Measuring operational risk in financial institutions: Contribution of credit risk modelling

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ABSTRACT

The scarcity of internal loss databases tends to hinder the use of the advanced approaches for operational risk measurement (AMA) in financial institutions. As there is a greater variety in credit risk modelling, this paper explores the applicability of a modified version of CreditRisk+ to operational loss data. Our adapted model, OpRisk+, works out very satisfying Values-at-Risk at 95% level as compared with estimates drawn from sophisticated AMA models. OpRisk+ proves to be especially worthy in the case of small samples, where more complex methods cannot be applied. OpRisk+ could therefore be used to fit the body of the distribution of operational losses up to the 95%-percentile, while Extreme Value Theory or external databases should be used beyond this quantile.

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1. Introduction

Over the past decade, financial institutions have experienced several large operational loss events leading to big banking failures. Memorable examples include the Barings' bankruptcy in 1995, the \$691 million trading loss at Allfirst Financial, or the \$140 million loss at the Bank of New York due to September 11th, 2001. These events, as well as developments such as the growth of e-commerce, changes in banks' risks management or the use of more highly automated technology, have led regulators and the banking industry to recognize the importance of operational risk in shaping the risk profiles of financial institutions.

Reflecting this recognition, the Basel Committee on Banking Supervision, in its proposal for A New Capital Accord, has incorporated into its proposed capital framework an explicit capital requirement for operational risk, defined as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but not strategic and reputational risks. As for credit risk, the Basel Committee does not believe in a "one-size-fits-all" approach to capital adequacy and proposes three distinct options for the calculation of the capital charge for operational risk: the basic indicator approach, the standardized approach and the advanced measurement approaches (AMA). The use of these approaches of increasing risk sensitivity is determined according to the risk management systems of the banks. The first two methods are a function of gross income, while the advanced methods are based on internal loss data, external loss data, scenario analysis, business environment and internal control factors.

In 2001, the Basel Committee was encouraging two specific AMA methods: (i) the Loss Distribution Approach (LDA) and (ii) an Internal Measurement Approach (IMA) developing a linear relationship between unexpected loss and expected loss to extrapolate credit-risk's internal rating based (IRB) approach to operational risk. However, prior to its 2003 Accord, the Basel Committee dropped formal mention of the IMA leaving the LDA as the only specifically recommended AMA method.

Even though the Basel Accord formally dropped Internal Measurement Approaches in favour of Value-at-Risk approaches, it is still legitimate to be inspired by modelling approaches for credit risk in order to model the distribution of operational loss data. Indeed, both definitions of risks have similar features, such as their focus on a one-year measurement horizon or their use of an aggregated loss distribution skewed towards zero with a long right-tail.

This paper explores the possibility of adapting one of the current proposed industry credit-risk models to perform much of the functionality of an actuarial LDA model. We identified CreditRisk+, the model developed by Credit Suisse, as an actuarial-based model whose characteristics can be adapted to fit the Loss Distribution Approach, which is explicitly mentioned in the Basel II Accord as eligible among the Advanced Measurement Approaches (AMA) to estimate risk capital, and has unambiguously emerged as the standard industry practice (see Sahay et al. (2007) or Degen et al. (2007)) . After some adjustment, we construct a distribution of operational losses through the adapted "OpRisk+" model. As this model calibrates the whole distribution, not only can we retrieve the quantiles of the operational loss distribution, but also an estimate of its expectation, needed for the computation of the economic capital.

Our research is aimed at answering the following question: how would the adaptation of CreditRisk+ model perform compared to sophisticated models such as the approach developed by Chapelle, Crama, Hübner and Peters (2008) (henceforth CCHP) or Moscadelli (2004) among others or compared to an extended IMA approach such as Alexander (2003)?

We address the question with an experiment based on generated databases using three different Pareto distributions. We study the behaviour of OpRisk+, together with an alternative characterization that specifically aims at modelling operational losses, when all these models are confronted to either fat, medium or thin tails for the loss distributions. Furthermore, we assess the influence of the number of losses recorded in the database on the quality of the estimation. The knowledge of the true distribution of losses is necessary to assess the quality of the different fitting methods. Had a real data set been used instead of controlled numerical simulations, we would not be able to benchmark the observed results against the true loss distribution and therefore we could not assess the performance of OpRisk+ for different loss generating processes and sample sizes.

We also test our new adapted IRB model against Alexander's existing "lower-bound" improvement to the basic IMA formula to see if our model outperforms. The lower bound is effectively a quantile value from a normal distribution table which allows identification of the unexpected loss if you know the mean and variance of the loss severity distribution and the mean of the frequency distribution. The multiplier is called the "lower-bound" because it will have its lowest value (approximately equal to 3.1) when the mean of the frequency distribution corresponds to a high-frequency type of loss (e.g. more than 100 losses per year).

Our main findings are twofold. First, we note that the precision of OpRisk+ is not satisfactory to estimate the very far end of the loss distribution, such as the Value-at-Risk (VaR)[1] at the 99.9% confidence level. Yet, our model works out very satisfying quantile estimates, especially for thin-tailed Pareto-distributions, up to a 95% confidence level for the computation of the VaR. The estimation error is relatively small and stable across distributions. Secondly, the simplicity of our model makes it applicable to "problematic" business lines, that is, with very few occurrences of events, and with very few years of data. Procedures that rely on extreme-value theory, by contrast, are very data-consuming, and yield very poor results when used with small databases.

These findings make the OpRisk+ approach clearly not an effective substitute, but indeed a very useful complement to approaches that specifically target the extreme tail of the loss distribution. In particular, the body of the loss distribution can be safely assessed with our method, while external data, as specifically mentioned in the Accord, can be used to estimate the tail. Being able to simultaneously rely on the body and the tail of the distribution is crucial for the operation risk capital estimation, because one needs the full distribution of losses in order to capture the expected loss that enters the regulatory capital estimate.

The paper is organized as follows: Section 2 describes the adjustment needed in order to apply CreditRisk+ model to operational loss data and presents two alternative methods to calibrate a Value-at-Risk on operational loss data (OpVaR). Section 3 describes our database, presents our results and compares them to the other approaches' results. Section 4 concludes.

2. Alternative Approaches for the Measurement of Operational Risk

This section first presents three alternative ways to calibrate a Value-at-Risk on operational loss data. The first one represents an adaptation of the CreditRisk+ framework, while the second one proposes an adaptation of the Loss Distribution Approach (LDA) in the context of operational losses with the use of Extreme Value Theory (EVT). Finally, we introduce a lower bound for the Value-at-Risk derived from a model developed by Alexander (2003).

2.1 OpRisk+: Application of CreditRisk+ to Operational Loss Data

CreditRisk+ developed by Credit Suisse First Boston is an actuarial model derived from insurance losses models. It models the default risk of a bond portfolio through the Poisson distribution. Its basic building block is simply the probability of default of a counterparty. In this model, no assumptions are made about the causes of default: an obligor is either in default with a probability P_A, or not in default with a probability $1-P_A$. Although operational losses do not depend on a particular counterparty, this characteristic already simplifies the adaptation of our model, as we do not need to make assumptions on the causes of the loss.

CreditRisk+ determines the distribution of default losses in three steps: the determination of the frequency of defaults, approximated by a standard Poisson distribution, the determination of the severity of the losses and the determination of the distribution of default losses.

The determination of the frequency of events leading to operational losses can be modelled through the Poisson distribution as for the probability of default in CreditRisk+:

$$P(N = n) = \frac{\mu^n e^{-\mu}}{n!} \text{ for } n = 0,1,2,\ldots \qquad (1)$$

where μ is the average number of defaults per period, and N is a stochastic variable with mean μ, and standard deviation $\sqrt{\mu}$.

CreditRisk+ computes the parameter μ by adding the probability of default of each obligor, supplied, for instance, by rating agencies. However, operational losses do not depend on a particular obligor. Therefore, instead of being defined as a sum of probabilities of default depending on the characteristics of a counterpart, μ can be interpreted as the average number of loss events of one type occurring in a specific business line during one period.

