, Y) = 0$ which implies that the country consumes a constant fraction A_1 of

[12]We prefer to omit the details of the resulting differential equation, because it is messy and uninformative.

[13]The change of variables from $x(t)$ to $z(t)$ is not strictly necessary, but it clarifies the idea that we are expanding with respect to the *initial* natural exports to capital ratio, x_0.

its capital and the positions in the futures contracts are proportional to $\gamma^{-1}\sigma_Y^{-1}\lambda_Y$.

The approximated solution is valid as long η and x_0 stay relatively close to zero. As we will see later, even for small values of η and x_0, there is a lot of action in our model and the consumption and hedging strategies differ significantly from the Merton solutions. Moreover, these assumptions have reasonable economic foundations. We expect the ratio between the natural exports and capital to be a small figure even for less developed countries. For example, for Chile whose economy depends heavily on its copper exports, we estimate that the copper exports to capital ratio is less than 1%. The same happens with η. Recent RBC studies that include energy as a production factor use values around 4% for the oil share of income, η (see Finn (2000) and Wei (2003)).

We show the approximation technique for a first-order dual-expansion, but this methodology can be implemented for higher-order expansions. We assume the following structure for the solution of ODE in equation (24):

$$h(x, Y) = h^{\eta}(z, Y)\,\eta + h^{x_0}(z, Y)\,x_0\,(1 + h^{\eta x_0}(z, Y)\,\eta) + O(\eta^2 + x_0^2) \tag{25}$$

We replace this solution in the ODE and pursue a first-order Taylor expansion of equation (24) around $\eta = 0$ and $x_0 = 0$:

$$\begin{aligned} 0 &= ode(x, Y; \eta, x_0) \\ &= ode^{\eta}(z, Y; 0, 0)\,\eta + ode^{x_0}(z, Y; 0, 0)\,x_0 + ode^{\eta x_0}(z, Y; 0, 0)\,\eta\,x_0 + O(\eta^2 + x_0^2) \end{aligned} \tag{26}$$

We seek for the functions $h^{\eta}(z, Y)$, $h^{x_0}(z, Y)$ and $h^{\eta x_0}(z, Y)$ such that $ode^{\eta}(z, Y; 0, 0) = ode^{x_0}(z, Y; 0, 0) = ode^{\eta x_0}(z, Y; 0, 0) = 0$. Interestingly, these expressions are very simple even for the general n-factor crude oil price process. We find that they are affine functions in the state variables $z(t)$ and $Y(t)$. The next proposition shows the results after the first-order expansion has been performed.[14]

[14]Technically speaking, taking advantage of the homogeneity of the problem is not necessary to get the

PROPOSITION 2: *Suppose that*

$$A_2 = \alpha + \phi + \sigma_X \rho_Y^\top \lambda_Y > 0. \tag{27}$$

The approximated solution of equation (24) using a first-order dual-asymptotic expansion in (η, x_0) *around the origin is given by equation (25) where*

$$h^\eta(z, Y) = M_0 - M_Y^\top Y \tag{28}$$

$$h^{x_0}(z, Y) = A_2^{-1} z \tag{29}$$

$$h^{\eta x_0}(z, Y) = N_0 - N_Y^\top Y \tag{30}$$

and the M's and N's are constants depending on the fundamental parameters of the model.

For the following we shall assume that conditions (23) and (27) are satisfied.

3.2 Characterizing the Optimal Controls

Now that we have an approximation for the value function $J(t)$ we are ready to revisit the optimal controls from Proposition 1. If we use Proposition 2 and replace equations (22) and (25) in the optimal controls we obtain complex expressions that are difficult to interpret. Instead, we follow Kogan and Uppal (2003) and present the approximations in a asymptotically equivalent representation by applying a new Taylor expansion to the approximated optimal decisions of the country (consumption, holdings, etc). Note that we do not need any extra assumption, because we are already considering that η and x_0 are *small*. The new expansions are asymptotically equivalent to the original ones in the sense that both converge to Merton's solutions in the limit.

approximated solution. It was useful though to understand that the solution in Proposition 2 was a function of $K^{-1}X$.

We need a measure of the total wealth of the country to better contrast our results with the ones in Merton's model. Indeed, in Merton's model the agent consumes a constant fraction of its wealth, so a fair comparison is to analyze the consumption-wealth ratio in our country. Here, the developing country's wealth is composed by it's capital and the present value of future exports of the local commodity. We use utility indifference pricing to obtain the value an extra unit of natural exports in terms of the numeraire $E(t)$, thus

$$E(t) = \frac{J_X(t)}{J_K(t)} \tag{31}$$

Note that the price $E(t)$ already considers the present value of future increments in the natural exports due to the extra unit today.[15] Let us define the total wealth of the country as

$$W(t) \equiv K(t) + E(t)X(t) \tag{32}$$

The definition of $W(t)$ is correct as long as the marginal price of the natural exports $E(t)$ corresponds to the average price. This is valid if $E(t)$ is independent from $X(t)$, which is true at least to a first-order expansion since

$$\frac{W(t)}{K(t)} = 1 + A_2^{-1} x(t) + O(\eta^2 + \eta\, x_0 + x_0^2) \tag{33}$$

Moreover, A_2^{-1} acts as a discount factor for the perpetual flow of natural exports, which is why we restrict A_2 to be positive in equation (27).

The next proposition shows the asymptotically equivalent expansions for the consumption-wealth ratio and the market value of the hedging positions to capital ratio.

PROPOSITION 3: *Let us define c^* as the consumption-wealth ratio (i.e. $c^* \equiv W^{-1}C^*$), and π^* as the ratio of the dollar amount invested in the futures contracts to the capital stock (i.e.*

[15]The GBM specification for the natural exports in equation (2) means that changes in the exports are permanent. This implies that an increase of 1% in today's exports generates an increase of 1% in future exports as well.

$\pi^* \equiv I_F K^{-1} p^*$).

Asymptotically equivalent expressions for optimal consumption and hedging strategies in the developing country are given by

$$c^* = A_1 \left(1 + \frac{1-\gamma}{\gamma} \eta (-M_0 + M_Y^\top Y) \right) + O(\eta^2 + \eta\, x_0 + x_0^2) \tag{34}$$

and

$$\begin{aligned} \pi^* &= \left(\sigma_Y \sigma_F^{-1}\right)^\top \times \\ &\left((1 + A_2^{-1} x) \frac{\sigma_Y^{-1} \lambda_Y}{\gamma} - \sigma_X\, \sigma_Y^{-1} \rho_Y\, A_2^{-1} x - \frac{1-\gamma}{\gamma} \eta\, M_Y + O(\eta^2 + \eta\, x_0 + x_0^2) \right). \end{aligned} \tag{35}$$

The analysis that follows is based on the approximated results, so the conclusions that we derive are valid only to a first order degree.

3.2.1 Consumption Strategy

Equation (34) shows that the consumption-wealth ratio is independent from $X(t)$, which means that the main effect of the natural exports in consumption is through the wealth of the country. A positive shock to the exports increases the total wealth, and consumption increases proportionally to the wealth.

