The hedging effectiveness of electricity futures

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Abstract

With the recent popularity of commodity (electricity) and financial futures, the academic and financial communities have seen a renewed interest in hedging theories. Hedgers in electricity markets use derivative securities, namely futures, to reduce the risk from variations in the spot market.

Hedging aims at reducing the risk involved in holding a financial asset by taking an exactly offsetting position, since it reduces the exposure to fluctuations in commodity prices. Therefore, the use of weekly and monthly futures contracts as a hedging instrument in 3 European electricity markets [German (EEX), French (Powernext) and Scandinavian (Nord Pool)], is the focus of this research. Weekly spot price risk is hedged with weekly futures for the Nordic Electricity market, month spot risk is hedged with month futures in EEX and Powernext, and we also take into account the fact that futures contracts and spot prices are made distinguishable in these markets.

The special features describing electricity markets may imply low correlation between spot and futures prices and condition the effectiveness of the hedging strategies. In this work minimum variance hedge ratios are conditionally estimated with the multivariate GARCH-BEKK model, and unconditionally by OLS and the Naive strategy. Empirical results indicate that dynamic hedging provides superior gains compared to those obtained from static hedging, and that multivariate GARCH models are successful in reducing the variance portfolio. With static hedging, several times, we were able to obtain hedging variance increases.

Results are also compared in and out of sample, and the effectiveness of multivariate GARCH models has to be recognized in terms of variance reduction, as confirmed in the empirical part of the present work.

EFM classification: 420 and 450

Keywords: Futures prices, spot prices, restructured electricity markets, dynamic and static hedging, multivariate GARCH, hedging effectiveness
1 Introduction

With the recent popularity of commodity (electricity) and financial futures, the academic and financial communities have seen a renewed interest in hedging theories. Hedgers in electricity markets use derivative securities, namely futures, to reduce the risk from variations in the spot market. They usually short an amount of futures contracts if they hold the long position on the underlying asset and vice-versa. An important question is how many futures contracts are needed. In other words, investors have to decide on the optimal hedge ratio, that is how many futures contracts should be held for each unit of the underlying assets. The hedge ratio is defined by Hull (2006) as "the ratio of the size of the portfolio taken in futures contracts to the size of the exposure".

Several have been the attempts to model the hedge ratio in the literature. Moschini and Myers (2002) assumed that the investor takes out futures positions and holds the position for a week (the nearby contract is typically the most actively traded, and this liquidity makes it attractive to potential hedgers.). At the end of the week, the investor reevaluates the futures position and chooses a new hedge ratio for the following week. Hence, the hedge ratio must be adjusted every week to reflect time varying volatility. Gagnon, Lypny and McCurdy (1998) consider a situation where an N-asset portfolio is hedged with an N-futures contract portfolio in a dynamic setting. Use daily spot and futures prices for the Deutsche Mark (DM), the Swiss Franc (SF) and the Japanese Yen (JY) from 1985 to 1990. They employ a multivariate GARCH (trivariate GARCH(1,1) system using the full BEKK parameterization.

Laws and Thompson (2005) use futures stock indices weekly data (FTSE100 and FTSE250) and point out that the exponential weighted moving average method of estimation provided the best estimate of the optimal hedge. These use the minimum variance hedge ratio and OLS, GARCH and EWMA frameworks. Moschini and Myers (2002) reject the null of a constant hedge ratio and that time variation in optimal hedge ratios can solely be explained by deterministic seasonality and time to maturity effects, using weekly corn cash and futures prices. They develop modified BEKK parameterization for the Bivariate GARCH(q,r) model.

Kumar, Singh and Pandey (2008) examine hedging effectiveness of futures in Indian markets. Estimate dynamic (VAR-MGARCH) and constant (OLS, VAR and VECM) hedge ratio for S&P CNX Nifty index futures, gold futures and soybean futures. Ripple and Moosa (2005) examine the effect of the maturity of the futures contract used as the hedging instrument on the effectiveness of futures hedging, using daily and monthly data on the WTI crude oil futures and spot prices (NYMEX). Use as measures of hedging effectiveness the near-month contract and the six-month contract, to conclude that futures hedging is more effective when the near month contract is used and that hedge ratios are lower for near-month hedging.

Hua (2007) estimates the constant and dynamic hedge ratios from 3 alternative modeling frameworks: OLS, VEC and MGARCH for Chinese copper futures markets, to conclude that the Multivariate GARCH dynamic hedge ra-
tios are superior to other hedge ratio estimates in terms of portfolio variance reduction. Pen and Sévi (2007) use as objective function the minimum variance hedge ratio and model the dynamic and distributional properties of daily spot and forward electricity prices across European wholesale markets. They doubt of the potential of forward markets for hedging purpose.

Modelling the asymmetric behavior of the covariance matrix in a multivariate setting and studying its consequences in three electricity spot-future systems is the main object of this paper. We also try to see if the different hedging specifications imply differences in terms of performance. As such, hedging length periods, hedging duration, different temporal futures contract specifications (month vs weekly), different European electricity markets and different "hours of the day" contracts (base vs peak) are analyzed in this paper in order to take some conclusions.

The appropriate way to calculate hedge ratios remains a controversial issue in the literature. The major methodologies for hedging with futures contracts have been OLS, VAR, VECM and multivariate GARCH. In this paper we use the first and the last one to calculate hedge ratios.

A special feature of multivariate GARCH models is that, with more or less precision, they always produce good results in terms of hedging. Different authors use different specifications and use valid arguments to justify one or the other (Byström, 2003, Torró, 2008, among others).

Multivariate GARCH models capture the dynamic evolution of the variance covariance matrix and construct an estimate of the optimal hedge ratio using the conditional variances and covariances of spot and futures returns.

In order to capture the dynamic structure of second moments conditional on the underlying and price variations, recent studies have concentrated in the development of hedging ratios changing through time using modelling techniques based on conditional heteroskedasticity.

The conditional heteroskedastic autoregressive specification (ARCH) was first presented by Engle (1982). It has been extended by Bollerslev (1986) to the generalized conditional heteroskedastic specification (GARCH). In fact, the great part of financial series contradict the constant correlation hypothesis as explored by Tse and Tsui (2002). In order to capture the different conditional correlation characteristics between rates, Engle and Kroner (1995) develop the BEKK procedure for the multivariate GARCH estimation. The BEKK algorithm allows changes through time of the conditional covariance which assumes the positiveness of the conditional variance covariance matrix.

Torró (2008) obtained an acceptable performance by increasing hedging duration and closing futures positions as near as possible to their final settlement. He uses weekly futures contracts and the weekly spot price (the average spot price for the 7 days in the week) for the period 1998 to 2007 in the Scandinavian electricity market; several combinations of hedging period lengths (one to 3 weeks) and times to maturity when futures positions are cancelled (one to 3 weeks) are examined. He argues that the poor effectiveness of hedging strategies reported by Moulton (2005) was due to the mismatch between the hedging period of the spot position (one day) and the underlying settlement period in the
futures used as a hedging vehicle (one month). In Moulton (2005), the underlying spot to the Californian futures was the average of peak hours spot prices in a month. On the other hand, Byström (2003) uses weekly spot price risk, hedged with weekly futures but only a one-week hedges duration were considered in the NordPool market. The intention is to study the short term hedging performance (one-week holding period), and since short-term future contracts are more liquid as well as more correlated with the underlying spot prices than the longer term contracts, futures with three weeks left to maturity are chosen for the hedging investigations.