CreditRisk+ adds the assumption that the mean default rate is itself stochastic in order to take into account the fat right tail of the distribution of defaults. Nevertheless, CCHP (2008) show that, as far as operational risk is concerned, using the Poisson distribution to model the frequency of operational losses does not lead to a substantial measurement error. Hence, we keep on assuming that the number of operational loss events follows a Poisson distribution with a fixed mean μ.

In order to perform its calculations, CreditRisk+ proposes to express the exposure (here, the losses) in a unit amount of exposure L2. The key step is then to round up each exposure size to the nearest whole number, in order to reduce the number of possible values and to distribute them into different bands. Each band is characterized by an average exposure, νj and an expected loss, εj, equal to the sum of the expected losses of all the obligors belonging to the band. Table 1 shows an example of this procedure.

[TABLE 1]

CreditRisk+ posits that

$$\varepsilon_j = \nu_j \mu_j \qquad (2)$$

where ε_j is the expected loss in band j, ν_j is the common exposure in band j, and μ_j is the expected number of defaults in band j.

As the operational losses do not depend on a particular transaction, we slightly modify the definition of these variables. The aim is to calculate the expected aggregate loss. We will therefore keep the definition of εj unchanged. However, as noted earlier, μ_j is not an aggregate expected number of defaults anymore but simply the (observed) average number of operational loss events occurring in one year. Consequently, in order to satisfy equation (2), v_j must be defined as the average loss amount per event for band j. The following table illustrates the reprocessing of the data:

[TABLE 2]

Each band is viewed as a portfolio of exposures by itself. Because some defaults lead to larger losses than others through the variation in exposure amounts, the loss given default involves a second element of randomness, which is mathematically described through its probability generating function. Thus, let G(z) be the probability generating function for losses expressed in multiples of the unit L of exposure:

$$G_j(z) = \sum_{n=0}^{\infty} P(\text{loss} = nL) z^n = \sum_{n=0}^{\infty} P(n \text{ defaults}) z^{n v_j} \tag{3}$$

As the number of defaults follows a Poisson distribution, this is equal to:

$$G_j(z) = \sum_{n=0}^{\infty} \frac{e^{-\mu_j} \mu_j^n}{n!} z^{n v_j} = e^{-\mu_j + \mu_j z^{v_j}} \tag{4}$$

As far as operational losses are concerned, a band can no more be considered as a portfolio but will simply be seen as a category of loss size. This also simplifies the model, as we do not distinguish exposure and expected loss anymore. For credit losses, exposures are first sorted, and then the expected loss is calculated, by multiplying the exposures by their probability of default. As far as operational losses are concerned, the loss amounts are directly sorted by size. Consequently, the second element of randomness is not necessary anymore. This has no consequences on the following results except simplifying the model.

Whereas CreditRisk+ assumed the exposures in the portfolio to be independent, OpRisk+ will assume the independence of the different loss amounts. Thanks to this assumption, the probability generating function for losses of one type for a specific business line is given by the product of the probability generating function for each band:

$$G(z) = \prod_{j=1}^{m} e^{-\mu_j + \mu_j z^{\nu_j}} = e^{-\sum_{j=1}^{m} \mu_j + \sum_{j=1}^{m} \mu_j z^{\nu_j}} \tag{5}$$

Finally, the loss distribution of the entire portfolio is given by:

$$P(\text{loss of } nL) = \frac{1}{n!} \left. \frac{d^n G(z)}{dz^n} \right|_{z=0} \quad \text{for } n = 1,2,\ldots \tag{6}$$

Note that this equation allows only computing the probability of losses and size 0, L, 2L and so on. This probability of loss of *nL* will further be denoted A_n.

Then, under the simplified assumption of fixed default rates, Credit Suisse has developed the following recursive equation[3]:

$$A_n = \sum_{j:\nu_j \le n} \frac{\varepsilon_j}{n} A_{n-\nu_j} \tag{7}$$

where $A_0 = G(0) = e^{-\mu} = e^{-\sum_{j=1}^{m} \frac{\varepsilon_j}{\nu_j}}$.

The calculation depends only on 2 sets of parameters: ν_j and ε_j, derived from μ_j, the number of events of each range, *j*, observed. With operational data, A_0 is derived directly from $A_0 = e^{-\mu}$.

To illustrate this recurrence, suppose your database contains 20 losses, 3 (resp. 2) of which having a size of 1L (resp. 2L):

$$A_0 = e^{-20} = 2.06.10^{-9}$$

$$A_1 = \sum_{j:\nu_j \le 1} \frac{\varepsilon_j}{1} A_{1-\nu_j} = \varepsilon_1 A_0 = 3 \text{ x } 2.06.10^{-9} = 6.18.10^{-9}$$

$$A_2 = \sum_{j:\nu_j \le 2} \frac{\varepsilon_j}{2} A_{2-\nu_j} = \frac{1}{2}\left(\varepsilon_1 A_1 + \varepsilon_2 A_0\right) = \frac{1}{2}\left(3 \text{ x } 6.18.10^{-9} + 2 \text{ x } 2.06.10^{-9}\right) = 1.13.10^{-8}$$

Therefore, the probability of having a loss of size resp. 0, 1L and 2L is resp. $2.06.10^{-9}$, $6.18.10^{-9}$ and $1.13.10^{-8}$, and so on.

From there, you can re-construct the distribution of the loss of size *nL*.

The procedure can be summarized as follows:

1. Choose a unit amount of loss *L*[4].
2. Divide the losses of the available database by *L* and round up these numbers.

3. Allocate the losses of different sizes to their band and compute the expected loss per band, equal to the observed number of losses per band multiplied by the average loss amount per band, equal to *j*.
4. Compute the probability of zero losses equal to $A_0 = e^{-\mu}$, where μ is the total number of losses observed per period.
5. For each band j = 1 to n, compute the probability of losses of size n by using equation (7).
6. Cumulate the probabilities in order to find the $OpVaR_{99.9}$.
7. Repeat the procedure for each year of data.

2.2 The Loss Distribution Approach adapted to Operational Risk

Among the Advanced Measurement Approaches (AMA) developed over the recent years to model operational risk, the most common one is the Loss Distribution Approach (LDA), which is derived from actuarial techniques (see Frachot, Georges and Roncalli, 2001 for an introduction).

By means of convolution, this technique derives the aggregated loss distribution (ALD) through the combination of the frequency distribution of loss events and the severity distribution of a loss given event[5]. The operational Value-at-Risk is then simply the 99.9th percentile of the ALD. As an analytical solution is very difficult to compute with this type of convolution, Monte Carlo simulations are usually used to do the job. Using the CCHP procedure with a Poisson distribution with a parameter μ equal to the number of observed losses during the whole period to model the frequency[6], we have:

1. Generate a large number M of Poisson(μ) random variables (say, 10000). These M values represent the number of events for each of the M simulated periods.
2. For each period, generate the required number of severity random variables (that is, if the simulated number of events for period m is x, then simulate x severity losses) and add them to get the aggregated loss for the period.
3. The obtained vector represents M simulated periods. If M = 10000, when sorted, the smallest value thus represents the 0.0001 quantile, the second the 0.0002 quantile, etc., which makes the OpVaRs very easy to calculate.

Examples of empirical studies using this technique for operational risk include Moscadelli (2004) on loss data collected from the Quantitative Impact Study (QIS) of the Basel Committee, de Fontnouvelle and Rosengren (2004) on loss data from the 2002 Risk Loss Data Collection Exercise initiated by the Risk Management Group of the Basel Committee or CCHP with loss data coming from a large European bank.

The latter underline that, in front of an actual database of internal operational losses, mixing two distributions fit more adequately the empirical severity distribution than a single distribution. Therefore, they divide the sample into two parts: a first one with losses below a selected threshold, considered as the "normal" losses, and a second one, including the "large" losses. To model the "normal" losses, CCHP compare several classic continuous distributions such as gamma, lognormal or Pareto.

