Measuring Event Risk

Peter Nyberg* Anders Wilhelmsson†

January 19, 2009

*Corresponding author. Swedish School of Economics and Business Administration and the Graduate School of Finance. Dept. of Finance and Statistics, P.O. Box 479, 00101 Helsinki, Finland. Phone: +35894313 3380, Fax: +358943133393. Email: peter.nyberg@hanken.fi

†School of Economics and Management, Lund University. Dept. of Economics. Lund, Sweden.

Abstract

This paper decomposes the popular risk measure Value-at-Risk (VaR) into one jump- and one continuous component. The continuous component corresponds to general market risk and the jump component is proportional to the event risk as defined in the Basel II accord. We find that event risk, which is currently not incorporated into most banks' VaR models, comprises a substantial part of total VaR. It constitutes 30% of the risk for a portfolio of small cap stocks but less than 1% for a portfolio of large cap stocks. The national supervising agency in each membership country is advised by the Basel rules to add an additional capital charge to a bank whose models do not capture event risk. The large variation in event risk, also found across 10 individual stocks, suggests that an approach that varies the capital surcharge, based on the type of asset, should be used by the supervisors.

Keywords: Value-at-Risk, Event Risk, NIG distribution, Jumps

JEL G21, G28, C22

1 Introduction

Because of its conceptual simplicity and prominent role in the Basel II accord (Basel, 2006) Value-at-Risk (VaR) is today one of the leading measures of market risk. VaR summarizes the market risk into a single number that expresses the largest expected loss in an investor's portfolio for a given level of confidence and target horizon. There is currently a very active literature concerning the estimation and evaluation of VaR models, see for example Bali et al. (2008), Kuester et al. (2006) as well as Engle and Manganelli (2004).

This paper is the first to decompose Value-at-Risk into one jump- and one continuous component. The continuous component corresponds to general market risk and the jump component is suggested to measure the event risk, as defined in the Basel II accord (BIS 2006). The decomposition is achieved by a new model for financial returns that allows for discontinuous price movements (jumps) and time variation in the first four conditional moments.

Models with time varying higher moments have been proposed before by among others Hansen (1994), Harvey and Siddique (1999), Mittnik and Paolella (2003), Bali et al. (2008) as well as Lanne and Saikkonen (2007), however these models do not allow for jumps in the price process. The two existing models that are most closely related to the model we propose is the NIG-GARCH model of Forsberg and Bollerslev (2002) and the GARJI model of Maheu and McCurdy (2004). Our model nests both of these previous models as special cases and can be viewed as the NIG-GARCH model with an added jump component or as the GARJI model with a changed distribution assumption.

The motivation to change the distribution assumption in Maheu and McCurdy (2004) stems from the fact that their model can only accommodate non-zero skewness and excess conditional kurtosis by the jump intensity parameter. This means that their model will overestimate the jump component if the true price process has a continuous part with non-zero skewness and/or excess kurtosis.

The rules in the Basel II framework apply an additional capital charge when a bank's internal model used for VaR calculation cannot properly capture the event risk. They further state that few models used today are able to achieve this. Our model is of interest to banks since it incorporates the event risk and thus allows for VaR modelling without additional capital charges. We also believe that our model used in conjunction with our proposed quantitative measure of event risk is of interest to regulators since it allows for a direct measurement of the event risk in an asset. This should be a valuable tool when determining the magnitude of the additional capital charge added to models that cannot capture event risk. We show empirically that the proportion of event risk varies greatly between different assets. The total VaR of a portfolio of the 30% smallest companies of the market index is found to consist of about 1/3 event risk. In contrast to this, a portfolio of the 30% largest companies has an event risk proportion less than 1%. This suggests that the regulatory surcharge should differentiate between assets and not be a constant scale factor that is independent of the asset's actual event risk.

The rest of the paper proceeds as follows. Section 2 describes the econometric model and section 3 gives an overview of Value-at-Risk and proposes a measure for event risk. Section 4 presents the data and section 5 displays the results. Section 6 concludes.

2 Models for equity returns

There is now a large body of literature[1] that documents the presence of discontinuities in the sample paths of financial returns. It is also known (see Maheu and McCurdy (2004) and Andersen et al., 2007) that news that give rise to jumps in prices take shorter time to dissipate than price movements due to "normal" news. Consequently, it may be necessary to use two components, one measuring the impact of normal news and one measuring the impact of more extreme events, to correctly measure financial risk. Explicit modelling of the jump component (that captures the extreme events) is shown important in variance forecasting by Maheu and McCurdy (2004), Andersen et al. (2007) and Lanne (2007). Further there is evidence of time variation not only in the conditional variance but also in the conditional skewness and kurtosis of financial returns. Models that capture this higher order dependence have been suggested by for example Hansen (1994), Harvey and Siddique (1999, 2000) and recently in a Value-at-Risk setting by Bali et al. (2008) and Wilhelmsson (2009). Below we will develop a model that allows for all the empirical features of financial data described above.

2.1 The GARJI model

Maheu and McCurdy (2004) suggest an interesting model, called the GARJI model, where the return consists of a sum of a Poisson distributed number of jumps and a continuous residual. The continuous residual is given an interpretation as a return shock due to normal news and more extreme news are picked up by the Poisson jump component. Since the return is modelled as a sum of these two components, both components must be drawn from a distribution that is closed under convolution, such as the normal, in order to get a closed form expression for the conditional distribution of the returns.

We propose to change the normality assumption in Maheu and McCurdy (2004) because under this assumption the model can only accommodate non-zero conditional skewness and excess conditional kurtosis by the jump intensity parameter[2]. This means that the original GARJI model will overestimate the jump component if the true price process has a continuous part with non-zero skewness and/or excess kurtosis. We propose to instead use the normal inverse Gaussian (NIG) distribution both for the error term of the continuous part and as a distribution for the jump size. The NIG distribution is very flexible in accommodating varying levels of skewness and kurtosis. Furthermore, it is closed under convolution for fixed values of the skewness and kurtosis parameters. Changing the distributional assumption gives us a more flexible model while at the same time retaining the analytical properties of the original GARJI model.

2.2 The Normal Inverse Gaussian distribution

The density function of the NIG distribution using the location scale invariant parameterization, $\bar{\alpha} = \alpha\delta$ and $\bar{\beta} = \beta\delta$, is given by

$$f\left(x; \bar{\alpha}, \bar{\beta}, \mu, \delta\right) = \frac{\bar{\alpha}}{\pi\delta} \exp\left[\sqrt{\bar{\alpha}^2 - \bar{\beta}^2} + \bar{\beta}\frac{(x-\mu)}{\delta}\right] q\left(\frac{x-\mu}{\delta}\right)^{-1} \times K_1\left(\bar{\alpha} q\left(\frac{x-\mu}{\delta}\right)\right), \tag{1}$$

with $0 \leq \left|\bar{\beta}\right| < \bar{\alpha}$, $\delta > 0$ and $q(z) = \sqrt{1+z^2}$. Here, $K_1(\cdot)$ is the modified Bessel function of third order and index one. The parameter $\bar{\alpha}$ controls the kurtosis of the distribution and $\bar{\beta}$ the asymmetry. The location and scale of the distribution is decided by μ and δ, respectively. For financial applications of the NIG distribution see e.g. Eberlein and Keller (1995), Barndorff-Nielsen (1997) as well as Forsberg and Bollerslev (2002) and references therein. The NIG distribution nests several distributions including the normal distribution $N(\mu, \sigma^2)$, as can be seen by setting $\beta = 0, \alpha \to \infty$ and $\delta/\alpha = \sigma^2$.