Our study will concentrate on weekly (Nord Pool) and monthly (EEX and Powernext) data. Data selection is a very important aspect for several reasons. Not only due to a required large number of observations, but also because non-overlapping futures contracts are preferable to avoid artificially introducing autocorrelation in the data series. Therefore, the present study focus on weekly/monthly futures, taking one price per week/month, with a closing price each Friday, or the day before if non-tradable, in the NordPool/EEX,Powernext electricity markets, respectively. For NP we use only base data, but for EEX and Powernext, base and peak data are considered.

In this work, minimum variance hedge ratios are conditionally and unconditionally estimated with the multivariate GARCH model, and the OLS and Naive models. Empirical results indicate that dynamic hedging in the 3 electricity markets (NP, EEX and Powernext) provides superior gains compared to those obtained from static hedging.

The structure of the paper develops as follows: Section 2 presents the data and stationarity tests used, and results obtained; Section 3 presents the minimum variance hedging strategy; Section 4 talks about constant vs dynamic hedging and the models applied to these, and the multivariate GARCH-BEKK model to be explored here, in terms of hedging effectiveness; Section 5 presents the hedging performance metric used and empirical results obtained; Section 6 concludes and points out some ongoing developments.

2 Data

This section presents the special characteristics of the electricity markets, the data used in the work and the cointegration tests applied to spot and futures prices. Results indicate that all the series are stationary and as such workable for our purposes since heteroskedasticity of the series is evident from the summary statistics provided.

2.1 Electricity markets

The electricity industry was considered to be a natural monopoly throughout most of the 20th century, due to economics of scale in generation and problems related to separation of transmission and generation activities. Technological innovations in generation and improved transmission facilities decreased
economies of scale during the last decades of the century and indicated that unbundling of transmission, distribution and generation activities could be possible, provided that a series of institutional difficulties could be overcome at reasonable transaction costs. The current liberalization of electricity markets is still in a development phase. To become a successful experiment the market must provide a satisfactory balance between the three main requirements of economic efficiency, security of supply and environmental protection.

The design of electricity markets is complex due to a series of electricity characteristics that affect supply and demand. The physical characteristics of the electricity system complicates the design of electricity markets. Electricity is non-storable and a flow commodity, which is consumed within a tenth of a second after its production by virtually all consumers. The transmission system can be viewed as a shared pool with numerous entry and exit points, from which electricity can be injected or withdrawn. The supply and demand of power must be kept in a near continuous balance throughout the entire grid to avoid frequency and voltage fluctuations, which can damage generation and transmission equipment.

Extreme volatility, mean-reversion, skewness and kurtosis of returns, jumps and spikes, and the seasonal behavior of electricity prices (due to cooling and heating needs), differentiate the power market from all other commodity markets. The special features describing this type of markets may imply low correlation between spot and futures prices and condition effectiveness of the hedging strategies. Also, the volatility of both spot and futures returns change over time and the assumption of identically and independently normally distributed returns seems unrealistic.

It is well known that electricity demand exhibits seasonal fluctuations. The major factors that explain the seasonality of electricity prices are business activities and weather conditions. They mostly arise due to changing climate conditions, like temperature and the number of daylight hours. In some markets, and typically those countries that are heavily dependent on hydroelectric generation, such as Norway (where 99% of generation capacity is hydro), Sweden (with roughly 50% hydro), and Austria (69%), supply-side seasonality becomes important: spot prices on the Scandinavian Nord Pool exchange are affected by rainfall and snowmelt. These seasonal fluctuations in demand and supply translate into seasonal behavior of electricity prices, and spot prices in particular. In some markets, however, no clear annual seasonality is present and the spot prices behave similarly throughout the year with spikes occurring in all seasons (examples are Spain, Czech Republic, Poland where most of its spikes are negative, and Italy).

Apart from the annual “sinusoidal” behavior there is a substantial intraday variability. Higher than average prices are observed during the morning and evening peaks, while mid-day and night prices tend to be lower than average. The intra-week variability, related to the business day-weekend structure, is also nonnegligible. The price begins to increase at roughly 6h a.m., as the populace wakes and the workday begins. This price increase continues throughout the day as demand builds, peaking at 16h. Prices begin to fall thereafter as the
workday ends and demand shifts to primarily residential usage (that’s why we distinguish between peak and off-peak data in EEX and Powernext).

Higher prices appear from Tuesdays to Fridays, with the highest spikes occurring at Friday (weekly effects), and around 9 am to 12 am (daily effects). However, prices follow back to normal levels overnight. Cuaresma et al. (2004) report prices higher during weekdays, and intraday patterns and price spikes. The weekday prices are higher than those during the weekends, when major businesses are closed. These effects are all present in the data samples considered in the paper.

2.2 Data used

Our study will concentrate on weekly (Nord Pool) and monthly (EEX and Powernext) data for three reasons. Firstly the data is more stable than daily data. Secondly, the various studies suggest that weekly/monthly hedges are more efficient than daily hedges. Thirdly, for data restrictions.

Electricity futures prices and spot prices were directly obtained from Nord Pool’s FTP server files, from the French Powernext website and from data request to the EEX (German) electricity market. In the spot market, hourly power contracts are traded daily for physical delivery in the next 24 hour period. This price is known as the system price. There is a wide range of electricity derivatives contracts (forward, futures, options and contracts for difference) traded at Nord Pool. EEX trades futures and options, while Powernext only trades electricity futures. At the moment, the most important are: daily and weekly futures; monthly, quarterly and yearly forwards, in base prices, at Nord Pool (NP); monthly, quarterly and yearly futures based on peak and base data1 for both EEX and Powernext (the ones to be used in the empirical part of this work). The present study focus on weekly/monthly futures, taking one price per week/month, with a closing price each Friday, or the day before if non-tradable.

The data period analyzed is from 18 June 2004 until 15 July 2008, from 1 July 2002 to 27 March 2008 and from 25 September 1995 until 19 July 2007, for the German, French and Scandinavian markets respectively. During the sample period in Nord Pool the number of weekly futures contracts able to be traded has changed, from 4 to 5, to 6, to 7 and from 1998 onwards 8 weekly contracts were able to be traded daily, however in every period only the four contracts nearest to the delivery period are free from non-trading problems. With the four nearest to delivery weekly futures contracts, two data series of futures prices were built by maintaining the time to delivery constant. For French, the number of monthly contracts able to be traded has been kept constant, remaining on the three months after the contract has been settled. As such, four data series of futures prices were able to be constructed, using base and peak data. For the

1On-Peak data corresponds to the average daily price between 7 am and 19 p.m., and Off-Peak data is the daily average price between 00 am to 6 am (6h30m for UK) and 20 p.m. (19h30m for UK) to 24 p.m. Instead, "base data" is the average daily price for the 24 hours in the day. Distinguishing between on-peak and off-peak data is important for derivative contractual terms.
German electricity markets 7 monthly futures contracts can be traded daily, and as such we were able to construct 12 data series of futures contracts using base and peak data. However, similar to Ripple and Moosa (1995) we use only the near-month and the six-month contract in the EEX market.

