Analysing Contagion and Bailout Effects with Copulae - The Case of Germany’s IKB

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Abstract

In this paper, a new methodical framework for the analysis of bank contagion is proposed. By combining conventional event study and copula methodology, this framework allows for an analysis of bank contagion that directly assesses the changes of dependencies between banks instead of proxying contagion via abnormal returns. Furthermore, to the best knowledge of the author, this paper is the first one to analyse changes in the dependence structure of banks around bailout announcements. More precisely, the empirical study given in this paper tries to answer two questions: Firstly, did the announcement of severe unanticipated depreciations of Germany’s Deutsche Industriebank IKB AG (being the first bank in Germany whose existence was severely threatened by the adverse effects of the subprime crisis) lead to contagion effects proxied as a change in lower tail dependence across German banks? Secondly, did the rescue efforts of the state-owned KfW (IKB’s main shareholder) as a lender of last resort to the IKB limit or reverse these contagion effects?

The results show that significant contagion effects could be detected in the German banking sector after those announcements of crisis at IKB that were not accompanied by immediate bailout announcements by KfW. After the final bailout of IKB, lower tail dependence was effectively reduced while at the same time tail independence increased significantly. This shift in tail dependence indicates that the bailout announcement did not restore the pre-crisis dependence structure, but rather only decreased the likelihood of a joint crash of bank stocks.

KEY WORDS: Contagion Effects; Bailout; Event Study; Copula.

JEL Classification: C01, G14, G21, G28
1 Introduction

Contagion effects between banks have been a field of research since the 1930s when bank failures occurred in a domino-like fashion (see e.g. Calomiris and Mason, 1997, for a study of bank failures during the Great Depression). In the context of bank contagion, one usually distinguishes between bank runs and bank panics with the former being confined to one specific bank and the latter being an irrational and indiscriminate withdrawal of deposits from all banks (see Bhattacharya and Thakor, 1993; Kaufman (1994) describes this irrational form of a bank panic as pure contagion). More generally, bank contagion can also be defined as a transmission of information within the banking industry (see e.g. Gorton, 1985; Bessler and Nohel, 2000; Akhigbe and Madura, 2001). Aharony and Swary (1983) define noisy (or firm-specific) bank contagion as an adverse effect of a bank failure on banks due to correlations between banks whereas pure contagion is caused by problems which are uncorrelated across banks.

The existence of these contagion effects in banking is often explained by the presence of information asymmetries between banks and its stakeholders. As a single bank usually shares certain characteristics with its competitors (e.g. a similar customer base, credit portfolio or syndicated corporate loans), adverse effects on the bank could also implicate adverse effects on other banks and possibly other industries. Due to information asymmetries between banks and stakeholders, however, the latter might not be able to distinguish between affected and unaffected banks withdrawing deposits and repricing stocks indiscriminantly (see Bessler and Nohel, 2000). It is this danger of an irrational bank panic (possibly leading to a systemic risk) causing considerable costs to the financial sector that is often named as a justification for a state’s involvement as a lender of last resort and the necessity for regulation in banking (see e.g. James, 1991; and Goodhart and Huang, 2005).

Methodically, contagion effects in banking have often been studied by computing abnormal stock returns (see e.g. Akhigbe and Madura, 2001; Gropp and Moerman, 2004; Kabir and Hassan, 2005). In these studies, contagion is presumed to be present if negative abnormal returns or increased volatility can be detected in the post-crisis period after the event that is supposed to be causing the bank panic. In addition to this, some authors have tried to use extreme value theory to estimate the number of co-exceedances in order
to isolate contagion effects across banks (see Gropp and Moerman, 2004; and Gropp and Vesala, 2004). Simultaneously to the analysis of bank contagion, a different branch of research has concentrated on analysing contagion effects between financial markets in times of crisis (like e.g. in the Asian crisis of 1997). In this branch of research, early works concentrated on studying correlations between stock market indices (see e.g. Forbes and Rigobon, 2002) whereas recent work has focused on substituting a correlation-based analysis by a more general copula-based approach (see e.g. Rodriguez, 2007 and Chen and Poon, 2007).

The aim of this paper is to propose a new methodical framework for the analysis of bank contagion. By combining well-known event study and copula methodology, the framework proposed in this paper allows for an analysis of bank contagion that directly assesses the changes of dependencies between banks instead of proxying contagion via abnormal returns. Furthermore, in the empirical part of this paper, I focus on detecting contagion effects and effects by bailout announcements for the near-collapse of German Deutsche Industriebank IKB AG (IKB) as a result of the subprime crisis. Although several previous studies have focused on contagion effects in banking (see e.g. De Bandt and Hartmann, 2001, for a comprehensive overview of empirical studies), the effects of a state’s involvement as a lender of last resort on banking contagion have only scarcely been analysed in empirical studies. To the best knowledge of the author, this paper is the first one to analyse changes in the dependence structure of banks around bailout announcements. More precisely, the empirical study given in this paper tries to answer two questions: Firstly, did the announcement of severe unanticipated depreciations of Germany’s IKB (being the first bank in Germany whose existence was severely threatened by the adverse effects of the subprime crisis) lead to contagion effects across German banks? Secondly, did the rescue efforts of the state-owned Kreditanstalt fr Wiederaufbau (KfW), IKB’s main shareholder, as a lender of last resort to the IKB limit or reverse these contagion effects?

To answer these questions, abnormal stock returns are computed in a first step from a market model with standard event study methodology for all German banks listed in the DAX stock index. In a second step, contagion between banks is parameterised by two concepts based on copulae: Firstly, a convex combination of parametric copulae with different tail dependence characteristics is fitted to the abnormal returns with contagion being indicated by an
increase in the coefficient of the lower tail dependent Clayton copula. Secondly, the extension of the well-known bivariate tail dependence coefficient to multivariate copulae proposed by Schmid and Schmidt (2007) is computed for the data to examine changes directly in the coefficient between announcements by the IKB and the KfW.

The contributions of this article are numerous. Firstly, this paper extends the ongoing work on the empirical analysis of bank contagion by examining the success of bailouts in reversing contagion effects. By combining well-known event study and state-of-the-art copula methodology, this paper presents a new framework for directly assessing the impact of contagion and bailouts on the dependencies between banks. Furthermore, the results in the empirical study show that contagion could be observed in the period after the announcements of severe losses at IKB. In addition to this, the state’s announcements of a bailout did not simply reverse the changes in the dependence structure but lead to a persistent shift from lower tail dependence to tail independence between German banks indicating that bailout announcements are successful in decreasing the probability of extreme joint downward movements of returns while leaving the probability of extreme joint upward movements unchanged.