2.3 The NIG-GARJI model

Consider the return $r_t = (P_t - P_{t-1})/P_{t-1}$, with P_t being the price of a financial asset at time t. The return is modelled as

$$r_t - r_f = \mu + \sqrt{h_t \overline{\gamma} \rho} + \varepsilon_{1,.t} + \varepsilon_{2,.t}, \tag{2}$$

with r_f being the risk free rate. The parameter $\overline{\gamma} = \sqrt{\bar{\alpha}^2 - \overline{\beta}^2}$ can be interpreted as a tail thickness parameter. Furthermore, $\bar{\rho} = \bar{\beta}/\bar{\alpha} = \beta/\alpha$ is a measure of skewness. More details on these parameters are given in e.g. Barndorff-Nielsen and Prause (2001). The first part of the return, $\mu + \sqrt{h_t \overline{\gamma} \rho} + \varepsilon_{1,t}$, is equal to the specification of the NIG-S&ARCH model of Jensen and Lunde (2001). Here, μ is a constant compensating for risk and $\sqrt{\overline{\gamma}}\bar{\rho}$ compensates for the time varying (continuous) volatility risk, $\sqrt{h_t}$. The continuous return innovation, $\varepsilon_{1,.t}$, is given by $\varepsilon_{1,.t} = \sqrt{h_t} z_t$ and the jump innovation $\varepsilon_{2,t}$ is defined as $\varepsilon_{2,t} = \sum_{k=0}^{\eta_t} J_{k,t} - \lambda_t \left(\mu_j + \frac{\overline{\rho}\delta_j}{\sqrt{1-\overline{\rho}^2}} \right)$. Both innovations are conditionally mean zero. The distribution of the standardized residual, z_t, is $NIG(\overline{\alpha}, \overline{\beta}, -\sqrt{\overline{\gamma}\rho}, \overline{\gamma}^{3/2}/\overline{\alpha})$ giving mean zero and unit variance.

The conditional variance evolves according to

$$\begin{aligned} h_t &= \omega + \exp\left(\kappa_1 + \kappa_{1,j} * F_{t-1} + I\left(\kappa_{1,a} + \kappa_{1,j,a} * F_{t-1}\right)\right) \times \\ &\quad \left(\varepsilon_{1,t-1} + \varepsilon_{2,t-1}\right)^2 + \kappa_2 h_{t-1}, \end{aligned} \tag{3}$$

with

$$F_{t-1} = \frac{f\left(r_t | \eta_t = j, \Omega_{t-1}\right) \exp(-\lambda_t) \lambda_t^j / j!}{f\left(r_t | \Omega_{t-1}\right)} \; j = 0, 1, 2... \tag{4}$$

being the filtered number of jumps and λ_t is the jump intensity parameter specified in equation (5) below. I is an indicator function taking the value 1 if $\varepsilon_{1,t-1} + \varepsilon_{2,t-1} < 0$ and zero otherwise. The variance specification is equal to that of Maheu and McCurdy (2004) and allows for four different responses depending on if there is a jump and on the sign of the sum of the jump and the normal residuals. The effect of positive normal news is given by κ_1 and the effect of negative normal news are given by $\kappa_1 + \kappa_{1,a}$. The effect of positive news when there is one jump is given by $\kappa_1 + \kappa_{1,j}$ and finally the effect of negative news in the presence of one jump is given by $\kappa_1 + \kappa_{1,j} + \kappa_{1,a} + \kappa_{1,j,a}$.

2.3.1 Jump intensity

The dynamics for the jump intensity are given by

$$\lambda_t = \lambda_0 + \varphi \lambda_{t-1} + \phi \xi_{t-1}, \tag{5}$$

where $\xi_{t-1} = F_{t-1} - \lambda_{t-1}$ is the expected (filtered) number of jumps at time $t-1$ given time $t-1$ information minus the expected number of jumps at time $t-1$ given $t-2$ information. Hence, ξ_{t-1} has the interpretation of an innovation to the jump arrival process. The jump size J_t is distributed as $NIG(\overline{\alpha}, \overline{\beta}, \mu_j, \delta_j)$, meaning that the jump size has the same shape parameters as the GARCH type residual z_t but it is allowed to have different scale and location parameters. It should be emphasized that setting the shape parameters in the jump size distribution and in the GARCH type residual equal is less restrictive than assuming normality as was done in the original GARJI model. The specification for the jump dynamics (5) is identical to those of Maheu and McCurdy (2004). The GARJI model is obtained as the special case when $\bar{\beta} = 0, \bar{\alpha} \to \infty$ and $\delta_j/\alpha = \sigma^2$. The NIG-GARCH model is the special case obtained when $\kappa_{1,j} = \kappa_{1,a} = \kappa_{1,j,a} = \bar{\beta} = \lambda_0 = \varphi = \phi = \mu_j = \delta_j = 0$.

2.3.2 Conditional moments

The conditional moments are calculated using the moment results for the NIG distribution that are given in e.g. Jensen and Lunde (2001) together with appendix A in Das and Sundaram (1997). To simplify notation we denote the mean jump size, $\mu_j + \frac{\overline{\rho}\delta_j}{\sqrt{1-\overline{\rho}^2}}$, by μ_j^* and the jump variance, $\frac{\delta_j^2}{\bar{\alpha}\left(1-\overline{\rho}^2\right)^{1.5}}$, by δ_j^*. The first four conditional moments of the model are then given by

$$E[r_t|\Omega_{t-1}] = \mu + \overline{\rho}\sqrt{h_t\overline{\gamma}}, \tag{6}$$

$$Var[r_t|\Omega_{t-1}] = h_t + \lambda_t \left(\left(\mu_j^*\right)^2 + \delta_j^* \right), \tag{7}$$

$$Skew[r_t|\Omega_{t-1}] = \frac{3h_t^{1.5}\frac{\overline{\rho}}{\left(1-\overline{\rho}^2\right)^{0.25}\overline{\alpha}^{0.5}} + \lambda_t\left(\frac{3\delta_j^2\mu_j^*}{\bar{\alpha}^2\left(1-\overline{\rho}^2\right)^2} + 3\delta_j^*\mu_j^* + \left(\mu_j^*\right)^3\right)}{\left(h_t + \lambda_t\left(\left(\mu_j^*\right)^2 + \delta_j^*\right)\right)^{1.5}}, \tag{8}$$

$$Kurt[r_t|\Omega_{t-1}] \quad = \quad 3\left(\begin{array}{c} h_t^2\left(\frac{4\overline{\rho}^2+1}{\bar{\alpha}\left(1-\overline{\rho}^2\right)^{0.5}}\right) + \\ \lambda_t\left(\begin{array}{c}\frac{\delta_j^4\left(4\overline{\rho}^2+1\right)}{\bar{\alpha}^3\left(1-\overline{\rho}^2\right)^{3.5}} + \frac{4\delta_j^3\overline{\rho}\mu_j^*}{\bar{\alpha}^2\left(1-\overline{\rho}^2\right)^{2.5}} + \left(\delta_j^*\right)^2 \\ +2\delta_j^*\left(\mu_j^*\right)^2 + \left(\mu_j^*\right)^4/3\end{array}\right)\end{array}\right) \left(h_t + \lambda_t\left(\delta_j^* + \left(\mu_j^*\right)^2\right)\right)^{-2} + 3. \tag{9}$$

As seen above, the conditional variance can be divided into a continuous part h_t and a jump part that will be zero if $\lambda_t = 0$. For the case with no asymmetry in the distribution $\beta = 0 \Longrightarrow \overline{\rho} = 0$ so the mean jump size simplifies to μ_j and the jump variance to $\frac{\delta_j^2}{\alpha\delta}$. The expression for the conditional skewness shows that skewness can be attained from a non-zero $\overline{\rho}$, meaning that the return innovation distribution and jump size distribution are both skewed. Skewness can also be attained from a jump size, μ_j^*, different from zero. For the special case when the number of jumps is zero, the skewness simplifies to $\frac{3\overline{\rho}}{\left(1-\overline{\rho}^2\right)^{0.25}\overline{\alpha}^{0.5}}$ and the kurtosis to $3\left(\frac{4\overline{\rho}^2+1}{\bar{\alpha}\left(1-\overline{\rho}^2\right)^{0.5}} + 1\right)$ which are the same conditional moments as for the NIG-S&ARCH model of Jensen and Lunde (2001). Since $\overline{\rho}$ is constant during the sample period the time-variation in the conditional skewness and conditional kurtosis are induced by time-variation in the jump intensity parameter λ_t and, just as for the GARJI model, also by time-variation in the conditional variance h_t.