In the empirical application futures with different maturities (3 weeks, 2 months, and 6 months for NP, Powernext and EEX, respectively) are considered to hedge the spot price variation and a unique hedging lengths is considered: one week for NP, for EEX and Powernext. We have focused on these strategies since we were trying to understand if maturity effects were able to conduct to different results.

2.3 Tests Of Unit Roots

Augmented Dicky-Fuller (ADF) and Philips-Perron (PP) tests attempt to account for temporally dependent and heterogeneously distributed errors by including lagged sequences of first differences of the variable in its set of regressors. The null hypothesis for ADF and PP test is that the variables contain a unit root or they are non-stationary at a certain significant level. However, the power of standard unit root tests which have null hypothesis of non-stationarity has recently been questioned by Schwertz (1987) and DeJong and Whiteman (1991) in that these tests often tend to accept the null too frequently against a stationary alternative. It appears that the failure to reject the null may be simply due to the standard unit root tests having low power against stable autoregressive alternatives with roots near unity. In particular, this knife-edge assumption of an exact unit root could lead to substantial biases, even in large samples. In view of the growing controversy surrounding the general tests for unit root, a different series of tests– KPSS tests proposed by Kwiatkowski et al. (1992) can also be employed in the context.

In the KPSS tests, the null hypothesis is that a series is stationary around a deterministic trend (TS) and the alternative hypothesis is that the series is difference stationary (DS). The series is expressed as the sum of deterministic trend, random walk, and stationary error as:

$$ y_t = \xi_t + r_t + \varepsilon_t $$

where $r_t = r_{t-1} + u_t$ and $u_t$ is i.i.d. $(0, \sigma_u^2)$.

The test is the LM test of the hypothesis that $r_t$ has zero variance. If this happens, the above equation becomes a constant and thus the series $\{y_t\}$ is trend stationary.

We have performed ADF, PP and KPSS test for all markets and strategies considered. However, all of them, despite the above discussion of seasonality and electricity main characteristics, confirm the assumption of stationary electricity prices. As such, table 1 presents the tests applied to both the spot and futures price series (logarithms) in the sample (for the futures is just for one of the strategies in each market). We have seen no need to include all test results since we have reached similar results for all strategies and all the tests confirm
the same. As such, we are only presenting the KPSS test results. They all confirm stationarity with trend, with trend and drift, and with none.

ADF and PP tests, applied to the natural logs of prices indicate the rejection of the null hypothesis of stationarity (both for peak and base data in EEX and Powernext markets), while the KPSS tests accept the null hypothesis of stationarity. The conclusion that the series is I(0) is confirmed by the tests applied to the first differences. We may conclude that the log price series do not have a trend nor drift. As such, error correction due to cointegration is not expected to improve the behavior of GARCH models due to stationarity.

Table 1: Stationarity of log-prices over the sample period

<table>
<thead>
<tr>
<th>Variable</th>
<th>KPSS test (drift)</th>
<th>Critical values</th>
<th>KPSS test (drift + trend)</th>
<th>Critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln St</td>
<td>0.021704</td>
<td>0.739</td>
<td>0.463</td>
<td>0.020979</td>
</tr>
<tr>
<td>ln Ft</td>
<td>0.057923</td>
<td>0.739</td>
<td>0.463</td>
<td>0.057807</td>
</tr>
<tr>
<td>ln St</td>
<td>0.023396</td>
<td>0.739</td>
<td>0.463</td>
<td>0.02229</td>
</tr>
<tr>
<td>ln Ft</td>
<td>0.040866</td>
<td>0.739</td>
<td>0.463</td>
<td>0.040715</td>
</tr>
<tr>
<td>ln St</td>
<td>0.075158</td>
<td>0.739</td>
<td>0.463</td>
<td>0.044192</td>
</tr>
<tr>
<td>ln Ft</td>
<td>0.02531</td>
<td>0.739</td>
<td>0.463</td>
<td>0.025346</td>
</tr>
<tr>
<td>ln St</td>
<td>0.06189</td>
<td>0.739</td>
<td>0.463</td>
<td>0.036644</td>
</tr>
<tr>
<td>ln Ft</td>
<td>0.018747</td>
<td>0.739</td>
<td>0.463</td>
<td>0.018753</td>
</tr>
<tr>
<td>ln St</td>
<td>0.071547</td>
<td>0.739</td>
<td>0.463</td>
<td>0.054726</td>
</tr>
<tr>
<td>ln Ft</td>
<td>0.043127</td>
<td>0.739</td>
<td>0.463</td>
<td>0.044028</td>
</tr>
</tbody>
</table>
The univariate characteristics of spot (base or peak) and futures return series are summarized in table 2. The high excess kurtosis value suggests that we are in the presence of leptokurtic distributions, what means we have heteroskedasticity present in the data.

Table 2: Instantaneous returns summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Variable</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>J.-B.</th>
<th>p-value</th>
<th>Obsv.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Powernext Base</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s_t$</td>
<td>0.0009</td>
<td>0.1734</td>
<td>-0.1206</td>
<td>9.5501</td>
<td>1856</td>
<td>0.0000</td>
<td>1037</td>
</tr>
<tr>
<td></td>
<td>$f_t$</td>
<td>0.0009</td>
<td>0.0373</td>
<td>3.6543</td>
<td>30.9036</td>
<td>101460</td>
<td>0.0000</td>
<td>1037</td>
</tr>
<tr>
<td><strong>Powernext Peak</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s_t$</td>
<td>0.0008</td>
<td>0.2009</td>
<td>-0.0989</td>
<td>11.1443</td>
<td>2868</td>
<td>0.0000</td>
<td>1037</td>
</tr>
<tr>
<td></td>
<td>$f_t$</td>
<td>0.0009</td>
<td>0.0445</td>
<td>4.7800</td>
<td>84.7696</td>
<td>292865</td>
<td>0.0000</td>
<td>1037</td>
</tr>
<tr>
<td><strong>EEX Base</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s_t$</td>
<td>0.0002</td>
<td>0.2141</td>
<td>0.5912</td>
<td>18.5299</td>
<td>14666</td>
<td>0.0000</td>
<td>1451</td>
</tr>
<tr>
<td></td>
<td>$f_t$</td>
<td>0.0005</td>
<td>0.0292</td>
<td>3.8964</td>
<td>154.6940</td>
<td>1394883</td>
<td>0.0000</td>
<td>1451</td>
</tr>
<tr>
<td><strong>EEX Peak</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s_t$</td>
<td>0.0001</td>
<td>0.2568</td>
<td>0.4046</td>
<td>18.3625</td>
<td>14308</td>
<td>0.0000</td>
<td>1451</td>
</tr>
<tr>
<td></td>
<td>$f_t$</td>
<td>0.0005</td>
<td>0.0329</td>
<td>1.2414</td>
<td>96.6766</td>
<td>530913</td>
<td>0.0000</td>
<td>1451</td>
</tr>
<tr>
<td><strong>NP Base</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s_t$</td>
<td>-0.0002</td>
<td>0.0778</td>
<td>0.8775</td>
<td>34.9405</td>
<td>125437</td>
<td>0.0000</td>
<td>2942</td>
</tr>
<tr>
<td></td>
<td>$f_t$</td>
<td>0.0001</td>
<td>0.0404</td>
<td>1.0579</td>
<td>27.8902</td>
<td>76440</td>
<td>0.0000</td>
<td>2942</td>
</tr>
</tbody>
</table>

Notation $s_t$ and $f_t$ represent returns of the spot and future prices respectively. The excess kurtosis values takes the normal distribution as the basic one.