Crude oil impacts consumption because it is an input to the production technology. The effect of crude oil shocks $Y(t)$ in consumption depends on the risk aversion parameter γ. This is related to the standard income and substitution effects with respect to each one the crude oil factors. For the analysis it is convenient to separate the crude oil price from the other factors, because the oil price is observable and directly affects the productivity of capital. The derivative of the consumption-wealth ratio with respect to the crude oil (log) price $u(t)$ is:[16]

$$c_u^* \approx A_1 \frac{1-\gamma}{\gamma} \eta \, M_u \qquad \text{where} \qquad M_u = \frac{\alpha}{A_1 + \psi_u} > 0 \tag{36}$$

The crude oil price has two opposite effects in today's consumption-wealth ratio. The income effect in consumption is negative, because an increase in today's crude oil price has a negative impact in the capital accumulation process of the economy. On the other hand, the substitution effect in today's consumption is positive. The intuition is that the negative impact of crude oil in the economy decreases the *expected capital stock* even further because there is less capital to invest in every period. This shortage of expected capital increases the relative price of tomorrow's consumption, thus affecting today's consumption positively. Equation (36) shows that if $\gamma > 1$, the consumption-wealth ratio decreases with an increase in the crude oil price. Indeed, if the country is too worried about consumption smoothing (high γ), it will consume less, even if today's consumption becomes relatively cheaper. In this case, the negative income effect dominates the substitution effect. If $\gamma < 1$, the consumption-wealth ratio increases with crude oil shocks. The country is less concerned about the variability of consumption and takes advantage of the relatively lower price of today's consumption. Here, the positive substitution effect dominates the income effect. Both effects cancel out if risk aversion is unity which corresponds to the logarithmic utility case. In this case, the consumption-wealth ratio is constant.

The mean-reverting parameter ψ_u in (36) relates the spot price and the convenience yield, but also determines the persistence of the crude oil price shocks and the unconditional volatility of crude oil returns. The price shocks have a half-life of $\psi_u^{-1} \log(2)$. For values of ψ_u close to zero, the shocks are permanent and the impact in the economy is greater. An increase in the oil price persists for a long time in the economy and it affects the productivity of capital in every subsequent period of time. For high values of ψ_u, the price shocks are temporal, thus they only affect the short-term dynamics of crude oil prices. In this case, the

[16]For the moment we assume that $\psi_u \geq 0$. CCD shows in a three-factor model that for crude oil prices this parameter is positive and highly significant. We obtain the same result in the next section for a one-factor model.

effect in consumption is less important.

The general impact of the convenience yield factors $v(t)$ in consumption is less intuitive. The reason is that in the maximal model these factors not only affect the current convenience yield through ψ_v, but also their own dynamics (see equations (5) and (6)). For example, a positive shock to $v_j(t)$ modifies the expected change of the variables $\{v_1(t), \ldots, v_j(t)\}$, because κ_v is an upper triangular matrix. The overall effect of this shock in the expected crude oil price depends on ψ_v and on the elements of column j of κ_v. Fortunately, there is one simple case to analyze. Shocks to $v_1(t)$ affect the convenience yield and its own dynamics while leaving the other v's imperturbable. The derivative of the consumption-wealth ratio with respect $v_1(t)$ is:[17]

$$c^*_{v_1} \approx A_1 \frac{1-\gamma}{\gamma} \eta \, M_{v_1} \qquad \text{where} \qquad M_{v_1} = -\frac{\psi_{v_1}}{A_1 + \kappa_{v_{11}}} M_u < 0 \tag{37}$$

Here, ψ_{v_1} is the effect of $v_1(t)$ in the convenience yield and $\kappa_{v_{11}}$ is the (1,1) element of κ_v. $\kappa_{v_{11}}$ determines the persistence of the shocks to $v_1(t)$. The derivative $c^*_{v_1}$ has the opposite sign than c^*_u in equation (36). The reason is simple. A positive shock to $v_1(t)$ decreases the expected crude oil spot price, because the convenience yield has a negative effect in crude oil returns. It turn out also that the income effect of this variable is positive while its substitution effect is negative. These effects are the antithesis to the income and substitution effects with respect to price shocks. For $\gamma > 1$ the consumption-wealth ratio increases because an increase in the convenience yield has a negative effect on prices, thus an overall positive effect in the economy (income effect). If $\gamma < 1$ today's consumption decreases, because it becomes relatively more expensive with respect to tomorrow's consumption (substitution effect).

[17] Without loss of generality, we assume that $\psi_{v_1} > 0$. Again, we use the results of CCD to consider that $\kappa_{v_{11}} > 0$.

3.2.2 Hedging Strategy

For the hedging strategy we use the dollar amount invested in the futures contracts to the capital stock instead of the number of contracts. This measure is better for the analysis because it controls for the size of the country given by $K(t)$. The hedging strategy in (35) has exactly the same three components as p^* in Proposition 1. The myopic demand is positive as long as the Sharpe ratio is positive. As expected, it is decreasing in the degree of risk-aversion γ, which implies that more risk-averse countries seek less exposure to the crude oil risk factors. Also, the myopic demand is proportional to the total wealth of the country.[18] The exports of the local commodity increases the total wealth and allows the country to increase its investment in futures contracts. The second term of the hedging strategy is the statistical hedging demand. This demand is negative for those crude oil factors that have a positive correlation with the natural exports and viceversa. Indeed, a higher correlation of the exports with a particular factor, means that a portfolio of futures that is perfectly correlated with this factor works better as a hedge against shocks in the exports. This implies that fewer units of this portfolio are necessary for the hedge.

The third term in (35) has the productive hedging demands. It is not surprising that these demands have a similar structure than the sensitivity of consumption with respect to the crude oil shocks (i.e. c^*_u and $c^*_{v_1}$). These demands are proportional to M_Y, because the country hedges against those crude oil shocks that have some impact on consumption. Crude oil shocks are transferred to consumption through the productivity of capital. Again the sign depends on the risk-aversion of the country. Consider a portfolio of futures contracts, f_u, that is perfectly correlated with the shocks to the crude oil (log) price, $u(t)$. An increase in the crude oil price, has a negative effect on today's consumption if $\gamma > 1$ and a positive effect if $\gamma < 1$ (see equation (36)). Clearly, the country chooses a strategy that minimizes the effect of these shocks in consumption by taking a long position in f_u if $\gamma > 1$ or a short position in f_u if $\gamma < 1$. The effects on consumption are compensated by the payoff from the marking-to-

[18] Recall that the first-order approximation to the total wealth to capital ratio is $1 + A_2^{-1}x$.

market of f_u. If these shocks are persistent (low ψ_u, high M_u), the country is more worried about this type of uncertainty and takes a larger position in f_u. The converse occurs with the convenience yield shocks through the factor $v_1(t)$. An increase in the convenience yield has a positive effect on the capital accumulation process. It has a positive effect on today's consumption if $\gamma > 1$ and a negative effect if $\gamma < 1$ (see equation (37)). If $\gamma > 1$, the country chooses a short position in a portfolio of futures f_{v_1}, that is perfectly correlated with $v_1(t)$. If $\gamma < 1$, the country takes a long position in this portfolio.