2.4 Likelihood function

Given that $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are contemporaneously independent then conditional on j jumps occurring it is easily found using the convolution property of the NIG distribution that

$$r_t\left(|\eta_t=j,\Omega_{t-1}\right)\sim NIG(\overline{\alpha},\overline{\beta},\mu+(j-\lambda_t)\left(\frac{\overline{\rho}\delta_j}{\sqrt{1-\overline{\rho}^2}}+\mu_j\right),\sqrt{h_t}\overline{\gamma}^{3/2}/\overline{\alpha}+j\delta_j),\quad (10)$$

so the conditional distribution of r_t for a given number of jumps is

$$\begin{aligned} f\left(r_t|\eta_t=j,\Omega_{t-1}\right) &= \frac{\overline{\alpha}}{\pi\delta_t^*}\exp\left(\overline{\gamma}+\frac{\overline{\beta}\left(r_t-\mu_t^*\right)}{\delta_t^*}\right)\times \\ & q\left(\frac{r_t-\mu_t^*}{\delta_t^*}\right)^{-1}\times K_1\left(\overline{\alpha}q\left(\frac{r_t-\mu_t^*}{\delta_t^*}\right)\right), \end{aligned} \quad (11)$$

with $\mu_t^*=\mu+(j-\lambda_t)\left(\frac{\overline{\rho}\delta_j}{\sqrt{1-\overline{\rho}^2}}+\mu_j\right)$ and $\delta_t^*=\sqrt{h_t}\overline{\gamma}^{3/2}/\overline{\alpha}+j\delta_j$. The number of jumps j is latent but with a known distribution so we can just integrate it out of the expression, resulting in the likelihood function

$$f\left(r_t|\Omega_{t-1}\right)=\sum_{j=0}^{\infty}f\left(r_t|\eta_t=j,\Omega_{t-1}\right)*\exp(-\lambda_t)\lambda_t^j/j!. \quad (12)$$

In the empirical part we find that the contribution to the likelihood is negligible for $j>8$ so we truncate the infinite sum in (12) at $j=8$. Gauss code for estimation of the model is available from the authors.

3 Value-at-Risk

Value-at-Risk is the maximum loss expected to incur over a certain time period (h) with a given probability α. Statistically, $VaR_t\left(\alpha,h\right)=F_{t+h}^{-1}\left(\alpha\right)|\Omega_t$, where F_{t+h}^{-1} is the h-step conditional forecast of the inverse cumulative distribution function (CDF) of the return.

There is much interest in the measure because of the ongoing adoption of Basel II, which allows banks to use internal VaR models for the purpose of regulating capital requirements. For a survey, see for example Duffie and Pan (1997) or the textbook treatment in Jorion

(2000).

3.1 VaR computation and decomposition

Using the fact that the conditional distributions of the GARJI and NIG-GARJI models are probability weighted sums of the conditional distributions for fixed values of j and by exchanging the order of summation and integration , the CDFs for the two models can be expressed as

$$F_{Total}(x) = \sum_{j=0}^{\infty} \int_{-\infty}^{x} f\left(r_t|\eta_t = j, \Omega_{t-1}\right) * \exp(-\lambda_t)\lambda_t^j/j!. \tag{13}$$

The contribution to the total CDF from the continuous component is given by setting $j = 0$ so that

$$F_{Cont}(x) = \int_{-\infty}^{x} f\left(r_t|\eta_t = 0, \Omega_{t-1}\right) * \exp(-\lambda_t). \tag{14}$$

For the NIG-GARJI model $f\left(r_t|\eta_t = j, \Omega_{t-1}\right)$ is given in (11) and for the GARJI model by

$$f\left(r_t|\eta_t = j, \Omega_{t-1}\right) = \frac{1}{\sqrt{2\pi\left(\sigma_t^2 + j\delta_j^2\right)}} \exp\left(-\frac{r_t - \mu + \theta\lambda_t - \theta j}{2\left(\sigma_t^2 + j\delta_j^2\right)}\right). \tag{15}$$

Total Value-at-Risk (VaR_{Total}) is computed by numerical inversion of (13) and continuous Value-at-Risk (VaR_{Cont}) is computed by inverting the CDF (numerically) conditioned on the number of jumps being equal to zero and then multiplying with the probability of getting zero jumps ($\exp(-\lambda_t)$). Trunctation of the sum is made at $j = 8$. The jump component of Value-at-Risk is defined as $VaR_{jump} = VaR_{Total} - VaR_{Cont}$. Matlab code for the VaR computation is available from the authors.

3.2 Specific risk and event risk

The Basel accord (Basel 2006) distinguishes between two types of market risk: general and specific with the event risk being a component of the specific risk. It further states that most banks use models that only incorporate general risk and they should therefore have a separate capital charge added for the specific risk.

Specific risk is defined as *"... the risk that an individual debt or equity security moves by more or less than the general market in day-to-day trading ... "* and event risk is defined as *"where the price of an individual debt or equity security moves precipitously relative to the general market"* Basel (2006) page 163. Event risk can be thought of as jump risk as pointed out by Gibson (2001). However, Gibson argues that jump risk and event risk are the same thing. Since event risk is defined as a precipitous move *in relation* to the market, this only holds true if there are no market wide jumps that affect all or most stocks. Contrary to this, we find in the empirical part of this paper that almost 1/3 of the VaR in the market index is due to the jump component. If jumps were purely idiosyncratic (asset specific), their effect would be diversified away in the market portfolio. We therefore propose to measure the average proportion of event risk in a position i over the period $[t, T]$ as

$$VaR_{event,i} = \frac{\sum\limits_{t=1}^{T} \max\left(VaR_{jump,i,t} - VaR_{jump,m,t}, 0\right)}{\sum\limits_{t=1}^{T} VaR_{total,i,t}} \in [0,1]\,, \tag{16}$$

with $VaR_{jump,m,t}$ being the VaR due to the jump component for the market portfolio on day t. This quantitative definition, in accordance with the qualitative definition in Basel II, implies that the market as a whole cannot have any event risk. Furthermore, we only measure event risk as VaR jump risk that is greater than the market's jump risk each day. We do this since the VaR jump contribution of an asset consists of both the VaR risk from asset specific jumps and from jump risk common to the whole market.

Subtracting the market's VaR jump risk gives us a measure that is in better accordance with the Basel definition of the event risk. If the market-wide jump risk is greater than the jump risk of the asset for a given day we set the asset-specific event risk to zero since negative event risk lacks any natural interpretation. The model in this paper can be used in conjunction with the above definition to measure the proportion (or absolute level if one prefers) of the event risk in an asset. However, the suggested definition in (16) is more general and not dependent on the particular model being used.

It is worth pointing out that since VaR_{event} is always less than or equal to VaR_{jump} we will have the relationship $VaR_{cont} + VaR_{event} \leq VaR_{Total}$, so we do not propose that capital adequacy should be computed from $VaR_{cont}+VaR_{event}$. It should still be computed from VaR_{Total}, but being able to measure the part of total VaR that stems from event risk is useful both for regulators and banks. For regulators, who have to impose additional capital charges on those bank's whose models do not capture event risk, measuring the event risk will help to determine an appropriate magnitude of the surcharge. For a bank, showing that it uses a model that can properly measure event risk, lets the bank avoid the additional capital charge. Further, for a bank to hedge its positions, it may be helpful to know how much of its jump risk exposure stems from the market component (which can be hedged by a short position in the market) and how much stems from the event risk component which cannot easily be hedged.