The remainder of this article is structured as follows. Section 2 discusses the theory on contagion effects and bailouts. In section 3, the methodology and model specifications are described. Section 4 exhibits the data and presents the empirical findings. Concluding remarks are given in Section 5.

2 Bank contagion and lenders of last resort

Announcements of adverse effects on a bank (be they illiquidity, insolvency or impending failure) can cause both a rational information effect as well as irrational pure contagion. The former is presumed to be caused by a correct measurement of the direction and strength of the correlation between the asset and loan portfolios of the failed bank and its competitors (see Diamond, 1984, 1991; and Bessler and Nohel, 2000). If this measurement by the economic agents is not correct, an indiscriminate repricing of the bank’s shares and bank panics jointly known as the phenomenon of pure contagion will ensue.

Contagion effects following bank failures have been discussed at great length in literature (see Kaufman, 1994, for a summary). Moreover, there exists a vast literature on the theory of the propagation channels for contagion ef-
fects with interbank lending, common customer bases and payments/settlements systems being considered to be the prime causes of (rational) contagion (see Allen and Gale, 2000; Freixas et al., 1998; Bessler and Nohel, 2000; and Goodhart and Huang, 2005). Concerning the economic consequences, pure contagion is widely regarded to be more dangerous to the stability of the financial system as all banks are affected by the adverse effect irrespective of their specific portfolio. Consequently, the possibility of a market failure manifested in a bank panic, i.e. pure contagion, is often stated as the fundamental justification for a state’s intervention as a lender of last resort (see Bagehot, 1873; Lerrick and Meltzer, 2003; and Goodhart and Huang, 2005). In the view of some authors, systemic risks can even be considered to be one of the fundamental reasons for the existence of central banking (see Gorton and Huang, 2006).

Most of the empirical work on banking contagion finds that rational contagion effects seem to prevail while pure contagion seems to be the exception. One explanation for the little empirical evidence for the existence of pure contagion after bank failures is the notion that contagion effects were limited by the actions undertaken by states as lenders of last resort (see Hasman and Samartín, 2008). From this, one could hypothesise that rescue measures made by lenders of last resort have been successful in preventing or reversing pure contagion. Until now, however, only little empirical work has focused on analysing the success of rescue measures by the state acting as a lender of last resort. One of the few examples can be found in Butkiewicz (1995), where an initial reversion of contagion effects is observed for the time of the Great Depression. Similar results, namely positive abnormal returns of banks, are found by Yorulmazer (2008) where the effects of the bank run at Northern Rock and the subsequent bailout announcement by the Bank of England are analysed. The empirical analysis there, however, is based purely on comparing abnormal returns of British banks. As bank contagion is regularly interpreted as a change of dependencies between banks, an empirical analysis of contagion and bailout announcements should ideally be based on some operationalisation of stochastic dependence. This analysis of the dependence structure, however, should only be based on filtered returns estimated from a market model in order to eliminate a possible bias induced by the market return or conditional heteroscedasticity. Moreover, conventional event studies like Yorulmazer (2008) usually only consider small time windows around events. In case market reaction to the event is lagged or information on the
event was available beforehand, small time windows can thus lead to biased results especially when analysing the dependence structures of (abnormal) returns thus requiring the analysis of longer time periods around events.

Though interesting in its own, this paper will concentrate neither on the mechanisms of propagation nor on a separation of rational and irrational contagion effects. Instead, the empirical analysis will focus on the success of rescue actions undertaken by the state as a lender of last resort with respect to reversing overall banking contagion parameterised by dependence in the lower tails of the banks’ return distributions. The reason for this approach is that the decision by a state to act as a lender of last resort will often be influenced by political considerations. In a situation of a heated public discussion and panicky investors, the state’s decision will thus often be made regardless of the type of contagion effects the banking industry is experiencing. Nevertheless, pure contagion effects will be presumed to be present due to the state’s role as both a lender of last resort and a regulating authority. If the state is in possession of near-complete information on the correlations between the banks’ portfolios due to its regulating function, the state will act as a lender of last resort only in the presence of the more dangerous pure contagion.

3 Methodology

3.1 GARCH-filtering and abnormal returns

The analysis of the contagion effects in this paper will be based on the daily stock returns of German banks. This approach follows several other studies like e.g. Bessler and Nohel (2000), Akhigbe and Madura (2001), Lau and McInish (2003), Kabir and Hassan (2005) and Yorulmazer (2008) in which different market models of stock returns are estimated. Different approaches like e.g. computing metrics like distances to default as it is done in Gropp and Moerman (2004) are not considered due to two reasons: First, transmitted adverse effects will always result in a devaluation of a bank, and thus, the adverse effect should be reflected in a repricing of the bank’s stock. Second, the use of additional data from the banks’ financial statements would require breaking down quarterly into diurnal data. Such methods (like e.g. cubic spline interpolation, see Gropp and Moerman, 2004) can only be seen as coarse approximations thus introducing an unnecessary bias in the data.

In contrast to the aforementioned papers, this paper proposes a new frame-
work for detecting contagion effects and market reactions to bailout announce-
ments. Instead of simply comparing abnormal returns, copula functions are
fitted to abnormal returns in order to analyse the dependence structure of a
struggling bank’s competitors. To minimise the effects of the market return
on the banks’ stock returns, abnormal returns rather than observable returns
(like it is done e.g. in Rodriguez, 2007) are used. Abnormal returns are esti-
mated from a market model that includes the German stock index DAX as a
proxy of the market return and the daily Euribor-1-month reference rate.

A stylised fact about financial data is the presence of conditional het-
eroscedasticity in stock returns. As the presence of conditional heteroscedastic-
ity could bias the results and as the copula models described below require the
input of i.i.d. data, I fit ARMA-GARCH models to the univariate marginals to
account for time-varying volatility. In particular, I model the stochastic pro-
cess \((r_t)_{t\in \mathbb{Z}}\) of the log-returns for each bank as an ARMA\((p_1,q_1)\)-GARCH\((p_2,q_2)\)
process with

\[
\begin{align*}
    r_t &= \mu_t + \epsilon_t \\
    \mu_t &= \mu + \sum_{i=1}^{p_1} \phi_i (r_{t-i} - \mu) + \sum_{j=1}^{q_1} \theta_j \epsilon_{t-j} \\
    \epsilon_t &= \sigma_t \varepsilon_t \\
    \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{p_2} \alpha_i (|\epsilon_{t-i}| + \gamma_i \epsilon_{t-i})^2 + \sum_{j=1}^{q_2} \beta_j \sigma_{t-j}^2
\end{align*}
\]

and \(\alpha_0 > 0, \alpha_i, \beta_j \geq 0\) for all \(i = 1, 2, ..., p_2\) and \(j = 1, 2, ..., q_2\) and \(\varepsilon_t\) being
a SWN\((0,1)\)-process (see Bollerslev, 1986, and Bollerslev et al., 1992). The
choice of distribution for the innovations \(\varepsilon_t\) as well as the exact specifications
for the volatility models and the ML-estimates for the parameters are given
later in the presentation of the results. After the models have been estimated,
the log return \(r_{i,t}\) of the ith univariate return series at time \(t\) is filtered accord-
ing to

\[
\hat{r}_{i,t} := \frac{r_{i,t} - \hat{\mu}_{i,t}}{\hat{\sigma}_{i,t}}, \ i \in \mathbb{N}, \ t = 1, 2, ..., T
\]