To take extreme and very rare losses into account (i.e. the "large" losses), the authors apply the Extreme Value Theory (EVT) on their results[7]. The advantage of EVT is that it provides a tool to estimate rare and not-yet-recorded events for a given database[8]. The use of EVT works out very similar OpVaRs, except for the very high percentile, that is the 99.99th. As the Basel Accord requires a confidence level of 99.90, the authors obtain that their Monte Carlo simulation allows them to compute a sufficiently complete sample, including most of the extreme cases.

2.3 Comparison with a lower bound

The basic formula of the Internal Measurement Approach (IMA) included in the Advanced Measurement Approaches of Basel II is:

$$UL = \gamma EL \tag{8}$$

where UL = unexpected loss, determining the operational risk requirement[9], and γ is a multiplier.

Gamma factors are not easy to evaluate as no indication of their possible range has been given by the Basel Committee. Therefore, Alexander (2003) suggests that instead of writing the unexpected loss as a multiple (γ) of expected loss, one writes unexpected loss as a multiple (Φ) of the loss standard deviation. Using the definition of the expected loss, she gets the expression for Φ:

$$\Phi = \frac{VaR_{99.9} - EL}{\sigma} \tag{9}$$

The advantages of this parameter are that it can be easily calibrated and that it has a lower bound.

The basic IMA formula is based on the binomial loss frequency distribution, with no variability in loss severity. For very high-frequency risks, Alexander notes that the normal distribution could be used as an approximation of the binomial loss distribution, providing for Φ a lower bound equal to 3.1 (as can be found from standard normal tables when the number of losses goes to infinity). She also suggests that the Poisson distribution should be preferred to the binomial as the number of transactions is generally difficult to quantify.

Alexander (2003) also shows that Φ, as a function of the parameter μ of the Poisson distribution, must be in a fairly narrow range: from about 3.2 for medium-to high frequency risks (20 to 100 loss events per year) to about 3.9 for low frequency risks (one loss event every one or two years) and only above 4 for very rare events that may happen only once every five years or so. Table 3 illustrates the wide range for the gammas by opposition to the narrow range of the phi's values.

[TABLE 3]

Then, assuming the loss severity to be random, i.e. with mean μ_L and standard deviation σ_L, and independent of the loss frequency, Alexander writes the Φ parameter as:

$$\Phi = \frac{VaR_{999} - \lambda\mu_L}{\sqrt{\left[\lambda\left(\mu_L^2 + \sigma_L^2\right)\right]}} \tag{10}$$

Where λ is the average number of losses.

For $\sigma_L > 0$, this formula produces slightly lower Φ than with no severity uncertainty, but it is still bounded below by the value 3.1.

For a comparison purpose, we will use the following value for the lower bound of the needed $OpVaR_{99.9}$, derived from equation (10) in which we replaced Φ by 3.1 (asymptotic value for the ratio of the difference between the 99th quantile and the parameter of the Poisson distribution to its standard deviation) :

$$LB = 3.1\sqrt{\left[\lambda\left(\mu_L^2 + \sigma_L^2\right)\right]} + \lambda\mu_L \tag{11}$$

Our model should therefore produce operational economic capital at least equal or higher than this lower bound.

3. An Experiment on Simulated Losses

3.1 Data

OpRisk+ makes the traditional statistical tests impossible, as it uses no parametrical form but a purely numerical procedure. Therefore, in order to perform tests of the calibrating performance of OpRisk+ on any distribution of loss severity, we simulate databases to obtain an exhaustive picture of the capabilities of the approach, that is, on the basis of three different kinds of distributions: a heavy-tail, a medium-tail and a thin-tail Pareto distribution. Indeed, Moscadelli (2004) and de Fontnouvelle and

Rosengren (2004) have shown that loss data for most business lines and event types may be well modelled by a Pareto-type distribution.

A Pareto distribution is a right-skewed distribution parameterized by two quantities: a minimum possible value or location parameter, x_m, and a tail index or shape parameter, ξ. Therefore, if X is a random variable with a Pareto distribution, the probability that X is greater than some number x is given by:

$$\Pr(X > x) = \left(\frac{x}{x_m}\right)^{-k}$$

for all $x \geq x_m$., and for x_m and $k = 1/\xi > 0$.

The parameters of our distributions are Pareto(100;0.3), Pareto(100;0.5) and Pareto(100;0.7): the larger the value of the tail index, the fatter the tail of the distribution. The choice of these functions has been found to be reasonable with a sample of real data obtained from a large European institution.

We ran three simulations: one for the thin-tailed Pareto severity distribution case, one for the medium-tailed Pareto severity distribution case and one for the fat-tailed Pareto severity distribution case. For each of these cases, we simulated two sets of 1000 years of 20 and respectively 50 operational losses and two sets of 100[10] series of 200 losses and 300, respectively. For each of the 6600 simulated years (3 x 2 x 1100), the aggregated loss distribution has been computed with the algorithm described in Section 2.2.

Table 4 gives the characteristics of each of the twelve databases (each thus comprising 1000 or 100 simulated aggregate loss distributions) constructed in order to implement OpRisk+. For each series of operational losses we computed the expected loss, that is, the mean loss multiplied by the number of losses. The five last lines present the mean, standard deviation, median, maximum and minimum of these expected losses.

[TABLE 4]

These results clearly show that data generated with a thin-tailed Pareto-distribution exhibit characteristics that make the samples quite reliable. The mean loss is very close to its theoretical level even for 20 draws. Furthermore, we observe a standard deviation of aggregate loss that is very limited, from less than 10% of the average for N=20 to less than 3% for N=200. The median loss is also close to the theoretical value. For a tail index of 0.5 (medium-tailed), the mean loss still stays close to the theoretical value but the standard deviation increases. Thus, we can start to question the stability of the loss estimate.

When the tail index increases, all these nice properties collapse. The mean aggregate loss becomes systematically lower than the theoretical mean, and this effect aggravates when one takes a lower number of simulations (100 drawings) with a larger sample. The standard deviation and range become extremely large, making inference based on a given set of loss observations extremely adventurous.

3.2. Application of OpRisk+

To apply OpRisk+ to these data, the first step consists of computing $A_0 = e^{-\mu}$, where μ is the average number of loss events. For instance, for N=200, this gives the following value: $A_0 = e^{-200} = 1.38 \cdot 10^{-87}$. Then, in order to assess the loss distribution of the entire population of operational risk events, we use the recursive equation (7) to compute A_1, A_2 etc.

Once the different probabilities A_n for the different sizes of losses are computed, we can plot the aggregated loss distribution as illustrated in Figure 1.

[FIGURE 1]

With this information, we can compute the different Operational Values-at-Risk (OpVaR). This is done by calculating the cumulated probabilities for each amount of loss. The loss for which the cumulated probability is equal to *p*% gives us the OpVaR at percentile *p*.

The average values for the different OpVaRs are given in Tables 4 and 5. Table 4 compares the OpVaRs obtained using OpRisk+ with the simulated data for the small databases. The first column represent the average observed quantiles of the aggregated distribution when simulating 25000 years with a Poisson(mu) distribution for the frequency and a Pareto(100, ξ.) for the severity. The tables also gives the minimum, maximum and standard deviation of the 100(0) OpVaRs produced by OpRisk+.

Panel A of Table 5 shows that OpRisk+ achieves very satisfactory OpVaRs for the Pareto-distribution with thin tail. The mean OpVaRs obtained for both the samples of 20 and 50 observations stays within a 4% distance from the true value. Even at the level of 99.9% required by Basel II, the OpRisk+

values stay within a very narrow range, while the standard deviation of the estimates is kept within 10% around the good value.

[TABLE 5]

The results obtained with the OpRisk+ procedure with medium and fat tails tend to deteriorate, which is actually not surprising as the adaptation of the credit risk model strictly uses observed data and does necessarily underestimate the fatness of the tails. However, we still have very good estimation for $OpVaR_{95.}$ It mismatches the true 95% quantile by 2% to 7% for the medium and fat tailed Pareto-distribution, while the standard deviation tends – naturally – to increase very fast.