4 Data

Evaluation of VaR is a study of extreme events which makes it important to use a long series of data. As a proxy for the market we use the value weighted market index from the CRSP record available from the data library at Professor Kenneth French's homepage. From the same source we also get the risk free rate proxied by the 30 day T-bill. Furthermore, we use three portfolios sorted on market capitalization, Size 1 (smallest 30% of the market), Size 2 (Middle 40% of the market) and Size 3 (largest 30% of the market). These series are also available from Professor French's data library.

In addition to this we use data on the 10 individual stocks with start dates in parenthesis: Amgen (17 June, 1983), Apple (December 12, 1980), Coca Cola (KO, July 1, 1963), General Motors (GM, July 1, 1963), Home Depot (HD, September 22, 1981), Hewlett-Packard (HWP, July 1, 1963), Intel (December 14, 1972), Johnson & Johnson (J&J, July 1, 1963), Motorola (MOT, July 1, 1963) and Texaco (July 1, 1963). The end date for all the individual stocks is June 29, 2007 except for Texaco which ends October 9, 2001. The market index data is from July 1, 1963 to September 28, 2007. For the three size portfolios the start date is July 1, 1963 and the end date is August 31, 2007. As seen in table 1, normality is clearly rejected with p-values from the Jarque and Bera (1987) test less than 0.001 for all the data series. The market and the three size sorted portfolios all have considerable negative skewness and excess kurtosis. While most of the individual stocks also have negative skewness, all are less left skewed than the market portfolio. It is commonly the case that stock indices are more left skewed than individual stocks as documented in Kim and Kon (1994).

[Insert table 1 here]

5 Results

The interpretation of the parameter estimation results focus on the market index for brevity but the parameter estimates for the three size sorted portfolios are also displayed in table 2. The parameter estimates and residual diagnostics for the 10 individual stocks are readily available from the authors upon request.

5.1 Variance equation

The effects of jumps and the effect of the sign of the return innovation on the volatility process can be seen from the four parameters in the variance equation. The effect of a positive return innovation when there is no jump is given by κ_1 $\left(e^{-2.8906} = 0.05\,6\right)$ for the

NIG-GARJI model and 0.015 for the GARJI model. For a negative return innovation with no jump the effect for the NIG-GARJI model is $\kappa_1 + \kappa_{1,a}$ $\left(e^{-2.8906+1.2570} = 0.1952\right)$ and 0.076 for the GARJI model showing that the leverage effect (Black, 1976) is present in the sample. The effect of a positive return innovation when there is one jump is given by $\kappa_1 + \kappa_{1,j}$ $\left(e^{-2.8906+(-12.0955)} = 3.1 \times 10^{-7}\right)$ and by 9.3×10^{-3} for the GARJI model showing that positive jumps does not lead to higher future volatility. For a negative return innovation when there is one jump the effect is given by $\kappa_1 + \kappa_{1,a} + \kappa_{1,j} + \kappa_{1,j,a}$ $\left(e^{-2.8906+1.2570+(-12.0955)+10.9816} = 0.064\right)$ and for the GARJI model 0.030. From this we can see that the effect on the squared residual is lower (the mean reversion rate is higher) when we have a jump, leading to lower persistence in the jump component consistent with the findings in Maheu and McCurdy (2004) and Andersen et al. (2007).

[Insert table 2 here]

5.2 Shape parameters and jump equation

Including jumps in the models seems justified from the drastic improvement in log likelihood value from the NIG-GARCH to the NIG-GARJI model. A likelihood ration test shows that the NIG-GARJI model is favored over the NIG-GARCH model for all the four reported data sets with p-values less than 0.001.

In the NIG-GARJI model the observed sample skewness of -0.74 and excess kurtosis of 17.83 can both be accommodated by a fat tailed and skewed error distribution or by the jump component. This makes it possible to examine if skewness and excess kurtosis is due to more extreme news events (picked up by the jump component) or if they are due to normal news (picked up by the GARCH residual). In the original GARJI model this cannot be examined since all skewness and excess conditional kurtosis must per construction be modeled by the jump component.

The skewness parameter $\bar{\beta} = -0.0467$ is insignificant but the jump size location parameter μ_j is equal to -0.51 and significant, showing that skewness "prefers" to be modeled by the jump component. It thus appears that skewness is better modeled as a result of rare and more extreme return innovations than as a result of normal news innovations being drawn from a skewed distribution. This cannot be accommodated by other popular models with time varying skewness such as the Autoregressive Conditional Density model of Hansen (1994).

The tail thickness parameter $\bar{\alpha} = 3.34$ (implies a kurtosis of around 4 in the standardized residual z_t) indicates that some of the excess kurtosis "prefers" to be modeled by the GARCH residual. One caveat to the above interpretation of the shape parameters is that the jumpsize distribution and the normal news residual have to share the same shape parameters.

The ARMA parameters, φ and ϕ, in the jump equation are significant in both the GARJI and NIG-GARJI models showing that assuming a constant jump intensity is too restrictive. The unconditional average number of jumps is $\lambda_0/(1-\varphi) = 0.17$ for the NIG-GARJI model and 0.13 for the GARJI model, this would at first seem contrary to the fact that more of the tails of the distribution can be captured by the normal news innovation in the NIG-GARJI model, as seen from figure 1. The reason for this, as can also be seen in figure 1, is that the jump distributions for the GARJI and NIG-GARJI models are very different. The GARJI model has much higher jump variance which leads the jump proportion of both the standard deviation (45% for the GARJI model and 31% for the NIG-GARJI model) and the jump proportion of VaR at the 0.5% level (38% instead of 30%) to be higher for the GARJI model.

[Insert figure 1 here]

5.3 Higher moment dynamics

During the sample period the conditional skewness varies from -0.86 to -0.02 for the NIG-GARJI model and from -1.84 to -0.03 for the GARJI model with the lowest (most negative) skewness during the 1960s for both models. The conditional kurtosis varies from 3.52 to

5.42 for the NIG-GARJI model and from 3.05 to 23.35 for the GARJI model, the highest kurtosis is also found during the 1960s when the conditional variance for both models was very low.[3].

We compute the conditional skewness and kurtosis with parameter values from the estimation on the CRSP market index to study if the conditional skewness and conditional kurtosis of the NIG-GARJI model is primarily affected by changes in the conditional variance or by changes in the jump intensity. By using equation (8) and by setting λ_t equal to its average value, we find that the skewness varies from -1.03 when $h_t = 0.04$ (the lowest value in the sample) to -0.02 when $h_t = 15.61$ (the highest value in the sample). By using equation (9) we see that the kurtosis changes from 5.16 to 3.89 for the same changes in conditional variance. The size of these changes can be compared to the effects of changes in λ_t. For the CRSP market index, λ_t varies from a minimum of 0.07 to a maximum of 1.01 which results in changes in skewness from -0.04 to -0.17 and in kurtosis from 3.89 to 3.55 when keeping h_t fixed on its average value. Interestingly, the conditional kurtosis can be both increasing and decreasing in λ_t depending primarily on the size of the variance parameter δ_j in the jump size distribution.

5.4 Model diagnostics

The residual diagnostics in table 3 show the heteroscedasticity adjusted Ljung-Box test of West and Cho (1995) for remaining serial correlation in the squared standardized residual z_t and in the jump residual ξ_t. The GARJI model shows no significant remaining autocorrelation in the squared residual, $\frac{(\varepsilon_{1,.t}+\varepsilon_{2,.t})^2}{VaR(r_t|\Omega_{t-1})}$, when fitted to the market index and to the large cap portfolio (Size 3) but for the two smaller size sorted portfolios there is remaining structure in the variance evident from the significant LB statistics at 5 lags. The pattern is the same for the NIG-GARJI model.