After the filtered returns have been computed, we need to exclude shocks
common to all market participants that might bias the results. Therefore, it
is assumed that the filtered log returns are generated by the following market
model:

\[
\hat{r}_{i,t} = \alpha_i + \beta_i r_{M,t} + \gamma_i I_t + \varepsilon_{i,t},
\]
with \( i = 1, 2, \ldots, n \) representing \( n \) banks whose dependence structure will be analysed, \( t = 1, 2, \ldots, T \) being a time index, \( r_{M,t} \) being the (GARCH-filtered) return on the market portfolio proxied by the German DAX stock index on day \( t \), \( I_t \) being the daily Euribor-1-month reference rate and \( \varepsilon_{i,t} \) being a random disturbance term for bank \( i \) at time \( t \).

### 3.2 Some preliminary copula theory

In order to detect contagion effects and possible remedies induced by the lender of last resort, the dependence structure inherent in the abnormal filtered returns \( \hat{r}_{i,t} \) of a set of banks is modelled by the use of copula functions. In the following, some basic results on copulae will be reviewed.

Consider the marginal distributions of a random vector \( X \) of length \( n \) to be previously specified, the process of aggregating these distributions to their joint distribution is reduced to choosing or estimating a copula that reflects the dependence structure between the marginals. The mathematical basis for the analysis of copulae was founded by Sklar (1959) and Hoeffding (1940). In the following, a basic definition of a copula and Sklar’s theorem are described (for a more detailed description of copulae see Nelsen, 2006 or Joe, 1997).

Let \( F_i \) be the \( i \)th marginal cumulative distribution function (cdf) of the random vector \( X \). An \( n \)-dimensional copula is a \( n \)-variate cumulative distribution function \( C : [0; 1]^n \rightarrow [0; 1] \) with uniformly distributed marginals (hereafter called \( n \)-copula). The central result in copula theory is Sklar’s theorem which ensures the existence of a unique copula under relatively weak conditions:

**Theorem 1 (Sklar)** Let \( F \) be a joint cumulative distribution function with \( n \) marginals \( F_i \). Then there exists an \( n \)-dimensional Copula \( C \) such that for all \( x \in \mathbb{R}^n \),

\[
F(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)).
\]  

If all marginals \( F_i \) are continuous, then the Copula \( C \) is unique.

Vice versa, if an \( n \)-Copula \( C \) and \( n \) cumulative distribution functions \( F_i \) are given then (4) yields an \( n \)-variate cumulative distribution function with marginals \( F_i \).

In contrast to traditional concepts of dependence like Kendall’s Tau or Spearman’s Rho, a copula captures the whole dependence between the marginals (see Chen and Huang, 2007) thus further explaining the surge in interest in copulae.
As the copula directly describes the dependence structure inherent in a random vector, it is not surprising that certain measures of dependence and concordance are closely linked with the copula concept. The most important dependence measure with respect to the analysis of financial contagion is the concept of asymptotic tail dependence which will be described in detail in the following. Tail dependence can synonymously be described as the extremal dependence of two random variables, i.e. the dependence in the tails of a bivariate distribution (see McNeil et al., 2005). For our purposes, asymptotic tail dependence is especially well suited for the analysis of financial contagion because it allows a differentiated analysis of the symmetric or asymmetric extremal dependence between two markets, or, as described by Rodriguez (2007), their propensity to crash (and/or to boom) together.

**Definition 3.1 (Upper tail dependence)** Let $X_1$ and $X_2$ be two random variables with cdfs $F_1$ and $F_2$. Then the upper tail dependence coefficient of the random vector $(X_1, X_2)$ is defined as (see McNeil et al., 2005)

$$\lambda_U := \lambda_U(X_1, X_2) = \lim_{u \uparrow 1} P\{X_2 > F_2^{-1}(u) | X_1 > F_1^{-1}(u)\}$$

provided that a limit of $\lambda_U$ exists in $[0; 1]$ with $F_i^{-1}$ being the quantile function of the cdf $F_i$ for $i \in \{1; 2\}$. For $\lambda_U \in (0; 1]$ the random variables are said to be upper tail dependent. For $\lambda_U = 0$, $X_1$ and $X_2$ are said to be asymptotically upper tail independent.

As said earlier, the notion of tail dependence is strongly linked with the concept of copulae. To be precise, for a bivariate random vector with continuous marginal cdfs $F_1$ and $F_2$, the coefficient of upper tail dependence (if it exists) can be expressed in terms of the underlying (unique) copula $C$:

$$\lambda_U = \lim_{u \uparrow 1} \frac{1 - 2u + C(u, u)}{1 - u}$$

(6)

Analogously, the coefficient of lower tail dependence is defined as

**Definition 3.2 (Lower tail dependence)**

$$\lambda_L := \lambda_L(X_1, X_2) = \lim_{u \downarrow 0} P\{X_2 \leq F_2^{-1}(u) | X_1 \leq F_1^{-1}(u)\}$$

(7)

again provided that a limit of $\lambda_U$ exists in $[0; 1]$. For $\lambda_U \in (0; 1]$ and $\lambda_U = 0$ we have lower tail dependent and asymptotically lower tail independent random variables respectively.
If the limit exists and $F_1$ and $F_2$ are continuous, we can express the coefficient in terms of the copula:

$$
\lambda_L = \lim_{u \downarrow 0} \frac{C(u, u)}{u} \quad (8)
$$

The definition of lower tail dependence given above only allows a bivariate comparison of random variables. In addition to the bivariate models explained later I make further use of a generalisation of the notion of lower tail dependence to multivariate random variables that has been proposed recently by Schmid and Schmidt (2007). A multivariate measure for lower tail dependence is given by

**Definition 3.3 (Multivariate lower tail dependence, Schmidt and Schmid)**

$$
\lambda^M_L := \lim_{p \downarrow 0} \rho(p) = \lim_{p \downarrow 0} \frac{n+1}{p^{n+1}} \int_{[0,p]^n} C(u) du 
$$

with

$$
\rho(p) := \frac{\int_{[0,p]^n} C(u) du - \left( \frac{p^2}{2} \right)^n}{\frac{p^{n+1}}{n+1} - \left( \frac{p^2}{2} \right)^n} \quad (10)
$$

being an $n$-dimensional conditional version of Spearman’s rho for $0 < p \leq 1$.