Both futures returns and the spot returns have means very close to zero, and we may say that the unconditional distribution of spot returns and in particular future returns are non-normal, as evidenced by skewness, high excess kurtosis and high values for the Jarque-Bera test statistic.

Evidence shows that asset storability does not affect the existence of cointegration between cash and futures prices and the usefulness of future markets in predicting future cash prices. However, it may affect the magnitude of bias of futures markets’ estimates (or predictions) for future cash prices. In fact, researchers have found mixed evidence of cointegration between cash and futures prices for storable commodities. Some (Covey and Bessler, 1995) have argued that cointegration may depend on asset storability. These authors have argued that researchers should not expect cointegration between cash and futures prices for nonstorable commodities but should expect it for storable commodities.

We wanted also to check this, and to test cointegration, the Johansen’s test was used, and although results are not presented here, this correlation varies.
between 0.7 and 0.86. The highest correlation between spot and futures is obtained for those futures positions held until maturity. As such, only these will ensure a good risk reduction for hedgers.

3 Hedging specification

In this section, the minimum variance hedge ratio is estimated. Torró (2008) uses Minimum Variance Hedge Ratio estimated by OLS and Multivariate GARCH with a bivariate t-student distribution. Moulton (2005) and Byström (2003) also use this as the main objective function. Lien and Tse (2000) consider the optimal strategy for hedging the downside risk measured by the lower partial moments. Lien and Tse (2002) evaluate constant hedge ratios and time-varying hedge ratios, exploring different econometric implementations. They provide a survey that reviews some recent developments in futures hedging.

Harris and Shen (2006) point out that the use of variance as a measure of portfolio risk - and hence the use of minimum variance hedging as a method of minimizing risk - is justified by assuming either that investors have quadratic utility functions or that asset returns are drawn from a multivariate elliptical distribution. Conclude that Minimum-VaR hedging yields hedge portfolios that are typically less negatively skewed and/or less leptokurtic than minimum-variance hedging both in sample and out-of-sample (similar conclusions stand for Minimum CVaR). Minimum VaR and Minimum CVaR hedge ratios are typically lower than minimum-variance hedge ratios, suggesting that smaller short positions are typically required to minimize VaR or CVaR than to minimize variance.

Mattos, Garcia and Nelson (2005) calculate hedge ratios under utility maximization based on a constant relative risk aversion (CRRA) utility function which allows the absolute level of risk aversion to change with wealth. Eftekhari (1998) minimizes the lower partial moment of order two (LPM2) with target set to zero to calculate the optimal hedge ratio for the FTSE-100 stock index from 1985 to 1994. The general result is that minimum-LPM hedge ratios are slightly smaller and tend to yield a better risk/return combination than the minimum-variance hedge ratios. Similarly, Lien and Tse (2000) calculated the minimum-LPM and the minimum-variance hedge ratios for the Nikkei Stock Average index over 1-week hedging horizons from January 1988 to August 1996. Three orders of the LPM were used (1, 2, and 3), and the target returns ranged from −1.5% to +1.5%. Their findings suggest that the minimum-LPM and the minimum-variance hedge ratios may differ sharply, particularly when the hedger is willingly to absorb small losses and very cautious about large losses, i.e., when the target return is small and the order of the LPM is large. Turvey and Nayak (2003) found that minimum-semivariance hedge ratios were usually smaller than the minimum-variance hedge ratios, but the difference between the two ratios varied depending on the target and the distribution of risk for the wheat market. Moreover, the minimum-semivariance hedge was found to offer a better protection against downside risk than the minimum-variance hedge. A differ-
ent approach was followed by Chen, Lee and Shrestha (2001), who adopted a mean-downside risk framework to estimate optimal hedge ratios. They argued that these hedge ratios should be calculated using utility maximization in a mean-risk framework.

As such, some avenues for future research and investigation are implicit in this brief review, and are being subject of current research. Next we concentrate on the most used one: the Minimum Variance Hedge Ratio.

3.1 Minimum-variance hedge ratio

Let’s illustrate hedging decisions with a one-period model. At the beginning of the period, that is, $t=0$, an individual is committed to a given spot position, $Q$, on a specific asset. A futures market for the security is available with different maturities. To reduce the risk exposure, the individual may choose to go short in the futures market. Due to liquidity and other concerns, let’s assume that he trades only in the "nearby" futures contract (that is, the contract the maturity of which is closest to the current date). With the futures trading, the individual becomes a short hedger. Let $X$ denote the futures position. At the end of the period, say, $t=1$, the hedger’s return, $r$, is calculated as follows:

$$r = \left( sQ - fX \right) / Q$$

where $s_t$ is the return of the spot position and $f_t$ is the return of the futures position, both at time $t$. As both spot and futures returns are unknown at $t=0$, $r$ is a random variable. The hedger will choose $X$ to minimize the risk (or uncertainty) associated with the random return.

In the finance literature, the risk of a random variable is usually measured by the variance (or standard deviation) conditional on the available information. Let $\phi$ denote the information set at $t=0$. Then the hedger’s risk is summarized by the conditional variance of $r$, $\text{Var}(r|\phi)$.

$$\text{Var}(r|\phi) = \left[ \text{Var}(s|\phi)Q^2 - 2\text{Cov}(s,f|\phi)XQ + \text{Var}(f|\phi)X^2 \right] / Q^2$$

The optimal futures position $X^*$ is chosen to minimize $\text{Var}(r|\Phi)$. Thus,

$$X^* = \left[ \text{Cov}(s,f|\phi) / \text{Var}(f|\phi) \right] Q = hQ$$

where $h = [\text{Cov}(s,f|\phi) / \text{Var}(f|\phi)]$ is the minimum-variance hedge ratio.

3.2 The expected utility framework

A more general approach to the hedging problem relies upon the expected-utility framework. Suppose that the hedger is endowed with a von-Neumann Morgenstern utility function $U(.)$ such that $U'(.) > 0$ and $U''(.) < 0$. Let $E(.)$ denote the expectation operator with respect to the joint distribution of $s$ and $f$. The optimal futures position, $X^\varepsilon$, is chosen to maximize the (conditional) expected utility $E \{ U(r|\phi) \}$. That is, $X^\varepsilon$ must satisfy the following condition
or alternatively,

\[ \text{Cov} \left( U' \left( \frac{sQ - fX^e}{Q} \right)|f|\phi \right) + \mathbb{E} \left\{ U' \left( \frac{sQ - fX^e}{Q} \right) \right\} \mathbb{E} \left\{ \frac{f}{Q}|\phi \right\} = 0 \]

Assuming that \( s = \alpha (\phi) + \beta (\phi) f + \varepsilon \), where \( f \) and \( \varepsilon \) are stochastically independent. Considering that \( E (f|\phi) = 0 \) (that is, the futures price is unbiased), the second term on the left-hand-side of the above equation vanishes. Moreover, when \( X^e = \beta (\phi) Q \) we have \( sQ - fX^e = \alpha (\phi) Q + \varepsilon Q \), which is stochastically independent of \( f \). Thus, the first term on the left-hand-side of the equation is also zero. In other words, the optimal solution is \( X^e = \beta (\phi) Q \), which in turn equal to \( \text{Cov} (s, f|\phi) Q / \text{Var} (f|\phi) \). Therefore, the optimal hedge ratio as defined by \( X^e / Q \) and derived from a general utility function is equal to the minimum variance hedge ratio.