The jump innovations show significant remaining autocorrelation for the NIG-GARJI model for three of the four test portfolios and for the GARJI model for the Size 1 and

Size 2 portfolios. We have tried different specifications for the jump equation without being able to mitigate this problem. For the ten individual stock returns (unreported) the residual diagnostics generally show much less autocorrelation indicating that both the NIG-GARJI and the GARJI model are better at capturing the variance and jump dynamics in individual stocks than in portfolios of stocks.

[Insert table 3 here]

To test if the models can produce realistic VaR estimates we use the Likelihood Ratio tests of Christoffersen (1998). These three tests allow us to ascertain if the number of VaR violations are correct (LR_{unc}), if the violations are independently distributed over time (LR_{ind}) and finally we have a joint test for independence and correct number of violations (LR_{cc}). The tests are performed on the indicator series I_t defined as

$$I_t = \begin{cases} 1, \ if \, r_t > VaR_{\alpha,t} \, |\Omega_{t-1} \\ 0 \ Otherwise \end{cases} \tag{17}$$

with t being a time subscript and α being the VaR level. This means that I_t will be 0 each time there is a violation (the loss is larger than the VaR level) and otherwise 1. The test statistics (LR_{unc}) and (LR_{ind}) are asymptotically distributed $\chi^2(1)$ and (LR_{cc}) is distributed $\chi^2(2)$. We perform these test for the VaR levels 0.5%, 1%, 2%, 3%, 4%, and 5% for both long and short positions (left and right tails of the distribution) in the CRSP market index. The results are displayed in table 4.

[Insert table 4 here]

Even with 11,138 observations the expected number of VaR violations is rather small at the 0.5% level (55.7 violations) so we simulate the distribution of the test statistics instead of relying on the asymptotics. This is easily done since the indicator series is *iid* Bernoulli$(1-\alpha)$ under the null hypothesis. The reader is referred to Christoffersen (1998) for further details on the tests and to Christoffersen and Pelletier (2004) for details of the simulation design.

Both models perform very well with the GARJI model perhaps performing somewhat better for long position and the NIG-GARJI model performs somewhat better for short positions. The NIG-GARJI model cannot be rejected as having the wrong number of rejections for any of the VaR levels at the 1% significance level and the GARJI model can only be rejected at the 0.5% VaR level.

The results from the independence tests are mixed but at least at the higher VaR levels (4% and 5%) both models seem to produce VaR violations that are clustered over time. This can be expected given the residual diagnostics test which showed remaining dependence in the innovations to the jump process.

By using the LR_{cc} test we examine if the models can produce a correct number of violations that are also independent over time. The GARJI model performs better and can only be rejected at the 0.5% VaR level at the 1% significance level. At this significance level the NIG-GARJI model can be rejected at 3 of the 12 VaR levels.

5.5 Volatility decomposition

The higher jump variance in the GARJI model results in an average 45.47% of the total standard deviation being due to jumps compared with 31.12% for the NIG-GARJI model, see figure 2. From the figure it can be seen that the jump component of the GARJI model varies considerably more over the sample period whereas for the NIG-GARJI model it is rather stable at around 3.4% expressed as yearly standard deviation. However, the proportion of jump risk decreases slightly over time. The average level of total volatility is close for the two models with 12.68% for the GARJI and 12.37% for the NIG-GARJI model compared with the sample standard deviation of 13.97%.

[Insert figure 2 here]

5.6 Value-at-Risk decomposition

We use the NIG-GARJI and GARJI models to decompose the proportion of the total VaR that is attributable to the jump component. The results can be seen in figure 3 and table 5. For the CRSP market index the 1% VaR from a long position consists of 29.38% jump risk compared with 30.07% for the 5% VaR. The jump risk proportion behaves much more erratically over time for the GARJI model because of the higher estimated jump variance in this model.

[Insert table 5 here]

[Insert figure 3 here]

For a short position, the jump proportion of the VaR is considerably smaller with 15.74% for the 1% VaR and 14.61% for the 5% VaR. The reason for this asymmetry is the negative average jump size of -0.51.

For the three size sorted portfolios we see a very clear pattern: the jump VaR is decreasing when the size of the firms is increasing. The smallest 30% of the stocks in the CRSP database (Size 1) have 60.02% jump VaR for the 1% VaR level. This can be compared to 42.30% for Size 2 and with 6.35% for the largest 30% of the companies. For a short position, the pattern is the same but the proportion of jump VaR is smaller because all the three size portfolios have negative average jump sizes.

Since the market portfolio holds a rather large degree of jump risk we would overestimate the event risk (in the Basel sense) in individual assets by equating jump risk and event risk. We therefore instead use our definition in (16) to calculate the event risk. The event risk proportions for each day are displayed in figures 4, 5 and 6 for the size sorted portfolios and the average event risk percentage for the size portfolios and for the 10 individual stocks are given in table 6.

[Insert table 6 here]

[Insert figures 4,5 and 6 here]

The proportion event risk varies greatly between the different assets. The small stock portfolio has an event risk of roughly 30% of the total VaR, the mid cap portfolio has around 13-14% event risk and the large cap portfolio has almost no event risk with a proportion less than 1%. We would like to emphasize that the large cap portfolio constitutes 30% of the value of the market portfolio and so does the portfolio of small cap stocks. The inverse relation between event risk and market size can therefore not simply be a mechanical artifact of subtracting the market's jump induced VaR.

The individual stocks have large variations in event risk. Motorola and General Motors have an event risk of 1%-2% whereas for Apple, Hewlett Packard and J&J the event risk is around 30% of the total VaR. For all the individual stocks the event risk is higher for short than for long positions even though the jump risk is lower for short than long positions for some of the stocks. This happens because even though some of the individual stock returns are left skewed (explaining the lower jump proportion for short positions) they are less left skewed than the market resulting in a higher event risk for short than long positions.

6 Conclusions

Models such as the NIG-GARJI that can capture and measure event risk are important for banks since they can use the models to better calculate their VaR with the added benefit of avoiding capital surcharges from regulators. Such models should also be of interest to supervisors since they have to be able to quantify the event risk in an asset or portfolio of assets to add a capital surcharge of appropriate magnitude.

We show that the Value-at-Risk of the market consists of about 30% jump risk and hence equating event risk and jump risk as suggested by Gibson (2001) would seriously overestimate the event risk since it is defined as a precipitous move *relative* to the market. We further show that the event risk varies much across different assets. The total VaR of a portfolio of the 30% smallest companies of the market index is found to consist of around 30% event risk. In contrast to this, a portfolio of the 30% largest companies has an event risk proportion less than 1%.

Also for individual stocks we find a large variation in the event risk across stocks but a rather small variation in the event risk for a given stock over time. The observed variation highlights the importance for the regulator to vary the capital surcharge, based on the type of assets, imposed on banks that do not properly model the event risk. Furthermore, the finding that the event risk for some assets constitutes nearly 1/3 of the total VaR shows it to be very important for banks to incorporate the event risk in their internal VaR models.

Legends

Legend 1: This figure shows the probability density plots for the jump size distribution (top left) and for the standardized return innovation distribution (top right), for the NIG-GARJI and GARJI models. The bottom left plot magnifies the left tail and the bottom right plot magnifies the right tail of the standardized return innovation distribution.

Legend 2: This figure shows the jump component in the standard deviation (top), the total standard deviation (middle) and the jump proportion (bottom) of the CRSP market portfolio from July 1, 1963 to September 28, 2007. The results for the NIG-GARJI model are to the left and for the GARJI model to the right.

Legend 3: This figure shows the jump proportion of the total Value-at-risk for the 1% VaR (top) and 5% VaR (bottom) of the CRSP market portfolio from July 1, 1963 to September 28, 2007. The results for the NIG-GARJI model are to the left and for the GARJI model to the right.

Legend 4: This figure shows the jump proportion of the total Value-at-risk for the 1% VaR long position (top) and 1% VaR short position (bottom) in the small cap portfolio from July 1, 1963 to August 31, 2007 for the NIG-GARJI model.