Empirical estimators for $\lambda^M_L$ and $\rho(p)$ based on a sample of size $T$ are given by

$$
\hat{\rho}_T(p) := \left\{ \frac{1}{T} \sum_{t=1}^{T} \left( p - \hat{U}_{i,t,T} \right)^+ - \left( \frac{p^2}{2} \right)^n \right\} / \left\{ \frac{p^{n+1}}{n+1} - \left( \frac{p^2}{2} \right)^n \right\} \quad (11)
$$

and

$$
\hat{\lambda}^M_{L,T}(p) := \hat{\rho}_T(k/T) \quad (12)
$$

respectively, where $\hat{U}_{i,t,T}$ are the pseudo-observations from the copula (see Schmid and Schmidt, 2007; or McNeil et al., 2005) given by

$$
\hat{U}_{i,t,T} := \frac{1}{T} (rank(X_{it}) \text{ in } X_{i1}, \ldots, X_{iT}) \quad (13)
$$

and $k \in \{1, 2, \ldots, T\}$ is a prespecified parameter.

In the following, the different copulae that are to be used in the empirical study shall be briefly discussed. One of the most basic copulae is the Gaussian copula given by the cdf

$$
C_n^\Phi(u; \Sigma) = \Phi^{(n)}(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n)) \quad (14)
$$
with \( \mathbf{u} =^t (u_1, u_2, \ldots, u_n) \in [0; 1]^n \). It can be obtained by applying the inversion method on an \( n \)-variate standard Gaussian distribution \( \Phi^{(n)} \) with correlation matrix \( \Sigma \) and \( n \) univariate standard Gaussian distributions as marginals (see Nelsen, 2006). For imperfectly correlated marginals the Gaussian copula \( C_{n}^\Phi \) is tail independent (see e.g. Sibuya, 1960; and Resnick, 1987).

Similarly as the Gaussian copula can be derived from a multivariate Gaussian distribution, the \( t \)-copula can be obtained from a (non-singular) \( n \)-dimensional Student’s \( t \)-distribution \( T_{d}(\mu; \Sigma; \nu) \) with density

\[
f(x) = \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\pi \nu)^n|\Sigma|}} \left(1 + \frac{(x - \mu)\Sigma^{-1}(x - \mu)}{\nu}\right)^{-\frac{\nu+n}{2}},
\]

\( \nu \) degrees of freedom, mean vector \( \mu \) and dispersion matrix \( \Omega \) (note that the dispersion matrix does not equal the covariance matrix in this case, see Demarta and McNeil, 2005). As copulae are invariant under strictly increasing transformations of the marginals, we can obtain the \( t \)-copula from the standardised \( n \)-dimensional \( t \)-distribution \( T_{n}(0; \Sigma; \nu) \) yielding

\[
C_{n}^T(\mathbf{u}; \nu; \Sigma) = \int_{-\infty}^{t_{\nu^{-1}}(u_1)} \ldots \int_{-\infty}^{t_{\nu^{-1}}(u_n)} \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\pi \nu)^n|\Sigma|}} \left(1 + \frac{x^\prime \Sigma^{-1} x}{\nu}\right)^{-\frac{\nu+n}{2}} dx,
\]

with \( t_{\nu^{-1}} \) being the inverted cdf of a standard univariate Student’s \( t \)-distribution with \( \nu \) degrees of freedom. The \( t \)-copula is symmetrically tail dependent and converges to the Gaussian copula for \( \nu \to \infty \).

Another symmetrically tail independent copula that will be implemented in the empirical study is the Frank copula given by

\[
C_{n}^F(\mathbf{u}; \delta) = -\frac{1}{\delta} \log \left(1 + \prod_{i=1}^{n} \frac{\exp(-\delta u_i) - 1}{\exp(-\delta) - 1}^{-1}\right),
\]

with parameter \( \delta \in \mathbb{R}^+ \) (for some properties of the bivariate Frank copula see Genest, 1987).

The aforementioned copulae exhibit tail independence (Gaussian and Frank) and symmetric tail dependence (Student’s \( t \)), respectively. For the purpose of capturing different patterns of tail dependence, the Gumbel copula which is asymmetrically tail dependent (upper tail dependence and lower tail independence) shall be considered in the empirical analysis as well. Its cdf is given by

\[
C_{n}^G(\mathbf{u}; \lambda) = \exp \left[- \left(\sum_{i=1}^{n} -(\log u_i)^\lambda\right)^\frac{1}{\lambda}\right],
\]
where the parameter $\lambda$ satisfies $\lambda \geq 1$. Here, I use the standard definition of the Gumbel copula (for a recursively defined definition that simplifies the computation see Bouyé, 2003).

The last parametric copula exhibiting lower tail dependence that will be considered in the empirical study is the Clayton copula (sometimes also called the Cook-Johnson or Pareto copula, see Genest and MacKay, 1986; and Hutchinson and Lai, 1990). The Clayton copula is given by

$$C_n^C(u; \theta) = \left( u_1^{-\theta} + \cdots + u_n^{-\theta} - n + 1 \right)^{-1/\theta}, \quad (19)$$

with $\theta \geq 0$ with the independence copula being the limiting case for $\theta \to 0$.

Parameter estimation for these copula functions is usually achieved by Maximum-Likelihood with the marginals being specified either parametrically or nonparametrically yielding the so-called Inference-for-margins (IFM) method and canonical Maximum-Likelihood respectively. The ML-estimators are consistent and asymptotically normal under some regularity conditions (see Genest et al., 1995). The asymptotic behaviour of these estimators, however, only holds for i.i.d. data used for estimating the copula parameters thus emphasising the need to apply GARCH-filters before modelling the dependence structure.