The bad news is that the procedure alone is not sufficient to provide the $OpVaR_{99.9}$ required by Basel II. It severely underestimates the true quantile, even though this true value is included in the range of the observed values of the loss estimates.

Table 6 displays the results of the simulations when a large sample size is used.

[TABLE 6]

Table 6, Panel A already delivers some rather surprising results. The OpRisk+ procedure seems to overestimates the true operational risk exposure for all confidence levels; this effect aggravates for a high number of losses in the database. This phenomenon is probably due to an intervalling effect, where losses belonging to a given band are given the value of the band. Given that extreme losses are likely to occur in the lower part of the band, as the distribution is characterized by a thin tail Pareto-distribution, taking the upper bound limit value for aggregation seems to deteriorate the estimation, making it too conservative. Nevertheless, the bias is almost constant in relative terms, indicating that its seriousness does not aggravate as the estimation gets far in the tail of the distribution. Sub-section 5.4. will go further in this assumption.

This intervalling phenomenon explains the behaviour of the estimation for larger values of the tail index. In Panel B, the adapted credit risk model still overestimates the distribution of losses up to a confidence level of 99%, but then does not capture to distribution at the extreme end of the tail

(99.9%). In Panel C, the underestimation starts earlier, around the 90% percentile of the distribution. The procedure, using only observed data, is still totally unable to capture the fat tailedness of the distribution of aggregated losses.

Nevertheless, from panels B and C altogether, the performance of OpRisk+ still stays honourable when the confidence level of 95% is adopted. The standard deviation of the estimates also remains within 15% (with the tail index of 0.5) and 30% of the mean (with a tail index of 0.7), which is fairly large but mostly driven by large outliers as witnessed in the last column of each panel.

A correct mean estimate of the $OpVaR_{95}$ would apply to a tail index between 0.5 and 0.7, which corresponds to a distribution with a fairly large tail index. Only when the tail of the Pareto-distribution is actually thin, one observes that the intervalling effect induces a large discrepancy between the theoretical and observed values.

It remains to be mentioned that the good application of OpRisk+ does not depend on the number of observed losses as it only affects the first term of the recurrence, namely A_0.

3.3. Comparison with the CCHP approach and the Lower-Bound of Alexander.

These results, if their economic and statistical significance have to be assessed, have to be compared with a method that aims at specifically addressing the issue of operational losses in the Advanced Measurement Approaches setup. We choose the CCHP approach, which is by definition more sensitive to extreme events than OpRisk+, but has the drawback of requiring a large number of events to properly derive the severity distributions of “normal” and “large” losses.

The graphs from Figure 2 display the OpVaRs (with confidence levels of 90, 95, 99 and 99.9%) generated from three different kind of approaches, that is the sophisticated CCHP approach, OpRisk+ and the simpler Alexander (2003) approach (See Section 4.3) for three small databases of 20 and of 50 loss events, and for three larger databases of 200 and of 300 events.

[FIGURE 2]

From the graphs in Figure 2, we can see that for most databases, OpRisk+ is working out a capital requirement higher than the lower bound, but smaller than the CCHP approach. This last result could be expected as CCHP is more sensitive to extreme events. We will discuss the fact that the database with 300 observations shows higher OpVaRs for OpRisk+ than CCHP in the last sub-section. However, we can already conclude that our model is more risk sensitive than a simple IMA approach.

Considering the thin tailed Pareto-distribution in Panel A, we can observe that OpRisk+ produces the best estimations for the small database. Indeed, those are very close to the theoretical OpVaRs for all confidence level. However, for the large database, it is producing too cautious (large) OpVaRs. The comparison with other methods sheds new light on the results obtained with Panel A of Table 6: OpRisk+ overestimates the true VaR, but the CCHP model, especially dedicated to the measurement of operational risk, does even worse. Actually, Alexander's (2003) lower bound, also using observed data but not suffering from an intervalling effect, works out very satisfactory results when the standard deviation of loss is a good proxy of the variability of the distribution.

For the medium and fat tailed Pareto-distributions, neither of the models is sensitive enough for OpVaRs of 99% and more. However, as far as the small databases are concerned, it is interesting to note that OpRisk+ is producing the best estimations for $OpVaR_{95}$.

Figure 3 compares the different values for the $OpVaR_{90}$, $OpVaR_{95}$, $OpVaR_{99}$ and $OpVaR_{99.9,}$ for one of the small samples and one of the large samples. It shows that the CCHP model produces a heavier tail than OpRisk+ does, except for the Pareto(200;0,7), where they are quite similar. However, OpRisk+ yields OpVaRs that are much closer to the theoretical ones as far as small databases are concerned. It even consistently produces very good OpVaR95 in all cases. However, at the level of confidence required by Basel II, this procedure leaves estimates that are still far from the expected value, especially when the tail of the Pareto-distribution gets bigger.

[FIGURE 3]

3.4. Comparison with OpRisk+ taking an average value of loss for each band.

As shown above, taking the upper bound limit value for aggregation as described in the CreditRisk+ model tends to overestimate the true operational risk exposure for all confidence levels; especially with larger databases. A solution could be to take the average value of losses for each band[11]. Table 7 displays the results of the simulations when a relatively large sample size is used.

Panel A of Table 7 shows that OpRisk+ achieves very good results for the Pareto-distribution characterized by a thin tail when using an average value for each band ("round" column). The OpVaR values obtained for the sample of 200 observations is very close from the theoretical value, whereas it stays within a 6% range from the "true" value with a 300 observations sample, including at the Basel II level of 99.9%.

When the loss Pareto-distributions are medium-tailed, the results obtained with the OpRisk+ procedure with the databases are very good for quantiles up to 95% but deteriorate for more sensitive OpVaRs. OpRisk+ is still totally unable to capture the tailedness of the distribution of aggregated losses for very high confidence interval, such as the Basel II requirement.

[TABLE 7]

Table 8 compares the two methods when applied to small databases of 20 and 50 observations. In such cases, OpRisk+ provides better results with the “round up” solution than with the “round” one. This bias could be due to the fact that with the second method we tend to loose the “extreme value theory” aspect of the model. Small databases tend indeed to lack extreme losses and taking the upper bound limit value for the aggregation makes the resulting distribution’s tail fatter.

[TABLE 8]

4. Conclusions

This paper introduces a structural operational risk model, named OpRisk+, that has been inspired from the well known credit risk model, CreditRisk+, which had characteristics transposable to the operational risk modelling.

In a simulations setup, we work out aggregated loss distributions and operational Value-at-Risks (OpVaR) corresponding to the confidence level required by Basel II. The performance of our model is assessed by comparing our results to theoretical OpVaRs, to a lower bound issued from a simpler approach, that is, the IMA approach of Alexander (2003), and to a more sophisticated approach using two distributions in order to model the severity distributions of “normal” and “extreme” losses proposed in Chapelle *et al.* (2008), or “CCHP” approach.

The results show that OpRisk+ produces higher OpVaRs than the lower bound of Alexander (2003), but that it is not receptive enough to extreme events. On the other hand, our goal is not to produce an adequate economic capital, but to try to propose a first solution to the lack of operational risk models. Besides, whereas the CCHP approach has better sensitivity to very extreme losses, the simplicity of OpRisk+ gives the model the advantage of requiring no large database in order to be implemented.

Specifically, we view the value-added of the OpRisk+ procedure as twofold. Firstly, it produces average estimates of operational risk exposures that are very satisfactory at the 95% level, which makes it a very useful complement to approaches that specifically target the extreme tail of the loss distribution. Indeed, even though the performance of OpRisk+ is clearly not sufficient for the measurement of unexpected operational losses as defined by the Basel II Accord (the VaR should be measured with a 99.9% confidence level), it could be thought of as a sound basis for the measurement of the body of losses; another more appropriate method must relay OpRisk+ for the measurement of the far end of the distribution.