In the next section we take our model to the data. We study the decision of a developing country assuming that crude oil prices are driven by a one-factor Gaussian model. This simple framework will help us quantify the aggregate effect of the crude oil shocks and the local commodity exports on the country's decisions.

4 Empirical Results for a One-Factor Model

In this section we estimate a one-factor model for crude oil prices and analyze the consumption and hedging strategies of a developing country. One futures contract is enough to span the whole futures curve in a one-factor model. In this model it is also easier to connect the empirical results with the theoretical results discussed in the previous section. Despite its simplicity, the model has a time-varying convenience yield and is able to generate both, contango and backwardation, in the futures curves. The relation between the spot price and the convenience yield, ψ_u, also regulates the persistence of the crude oil shocks.

We first estimate the crude oil pricing model and calibrate the parameters for the country technologies and utility functions. Then we discuss the effect of the natural exports and its correlation with crude oil prices in consumption and the hedging strategy. Finally, we look at the particular effect of risk-aversion of the country and mean-reversion and volatility of crude oil prices in the optimal controls.

4.1 Crude Oil Estimates and Developing Country Parameters

We estimate a single factor model for the crude oil spot (log) price $u(t)$. The model is equivalent to the one in equations (4) and (5) with the extra restrictions that $\varsigma = \psi_v = 0$. Under the historical measure, the dynamics of the futures price on a contract that matures τ_1 periods from now is:

$$dF_1 = F_1 B_{u,1} \sigma_u \lambda_u dt + F_1 B_{u,1} \sigma_u dZ_u \tag{38}$$

with $B_{u,1} = e^{-\psi_u \tau_1}$. The convenience yield parameter ψ_u affects the volatility of the futures returns, and therefore, the futures risk premia.

The dataset consist on weekly crude oil futures prices from 1/2/1990 to 12/31/2005 from NYMEX with maturities of 1, 3, 6, 9, 12, 15, 18, 24, 30 and 36 months. We use maximum-likelihood estimation using both time-series and cross-sectional data. We pursue the approach of Chen and Scott (1993) and Pearson and Sun (1994) and assume that part of the data is observed with no error. In particular, we follow Collin-Dufresne, Goldstein, and Jones (2008) and CCD and choose to perfectly fit the first principal component of the futures curve. Since the principal component remains affine in the state variable, it can easily be inverted to obtain the state variable $u(t)$. The remaining principal components of futures prices are then over-identified and observed with "measurement errors," which we assume follow an AR(1) process.

Table 1 presents four groups of maximum-likelihood estimates for the single factor model. We set the interest rate to $r = 0.03$ without loosing any generality, because we estimate the convenience yield parameter ψ_0 (see footnote 7). The first group (Set 0) has the unconstrained parameter estimates. We obtain interesting results that are consistent with the findings of CCD. First, we detect that the convenience yield parameters and the volatility of crude oil returns are all significantly different from zero. Moreover, the mean-reverting parameter that plays a particular role in the consumption and hedging strategies, is positive and highly significant ($\psi_u = 0.076$). We also find that the risk premium parameter λ_u

is not significant. Interestingly, CCD found that even for a richer structure, the crude oil risk-premia parameters are less significant than those for copper, silver and gold. The most important effect of λ_u is the existence of the myopic demand, a result that is well-known and broadly documented in the portfolio selection literature. Therefore, we decide to drop this parameter. The second group of parameters (Set 1) has the estimates assuming that $\lambda_u = 0$. We find that the other estimates are not affected by this assumption and that the change in the log-likelihood is not significant.[19] We use this parameter set for the analysis of the strategies below. The third and forth parameter groups in Table 1 (Sets 2 & 3) have the estimates assuming a particular value for the mean-reversion parameter ψ_u. To analyze the effect of the parameter ψ_u, it is important to consider a realistic pricing model, thus the other parameters need to adjust to changes in ψ_u. For example, note that the parameter ψ_0 changes radically for the different assumptions for ψ_u. This occurs, because the parameters ψ_0 and ψ_u jointly regulate the slope of the futures curve. It doesn't make sense to change ψ_u while keeping ψ_0 constant.

We choose Chile as the benchmark developing country, because it's economy has similar characteristics to the ones considered for our representative country. Chile's exports are mostly from the mining industry and it's the world's largest copper producer. According to the U.S. Geological Survey, Chile accounted for 35% of the world's copper production in 2006 followed by the U.S. with 8% of the global production (see USGS (2008)). Chile has also more than 31% of the world's known copper reserves. In the last decades, Chile has had a stable economy absent from major governability problems, which are sometimes common in emerging economies. This means that its economic data is more related to the productivity parameters in our model, than to other political factors that are not considered in this study.

Table 2 shows the parameters that we use for the benchmark developing economy. We first calibrate the marginal productivity of capital (MPR) which in our case is $\alpha(1-\eta)$.

[19] The chi-squared statistic with one degree of freedom for the LR test is 2.8, while the critical value for a 5% significance level is 3.84 (i.e. $\text{Prob}\{\chi_1^2 \geq 3.84\} = 0.05$).

We follow Caselli and Feyrer (2007) and use 9% for Chile. This paper estimates the MPR for various countries using a measure that accounts for natural capital adjustments and differences in prices of capital and consumption goods. Natural capital adjustments result in that the natural capital accounts, such as land and natural resources, are deducted from the national wealth, because only the payments to reproducible capital are relevant to estimate the MPR. For the oil share of income in Chile, η, we use 3%. Recent studies such as Finn (2000) and Wei (2003) use an energy share of 4%, but oil consumption accounts only for fraction of the total energy consumption of the country. Considering the MPR and the oil share of income, we calibrate a total factor productivity, α, of $\frac{9\%}{1-3\%} = 9.3\%$. We normalize ω to 1 for the efficiency of oil parameter in Chile. The International Energy Agency (2004) report documents that on average, oil-importing developing countries use more than twice of the oil than OECD countries to produce a unit of economic output. This means that we should consider a higher efficiency parameter, say $\omega = 2$, if we want to consider a more developed country than Chile.

For the initial natural exports to capital ratio, x_0, we need an estimate of $X(0)$ and $K(0)$. The total copper exports for Chile were $X(0) =$ US$ 14.9 billions in 2005. For the initial capital stock, we find $K(0)$ such that the output in equation (1) is the Chilean GDP in 2005. The output of the country considers the optimal demand of oil from Proposition 1 and needs an estimate for the crude oil price. Using that the GDP was US$ 118.9 billions in 2005 and that the average crude oil WTI price that year was US$ 56.5 per barrel, we obtain an estimate for the Chilean capital stock of US$ 2015.5 billions. These estimates yield an initial natural exports to capital ratio of $x_0 = 0.7\%$. Interestingly, these figures imply that the optimal imports of crude oil, $S(0)Q^*(0)$, is US$ 4.1 billions which is very close to the Chilean fuel and energy imports of US$ 3.6 billions in 2005. To estimate the volatility of the natural exports returns, σ_X, and it's correlation with the crude oil shocks, ρ_u, we assume that the Chilean copper production changes at a constant rate. This implies that the second moments are due only to variations in copper prices. We consider the closest maturity futures price to be a proxy for the spot price for both, copper and crude oil. Using monthly data from 1995 to 2007, we find that the annualized volatility of copper returns is 21.2% and the correlation between copper and crude oil shocks is 28.9%. The selection of the export's depreciation rate, ϕ, is the most arbitrary one. We choose 5% as the annual depreciation rate, but try different values later. A positive rate captures that the natural resource is exhaustible and that the developing country diversifies its exports over time. Finally, we assume that the country's risk-aversion parameter, γ, is 5.0, and that its impatience parameters, β, is 5%. These values are standard in the literature, but given that the country's risk-aversion has a great repercussion in consumption and in the productive hedging demands, we do a sensitivity analysis with respect to it.