### 3.3 Detecting contagion effects with copulae

Following Patton (2002), Jondeau and Rockinger (2006), Rodriguez (2007) and Chen and Poon (2007) I try to capture any change in the dependence structure of abnormal bank returns by analysing the changes in the parametric form and the parameters of various copulae. Unlike these studies, however, I apply their methodology in an event study to analyse changes in the dependence structure between announcements of the IKB and its lender of last resort. To be precise, I analyse the time-variation of the fitted copulae conditional on the set of given past information represented by the sub-$\sigma$-algebra $\mathcal{G}$. For a given date $t$ and an information set $\mathcal{G}_t := \sigma(\{r_{i,j}|j = 1, 2, ..., t-1\})$ for the $i$th bank, Sklar’s theorem becomes

$$F_t(x_1, x_2, ..., x_n|\mathcal{G}_t) = C^C_{n,t}(F_{1,t}(x_1|\mathcal{G}_t), F_{2,t}(x_2|\mathcal{G}_t), ..., F_{n,t}(x_n|\mathcal{G}_t)|\mathcal{G}_t) \quad (20)$$

with $C^C_{n,t}(u|\mathcal{G}_t)$ being the $n$-dimensional conditional copula, $F_{i,t}(x_i|\mathcal{G}_t)$ being the conditional c.d.f. of the $i$th univariate marginal and $F_t(x_1, ..., x_n|\mathcal{G}_t)$ being the joint c.d.f. of the random vector (see Patton, 2002 for an introduction into
the theory of conditional copulae and the respective parameter estimation). The merit of using abnormal returns rather than observed returns is that the analysis of the dependence structure will not be biased by the influence of the market return proxied by a stock index on the banks’ returns.

In the event study framework of this paper, I assume that the sub-$\sigma$-algebra $G_t \equiv G_{p_q} = \sigma(\Omega)$ is generated by the subset $\Omega := \{r_{i,j} | j = e_1, e_1 + 1, \ldots, e_2 - 1, e_2\}$ containing all available information of the time window $p_q$ between two events $e_1$ and $e_2$ where $q = 1, 2, \ldots$ is the index of the time window.

The model for capturing changes in the dependence structure extends the ideas of Rodriguez (2007) to detect changes in the parametric form of the copula by estimating mixtures of different parametric copulae. It is common knowledge that a convex linear combination of a finite set of copulae is again a copula (see Nelsen, 2006). The analysis of the time-variance in the dependence structure (restricted to a change in the parametric form) can thus be observed in the changes in the weights of the convex combination over time. For each time window $p_q$, a convex combination

$$C_{n,p_q}^{mix}(u; \nu, \rho, \delta, \theta, \lambda | G_{p_q}) \equiv \pi_T^{p_q} C_{n,p_q}^{T}(u; \nu, \rho | G_{p_q}) + \pi_F^{p_q} C_{n,p_q}^{F}(u; \delta | G_{p_q})$$

+ $\pi_C^{p_q} C_{n,p_q}^{C}(u; \theta | G_{p_q}) + (1 - \pi_T^{p_q} - \pi_F^{p_q} - \pi_C^{p_q}) C_{n,p_q}^{G}(u; \lambda | G_{p_q})$

with $\pi_T^{p_q}, \pi_F^{p_q}, \pi_C^{p_q} \in (0; 1)$ and $\pi_T^{p_q} + \pi_F^{p_q} + \pi_C^{p_q} \leq 1$

In the next step, the goodness-of-fit of each configuration of the copula mixture is assessed in order to prevent the models from overfitting the data and to check the model specification. The first metric that will be used is Akaike’s Information Criterion which is given by

$$AIC := 2k - 2L(\hat{\eta}),$$
where \( k \) is the number of model parameters and \( L(\hat{\eta}) \) is the maximised Loglikelihood at the estimate of the parameter vector \( \hat{\eta} \). Furthermore, goodness-of-fit test procedures specially adapted to copula models can be employed for choosing the optimal copula model. An example for such a metric is given by the Cramér-von-Mises statistic

\[
\xi = T \int_{[0;1]^n} \left\{ C_{n,T}^{emp}(u) - C_n(u, \eta) \right\}^2 dC_{n,T}^{emp}(u).
\]  

(23)

which measures the distance between the parametric copula and the empirical copula \( C_{n,T}^{emp} \) estimated from a sample of size \( T \) (see Fermanian, 2005, for an early mentioning and Genest et al., 2008, for an extensive analysis). The empirical version of \( \xi \) is given by (see Genest et al., 2008)

\[
\hat{\xi} = \sum_{t=1}^{T} \left\{ C_{n,T}^{emp}(u_t) - C_n(u_t, \eta) \right\}^2
\]  

(24)

with \( u_t \) being the \( t \)-th sample from the copula. For both metrics, the copula configuration yielding the lowest value will be considered to be optimal. However, one has to be careful when using the GoF-metric in (24) as it does not account for the number of parameters estimated thus possibly leading to overfitting of the data.

After an optimal copula model has been found, I fit the optimal convex combination of copulae to the abnormal filtered returns of each pair of the initial distressed bank’s competitors before the initial and between subsequent events. The null hypothesis then is that \( \pi_{C_{pq}} \) will increase for all pairs of competitors after negative announcements while positive announcements concerning a bailout by the lender of last resort will result in a decrease of the parameters \( \pi_{C_{pq}} \).

Furthermore, I estimate the measure for multivariate lower tail dependence given by (12) in order both to extend the empirical study to a multivariate analysis of possible contagion effects as well as to check the robustness of the (bivariate) results. A summary of the complete framework is given below:

1. Choose the events \( e_1, e_2, \cdots \) on which bank failures (or impending failures) and bailouts became publicised. Identify the first (critical) event as \( t_0 \).

2. Estimate the filtered returns \( \hat{r}_{i,t} \) from ARMA-GARCH-models to control for heteroscedasticity and serial correlation.
3. Estimate the market model \( \hat{r}_{i,t} = \alpha_{i} + \beta_{i} r_{M,t} + \gamma_{i} I_{t} + \varepsilon_{i,t} \) based on stock returns \( r_{i,t} \) observed in the time window \([t_{0} - 400; t_{0} - 100]\) and compute abnormal returns by the identity \( AR_{i,t} := r_{i,t} - \hat{\alpha}_{i} - \hat{\beta}_{i} r_{M,t} - \hat{\gamma}_{i} I_{t} \).

4. Identify the optimal configuration of the copula model in each time window by the use of AIC and additional copula-GoF metrics.

5. Fit the optimal convex combination of copulae to all pairs of banks excluding the initial contagious bank for the time window \([t_{0} - 100; t_{0}]\), to which I will refer to as the pre-crisis period, and any other time window with length \( \geq 50 \) between two events.

6. Conduct significance tests on the null hypothesis of constant parameters between two events.

7. Compute the multivariate lower tail dependence coefficient given by (12) for the pre-crisis and subsequent time windows.

In the following section, the data and chosen events are presented.