Secondly, despite the fact that we can not conclude that OpRisk+ is an adequate model to quantify the economic capital associated to the bank's operational risk in the LDA approach, its applicability to approximate the loss distribution with small databases is proven. Even for such a small database as one comprising 20 observations, the estimation could make it attractive as a complement to more sophisticated approaches requiring large numbers of data per period. The fit is almost perfect when the Pareto-distribution has a thin tail, and the $OpVaR_{95}$ is the closest among the three specifications tested when the tail gets fatter.

Of course, this approach is still subject to refinements, and could be improved in many ways. Indeed, internal data rarely includes very extreme events (banks suffering those losses probably would no more be there to tell us), whereas the last percentiles are very sensitive to the presence of those events. The problem would therefore be to determine which weight to place on the internal data and on the external ones. From our study, we could imagine that fitting a distribution calibrated with external data or relying on EVT beyond the 95% percentile would justify the simultaneous use of OpRisk+ preferably to other models. This advantage can prove to be crucial for business lines or event types where very few internal observations are available, and thus where most approaches such as the CCHP would be powerless.

5. Notes

[1] The Value-at-Risk (VaR) is the amount that losses will likely not exceed, within a predefined confidence level and over a given time-period.

[2] CreditRisk+'s authors argue that the exact amount of each loss cannot be critical in the determination of the global risk.

[3] See Appendix.

[4] Several unit amounts were tested and did not lead to significantly different results. The size of the unit amount should especially be chosen according to the number of losses: the bigger the number of losses in the database, the bigger the unit amount, in order to lighten the computations.

[5] More precisely the ALD is obtained through the n-fold convolution of the severity distribution with itself, n being a random variable following the frequency density function.

[6] While frequency could also be modelled with other discrete distributions such as the Negative Binomial for instance, many authors use the Poisson assumption (see de Fontnouvelle, DeJesus-Rueff, Jordan and Rosengren, 2003, for instance).

[7] This solution has been advocated by many other authors; see for instance King (2001), Cruz (2004), Moscadelli (2004), de Fontnouvelle and Rosengren (2004) or Chavez-Demoulin, Embrechts and Neslehova (2006).

[8] See Embrechts, Klüppelberg and Mikosch (1997) for a comprehensive overview of EVT.

[9] The unexpected loss is defined as the difference between the VaR at 99.9% and the expected loss.

[10] Only 100 years of data were simulated for high-frequency databases as the computation becomes too heavy for a too large number of data.

[11] That is, every loss between 15000 and 25000 would be in band 20, instead of every loss between 10,000 and 20,000 being in band 20.

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7. Tables

Loss Amount (LGE)	Loss in L	round-off loss v_j	band j
1 500	1.5	2.00	2
2 508	2.51	3.00	3
3 639	3.64	4.00	4
1 000	1.00	1.00	1
1 835	1.84	2.00	2
2 446	2.45	3.00	3
7 260	7.26	8.00	8

Table 1 - Allocating losses to bands.

v_j	μ_j	ε_j
1	9	9
2	121	242
3	78	234
4	27	108
5	17	85
6	15	90
7	8	56
8	4	32
⋮	⋮	⋮

Table 2 - Exposure, number of events and expected loss.

μ	*100*	*50*	*40*	*30*	*20*	*10*	*8*	*6*
$VaR_{99.9}$	131.81	72.75	60.45	47.81	34.71	20.66	17.63	14.45
Φ	**3.18**	**3.22**	**3.23**	**3.25**	**3.29**	**3.37**	**3.41**	**3.45**
γ	0.32	0.46	0.51	0.59	0.74	1.07	1.21	1.41
μ	*5*	*4*	*3*	*2*	*1*	*0.9*	*0.8*	*0.7*
$VaR_{99.9}$	12.77	10.96	9.13	7.11	4.87	4.55	4.23	3.91
Φ	**3.48**	**3.48**	**3.54**	**3.62**	**3.87**	**3.85**	**3.84**	**3.84**
γ	1.55	1.74	2.04	2.56	3.87	4.06	4.29	4.59
μ	*0.6*	*0.5*	*0.4*	*0.3*	*0.2*	*0.1*	*0.05*	*0.01*
$VaR_{99.9}$	3.58	3.26	2.91	2.49	2.07	1.42	1.07	0.90
Φ	**3.85**	**3.90**	**3.97**	**4.00**	**4.19**	**4.17**	**4.54**	**8.94**
γ	4.97	5.51	6.27	7.30	9.36	13.21	20.31	89.40

Table 3 - Gamma and phi values (no loss severity variability) (source: Alexander (2003), p151).

Panel A : Thin-tailed-Pareto distribution (shape parameter = 0.3)				
	Number of losses.			
Poisson parameter μ	20	50	200	300
Theoretical Mean	2,857	7,143	28,571	42,857
Mean	2,845	7,134	28,381	42,886
Standard deviation	287	472	847	1,118
Median	2,796	7,078	28,172	42,763
Maximum	4,683	9,026	30,766	45,582
Minimum	2,268	6,071	26,713	40,383
Number of simulated years	1000	1000	100	100

Panel B : Medium-tailed-Pareto distribution (shape parameter = 0.5)				
	Number of losses			
Poisson parameter μ	20	50	200	300
Theoretical Mean	4,000	10,000	40,000	60,000
Mean	3,924	9,913	39,871	59,431
Standard deviation	1,093	1,827	3,585	5,504
Median	3,676	9,594	39,777	57,947
Maximum	15,680	29,029	54,242	91,182
Minimum	2,567	7,097	33,428	52,436
Number of simulated years	1000	1000	100	100

Panel C : Fat-tailed-Pareto distribution (shape parameter = 0.7)				
	Number of losses.			
Poisson parameter μ	20	50	200	300
Theoretical Mean	6,667	16,667	66,667	100,000
Mean	6,264	16,165	61,711	93,724
Standard deviation	5,940	13,018	13,899	24,514
Median	5,180	13,721	57,713	87,646
Maximum	157,134	265,621	137,699	248,526
Minimum	2,646	8,304	45,315	69,991
Number of simulated years	1000	1000	100	100

Table 4 - Characteristics of Twelve Databases under OpRisk+.

Panel A : Thin-tailed-Pareto distribution (shape parameter = 0.3)

	N = 20						N = 50					
	Simulated	OpRisk+					Simulated	OpRisk+				
		Mean	Δ	S.D.	Min	Max		Mean	Δ	S.D.	Min	Max
$OpVaR_{90}$	3770	3880	3%	448	3080	7580	8573	8882	4%	706	7530	16980
$OpVaR_{95}$	4073	4173	2%	505	3290	8020	9030	9334	3%	769	7880	17010
$OpVaR_{99}$	4712	4744	1%	612	3710	8110	9942	10209	3%	906	8560	17020
$OpVaR_{99,9}$	5596	5410	-3%	717	3780	9800	11141	11250	1%	1241	9340	30010

Panel B : Medium-tailed-Pareto distribution (shape parameter = 0.5)

	N = 20						N = 50					
	Simulated	OpRisk+					Simulated	OpRisk+				
		Mean	Δ	S.D.	Min	Max		Mean	Δ	S.D.	Min	Max
$OpVaR_{90}$	5579	5672	2%	2209	3470	29360	12630	12855	2%	3102	8760	51800
$OpVaR_{95}$	6364	6247	-2%	2896	3720	40860	13862	13734	-1%	3838	9190	70420
$OpVaR_{99}$	8966	7329	-18%	3940	4190	53610	18051	15410	-15%	4900	10020	91180
$OpVaR_{99,9}$	18567	8626	-54%	5120	4750	66930	33554	17338	-48%	6013	10990	112940

Panel C : Fat-tailed-Pareto distribution (shape parameter = 0.7)

	N = 20						N = 50					
	Simulated	OpRisk+					Simulated	OpRisk+				
		Mean	Δ	S.D.	Min	Max		Mean	Δ	S.D.	Min	Max
$OpVaR_{90}$	9700	11410	18%	10214	4300	106700	22495	23992	7%	25901	11700	526900
$OpVaR_{95}$	12640	12931	2%	12441	4600	142450	28103	27089	-4%	37495	12300	777800
$OpVaR_{99}$	27261	15583	-43%	15736	5150	189050	55994	32020	-43%	49854	13450	1033000
$OpVaR_{99,9}$	114563	18726	-84%	19524	5850	236200	220650	38761	-82%	69020	14750	1290300

Table 5 - Values-at-Risk generated by OpRisk for small databases, with 20 and 50 loss events.