4.2 Consumption Strategy

One of the main objectives of the paper is to study the effect of crude oil in the country's consumption decisions. For this reason we concentrate on the parameters related to the crude oil dynamics and its impact in the economy, and their effect on the expanded consumption-wealth ratio from equation (34). It is important to remember that the relative size of the natural exports, x_0, has a direct effect on consumption through an increase in wealth, but it has no first order effect on the consumption-wealth ratio.

Figure 1 present the consumption strategy with respect to the technology parameters that determine the impact of the oil in the economy. The figure has three plots, each one for a different country's risk-aversion parameter. Each plot shows the consumption-wealth ratio, c^*, as a function of the oil share of input, η, for 4 different situations: one is the Merton case (i.e. $\eta = x_0 = 0$) and the others represent countries with different efficiency of oil usage, ω. The plots confirm that risk aversion has a decisive effect on consumption. For $\gamma < 1$, the oil share of input has a positive effect on today's consumption (upper plot), while for $\gamma > 1$ this effect is negative (lower plot). The intuition is that for our parameters, η has a negative effect on the productivity of capital.[20] Therefore, η has a negative income effect and a positive substitution effect. As always, the substitution effect dominates for $\gamma < 1$ and the income effect prevails if $\gamma > 1$. The opposite happens with respect to the efficiency parameter ω, because for higher ω less barrels of oil are demanded for production.[21] The empirical evidence supports the fact that more developed economies are more efficient in the usage of oil than less developed countries. This means that if $\gamma < 1$, a developed country consumes a lower fraction of its wealth than a developing economy (upper plot for a fixed η). The reverse occurs if $\gamma > 1$ (lower plot for a fixed η). Also, there's an overall effect of risk aversion in the level of c^* that can be observed by comparing the Merton cases across plots. Today's consumption is increasing on risk-aversion for the Merton case. This occurs because we have calibrated a total factor productivity, α, that is higher than the impatience parameter, β. A relatively high α implies a positive income effect and a negative substitution effect of this parameter, so the consumption-wealth ratio is higher for $\gamma > 1$. Of course, all income and substitution effects cancel out if $\gamma = 1$, implying that no variable changes the consumption-wealth ratio that is fixed at $\beta = 5\%$ (middle plot).

The effect of the crude oil price and its dynamics is shown in figure 2. We consider again different risk-aversion degrees for each one of the three plots. Each plot shows the consumption-wealth ratio, c^*, as a function of the crude oil price, S, for 3 different sets of parameters. As we mentioned before, each set has a different assumption for the mean-reversion parameter ψ_u. The plots confirm that crude oil prices have an effect on consumption that depends on risk-aversion and on the persistence of the shocks. Crude oil is an input to the production technology, therefore it has a negative income effect and a positive substitution effect. As before, this means that for $\gamma < 1$ today's consumption-wealth ratio increases (upper plot), and for $\gamma > 1$, consumption decreases (lower plot). Higher degrees

[20] We can show from equation (21) that the productivity of capital is decreasing in η, i.e. $\frac{\partial \mu^*}{\partial \eta} < 0$, if the input ratio $\frac{\omega Q}{K}$ is less than 1. For our parameters this condition is violated only for extremely low crude oil prices ($S < 0.00279$).

[21] Again, from equation (21) we get that $\frac{\partial \mu^*}{\partial \omega} > 0$. The income effect w.r.t ω is positive while its substitution effect is negative.

of mean-reversion (i.e. lower persistence) tend to decrease these effects because shocks are short lived and the price reverts faster to its long term mean. Finally, for a country with log utility, the net effect of these variables disappear.

4.3 Hedging Strategy

To study the hedging strategy we use the expanded solution for the ratio between the dollar amount invested in the futures contracts and the capital stock from equation (35). Because oil has only one risk factor, we consider that at every point in time the country takes positions in one futures contract. We assume that this contract expires 3 months from now and its position is rebalanced continuously.

Figure 3 shows the different sources of the hedging strategy as a function of the correlation between the crude oil and the natural exports shocks, ρ_u. The figure has 3 plots, each one for a different relative size of the exports, x_0. The myopic demand is represented by π_1 which is always zero, because the parameter Set 1 assumes that the oil price risk premium, λ_u, is zero. The productive hedging demand, π_3^*, is independent of the exports and the correlation ρ_u. This demand is positive in all plots ($\pi_3^* = 1.4\%$), because the risk-aversion degree for the country is greater than 1. This value implies that the country takes long positions in the futures contracts. The interesting term here is the pure hedging or statistical hedging component, π_2^*. As we noticed before, π_2^* is decreasing in the correlation ρ_u and proportional to the relative size of the exports, x_0. In particular, this demand is more negative for positive correlations and high exports, implying a larger short position in the futures contracts (see the lower plot for higher correlations). Figure 3 is also useful to understand up to what extent the natural exports can be used to hedge the crude oil risk. For example, consider the case of Chile which corresponds the plot in the middle ($x_0 = 0.7\%$). We have calibrated a value of 28.9% for ρ_u, which means that the net hedging position, π^*, is almost zero. The short positions due to this positive correlation offset the long positions in the contract from the productive hedging demand. Therefore, for this case, the exports are indeed a natural hedge against crude oil shocks.

Figure 4 shows the hedging strategy as a function of the oil share of input, η, for different parameter sets (Sets 1, 2 & 3 from Table 1). This figure is good to analyze the productive hedging demands, because this is the only demand that varies with η.[22] Each one of the three plots in the figure is for a different risk-aversion degree. The upper plot shows that the hedging strategy is decreasing in η if $\gamma < 1$. In this case, the productive hedging demand is negative. If $\gamma < 1$ a negative shock to crude oil prices decreases today's consumption (see figure 2). This is specially true if oil is more important for the economy (higher η's). This higher sensitivity forces the country to take a larger short position in the futures contract. The reverse happens for $\gamma > 1$ (lower plot). In both cases, the effect of η decreases for a higher mean-reversion degree, i.e. the slope of the curves becomes flatter. This occurs because the effect of the crude oil shock is less persistent, so fewer contracts are needed to hedge against this scenario. Finally, for the log case (middle plot), the productive hedging demand is zero and the hedging strategy is independent from η. In this case, the demands are different for each parameter set, because the oil returns volatilities and mean-reversion estimates changes. The statistical hedging demand is decreasing in σ_u and increasing in ψ_u through the volatility of the futures returns. For the middle plot the volatility effect dominates implying that the set with a higher volatility has a lower hedging strategy.