4 Data and empirical findings

4.1 Sample description and ARMA-GARCH-modelling

The data sample used in the analysis below consists of 638 daily observations of the logarithmic stock returns of the IKB and the three largest publicly traded German banks listed in the German DAX stock index, i.e. Deutsche Bank AG, Commerzbank AG and Deutsche Postbank AG, covering the period from January 3, 2006 to July 3, 2008. For all banks, returns are defined as the percentage logarithmic difference of the stock price, i.e. \( r_{i,t} \equiv 100 \cdot \ln(P_{i,t}/P_{i,t-1}) \) with \( P_{i,t} \) being the stock price of bank \( i \) at time \( t \). All daily observations of the stock prices were obtained from Thomson Financial Datastream. Following Jondeau and Rockinger (2006) and Bartram et al. (2007) holidays are excluded from the data sample in order to eliminate spurious correlation. Table 1 gives some descriptive statistics for the unfiltered return series.

Insert Table 1 here

Over the whole sample period, all bank stock indices yielded negligible mean daily log-returns. We can observe from the summary statistics that all
returns series are skewed with IKB and Deutsche Postbank being leptokurtic and the log-returns of the remaining banks being platykurtic. From this we can conclude that all return series are not normally distributed. The Jarque-Bera test confirms this conjecture by rejecting the hypothesis of a normal distribution for all five series. Note the very high extrema (-27% and +26%) of the returns of IKB. Furthermore, the return series are tested for ARCH effects with Engle’s LM test and serial correlation with the Ljung-Box test. The LM test of no ARCH effects is rejected for all five series indicating the presence of conditional heteroscedasticity in the data. The Ljung-Box could not be rejected for all banks. Finally, the Bravais-Pearson correlation coefficients for the return series indicate strong linear dependence between all five banks.

As the LM-test was rejected for all bank return series, the univariate marginal distributions are modelled according to the ARMA($p_1$, $q_1$)-GARCH($p_2$, $q_2$)-model described in (1) and fitted by Maximum-Likelihood. For the innovations $\varepsilon_t$ several different distributions have been proposed in the literature. For example Rodriguez assumes the innovations to be normally distributed, whereas Patton (2002) and Chen and Poon (2007) use a skewed Student’s $t$ distribution. In this study, the normal, skewed normal, Student’s $t$ and skewed Student’s $t$ distribution were considered as the conditional distribution of the innovations. The lags and distributions of the innovations necessary to remove GARCH-effects as well as the parameter estimates and results of the LM, Ljung-Box and Jarque-Bera tests on the filtered returns are given in Table 2.

**Insert Table 2 here**

The test statistics show that serial correlation and the observed ARCH effects could be removed from all filtered return series with all test results being significant at the 1% significance level.

### 4.2 Abnormal returns

In the analysis in this paper, I concentrate on three major events on which announcements concerning the financial stability of IKB were made by either IKB or KfW:

- **Initial crisis and immediate bailout** (July 30, 2007): IKB announces that funding of the conduit “Rhineland Funding” is endangered and issues a profit warning. State-owned KfW, IKB’s main stakeholder, provides an 8.1 billion Euro liquidity line for IKB.
• **Second crisis** (November 28, 2007): KfW announces that IKB requires another 2.3 billion Euro to cover its risks. The German government refuses to issue a debt guarantee that would have supported IKB and KfW.

• **Final Bailout** (March 28, 2008): KfW and IKB’s remaining stockholders agree on a recapitalisation amounting to 1.5 billion Euro.

To estimate abnormal returns, the parameters of the market model given by (3) are estimated by OLS using 300 observations from the time window \([t_0 - 400; t_0 - 101]\) with \(t_0\) being July 30, 2007. The parameter estimates are then used to compute abnormal daily returns for all banks for the time windows \([t_0 - 100; t_0 - 1]\) (referred to as the pre-crisis period), \([t_0; t_0 + 86]\) (being the period after first news on IKB’s losses were publicised and KfW announced its initial bailout), \([t_0 + 87; t_0 + 168]\) (referred to as the crisis period) and finally \([t_0 + 169; t_0 + 237]\) (referred to as the post-crisis period). An overview of the announcement dates and the time windows of interest are shown in Figure 1.

**Insert Figure 1 here**

In a first step, cumulative abnormal returns are estimated in the usual fashion over a 3-day time window centered around each announcement date for all banks excluding IKB. Significance of the obtained results is tested using standardised residual t-tests (see e.g. Fee and Thomas, 2004, for a similar approach and McWilliams and McWilliams, 2000, for a description of the test procedure). The results are reported in Table 3.

**Insert Table 3 here**

For the first announcement date (the advent of the crisis), all banks with the exception of Deutsche Bank earn (insignificant) negative abnormal returns. As all banks earn positive or insignificant negative abnormal returns, one can argue that KfW’s initial announcement of a bailout was partly successful in preventing contagion. Inconsistent with the hypothesis of increased contagion, however, significant positive abnormal returns can be found for Deutsche Bank and Commerzbank on the second announcement day while Deutsche Postbank earned insignificant positive abnormal returns. Consistent with the hypothesis of a reversion of contagion effects, significant (insignificant) positive abnormal returns could be observed for Commerzbank (Deutsche Postbank) after the
third announcement day while Deutsche Bank earned an insignificant negative abnormal return. Overall, one can see that these results are no indication for significant sector-wide contagion effects while the hypothesis of positive abnormal returns after the bailout cannot be rejected. As stated earlier, an analysis of contagion effects that is solely based on abnormal returns on selected trading days can yield only evidence on short-term market comovements rather than on changes in the sector’s dependence structure. Moreover, a simple comparison of abnormal returns suffers from the problem that we need to (arbitrarily) decide which abnormal returns should be considered extremal (and thus resulting from contagion). Therefore, to assess the question whether the announcements by IKB and KfW resulted in persistent changes of the extremal dependence inherent in the German banking sector, the copula models described above are fitted to the abnormal returns.

4.3 Detecting contagion effects by the use of copulae

Following Rodriguez (2007), I model contagion effects as a change in lower tail dependence between IKB’s rivals. In order to decide which convex mixture of parametric copulae is best suited for modelling the dependence structure, I first estimated each possible mixture of three or four parametric copulae and computed the corresponding Akaike’s Information Criterion.

**Insert Table 4 here**

The results given in Table 4 show that the Clayton-Frank-Gumbel mixture is the best choice in almost all cases according to Akaike’s criterion while the more flexible full mixture model which includes the Student’s t copula seems to overfit the data (see e.g. Rodriguez, 2007, or Dias and Embrechts, 2008, for a similar use of AIC). Additional Goodness-of-Fit tests using a Cramér-von-Mises criterion proposed by Genest et al. (2008) which is based on a comparison between the null hypothesis and the empirical copula only yielded inconclusive results. As the metric proposed by Genest et al. (2008) does not account for the number of parameters, however, and as overfitting is a severe problem in this setting considering the dynamic range of the number of parameters used in the models, AIC is much more favorable than GoF-metrics for model selection. Therefore, only the Clayton-Frank-Gumbel mixture will be considered in the following.
In the next step, I fitted the Clayton-Frank-Gumbel mixture to each time frame and each pair of return series to identify any change in the bivariate dependence structure of German banks. Results for the Clayton-Frank-Gumbel models are presented in Table 5.