Panel A : Thin-tailed-Pareto distribution (shape parameter = 0.3)

	N = 200						N = 300					
	Simulated	OpRisk+					Simulated	OpRisk+				
		Mean	Δ	S.D.	Min	Max		Mean	Δ	S.D.	Min	Max
$OpVaR_{90}$	31448	33853	7%	14819	31780	36640	46355	56470	22%	1240	53950	59800
$OpVaR_{95}$	32309	34728	7%	15435	32600	37780	47403	57683	22%	1305	55100	61200
$OpVaR_{99}$	33995	36397	7%	16677	34180	40340	49420	59992	21%	1431	57200	63950
$OpVaR_{99,9}$	36063	38310	6%	18197	36000	43740	51750	62628	21%	1589	59650	67150

Panel B : Medium-tailed-Pareto distribution (shape parameter = 0.5)

	N = 200						N = 300					
	Simulated	OpRisk+					Simulated	OpRisk+				
		Mean	Δ	S.D.	Min	Max		Mean	Δ	S.D.	Min	Max
$OpVaR_{90}$	45757	51836	13%	5875	43500	79650	67104	75723	13%	9683	66600	146300
$OpVaR_{95}$	48259	53816	12%	6935	44650	89600	70264	78161	11%	11515	68100	164700
$OpVaR_{99}$	55919	57668	3%	8940	46900	105300	79718	82817	4%	14571	70950	193300
$OpVaR_{99,9}$	83292	62237	-25%	11431	49450	123600	113560	88309	-22%	18259	74250	226700

Panel C : Fat-tailed-Pareto distribution (shape parameter = 0.7)

	N = 200						N = 300					
	Simulated	OpRisk+					Simulated	OpRisk+				
		Mean	Δ	S.D.	Min	Max		Mean	Δ	S.D.	Min	Max
$OpVaR_{90}$	82381	82539	0%	24959	57200	218600	120654	119943	-1%	34805	86100	364800
$OpVaR_{95}$	96971	88248	-9%	30247	58950	247000	139470	127037	-9%	42644	88400	436550
$OpVaR_{99}$	166962	98972	-41%	39404	62300	303400	234442	55846	-40%	55846	92850	550800
$OpVaR_{99,9}$	543597	111875	-79%	50432	66150	373100	733862	156642	-79%	71810	98000	687000

Table 6 - OpVaRs generated by OpRisk+ for databases with 200 and 300 loss events.

Panel A : Thin-tailed-Pareto distribution (shape parameter = 0.3)

	N = 200					N = 300				
	Simulated	OpRisk+				Simulated	OpRisk+			
		Roundup	Δ	Round	Δ		Roundup	Δ	Round	Δ
$OpVaR_{90}$	31448	33853	8%	30576	-3%	46355	56470	22%	43558	-6%
$OpVaR_{95}$	32309	34728	7%	31404	-3%	47403	57683	22%	44563	-6%
$OpVaR_{99}$	33995	36397	7%	32991	-3%	49420	59992	21%	46486	-6%
$OpVaR_{99,9}$	36063	38310	6%	34813	-3%	51750	62628	21%	48687	-6%

Panel B : Medium-tailed-Pareto distribution (shape parameter = 0.5)

	N = 200					N = 300				
	Simulated	OpRisk+				Simulated	OpRisk+			
		Roundup	Δ	Round	Δ		Roundup	Δ	Round	Δ
$OpVaR_{90}$	45757	51836	13%	44338	-3%	67104	75723	13%	64523	-4%
$OpVaR_{95}$	48259	53816	12%	46222	-4%	70264	78161	11%	66849	-5%
$OpVaR_{99}$	55919	57668	3%	49885	-11%	79718	82817	4%	71296	-11%
$OpVaR_{99,9}$	83292	62237	-25%	54257	-35%	113560	88309	-22%	76544	-33%

Panel C : Fat-tailed-Pareto distribution (shape parameter = 0.7)

	N = 200					N = 300				
	Simulated	OpRisk+				Simulated	OpRisk+			
		Roundup	Δ	Round	Δ		Roundup	Δ	Round	Δ
$OpVaR_{90}$	82381	82539	0%	75696	-8%	120654	119943	-1%	112596	-7%
$OpVaR_{95}$	96971	88248	-9%	81375	-16%	139470	127037	-9%	120850	-13%
$OpVaR_{99}$	166962	98972	-41%	91991	-45%	234442	55846	-76%	135481	-42%
$OpVaR_{99,9}$	543597	111875	-79%	104699	-81%	733862	156642	-79%	152904	-79%

Table 7 - Comparison of OpRisk+ with an upper bound limit value (rounded up) and an average value (rounded) for each band, for large databases.

Panel A : Thin-tailed-Pareto distribution (shape parameter = 0.3)

	N = 20					N = 50				
	Simulated	OpRisk+				Simulated	OpRisk+			
		Roundup	Δ	Round	Δ		Roundup	Δ	Round	Δ
$OpVaR_{90}$	3770	3880	3%	3535	-6%	8573	8882	4%	8074	-6%
$OpVaR_{95}$	4073	4173	2%	3815	-6%	9030	9334	3%	8501	-6%
$OpVaR_{99}$	4712	4744	1%	4363	-7%	9942	10209	3%	9332	-6%
$OpVaR_{99,9}$	5596	5410	-3%	5010	-10%	11141	11250	1%	10311	-7%

Panel B : Medium-tailed-Pareto distribution (shape parameter = 0.5)

	N = 20					N = 50				
	Simulated	OpRisk+				Simulated	OpRisk+			
		Roundup	Δ	Round	Δ		Roundup	Δ	Round	Δ
$OpVaR_{90}$	5579	5672	2%	5332	-4%	12630	12855	2%	11323	-10%
$OpVaR_{95}$	6364	6247	-2%	5901	-7%	13862	13734	-1%	12152	-12%
$OpVaR_{99}$	8966	7329	-18%	6945	-23%	18051	15410	-15%	13668	-24%
$OpVaR_{99,9}$	18567	8626	-54%	7904	-57%	33554	17338	-48%	14377	-57%

Panel C : Fat-tailed-Pareto distribution (shape parameter = 0.7)

	N = 20					N = 50				
	Simulated	OpRisk+				Simulated	OpRisk+			
		Roundup	Δ	Round	Δ		Roundup	Δ	Round	Δ
$OpVaR_{90}$	9700	11410	18%	9413	-3%	22495	23992	7%	25235	12%
$OpVaR_{95}$	12640	12931	2%	10914	-14%	28103	27089	-4%	28537	2%
$OpVaR_{99}$	27261	15583	-43%	13353	-51%	55994	32020	-43%	33837	-40%
$OpVaR_{99,9}$	114563	18726	-84%	16290	-86%	220650	38761	-82%	40024	-82%

Table 8 - Comparison of OpRisk+ with an upper bound limit value (round up) and an average value (round) for each band, for small databases.