In order to quantify if the size of the futures demands are significant or not, we express the strategy in terms of the imports of oil, $S\,Q^*$. We define θ^* as the dollar amount in the futures contracts over the dollar amount of the crude oil imports, i.e. $\theta^* \equiv I_F(S\,Q^*)^{-1}p^*$. For the one-factor model, the interpretation of this variable is simple. This hedge ratio represents the portion of the oil imports being hedged. Figure 5 shows the hedge ratio against the correlation ρ_u for different levels of relative natural exports, x_0. We have already presented the effect of these variables in the hedging strategy, so here we limit the discussion to the

[22] Recall from equation (35) that the productive hedging demands are proportional to η.

measurement of the hedge ratio. Let's consider the case when the natural exports are high ($x_0 = 1.4\%$). When the correlation between oil and exports shocks is high ($\rho_u \sim 0.5$), the hedging decision is to take a short position in the futures contract for approximately 30% of the imports of crude oil. If the correlation is zero, the optimal strategy is to hedge around 10% of the total oil imports. If the correlation is negative and large ($\rho_u \sim -0.5$), the hedge ratio can be as large as 55% of the oil imports.

Finally, figure 6 shows the hedging strategy with respect to the depreciation rate of the exports, ϕ. We do a sensitivity analysis with respect to this rate, because it was one of the few parameters that was arbitrarily chosen for the calibration. Of course, this parameter is only relevant for the case where $x_0 > 0$. The hedge ratio increases with ϕ, because the negative statistical hedging component decreases in absolute terms with this parameter.[23] This occurs because for a higher ϕ, the present value of the exports ($A_2^{-1}X(t)$) is lower, thus it decreases its weight in the economy.

5 Conclusions

We study the consumption and hedging strategies of an oil-importing developing country that confronts exogenous crude oil shocks in a dynamic framework. Developing countries differ from more developed economies in that their technologies are more intense and less efficient in the use of energy. Also, their economies typically rely on the export of a small number of primary commodities that can potentially be correlated with the crude oil shocks. We consider a multi-factor Gaussian model that correctly captures the behavior of crude oil prices.

The developing country optimally chooses consumption, the physical crude oil imports and the hedging strategy for the existing crude oil futures contracts. Less efficient countries

[23]The statistical hedging demand is negative, because we consider a positive ρ_u for the benchmark case.

with high degrees of risk aversion consume less than more developed countries, because inefficiencies generate a negative income effect. For countries that care less about consumption-smoothing the opposite may occur, because there exists a substitution effect that makes today's consumption relatively cheaper than tomorrow's consumption. The crude oil price has an overall negative effect for oil-importing economies, because of a lower productivity of capital. The income and substitution effects balance the effect of the crude oil in the economy. Countries with relative risk-aversion degrees greater than one, decrease their consumption if a positive price shock occurs. Shocks to other crude oil factors, such as the convenience yield, are also studied. The impact of these shocks on consumption is through their effect in the expected crude oil price. The more persistent these shocks are, the greater are their impact in the economy.

The long/short positions in the futures contracts are used for hedging purposes, but they also enhance the country's investment opportunity set. The demand for these contacts can be decomposed into the standard myopic demand, a pure hedging or statistical hedging component and the productive hedging demands. The relative size of the natural exports and their correlation with the crude oil shocks are essential for the pure hedging component. This hedging term helps us understand up to what extent the natural exports can be used to hedge the crude oil risk. We find productive hedging demands with respect to each crude oil factor. These demands hedge against future changes in the productivity of capital rather than against changes in the investment opportunity set as in Breeden (1979). The country's risk-aversion degree, the effect of each oil risk factor in consumption and the shocks persistence drive the size and direction of these demands. Finally, we choose Chile as a benchmark economy and estimate a one-factor model for crude oil prices. We find that the country's copper exports act as a significant natural hedge against oil shocks.

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Table 1: Maximum-Likelihood Estimates for the One-Factor Crude Oil Pricing Model

Maximum-likelihood estimates for the one-factor model for crude oil weekly prices from 1990 to 2005.

	Set 0	Set 1	Set 2	Set 3
Parameter	Crude Oil Estimate (Std. Error)	Crude Oil Estimate (Std. Error)	Crude Oil Estimate (Std. Error)	Crude Oil Estimate (Std. Error)
r	0.030	0.030	0.030	0.030
ψ_0	-0.207 (0.002)	-0.207 (0.001)	0.031 (0.000)	-2.963 (0.006)
ψ_u	0.076 (0.001)	0.076 (0.000)	0.000	1.000
λ_u	1.674 (0.973)	0.000	0.000	0.000
σ_u	0.247 (0.005)	0.248 (0.005)	0.245 (0.006)	0.427 (0.009)
error auto-corr.	0.809 (0.008)	0.809 (0.010)	0.840 (0.009)	0.883 (0.008)
Log-likelihood	26017.3	26015.9	25613.2	24980.8

Table 2: Developing Country Parameters

Parameters used for the benchmark country.

Parameter	Estimates
α	9.3%
η	3.0%
ω	1.0
x_0	0.7%
σ_X	21.2%
ρ_u	28.9%
ϕ	5.0%
γ	5.0
β	5.0%