If the futures price is biased such that \( E (f|\phi) \neq 0 \) (due to transaction costs, for example), then the optimal hedge ratio diverges from the minimum-variance hedge ratio. In this case, however there is a speculative motivation to trade so as to take advantage of the bias in the futures market. Consequently, \( X^e \) contains both hedging and speculative components. The former is characterized by the condition \( E (f|\phi) = 0 \). Thus, the hedging component of the optimal futures position is equal to the minimum-variance futures position. Assuming the hedger has a mean-variance utility function given by \( E (r|\phi) - (A/2) \text{Var}(r|\phi) \), where \( (A/2) \) is the Arrow-Pratt risk aversion coefficient, the optimal futures position \( X^e \) is \( E (-f|\phi) / A + [\text{Cov} (s, f|\phi) / \text{Var} (f|\phi)] Q \). The first component represents the speculative trading whereas the second is the usual optimal hedge position.

In the above derivation both minimum-variance and optimal hedge ratios are functions of the information set \( \phi \). As \( \phi \) changes, both hedge ratios change. Typical information sets include the historical spot and futures returns, the contract maturity and the hedge horizon. Whenever the spot and futures return distributions depend on the information, both optimal hedge and minimum-variance hedge strategies depend on the time-varying dynamic hedge ratios.

Transaction costs are not considered in the present paper when comparing hedging methods, as the hedging theoretical framework is a one-period model for all hedging methods. Furthermore, the individual is considered to take futures positions at the beginning of the period and cancel them at the end of the period. since hedging ratio values are quite similar within methods, these will have similar transaction costs.

Transaction costs are also ignored since Meneu and Torró (2002) concluded (using the IBEX35 index and futures contracts on this) that variance reduction is economically significant in the sample period but not in the out-of-sample period, where transaction costs in dynamic hedging absorb the small differences in risk reduction. Furthermore, they conclude that dynamic hedging with high transaction costs are delayed in updating information affecting the optimal
hedge ratio, meaning that hedge ratios and hedging strategies are insensitive to any asymmetric consideration. When transaction costs are included, they conclude that investors would prefer in some cases a static strategy because the small risk reduction achieved do not compensate the transaction costs needed to maintain a dynamic strategy.

Since transaction costs are not included into the analysis, the optimal hedge ratio will effectively coincide with the minimum variance hedge ratio and both theoretical versions of the same subject equal.

4 Constant versus Dynamic Hedging

As before, let $S_t$ and $F_t$ be the natural logarithm of spot and futures prices, respectively, and $s_t$ and $f_t$ denote the changes in the logarithms of spot and futures prices at time $t$. Next, the constant and dynamic hedge ratio estimation models to be used in this work are presented.

When static hedging strategies are used, each moment the agent faces decision whether to hedge with the current future price estimate or wait for new information. When we refer to static hedging we mean that once the hedge is created it is not changed after that. So, static hedging means that the hedging ratio $h$ remains constant over time.

Before discussing dynamic hedging strategies, the investor has to take a basic decision on the best static hedging ratio. This decision depends on several factors and upon certain decisions of the investor: choice of the foreign countries of the investment, frequency of investment changes, limits of risk and exposure in the dynamic hedging strategy,... The static hedging strategy determines the equilibrium point or neutral point of the dynamic hedging strategy. If the position taken in derivatives changes over time, the hedging strategy is dynamic.

We will assume that the market is incomplete, therefore not all the risks are hedgeable through trading the underlying stock. If the market were complete, given sufficient initial capital, all claims could be replicated by trading the stock dynamically. Static derivatives hedges do not add anything to dynamic hedges in complete markets, but of course they are very valuable tools in realistic incomplete market models, where there may be risk factors that cannot be eliminated just by dynamic trading of the underlying stock. By incorporating static hedges, we enlarge the set of feasible hedging strategies that the investor can choose from and allow for a better hedging performance.

4.1 Constant Hedge Ratio Estimation Model

The conventional method is the minimum variance (MV) ratio, which is $h^* = \frac{\text{cov}(s_t, f_t)}{\text{Var}(f_t)}$, i.e., the covariance between spot and futures returns relative to the variance of the futures return. Empirically, the one period hedge ratio is estimated by the slope from the following ordinary least squared (OLS) regression equation:
\[ s_t = \alpha + h^* f_t + \varepsilon_t \]

Where \( \varepsilon_t \) is the error term. The minimum hedge ratio is \( h^* \).

### 4.2 Time-varying Hedge Ratio Estimation Model

Developed by Engle (1982) and then Bollerslev (1986), the autoregressive conditional heteroskedasticity model (ARCH) sparked a substantial body of work which concerns with not only further examining the second moment of economic and financial time series, but also extending and generalizing the initial ARCH model to better fit the situation being investigated. Bollerslev, Engle and Wooldridge (1988) generalised the univariate GARCH to a multivariate dimension to simultaneously model the conditional variance and covariance of two interacted series. This multivariate GARCH model is thus applied to the calculation of dynamic hedge ratios that vary over time based on the conditional variance and covariance of the spot and futures prices. Engle and Kroner (1995) present various MGARCH models with variations to the conditional variance-covariance matrix of equations.

Generalised from GARCH(1,1), a standard M-GARCH(1,1) model is expressed as:

\[
\begin{bmatrix}
    h_{ss,t} \\
    h_{sf,t} \\
    h_{ff,t}
\end{bmatrix} =
\begin{bmatrix}
    c_{ss,t} \\
    c_{sf,t} \\
    c_{ff,t}
\end{bmatrix} +
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
\end{bmatrix} \times
\begin{bmatrix}
    \varepsilon_{s,t-1}^2 \\
    \varepsilon_{s,t-1} \varepsilon_{f,t-1} \\
    \varepsilon_{f,t-1}^2
\end{bmatrix} +
\begin{bmatrix}
    b_{11} & b_{12} & b_{13} \\
    b_{21} & b_{22} & b_{23} \\
    b_{31} & b_{32} & b_{33}
\end{bmatrix} \times
\begin{bmatrix}
    h_{ss,t-1} \\
    h_{sf,t-1} \\
    h_{ff,t-1}
\end{bmatrix}
\]

where \( h_{ss}, h_{ff} \) are the conditional variance of the errors \((\varepsilon_{s,t}, \varepsilon_{f,t})\) from the mean equations. Where we have that:

\[ \varepsilon_t|\phi_{t-1} \sim BN(0, H_t) \text{ with} \]

\[ \varepsilon_t = \begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix} \text{ and } H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{fs,t} & h_{ff,t} \end{bmatrix} \]