8. Figures

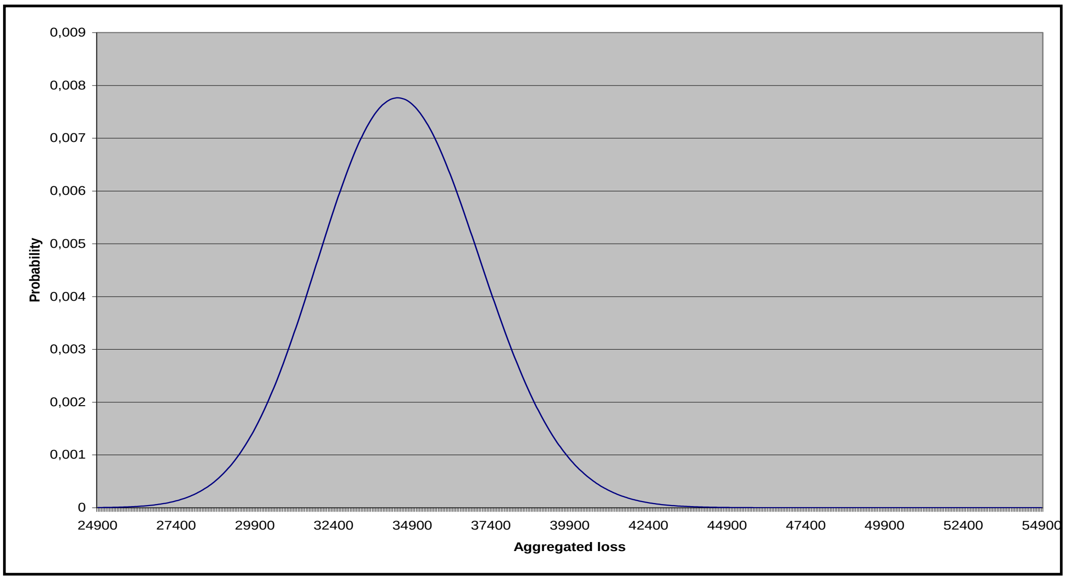
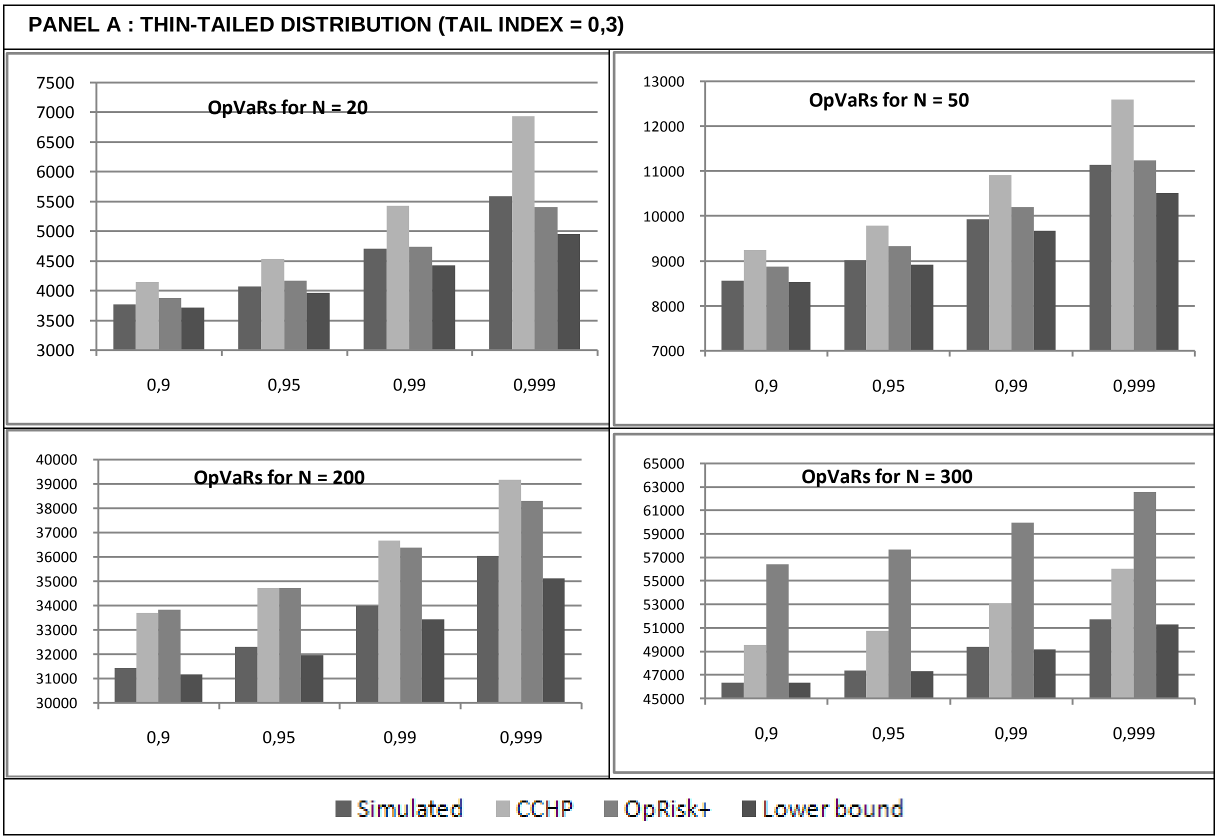


Figure 1. Aggregated loss distribution for a series of 200 loss events characterized by a Pareto(100;0.3).



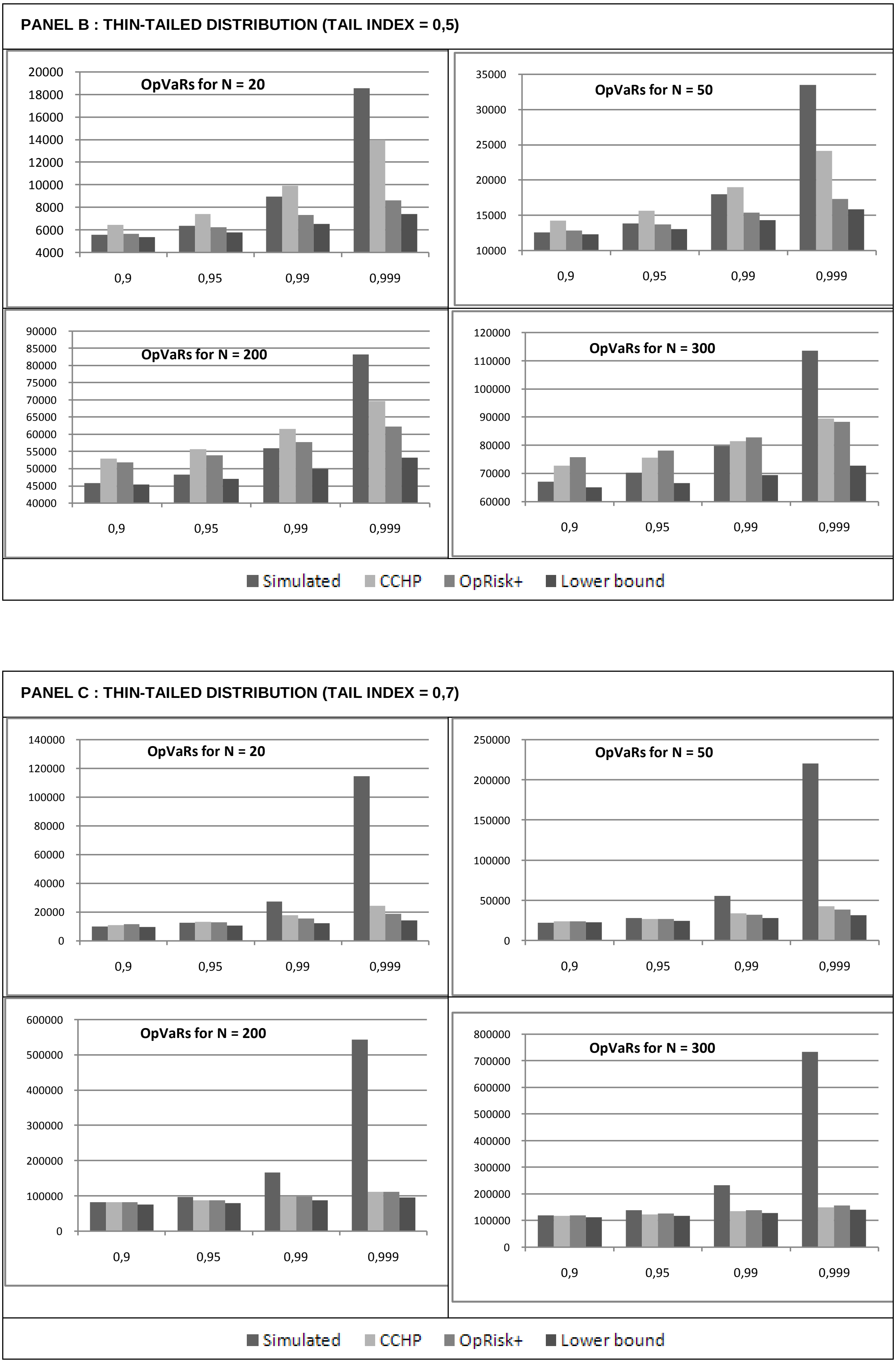


Figure 2 - Comparison of CCHP, OpRisk+ and the lower bound proposed by Alexander (2003).