Legend 5: This figure shows the jump proportion of the total Value-at-risk for the 1% VaR long position (top) and 1% VaR short position (bottom) in the medium cap portfolio from July 1, 1963 to August 31, 2007 for the NIG-GARJI model.

Legend 6: This figure shows the jump proportion of the total Value-at-risk for the 1% VaR long position (top) and 1% VaR short position (bottom) in the large cap portfolio from July 1, 1963 to August 31, 2007 for the NIG-GARJI model.

References

Andersen, T., Bollerslev, T. & Diebold, F. (2007), 'Roughing it up: Including jump components in the measurement, modeling and forecasting of return volatility', *Review of Economics and Statistics* **89**, 701–720.

Andersen, T. G., Bollerslev, T., Diebold, F. X. & Labys, P. (2003), 'Modeling and forecasting realized volatility', *Econometrica* **71**, 579–625.

Bali, T., Hengyong, M. & Tang, Y. (2008), 'The role of autoregressive conditional skewness and kurtosis in the estimation of conditonal VaR', *Journal of Banking and Finance* **32**, 269–282.

Barndorff-Nielsen, O. E. (1997), 'Normal inverse Gaussian distributions and stochastic volatility modelling', *Scandinavian Journal of Statistics* **24**, 1–14.

Barndorff-Nielsen, O. E. & Prause, K. (2001), 'Apparent scaling', *Finance and Stochastics* **5**, 103–113.

Barndorff-Nielsen, O. E. & Shephard, N. (2004), 'Power and bipower variation with stochastic volatility and jumps (with discussion)', *Journal of Financial Econometrics* **2**, 1–48.

Barndorff-Nielsen, O. E. & Shephard, N. (2006), 'Impact of jumps on returns and realised variances: econometric analysis of time-deformed Lévy processes', *Journal of Econometrics* **131**, 217–252.

BASEL (2006), 'International convergence of capital measurement and captial standards. A revised framework- Comprehensive version', *Basel Committee on Banking Supervision* .

Black, F. (1976), 'Studies of stock price volatility changes', *Proceedings of the Business and Economic Statistics Section, American Statistical Association* pp. 177–181.

Bollerslev, T. & Forsberg, L. (2002), 'Bridging the gap between the distribution of realized (ECU) volatility and ARCH modeling (of the Euro): The GARCH normal inverse Gaussian model', *Journal of Applied Econometrics* **17**, 535–548.

Christoffersen, P. (1998), 'Evaluating interval forecasts', *International Economic Review* **39**, 841–862.

Christoffersen, P. & Pelletier, D. (2004), 'Backtesting Value-at-Risk: A duration-based approach', *Journal of Financial Econometrics* **2**, 84–108.

Das, S. & Sundaram, R. (1997), 'Taming the skew: Higher-order moments in modeling asset price processes in finance', *Working Paper 5976, National Bureau of Economic Research* .

Duffie, D. & Pan, J. (1997), 'An overview of value at risk', *The Journal of Derivatives* **4**, 7–49.

Eberlein, E. & Keller, U. (1995), 'Hyperbolic distributions in finance', *Bernoulli* **1**, 281–299.

Engle, R. & Manganelli, S. (2004), 'CAViaR: Conditional autoregressive value at risk by regression quantiles', *Journal of Business and Economic Statistics* **22**, 367–381.

Eraker, B., Johannes, M. & Polson, N. G. (2003), 'The impact of jumps in returns and volatility', *Journal of Finance* **53**, 1269–1300.

Gibson, M. (2001), 'Incorporating event risk into value-at-risk', *FEDS Discussion Paper No. 2001-17* .

Hansen, B. E. (1994), 'Autoregressive conditional density estimation', *International Economic Review* **35**, 705–730.

Harvey, C. & Siddique, A. (1999), 'Autoregressive conditional skewness', *Journal of Financial and Quantitative Analysis* **34**, 465–488.

Harvey, C. & Siddique, A. (2000), 'Conditonal skewness in asset pricing tests', *Journal of Finance* **55**, 1263–1295.

Jarque, C. & Bera, A. (1987), 'A test for normality of observations and regression residuals', *International Statistical Review* **55**, 163–172.

Jensen, M. B. & Lunde, A. (2001), 'The NIG-S&ARCH model: a fat tailed, stochastic, and autoregressive conditional heteroskedastic volatility model', *Econometrics Journal* **4**, 319–342.

Jorion, P. (2000), *Value at Risk. The new benchmark for managing financial risk*, McGraw-Hill, New York.

Kim, D. & Kon, S. (1994), 'Alternative models for the conditional heteroskedasticity of stock returns', *Journal of Business* **67**, 563–598.

Kuester, K., Mittnik, S. & Paolella, M. (2006), 'Value-at-Risk prediction: a comparison of alternative strategies', *Journal of Financial Econometrics* **4**, 53–89.

Lanne, M. (2007), 'Forecasting realized volatility by decomposition', *International Journal of Forecasting* **23**, 307–320.

Lanne, M. & Saikkonen, P. (2007), 'Modeling conditional skewness in stock returns', *European Journal of Finance* **13**, 691–704.

Maheu, J. M. & McCurdy, T. H. (2004), 'News arrival, jump dynamics and volatility components for individual stock returns', *Journal of Finance* pp. 755–793.

Mittnik, S. & Paolella, M. (2003), 'Prediction of financial downside risk with heavy tailed conditional distributions', *Working paper, Johann Wolfgang Goethe-Universität* .

West, K. & Cho, D. (1995), 'The predictive ability of several models of exchange rate volatility', *Journal of Econometrics* **69**, 367–391.

Wilhelmsson, A. (2009), 'Value at Risk with time-varying variance, skewness and kurtosis - the NIG-ACD model', *The Econometrics Journal, forthcoming* .

Notes

[1] See e.g. Andersen et al. (2003), Eraker et al. (2003), Barndorff-Nielsen and Shephard (2004, 2006) and references therein.

[2] However, as pointed out by a referee, the GARJI model can accomodate non-zero unconditonal skewness through the asymmetry in the variance equation.

[3] Time series plots of the conditional moments can be obtained from the corresponding author but are left out because of space considerations.

Table 1

Descriptive statistics

This table shows the descriptive statistics for the daily CRSP market index for the sample period July 1, 1963 to September 28, 2007, the three size sorted portfolios from July 1, 1963 to August 31, 2007 and for 10 individual stocks with start dates in parenthesis: Amgen (17 June, 1983), Apple (12 December, 1980), Coca Cola (KO, July 1, 1963), General Motors (GM, July 1, 1963), Home Depot (HD, September 22, 1981), Hewlett-Packard (HWP, July 1, 1963), Intel (December 14, 1972), Johnson & Johnson (J&J, July 1, 1963), Motorola (MOT, July 1, 1963) and Texaco (July 1, 1963). The end date for all the individual stocks is June 29, 2007 except for Texaco which ends October 9, 2001. All returns are in percentage form in excess of the risk free rate. JB is the p-value from the Jarque and Bera (1987) test with the null hypothesis of normally distributed returns.

Measure	CRSP	Size1	Size2	Size3	Amgen	Apple	Ko	GM	HD	HWP	Intel	J&J	MOT	Texaco
# observations	11,138	11,119	11,119	11,119	6,063	6,698	11,075	11,075	6,500	11,075	8,716	11,075	11,075	9,635
Daily mean	0.022	0.028	0.028	0.021	0.103	0.084	0.042	0.017	0.117	0.058	0.090	0.048	0.049	0.026
Yearly standard deviation	13.97	13.02	13.68	14.47	43.81	50.55	24.19	27.279	37.54	35.74	45.70	24.30	38.05	24.28
Maximum	8.63	6.42	7.95	8.763	20.49	33.20	19.65	18.10	22.60	20.90	26.35	15.88	25.42	12.63
Minimum	-17.16	-10.67	-13.34	-18.85	-20.01	-51.89	-24.72	-21.05	-28.76	-20.32	-29.60	-18.38	-23.06	-13.21
Skewness	-0.74	-0.94	-0.66	-0.77	0.22	-0.38	-0.08	0.18	-0.37	0.08	-0.16	0.05	-0.04	0.17
Excess kurtosis	17.83	9.91	10.41	20.86	5.09	16.54	12.55	6.80	11.55	5.62	6.31	6.40	7.05	5.65
JB	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

Table 2

Estimation results on daily percentage returns

This table shows the maximum likelihood parameter estimates, with standard errors from the final update of the Hessian matrix below, from the estimation of the models on the CRSP market index returns from July 1, 1963 to September 28, 2007 (11,138 observations), and the three size sorted portfolios from July 1, 1963 to August 31, 2007. LL gives the log likelihood values (constant terms included) of the models.