**Insert Table 5 here**

The bivariate results show that between the pre-crisis period and the time after the initial crisis and bailout, no clear sign of sector-wide contagion can be found. While lower tail dependence (as indicated by the coefficient $\pi^C$ of the Clayton copula) between Deutsche Bank and Deutsche Postbank increases significantly, the coefficient decreases for Deutsche Bank and Commerzbank as well as Commerzbank and HypoVereinsbank. These nonuniform changes in lower tail dependence could be a result of the fact that the initial announcement of financial crisis at IKB was immediately accompanied by the bailout announcement by KfW thus averting sector-wide contagion.

Between the second and third time window, i.e. the time before and after the second announcement of severe financial crisis at IKB, results show an unequivocal picture: For all combinations of Deutsche Bank, Deutsche Postbank and Commerzbank, lower tail dependence rises significantly at the 5%-level. Consequently, the bailout by state-owned KfW was economically justified as the probability of a joint crash of German banks had increased sharply after the announcement by IKB (which in this case was not accompanied by an immediate bailout announcement). Moreover, the increases in lower tail dependence all coincide with decreases in upper tail dependence as indicated by the coefficient $\pi^G$ of the Gumbel copula. This result clearly underlines the dramatic change in the dependence structure that took place in the German banking sector after the second announcement of crisis.

After the final bailout, the results given in Table 5 show that for all banks lower tail dependence, i.e. the propensity of German banks to crash together, decreases. This is consistent with the hypothesis of contagion being successfully reversed as a result of KFW’s bailout of IKB. In addition to this, in all bivariate models the decrease in lower tail dependence is accompanied by a significant increase in the coefficient $\pi^F$ signalling tail independence. Economically, this means that after the final bailout announcement, joint extreme upward movements of German banks’ abnormal returns became less likely than before the advent of the crisis. In other words, the instrument of a
bailout does not seem to be suitable for completely reversing contagion but rather seems to transform lower tail dependence into tail independence. This finding is consistent with the economic intention of a bailout as it should not increase the sector’s propensity to boom together but should rather be limited to decreasing the probability of a joint crash.

4.4 Robustness checks

In order to extend the described bivariate comparisons to a multivariate analysis of the changes in the dependence structure of the German banking sector, I estimate the coefficient of multivariate lower tail dependence given by (9) based on the abnormal returns of Deutsche Bank, Commerzbank and Deutsche Postbank. The parameter $k$ in (9) is chosen to be 40. Unreported results with different parameter choices only resulted in marginal shifts in the level of the estimates thus leading to the assumption that this particular choice of the parameter did not alter the estimates of the coefficient of multivariate lower tail dependence. Results are given in Table 6.

**Insert Table 6 here**

From Table 6 one can see that the main results from the previous analysis also hold in the multivariate setting using a different methodology. After a first sharp increase in lower tail dependence after the crisis announcement, contagion effects slowly decrease until the final bailout announcement after which lower tail dependence is slightly reduced. Again the empirical results are consistent with both the hypothesis of contagion effects increasing after the announcement of crisis at IKB and the hypothesis that the bailout by KfW was (partly) successful in reducing contagion effects.

As event studies are often biased by confounding events (especially when using large time windows around events), we need to check the robustness of the previous results with respect to a sub-sample excluding confounding events that occurred during the four time windows. I therefore identified 16 confounding events of the three banks from the internet archive of the Financial Times comprising e.g. interim reports, announcement of mergers and profit warnings that were related to the subprime crisis. Following Foster (1980) I build a sub-sample excluding symmetric three-day intervals around each confounding event from the initial sample (if a confounding event occurred at any one of the three banks, the interval around the event was eliminated
from all banks’ return series). In total, 44 trading days are excluded from the initial sample of filtered abnormal returns. In the next step, the bivariate copula models as well as the coefficient of multivariate lower tail dependence are estimated from the sub-sample. Results for the bivariate models are given in Table 7.

**Insert Table 7 here**

The given results show that the analysis of lower tail dependence is robust even when confounding events that were not related to IKB’s announcement are excluded from the sample. This finding supports the notion that indeed the announcements by IKB and KfW were responsible for the changes in the dependence structure of the German banking sector. Additionally, the exclusion of confounding events did not change the results with respect to upper tail dependence. Thus, the finding of a persistent change from upper tail dependence to symmetric tail independence after the bailout holds for the sub-sample as well. In addition to the bivariate models, I also estimate the coefficients of multivariate lower tail dependence for the sub-sample. Results are given in Table 8.

**Insert Table 8 here**

Finally, the analysis of the coefficients of multivariate tail dependence given in Table 8 shows that the previous results remain almost unchanged by the exclusion of confounding events. Moreover, the reduction of contagion effects after the bailout is even more pronounced than in the previous analysis thus underlining the robustness of the given results.

## 5 Conclusion

In this paper a new framework for detecting effects of bank contagion and bailouts by the use of conventional event study and copula methodology was proposed. By estimating GARCH-filtered abnormal returns instead of observed returns, the dependence structure inherent in a banking sector is not biased by conditional heteroscedasticity in the variances or the influences of common factors like e.g. the market return. By using copula methodology instead of simply comparing abnormal returns, the contagion and bailout effects can be analysed directly as a change of the dependence structure.
The empirical study in this paper analysed the changes in the dependence structure of German banks around announcements of financial crisis of IKB as a result of the subprime crisis and the subsequent bailout by KfW. The results show that significant contagion effects could be detected in the German banking sector after those announcements of crisis at IKB that were not accompanied by immediate bailout announcements by KfW. After the final bailout of IKB, lower tail dependence was effectively reduced while at the same time tail independence increased significantly. All given results also hold in a multivariate setting and are robust to an exclusion of confounding events.

The described shift in tail dependence indicates that the bailout announcement did not restore the pre-crisis dependence structure, but rather only decreased the likelihood of a joint crash of bank stocks. One topic not addressed in this paper is the question which factors of the banking system determine the likelihood of contagion effects and the success of bailouts. To answer this question, more examples of bank contagion and bailouts need to be analysed in future research complemented by cross-sectional analyses.
References


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Table 1: Summary statistics for the unfiltered return series (for the hypothesis tests, p-values are given in parentheses)
Table 2: Model specifications, estimated ARMA($p_1,q_1$)-GARCH($p_2,q_2$) parameters, LM test and Ljung-Box test statistics (p-values) for the filtered returns. For the parameter estimates, standard errors are given in parentheses. For the DAX return series, both the LM and the Ljung-Box test could not be rejected.