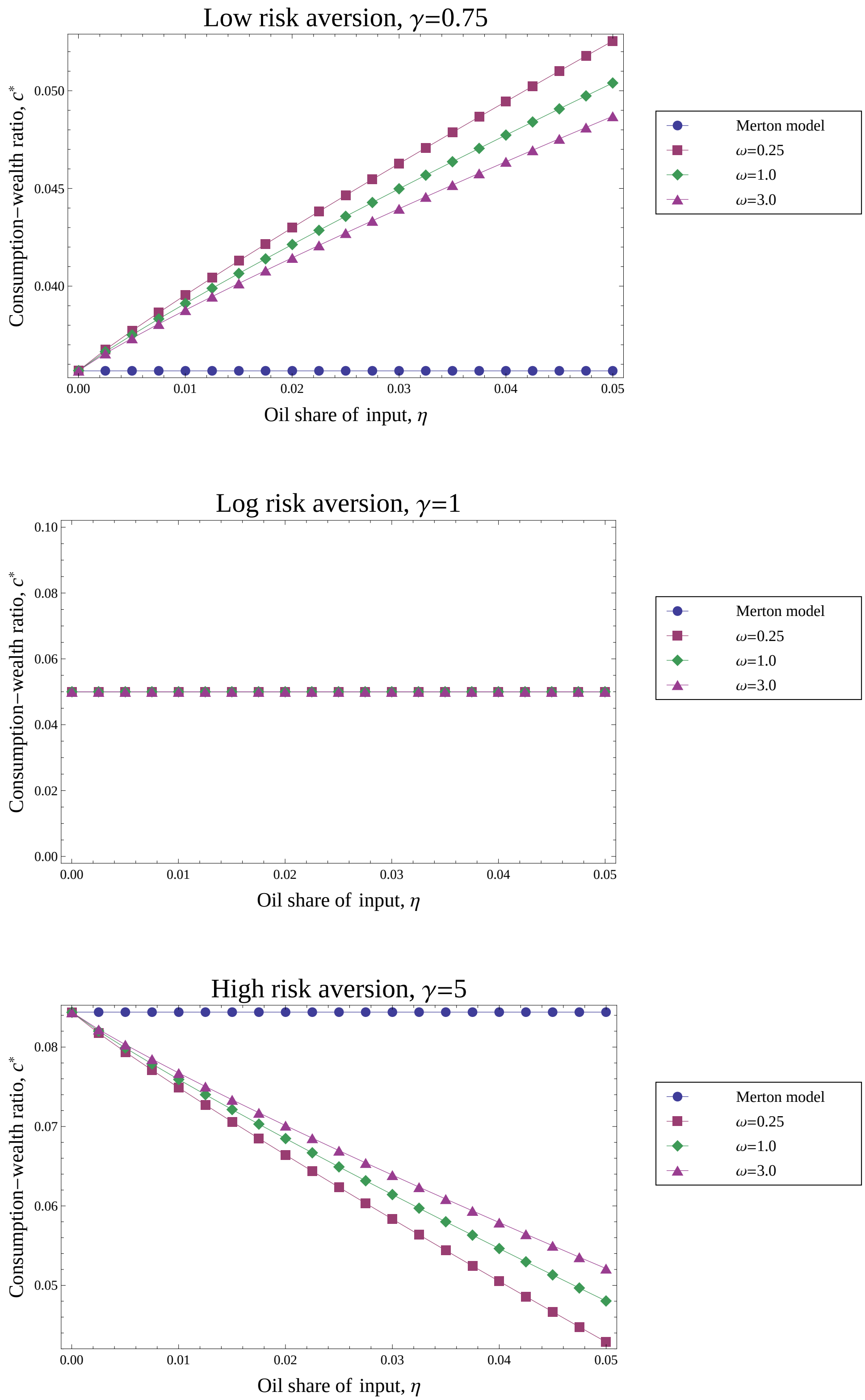

Figure 1: **Consumption-wealth ratio and technology parameters related to the crude oil.** Consumption-wealth ratio, c^*, as a function of the oil share of input, η, the efficiency of oil usage, ω, and the country's risk aversion, γ. The top figure is for $\gamma = 0.75$, the plot in the middle is for $\gamma = 1$ and the one below is for $\gamma = 5$. The *Merton model* lines correspond to the case where $\eta = x_0 = 0\%$. For the crude oil dynamics we use the parameters from Set 1 in Table 1 and for the country's technologies we use the parameters from Table 2.

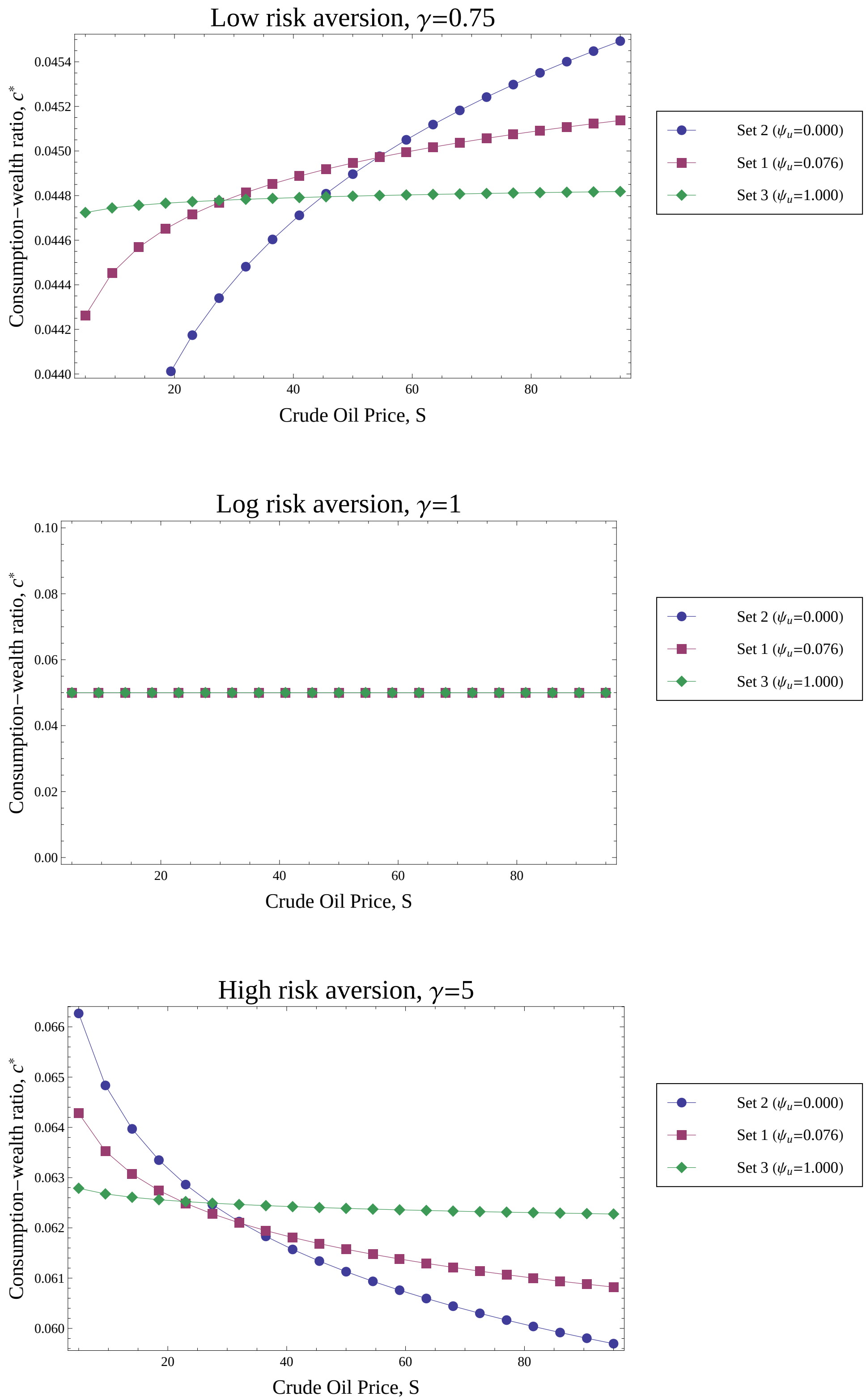

Figure 2: **Consumption-wealth ratio and crude oil prices.** Consumption-wealth ratio, c^*, as a function of the crude oil price, S, different sets of crude oil parameters and the country's risk aversion, γ. The top figure is for $\gamma = 0.75$, the plot in the middle is for $\gamma = 1$ and the one below is for $\gamma = 5$. Set 1 has the default crude oil parameters. To obtain Sets 2 and 3, we fix the mean reversion degree in $\psi_u = 0$ and $\psi_u = 1$, respectively, and estimate the other parameters. For the country's technologies we use the parameters from Table 2.