*By restriction.

	CRSP market index, value weighted			Size 1			Size 2			Size 3		
Parameter	NIG-GARJI	GARJI	GARCH-NIG	NIG-GARJI	GARJI	GARCH-NIG	NIG-GARJI	GARJI	GARCH-NIG	NIG-GARJI	GARJI	GARCH-NIG
μ	0.0470	0.0271	0.0513	0.1259	0.0521	0.0880	0.1074	0.052	0.0819	0.0394	0.0198	0.0405
	0.0147	0.0062	0.0059	0.0156	0.0061	0.0054	0.0160	0.0062	0.0059	0.0147	0.0065	0.0061
ω	0.000001*	0.000045	0.0048	0.0009	0.0034	0.0149	0.000001*	0.0012	0.0116	0.0026	0.0025	0.0038
	.	0.0003	0.0010	0.0010	0.0010	0.0019	.	0.0007	0.0018	0.0010	0.0006	0.0008
κ_1	-2.8906	-4.2308	0.0753	-1.7902	-2.1952	0.1901	-2.2570	-2.6949	0.1289	-3.5822	-4.1238	0.0667
	0.2176	0.2759	0.0059	0.2319	0.1209	0.0129	0.1687	0.1737	0.0093	0.1428	0.2162	0.0052
$\kappa_{1,j}$	-12.0955	-0.4419		-0.8999	-0.2399		-5.3097	-0.4263		-0.9687	0.1190	
	16.469	0.4918		0.4023	0.3506		4.0920	0.35429		0.6924	0.3341	
$\kappa_{1,a}$	1.2570	1.6603		0.8383	0.8082		1.1166	1.3548		1.3032	1.8322	
	0.2798	0.2633		0.2451	0.158567		0.19706	0.190594		0.1451	0.2090	
$\kappa_{1,j,a}$	10.9816	-0.4786		0.1451	-0.3949		4.3234	-0.2430		-0.5205	-1.3321	
	16.238	0.6043		0.41997	0.40159		3.9919	0.37241		0.8667	0.4313	
κ_2	0.9259	0.9584	0.9213	0.8177	0.8174	0.8034	0.8822	0.8629	0.8613	0.9349	0.9401	0.9309
	0.0060	0.0057	0.0059	0.0144	0.0138	0.0120	0.0076	0.0098	0.0095	0.0051	0.0045	0.0051
λ_0	0.0245	0.0090		0.3716	0.1767		0.0554	0.3194		0.0041	0.0072	
	0.0244	0.0021		0.1143	0.05078		0.089347	0.06637		0.00261	0.00273	
φ	0.8564	0.9319		0.3560	0.2624		0.8091	0.1303		0.6292	0.7610	
	0.0398	0.0165		0.1457	0.18546		0.64893	0.1210		0.3127	0.09289	
ϕ	0.2315	0.4132		0.7046	0.5126		0.2486	0.6249		0.6887	0.4046	
	0.07167	0.0685		0.1249	0.12742		0.05588	0.11048		0.2286	0.1129	

table 2 continued

	CRSP market index, value weighted			Size 1			Size 2			Size 3		
Parameter	NIG-GARJI	GARJI	GARCH-NIG	NIG-GARJI	GARJI	GARCH-NIG	NIG-GARJI	GARJI	GARCH-NIG	NIG-GARJI	GARJI	GARCH-NIG
μ_j	-0.5104	-0.4431		-0.1568	-0.3396		-0.4046	-0.2825		-0.7856	-0.4728	
	0.2138	0.05870		0.02858	0.044668		0.1433	0.035357		0.4552	0.1426	
δ_j	0.1933	1.0174		0.3187	0.6492		0.2524	0.5160		2.5288	1.7088	
	0.04556	0.0633		0.07712	0.04423		0.04198	0.033569		1.6643	0.2089	
$\overline{\alpha}$	3.3401		2.1139	4.1619		1.7024	4.3513		2.1281	4.0495		2.3223
	0.3911		0.1853	0.8900		0.1301	0.6489		0.18306	0.6479		0.2112
$\overline{\beta}$	-0.0467			-0.2585			-0.2038			-0.0417		
	0.044199			0.056895			0.5588			0.04486		
LL	-12,280.29	-12,361.195	-12,455.74	-11,012.39	-11,079.67	-11,214.90	-11,823.67	-11,878.32	-12,059.66	-12,722.99	-12,757.55	-12,840.36

Table 3

Residual diagnostics

This table shows heteroscedasticity adjusted Ljung-Box tests for remaining serial correlation in the squared standardized residual $\frac{(\varepsilon_{1,.t}+\varepsilon_{2,.t})^2}{VaR(r_t|\Omega_{t-1})}$ and in the jump residual $\xi_t = F_t - \lambda_t$ for 5 and 20 lags.

	CRSP market index		Size 1		Size 2		Size 3	
	NIG-GARJI	GARJI	NIG-GARJI	GARJI	NIG-GARJI	GARJI	NIG-GARJI	GARJI
$Q^2_{z_t}(5)$	5.71	3.05	14.36	17.41	15.66	11.94	2.67	1.29
	0.335	0.693	0.007	0.004	0.007	0.036	0.750	0.992
$Q^2_{z_t}(20)$	13.75	12.18	58.10	30.51	27.48	26.04	12.93	10.22
	0.843	0.910	<0.001	0.062	0.122	0.165	0.880	0.964
$Q_{\xi_t}(5)$	61.04	5.108	131.71	72.14	196.56	76.79	7.66	3.23
	<0.001	0.403	<0.001	<0.001	<0.001	<0.001	0.176	0.664
$Q_{\xi_t}(20)$	76.44	12.48	160.58	93.48	208.08	91.51	33.66	25.06
	<0.001	0.899	<0.001	<0.001	<0.001	<0.001	0.029	0.199

Table 4

Value-at-Risk performance

Panel A of this table reports the percentage of VaR violations from the 11,138 daily VaR estimates calculated from July 1, 1963 to September 28, 2007. Also reported are the test statistics,with simulated p-values below., for the unconditional (panel B), independence (panel C) and joint test (panel D) for the null of correct conditional coverage introduced by Christoffersen (1998). 10,000 replications are used in the simulations.