Karolyi (1995) suggests that the BEKK (Baba, Engle, Kraft and Kroner) model allows the conditional variance and covariance of the spot and futures prices to influence each other, and, at the same time, do not require the estimation of a large number of parameters to be employed. The model also ensures the condition of a positive semi-definite conditional variance-covariance matrix in the optimization process which is a necessary condition for the estimated variance to be zero or positive. The BEKK parameterization for the MGARCH(1,1) model is written as:
\[
\begin{bmatrix}
    h_{ss,t}^2 & h_{sf,t}^2 \\
    h_{fs,t}^2 & h_{ff,t}^2
\end{bmatrix}
= 
\begin{bmatrix}
    b_{ss} & b_{sf} \\
    0 & b_{ff}
\end{bmatrix}
\begin{bmatrix}
    b_{ss} & b_{sf} \\
    0 & b_{ff}
\end{bmatrix} + 
\begin{bmatrix}
    c_{ss} & c_{sf} \\
    c_{fs} & c_{ff}
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_{s,t-1}^2 & \varepsilon_{s,t-1} \varepsilon_{f,t-1} \\
    \varepsilon_{f,t-1} \varepsilon_{s,t-1} & \varepsilon_{f,t-1}^2
\end{bmatrix}
\begin{bmatrix}
    c_{ss} & c_{sf} \\
    c_{fs} & c_{ff}
\end{bmatrix} + 
\begin{bmatrix}
    g_{ss} & g_{sf} \\
    g_{fs} & g_{ff}
\end{bmatrix}
\begin{bmatrix}
    h_{ss,t-1}^2 & h_{sf,t-1}^2 \\
    h_{fs,t-1}^2 & h_{ff,t-1}^2
\end{bmatrix}
\begin{bmatrix}
    g_{ss} & g_{sf} \\
    g_{fs} & g_{ff}
\end{bmatrix}
\]

where \( h_{ss,t}^2, h_{sf,t}^2 \) and \( h_{ss,t}^2 \) are the conditional variance and covariance of the errors \((\varepsilon_{st}, \varepsilon_{ft})\) from mean equations. Conditional variance and covariance only depend on their own lagged squared residuals and lagged values. The MGARCH model incorporates a time-varying conditional covariance and variance between the spot and futures prices and hence generates more realistic time-varying hedge ratios.

Notice that the assumption of normality in electricity log-price variation is not a realistic one. Has we have seen in the summary statistics of the data, one fact that characterizes electricity price distribution is its leptokurtosis due to the presence of many extreme values (jumps and/or spikes). As such, as an alternative empirical distribution to the normal one we will also use the bivariate t-student distribution in the multivariate-GARCH BEKK model used here:

\[
\varepsilon_t | \Phi_{t-1} \sim t(0, H_t, v)
\]

where \( v \) is the degrees of freedom parameter of a conditional bivariate t-student distribution.

Bivariate GARCH modelling allows to model the conditional second moments, but also the cross moments, with special relevance, in our case, to the contemporaneous covariance between electricity spot and futures. That’s why the conditional, on time \( t - 1 \) available information, error term vector follows a bivariate normal law, and for the comparison purpose also a bivariate \( t \) distribution, being \( H_t \) the positive definite variance covariance matrix dependent on time.

## 5 Hedging effectiveness and Results

In this section we will present all the obtained results derived from the econometric techniques analyzed before. Furthermore, we present, test and show the obtained results using the variance hedging effectiveness metrics.

### 5.1 Hedging effectiveness metrics: the variance

The performance metric used to examine and compare the hedging performance of each strategy is the variance. The variance metric \((HE_1)\) measures the percentage reduction in the variance of a hedged portfolio as compared with the
variance of an unhedged portfolio. The hedged portfolios are calculated by using the OHR’s derived from the hedging models, with the best model being the one with the largest reduction in the variance. The performance metric is:

\[ HE_1 = 1 - \left( \frac{\text{Variance}_{	ext{hedged Portfolio}}}{\text{Variance}_{	ext{unhedged Portfolio}}} \right) \]

This gives us the percentage reduction in the variance of the hedged portfolio as compared with the unhedged portfolio. When the futures contract completely eliminates risk, we obtain \( HE_1 = 1 \) which indicates a 100% reduction in the variance, whereas we obtain \( HE_1 = 0 \) when hedging with the futures contract does not reduce risk. Therefore, a larger number indicates better hedging performance.

The variance is a standard measure of risk in finance and has become the dominant measure of hedging effectiveness used by hedgers. It has also been extensively applied in the literature on hedging and was used by Ederington (1979) to evaluate hedging effectiveness. The advantage of using the variance as a measure of performance is its ease of calculation and interpretation.

So, in order to measure hedging effectiveness we present the risk reduction measures, computing the variance of a hedge strategy as the variance of the hedged portfolio and the risk reduction achieved for each strategy is computed by comparison with the variance of the spot position (\( h_t = 0, \forall t \)).

Using unconditional probability distributions, the hedge ratio can be estimated from a linear relationship between spot and futures returns by ordinary least squares (OLS) by simply adding an intercept and a white noise to the equation:

\[ s_t = \alpha + h^* f_t + \varepsilon_t \]

Using this specification, the OLS estimator or \( h \) will be the unconditional definition of the optimal hedge ratio.

When a hedge where the futures position have the same size but the opposite sign than the position held in the spot market is considered, we have what is called a Naive hedge ratio (\( h_t = 1, \forall t \)), which will also be considered in the present work.

As such, following Park and Switzer (1995) we choose to evaluate to what extent the different hedges reduce the unconditional variance (the sample variance of the spot returns and hedge portfolio returns) over the test period.

The purpose of the hedging exercise has been to minimize the variance (uncertainty) of the hedge portfolio. For an infinitely risk avert trader or martingale futures, this is equal to utility maximizing as seen above.

The data sets available where all divided into an estimation period, and an out-of-the-sample period consisting of the last 100 observations for each market. For the dynamic model, the hedge ratio is updated each day in the test period.

To evaluate the hedging performance out-of-sample, one looks at the test period covering approximately the months February 2008 - July 2008, for EEX, the months October 2008 - March 2008, for Powernext and April 2007 - July
2007 for the Scandinavian market. All hedge ratios are predicted hedge ratios using predicted variances and covariances.

5.2 Empirical results

Table 3 displays the variance reduction of the hedging method. It contains data for the three different markets under analysis and displays, in each panel, the risk reduction achieved by each hedging strategy explored here: Naive, OLS and multivariate GARCH-BEKK (assuming the errors follow a bivariate normal - 4th row of each panel - and a bivariate t distribution - last row of each panel. The second column reports in-the-sample results and the third column reports out-of-the-sample results. For all electricity markets under analysis the out-of-sample results are taken for the last 100 observations of the sample periods. Panels 3A, 3B, 3C, and 3D are referred to the EEX electricity market data. Panels 3E, 3F are for the Powernext market (only the P2 - two months to expiration, one week before - and B1 - one month to expiration, one week before - strategies where considered). The rest of the panels (3G and 3H) are for the NordPool strategies (3,1: 3 weeks left to maturity, one week prior to expiration; 1,1: one week left to maturity, one week prior to expiration) and hedging variance reductions.