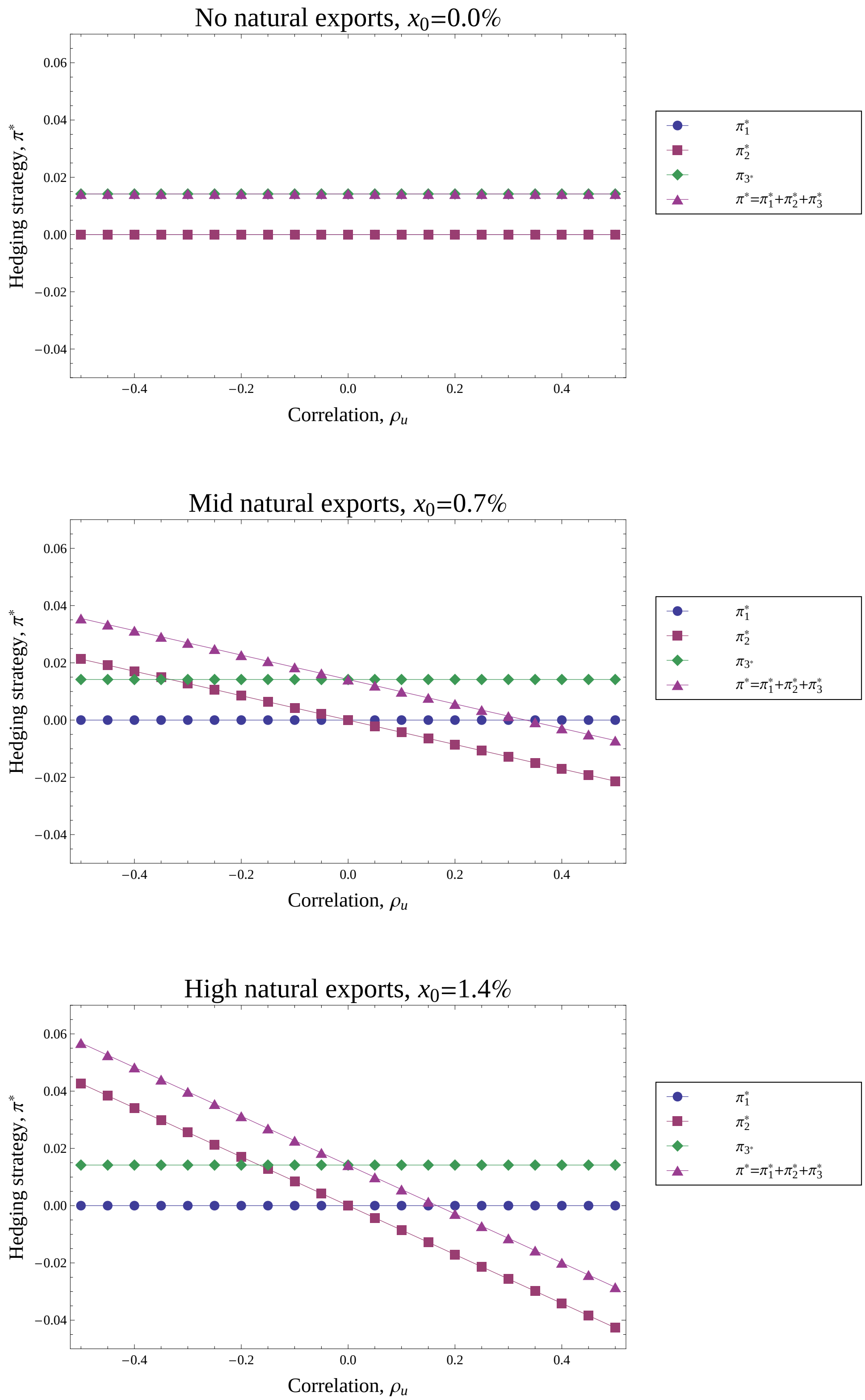

Figure 3: **Hedging strategy, correlation and exports.** Sources of the hedging strategy, π^*, as a function of the correlation between crude oil and natural exports shocks, ρ_u, and the relative size of the natural exports, x_0. π_1 is the myopic demand, π_2 is the statistical hedging component and π_3 is the productive hedging demand. The top figure is for $x_0 = 0.0\%$, the plot in the middle is for $x_0 = 0.7\%$ and the one below is for $x_0 = 1.4\%$. For the crude oil dynamics we use the parameters from Set 1 in Table 1 and for the country's technologies we use the parameters from Table 2.