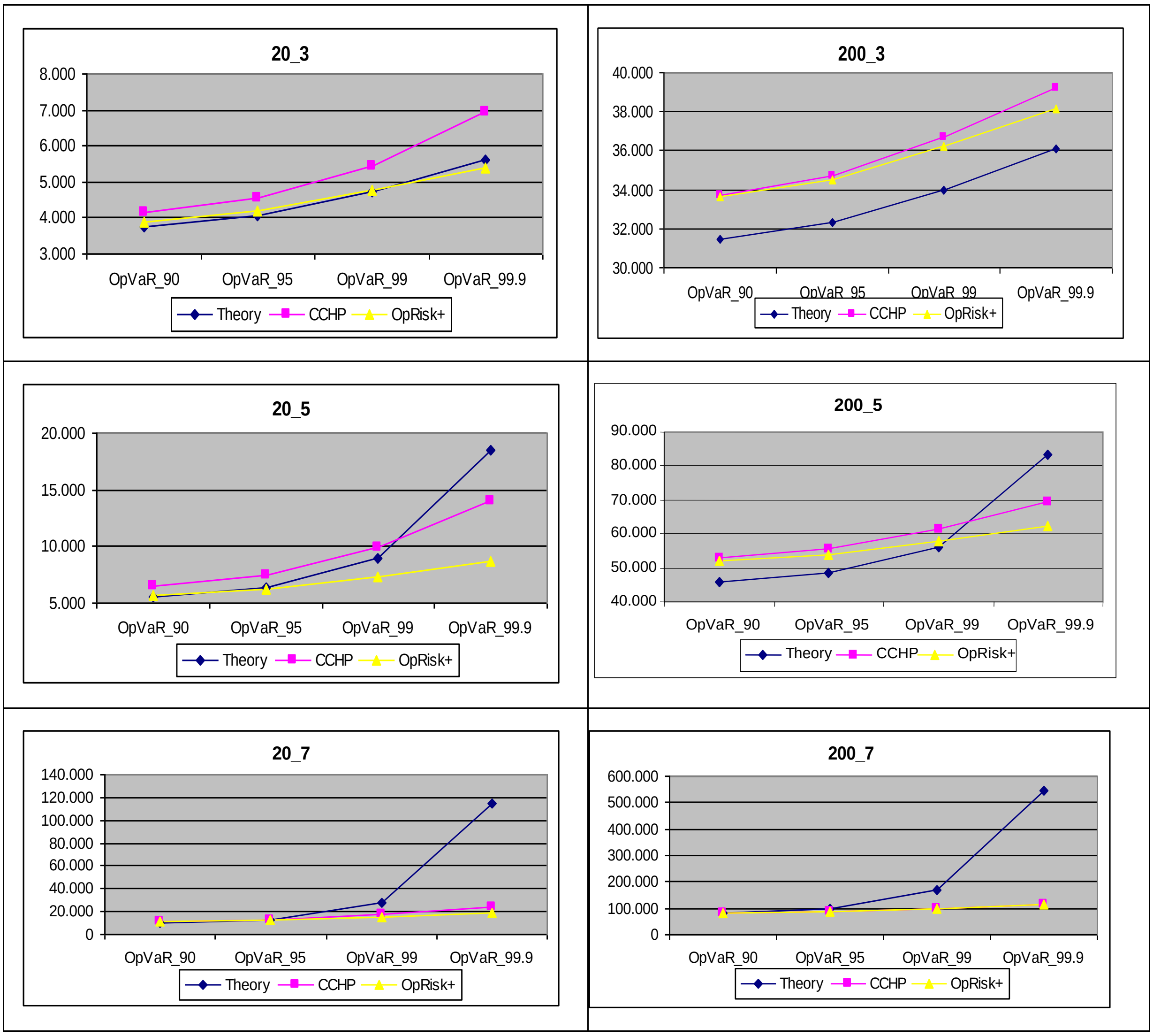


Figure 3. OpVaR levels generated by CCHP, OpRisk+ and theoretical OpVaR for databases of 20 events characterized by a Pareto distribution with parameters (100;0.3), (100;0.5) and (100.0.7), namely 20_3, 20_5 and 20_7, respectively, and for databases of 50 events characterized by the same distribution, that is respectively 200_3, 200_5 and 200_7.

9. Appendix - CreditRisk+: The distribution of Default losses Calculation procedure[i]

CreditRisk+ mathematically describes the random effect of the severity distribution through its probability generating function G(Z):

$$G(z) = \sum_{n=0}^{\infty} P(\text{aggregated loss} = \text{n x L})z^{n}$$

Comparing this definition with the Taylor series expansion for G(z), the probability of a loss of n x L, An, is given by:

$$P(\text{loss of nL}) = \frac{1}{n!}\frac{d^{n}G(z)}{dz^{n}}\bigg|_{z=0} = \mathrm{A}_{n}$$

In CreditRisk+, G(Z) is given in closed form by :

$$G(z) = \prod_{j=1}^{m} e^{-\mu_j + \mu_j z^{\nu_j}} = e^{-\sum_{j=1}^{m}\mu_j + \sum_{j=1}^{m}\mu_j z^{\nu_j}}$$

Therefore, using Leibniz formula we have:

$$\frac{1}{n!}\frac{d^{n}G(z)}{dz^{n}}\bigg|_{z=0} = \frac{1}{n!}\frac{d^{n-1}}{dz^{n-1}}\left(G(z).\frac{d}{dz}\sum_{j=1}^{m}\mu_j z^{\nu_j}\right)\bigg|_{z=0}$$

$$= \frac{1}{n!}\sum_{k=0}^{n-1}\binom{n-1}{k}\frac{d^{n-k-1}}{dz^{n-k-1}}G(z).\frac{d^{k+1}}{dz^{k+1}}\left(\sum_{j=1}^{m}\mu_j z^{\nu_j}\right)\bigg|_{z=0}$$

However

$$\frac{d^{k+1}}{dz^{k+1}}\left(\sum_{j=1}^{m}\mu_j z^{\nu_j}\right)\bigg|_{z=0} = \begin{cases} \mu_j (k+1)! & \text{if } k = \nu_j - 1 \text{ for some j} \\ 0 & \text{otherwise} \end{cases}$$

and by definition

$$\frac{d^{n-k-1}}{dz^{n-k-1}}G(z)\bigg|_{z=0} = (n-k-1)!\,\mathrm{A}_{n-k-1}$$

Therefore

$$\mathrm{A}_n = \sum_{\substack{k \le n-1 \\ k=\nu_j - 1 \text{ for some j}}} \frac{1}{n!}\binom{n-1}{k}(k+1)!(n-k-1)!\,\mu_j \mathrm{A}_{n-k-1} = \sum_{j:\nu_j \le n}^{m} \frac{\mu_j \nu_j}{n}\mathrm{A}_{n-\nu_j}$$

Using the relation $\varepsilon_j = \nu_j \cdot \mu_j$, the following recurrence relationship is obtained:

$$A_n = \sum_{j:\nu_j \le n} \frac{\varepsilon_j}{n} A_{n-\nu_j}$$

i Source : Credit Suisse (1997); "CreditRisk+ : A Credit Risk Management Framework", Credit Suisse Financial Products, Appendix A, Section A4, p36.