The spot variance reduction is computed by comparison with the unhedged spot position variance, in the first row of each panel. Those strategies with largest risk reduction are indicated with a plus (+) for ease of identification. Results obtained imply that the better statistical performance of the multivariate GARCH-BEKK model implies also a better hedging strategy performance. In the one week hedges, the Naive and OLS strategies clearly obtain the worst score, favouring the multivariate GARCH models which are those that in fact deliver the highest variance reduction.

Table 3: Hedging effectiveness

<table>
<thead>
<tr>
<th>Hedging</th>
<th>In the sample</th>
<th>Out of the sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naive (b = 1)</td>
<td>-3,0</td>
<td>3,5</td>
</tr>
<tr>
<td>OLS (b = \frac{h_{FE}}{h_{FE}})</td>
<td>0,3</td>
<td>4,3</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{\lambda_{FE}}{h_{FE,t}})</td>
<td>14,5</td>
<td>32,1</td>
</tr>
<tr>
<td>T-Diag-BEKK (b_t = \frac{\lambda_{FE,t}}{h_{FE,t}})</td>
<td>23,4</td>
<td>38,4^+</td>
</tr>
</tbody>
</table>

Table 3A. Hedging Effectiveness

This table displays the risk reduction achieved by each hedging strategy. NAIVE, OLS and BEKK

EEX - P1

| Spot variance (no hedging) \(b = 0\) | 0,06467 | 0,07921 |
| In the sample | | |
| Out of the sample | | |

<table>
<thead>
<tr>
<th>Hedging</th>
<th>Risk reduction (%)</th>
<th>Risk reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naive (b = 1)</td>
<td>-3,0</td>
<td>3,5</td>
</tr>
<tr>
<td>OLS (b = \frac{h_{FE}}{h_{FE}})</td>
<td>0,3</td>
<td>4,3</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{\lambda_{FE}}{h_{FE,t}})</td>
<td>14,5</td>
<td>32,1</td>
</tr>
<tr>
<td>T-Diag-BEKK (b_t = \frac{\lambda_{FE,t}}{h_{FE,t}})</td>
<td>23,4</td>
<td>38,4^+</td>
</tr>
</tbody>
</table>
### Table 3B. Hedging Effectiveness

This table displays the risk reduction achieved by each hedging strategy. NAIVE, OLS and BEKK

<table>
<thead>
<tr>
<th>Hedging</th>
<th>Risk reduction (%)</th>
<th>Risk reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the sample</td>
<td>Out of the sample</td>
<td></td>
</tr>
<tr>
<td>Spot variance (no hedging) (b = 0)</td>
<td>0.06467</td>
<td>0.07921</td>
</tr>
<tr>
<td>Naive (b = 1)</td>
<td>-2.9</td>
<td>-2.9</td>
</tr>
<tr>
<td>OLS (b = \frac{h_{F \times F}}{h_F})</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{h_{F \times F}}{h_{F \times F}})</td>
<td>17.4+</td>
<td>16.5</td>
</tr>
<tr>
<td>T-Diag-BEKK (b_t = \frac{h_{F \times F}}{h_{F \times F}})</td>
<td>12.6</td>
<td>13.3</td>
</tr>
</tbody>
</table>

### Table 3C. Hedging Effectiveness

This table displays the risk reduction achieved by each hedging strategy. NAIVE, OLS and BEKK

<table>
<thead>
<tr>
<th>Hedging</th>
<th>Risk reduction (%)</th>
<th>Risk reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the sample</td>
<td>Out of the sample</td>
<td></td>
</tr>
<tr>
<td>Spot variance (no hedging) (b = 0)</td>
<td>0.04457</td>
<td>0.06050</td>
</tr>
<tr>
<td>Naive (b = 1)</td>
<td>-3.1</td>
<td>5.3</td>
</tr>
<tr>
<td>OLS (b = \frac{h_{F \times F}}{h_F})</td>
<td>0.2</td>
<td>8.5</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{h_{F \times F}}{h_{F \times F}})</td>
<td>11.4</td>
<td>22.2</td>
</tr>
<tr>
<td>T-Diag-BEKK (b_t = \frac{h_{F \times F}}{h_{F \times F}})</td>
<td>17.8</td>
<td>33.2+</td>
</tr>
</tbody>
</table>

### Table 3D. Hedging Effectiveness

This table displays the risk reduction achieved by each hedging strategy. NAIVE, OLS and BEKK

<table>
<thead>
<tr>
<th>Hedging</th>
<th>Risk reduction (%)</th>
<th>Risk reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the sample</td>
<td>Out of the sample</td>
<td></td>
</tr>
<tr>
<td>Spot variance (no hedging) (b = 0)</td>
<td>0.04457</td>
<td>0.06050</td>
</tr>
<tr>
<td>Naive (b = 1)</td>
<td>-2.6</td>
<td>-4.7</td>
</tr>
<tr>
<td>OLS (b = \frac{h_{F \times F}}{h_F})</td>
<td>0.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{h_{F \times F}}{h_{F \times F}})</td>
<td>19.8</td>
<td>28.7+</td>
</tr>
<tr>
<td>T-Diag-BEKK (b_t = \frac{h_{F \times F}}{h_{F \times F}})</td>
<td>10.7</td>
<td>12.7</td>
</tr>
</tbody>
</table>
### Table 3E. Hedging Effectiveness
This table displays the risk reduction achieved by each hedging strategy. NAIVE, OLS and BEKK

<table>
<thead>
<tr>
<th>Hedging</th>
<th>In the sample</th>
<th>Out of the sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot variance (no hedging) (b = 0)</td>
<td>0.04317</td>
<td>0.01372</td>
</tr>
<tr>
<td><strong>Hedging</strong></td>
<td><strong>Risk reduction (%)</strong></td>
<td><strong>Risk reduction (%)</strong></td>
</tr>
<tr>
<td>Naive (b = 1)</td>
<td>-1.7</td>
<td>-6.2</td>
</tr>
<tr>
<td>OLS (b = \frac{h_{F,S}}{h_F})</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{h_{F,t}}{h_{F,t}})</td>
<td>15.5(^+)</td>
<td>5.6</td>
</tr>
<tr>
<td>T-Diag-BEKK (b_t = \frac{h_{F,t}}{h_{F,t}})</td>
<td>11.6</td>
<td>-2.7</td>
</tr>
</tbody>
</table>

### Table 3F. Hedging Effectiveness
This table displays the risk reduction achieved by each hedging strategy. NAIVE, OLS and BEKK

<table>
<thead>
<tr>
<th>Hedging</th>
<th>In the sample</th>
<th>Out of the sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot variance (no hedging) (b = 0)</td>
<td>0.03169</td>
<td>0.01427</td>
</tr>
<tr>
<td><strong>Hedging</strong></td>
<td><strong>Risk reduction (%)</strong></td>
<td><strong>Risk reduction (%)</strong></td>
</tr>
<tr>
<td>Naive (b = 1)</td>
<td>-0.7</td>
<td>-7.8</td>
</tr>
<tr>
<td>OLS (b = \frac{h_{F,S}}{h_F})</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{h_{F,t}}{h_{F,t}})</td>
<td>-2.4</td>
<td>-2.6</td>
</tr>
<tr>
<td>T-Diag-BEKK (b_t = \frac{h_{F,t}}{h_{F,t}})</td>
<td>2.1</td>
<td>2.4(^+)</td>
</tr>
</tbody>
</table>