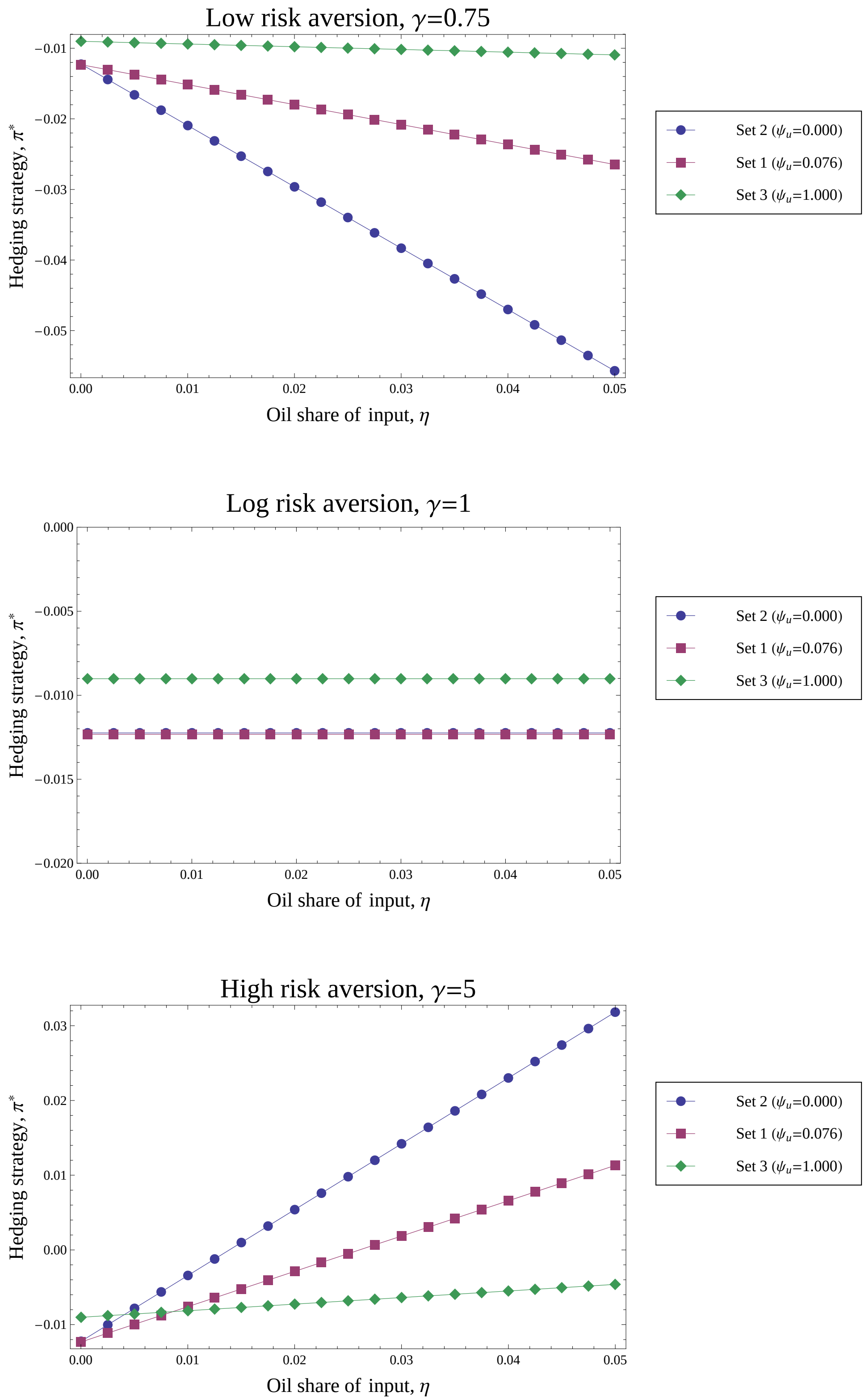

Figure 4: **Hedging strategy and crude oil.** Hedging strategy, π^*, as a function of the oil share of input, η, different sets of crude oil parameters and the country's risk aversion, γ. The top figure is for $\gamma = 0.75$, the plot in the middle is for $\gamma = 1$ and the one below is for $\gamma = 5$. Set 1 has the default crude oil parameters. To obtain Sets 2 and 3, we fix the mean reversion degree in $\psi_u = 0$ and $\psi_u = 1$, respectively, and estimate the other parameters. For the country's technologies we use the parameters from Table 2.

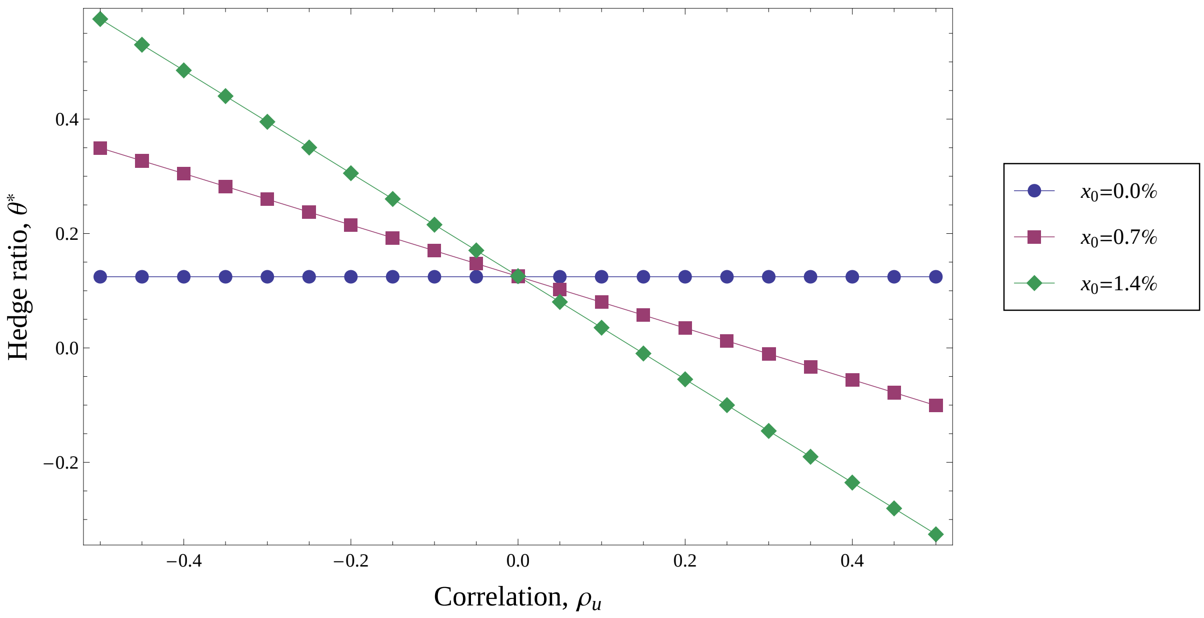

Figure 5: **Hedge ratio, correlation and exports.** Total fraction of crude oil imports being hedged, θ^*, as a function of the correlation between crude oil and natural exports shocks, ρ_u, and the relative size of the natural exports, x_0. For the crude oil dynamics we use the parameters from Set 1 in Table 1 and for the country's technologies we use the parameters from Table 2.

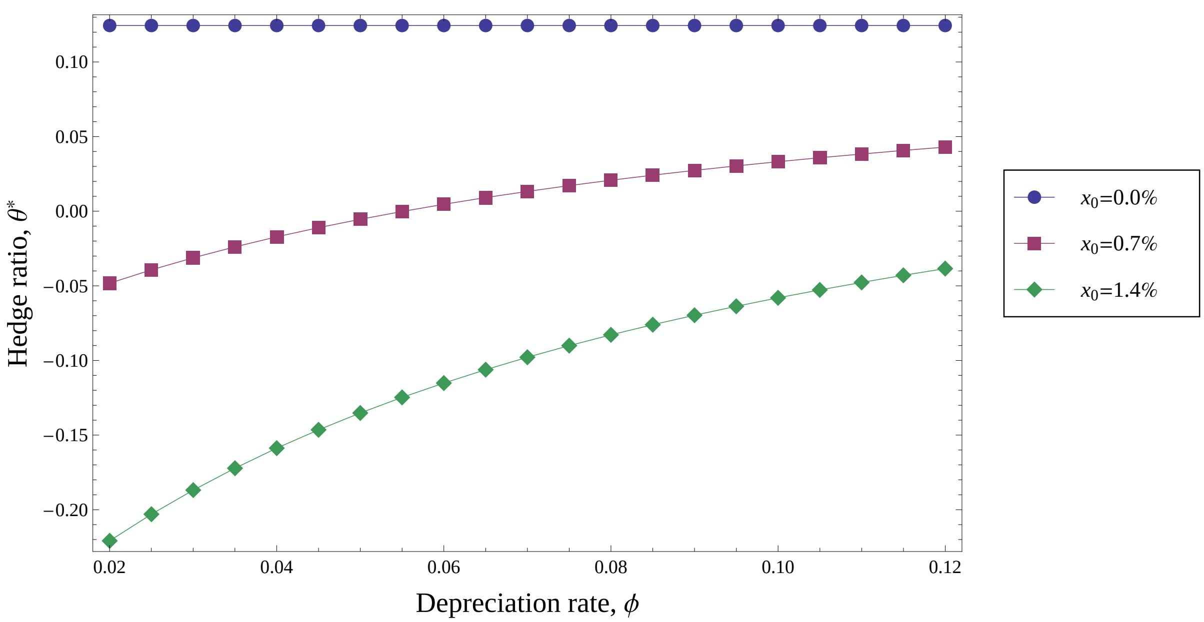

Figure 6: **Hedge ratio and depreciation.** Total fraction of crude oil imports being hedged, θ^*, as a function of the exports depreciation rate, ϕ, and the relative size of the natural exports, x_0. For the crude oil dynamics we use the parameters from Set 1 in Table 1 and for the country's technologies we use the parameters from Table 2.