*** Significant at the 0.1% level.
** Significant at the 1% level.
* Significant at the 5% level.
Table 3: Cumulative abnormal returns in per cent. t-statistics are given in parantheses.

<table>
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<tr>
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<td>Cumulative abnormal returns (%)</td>
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<tr>
<td>July 30, 2007</td>
<td>3.31 (2.726)***</td>
<td>-1.03 (-0.416)</td>
<td>-0.90 (-0.427)</td>
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<td>November 28, 2007</td>
<td>3.76 (3.111)***</td>
<td>6.90 (2.807)***</td>
<td>0.70 (0.331)</td>
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<tr>
<td>March 28, 2008</td>
<td>-0.16 (-0.130)</td>
<td>4.67 (1.898)**</td>
<td>2.59 (1.231)</td>
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*** Significant at the 1% level.
** Significant at the 5% level.
* Significant at the 10% level.
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Table 4: AIC for the different copula mixture models and time windows. T is the Student’s t copula, C the Clayton copula, F the Frank copula and G is the Gumbel copula. The best model according to AIC is written in bold.

The time windows are defined as follows:
- Pre-crisis=[t₀ − 100; t₀ − 1].
- Crisis and initial bailout=[t₀; t₀ + 86].
- Second crisis=[t₀ + 87; t₀ + 269].
- Final bailout=[t₀ + 270; t₀ + 338].
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</tr>
<tr>
<td>(\pi^C)</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.1573</td>
<td>0.0675</td>
</tr>
<tr>
<td><strong>p-value</strong></td>
<td>0.999</td>
<td>0.000*</td>
<td>0.000*</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Results of the bivariate Clayton-Frank-Gumbel models.
The table gives estimates of the coefficients \(\pi^G\), \(\pi^F\) and \(\pi^C\) of the convex combination.
p-values are estimated by a Likelihood ratio test for the null hypothesis of constant weights between two time windows. A p-value lower than 0.05 indicates that the change of weights is significant at the 5%-level (indicated by * in the table).
<table>
<thead>
<tr>
<th>Model</th>
<th>$[t_0 - 100; t_0 - 1]$</th>
<th>$[t_0; t_0 + 86]$</th>
<th>$[t_0 + 87; t_0 + 269]$</th>
<th>$[t_0 + 270; t_0 + 338]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multivariate lower tail dependence</td>
<td>0.0805</td>
<td>0.4432</td>
<td>0.2972</td>
<td>0.2205</td>
</tr>
</tbody>
</table>

Table 6: Multivariate lower tail dependence in the German banking sector.

The table gives estimates of the coefficient of multivariate lower tail dependence for the portfolio consisting of Deutsche Bank, Commerzbank and Deutsche Postbank as defined by Schmidt and Schmid (2007). The parameter $k$ was chosen to be 40 (see text for an explanation of this choice).
<table>
<thead>
<tr>
<th></th>
<th>$[t_0-100; t_0-1]^*$</th>
<th>$[t_0; t_0+86]^*$</th>
<th>$[t_0+87; t_0+269]^*$</th>
<th>$[t_0+270; t_0+338]^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DB+CB</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^G$</td>
<td>0.6687</td>
<td>0.7547</td>
<td>0.3574</td>
<td>0.4552</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>0.0003</td>
<td>0.2448</td>
<td>0.3244</td>
<td>0.5443</td>
</tr>
<tr>
<td>$\pi^C$</td>
<td>0.3310</td>
<td>0.0005</td>
<td>0.3182</td>
<td>0.0005</td>
</tr>
<tr>
<td>p-value</td>
<td>0.000*</td>
<td>0.000*</td>
<td></td>
<td>0.000*</td>
</tr>
<tr>
<td><strong>DB+PB</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^G$</td>
<td>0.4625</td>
<td>0.6682</td>
<td>0.0008</td>
<td>0.1606</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>0.5370</td>
<td>0.0008</td>
<td>0.8008</td>
<td>0.8411</td>
</tr>
<tr>
<td>$\pi^C$</td>
<td>0.0005</td>
<td>0.3310</td>
<td>0.1984</td>
<td>0.0013</td>
</tr>
<tr>
<td>p-value</td>
<td>0.000*</td>
<td>0.000*</td>
<td></td>
<td>0.000*</td>
</tr>
<tr>
<td><strong>CB+PB</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^G$</td>
<td>0.9982</td>
<td>0.1111</td>
<td>0.0890</td>
<td>0.0284</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>0.0009</td>
<td>0.8879</td>
<td>0.7679</td>
<td>0.8075</td>
</tr>
<tr>
<td>$\pi^C$</td>
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<td>0.0010</td>
<td>0.1431</td>
<td>0.1641</td>
</tr>
<tr>
<td>p-value</td>
<td>0.999</td>
<td>0.000*</td>
<td></td>
<td>0.000*</td>
</tr>
</tbody>
</table>

Table 7: Results of the robustness check for the bivariate Clayton-Frank-Gumbel models. The table gives estimates of the coefficients $\pi^G$, $\pi^F$ and $\pi^C$ of the convex combination. p-values are estimated by a Likelihood ratio test for the null hypothesis of constant weights between two time windows. A p-value lower than 0.05 indicates that the change of weights is significant at the 5%-level (indicated by * in the table). * Excluding three-day-windows symmetrically set around confounding events (see text).
The table gives estimates of the coefficient of multivariate lower tail dependence for the portfolio consisting of Deutsche Bank, Commerzbank and Deutsche Postbank as defined by Schmidt and Schmid (2007). The parameter $k$ again was chosen to be 40 (see text for an explanation of this choice).

* Excluding three-day-windows symmetrically set around confounding events (see text).

<table>
<thead>
<tr>
<th>Model</th>
<th>$[t_0 - 100; t_0 - 1]$</th>
<th>$[t_0; t_0 + 86]$</th>
<th>$[t_0 + 87; t_0 + 269]$</th>
<th>$[t_0 + 270; t_0 + 338]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multivariate lower tail dependence</td>
<td>0.1046</td>
<td>0.4762</td>
<td>0.3206</td>
<td>0.0274</td>
</tr>
</tbody>
</table>

Table 8: Results of the robustness check for the multivariate lower tail dependence in the German banking sector.
Figure 1: Timeline of the crisis and time windows