### Table 3G. Hedging Effectiveness
This table displays the risk reduction achieved by each hedging strategy. NAIVE, OLS and BEKK

<table>
<thead>
<tr>
<th>Hedging</th>
<th>In the sample</th>
<th>Out of the sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot variance (no hedging) (b = 0)</td>
<td>0.00613</td>
<td>0.00375</td>
</tr>
<tr>
<td><strong>Hedging</strong></td>
<td><strong>Risk reduction (%)</strong></td>
<td><strong>Risk reduction (%)</strong></td>
</tr>
<tr>
<td>Naive (b = 1)</td>
<td>-11.6</td>
<td>-7.8</td>
</tr>
<tr>
<td>OLS (b = \frac{h_{F,S}}{h_F})</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{h_{F,t}}{h_{F,t}})</td>
<td>20.6(^+)</td>
<td>13.2</td>
</tr>
</tbody>
</table>
Table 3H. Hedging Effectiveness

This table displays the risk reduction achieved by each hedging strategy. NAIVE, OLS and BEKK.

<table>
<thead>
<tr>
<th>Hedging</th>
<th>In the sample</th>
<th>Out of the sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot variance (no hedging) (b = 0)</td>
<td>0.00613</td>
<td>0.00375</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hedging</th>
<th>Risk reduction (%)</th>
<th>Risk reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naive (b = 1)</td>
<td>21.0</td>
<td>0.0</td>
</tr>
<tr>
<td>OLS (b = \frac{\hat{h}<em>{F,S}}{\hat{h}</em>{F}})</td>
<td>2.7</td>
<td>-3.1</td>
</tr>
<tr>
<td>Diag-BEKK (b_t = \frac{\hat{h}<em>{F,S,t}}{\hat{h}</em>{F,t}})</td>
<td>2.3</td>
<td>4.0+</td>
</tr>
</tbody>
</table>

We confirm the variance reduction of the different hedging methods in electricity markets, compared to Byström (2003), Moulton (2005) and Torró (2008). The spot variance reduction is computed comparing with the unhedged spot portfolio variance. The reduction obtained is about 15% in the sample and 20% in the out-of-sample part. In both parts, dynamic methods reduce the risk more than the static hedging method considered (OLS). Nevertheless, the differences are quite high in some markets like the EEX with monthly futures contracts, but very small depending on the strategies adopted. Moreover, results become inconclusive with respect to duration effects, but hedging with three week spot risk is better in terms of variance reduction in the Nord Pool market.

Results obtained for the French electricity market were even more difficult to interpret because we were not able to see a behavioral pattern given the results obtained. As such, we decided to present the results for only two of the strategies adopted and the t-distribution assumed improved the results for the base (B1) strategy "1 month to expiration". We attribute these inconsistencies to the fact that this is the most recent market from all the markets under analysis, and data availability may be influencing the results obtained.

As we are able to see, for most of the strategies, the Naive strategy results (in and out sample) indicate that in that case instead of variance reduction we might have variance increases, which contradicts the literature that defend unconditional hedges.

Not all hedges reduce the variance. The best hedge is in almost all cases the bivariate GARCH - BEKK hedge. Contrarily to Byström (2003) results, the dynamic hedge ratios perform better than the static ones, and as such there seems to be major gains from modelling spot and futures returns, despite if the spot is on the base or peak data, if we are talking of month or week futures contracts, or even despite the market under analysis and the strategy used (the exception is the NordPool market).

The finding that the OLS hedge performs even slightly better (in most of the cases) than the naive hedge, but that conditional hedges perform even better, is an example of how simpler models do not always work well (which contradicts Park and Switzer, 1995, and Byström, 2003). The theoretical analysis above suggests that unconditional hedges, the naive and OLS hedge, do not outperform...
the conditional hedges, since the conditional ones reduce the variance more. In order to update the hedge according to the dynamic model one has to buy or sell futures each day, and the cost of daily updating the hedge does not seem to influence. Once again, we reinforce the idea that transaction costs in the market are not the reason for not to hedge. So, when adding the costs and time spent on designing and estimating the dynamic hedges to the actual transaction costs we may not probably end up with a significant additional daily cost compared to the "buy and hold" OLS hedge.

A curious fact is the improvement obtained in variance reduction when conditional hedges are estimated under the t-distribution relative to the bivariate normal distribution assumed. However this is only confirmed relative to the closest to maturity hedging strategies (one month to maturity, one week before, being it relative to peak data - EEX P1 - or to base data - EEX B1 and FF B1. For the other markets, much more work remains to be done. We have also concluded that the risk reduction, relative to in-sample results for the EEX market, is higher for the six months to expiration, one week prior, relative to the one month to expiration, one week prior, but we need to extend the results to other strategies that can be adopted in order to confirm these previous findings, since the results obtained are not conclusive with this respect. Variance reduction is even higher when we consider only peak data, than that obtained for base data. This may be due to the fact that peak data is even more volatile and as such an even higher risk reduction should be expected in hedging the spot-peak with futures peak prices contracts, as it was the case.

Finally, we should reinforce the idea that differences in the adopted hedging strategies and period lengths must be considered when we try to compare our results to those obtained by other author’s in previous works (Byström, 2003 and Torró, 2008).

6 Conclusions

In this work we have tried to explore the hedging effectiveness of electricity futures for three different European electricity markets (Scandinavian, German and French markets). For this we employ minimum variance hedge ratios, which are conditionally estimated with the multivariate GARCH model. Empirical results obtained indicate that dynamic hedging provides superior gains compared to those obtained from static hedging, but even so results depend on which strategy and market is being analyzed.

For these results we use the traditional unconditional naive and OLS hedge and the dynamic conditional GARCH-BEKK hedges, where we conclude that, with more or less precision, they all reduce the variance of the hedge portfolio compared to the spot position. The errors distributional assumption is also an important aspect when we are talking about electricity markets since variance reduction is even more evident, but the market under analysis may still influence this results.

However, there are superior gains including heteroskedasticity and time-
varying variances in the calculation of hedge ratios. As such, multivariate GARCH models are useful in reducing the variance portfolio.

Lee et al. (1987) found that if the near-month contract is used as the hedging instrument, the hedge ratio will be higher than it would be if a contract with a longer maturity is used for this purpose. Another issue is the relation between the frequency of the data used to estimate the hedge ratio and the hedging horizon, particularly the question of whether or not the data frequency should be equal to the hedging horizon. Milliaris and Urrutia (1991) used weekly data to estimate the optimal hedge ratio and found hedging to be more effective when the hedging horizon was equal to the frequency of the data. Also by using weekly data, Benet (1992) found that shorter hedging horizons produced more effective hedging. And we were able to confirm this for the EEX market. However, Chen et al. (2003) stress the potential problem of matching the length of the hedging horizon with data frequency, which leads to the loss of data observations. A going on research will be focusing on this type of issues, that were not explored in detail in the present paper.

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References