Panel A: Percentage of returns below VaR

	Long position						Short position					
VaR level	0.5%	1%	2%	3%	4%	5%	0.5%	1%	2%	3%	4%	5%
NIG-GARJI	0.40	0.82	1.71	2.81	3.75	4.92	0.41	0.89	1.98	3.05	4.09	5.20
GARJI	0.32	0.83	1.96	2.99	4.05	5.11	0.60	1.11	2.23	3.19	4.18	5.06

Panel B: LR statistics and simulated p-values from the unconditional test for coverage

	Long position						Short position					
VaR level	0.5%	1%	2%	3%	4%	5%	0.5%	1%	2%	3%	4%	5%
NIG-GARJI	2.21	4.02	5.17	1.41	1.81	0.15	1.80	1.44	0.04	0.11	0.21	0.91
	0.13	0.04	0.02	0.24	0.18	0.70	0.19	0.23	0.85	0.76	0.66	0.34
GARJI	8.00	3.63	0.07	<0.01	0.07	0.28	2.17	1.39	2.82	1.32	0.97	0.09
	<0.01	0.06	0.80	0.95	0.80	0.59	0.15	0.25	0.10	0.25	0.33	0.75

Panel C: LR statistics and simulated p-values from the independence test

	Long position						Short position					
VaR level	0.5%	1%	2%	3%	4%	5%	0.5%	1%	2%	3%	4%	5%
NIG-GARJI	1.80	0.08	1.95	7.94	7.35	12.51	11.29	6.07	7.54	6.20	5.48	11.31
	0.04	0.71	0.17	<0.01	0.01	<0.01	<0.01	<0.01	0.01	0.01	0.02	<0.01
GARJI	2.58	0.07	0.62	1.56	3.18	7.47	3.29	3.38	3.09	1.85	2.83	7.12
	0.03	0.72	0.43	0.22	0.07	0.01	0.03	0.03	0.09	0.18	0.10	0.01

Panel D: LR statistics and simulated p-values from the joint test

	Long position						Short position					
VaR level	0.5%	1%	2%	3%	4%	5%	0.5%	1%	2%	3%	4%	5%
NIG-GARJI	4.01	4.10	7.13	9.35	9.15	12.67	13.09	7.52	7.58	6.30	5.69	12.22
	0.10	0.13	0.03	0.01	0.01	<0.01	<0.01	0.02	0.03	0.05	0.06	<0.01
GARJI	10.58	3.69	0.68	1.56	3.25	7.74	5.46	4.78	5.90	3.16	3.80	7.21
	<0.01	0.15	0.69	0.46	0.19	0.02	0.04	0.08	0.05	0.20	0.15	0.03

Table 5

Value-at-Risk decomposition - Proportion Jump risk

This table shows the average proportion of jump risk of the total VaR for the 1% and 5% VaR for both long and short positions in the assets. The results are for the CRSP market portfolio (Market) from July 1, 1963 to September 28, 2007, the three size sorted portfolios (Size 1, Size 2 and Size 3) from July 1, 1963 to August 31, 2007 and for 10 individual stocks. See the main text for details on dates and acronyms for the individual stocks.

	Proportion jump risk			
	1% VaR, long	5% VaR, long	1% VaR, short	5% VaR, short
Market	29.38%	30.07%	15.74%	14.61%
Size1	60.02%	59.88%	48.17%	46.11%
Size2	42.30%	43.05%	26.06%	24.53%
Size3	6.35%	3.83%	2.74%	1.96%
Amgen	23.14%	16.69%	25.15%	18.52%
Apple	36.25%	31.01%	44.78%	38.62%
KO	24.71%	20.65%	29.22%	24.14%
GM	0.89%	0.56%	3.94%	1.85%
HD	15.39%	10.03%	14.89%	9.96%
HWP	38.51%	34.93%	43.90%	40.31%
Intel	12.94%	7.94%	10.30%	6.67%
J&J	47.56%	43.97%	51.76%	48.33%
MOT	10.75%	6.28%	10.22%	6.09%
Texaco	19.47%	16.56%	28.04%	23.38%

Table 6

Value-at-Risk decomposition - Proportion Event risk

This table shows the average proportion event risk calculated according to (16) of the total VaR for the 1% and 5% VaR for both long and short positions in the assets. The results are for the CRSP market portfolio (Market) from July 1, 1963 to September 28, 2007, the three size sorted portfolios (Size 1, Size 2 and Size 3) from July 1, 1963 to August 31, 2007 and for 10 individual stocks. See the main text for details on dates and acronyms for the individual stocks.

	Proportion event risk			
	1% VaR, long	5% VaR, long	1% VaR, short	5% VaR, short
Market	0%	0%	0%	0%
Size1	29.95%	28.29%	31.18%	30.45%
Size2	13.70%	13.74%	10.09%	10.04%
Size3	0.95%	0.58%	0.47%	0.32%
Amgen	14.38%	8.74%	20.91%	14.25%
Apple	28.56%	22.86%	41.16%	34.86%
KO	9.44%	6.37%	21.57%	16.54%
GM	0.34%	0.21%	1.77%	0.92%
HD	6.56%	3.50%	10.10%	5.61%
HWP	27.49%	23.53%	38.87%	35.24%
Intel	5.11%	2.17%	6.00%	3.02%
J&J	31.26%	27.31%	44.30%	40.94%
MOT	2.30%	0.66%	5.56%	2.15%
Texaco	9.70%	7.55%	22.15%	17.84%

Figure 1

Probability density plots - NIG-GARJI and GARJI models

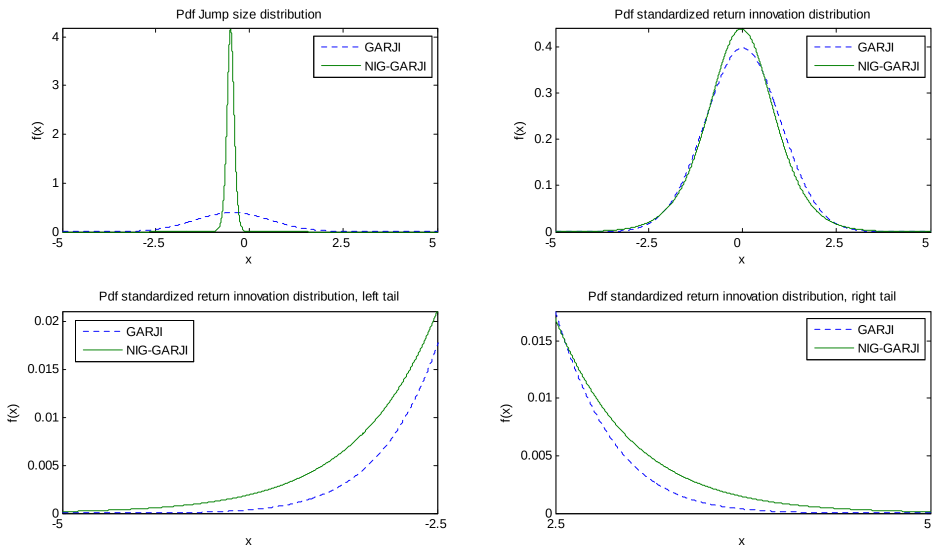


Figure 2

Variance decomposition market index - NIG-GARJI and GARJI models

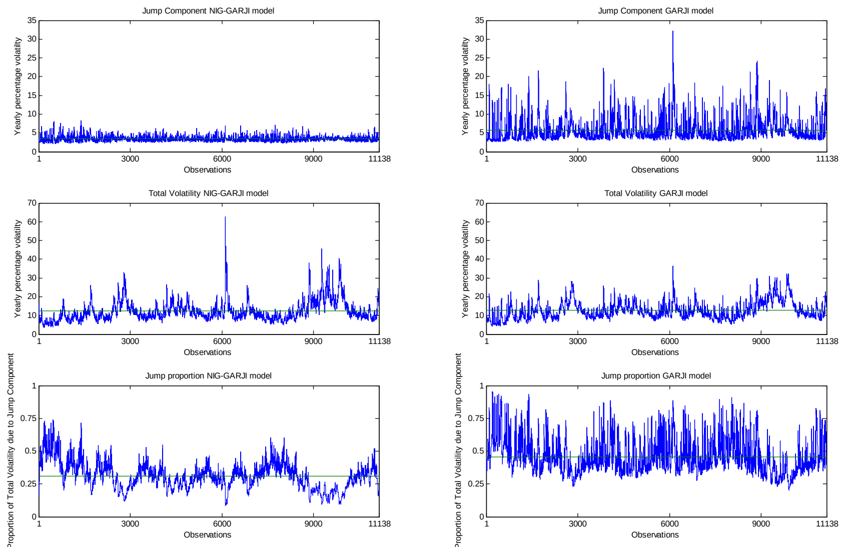


Figure 3

Value-at-Risk decomposition market index - NIG-GARJI and GARJI model

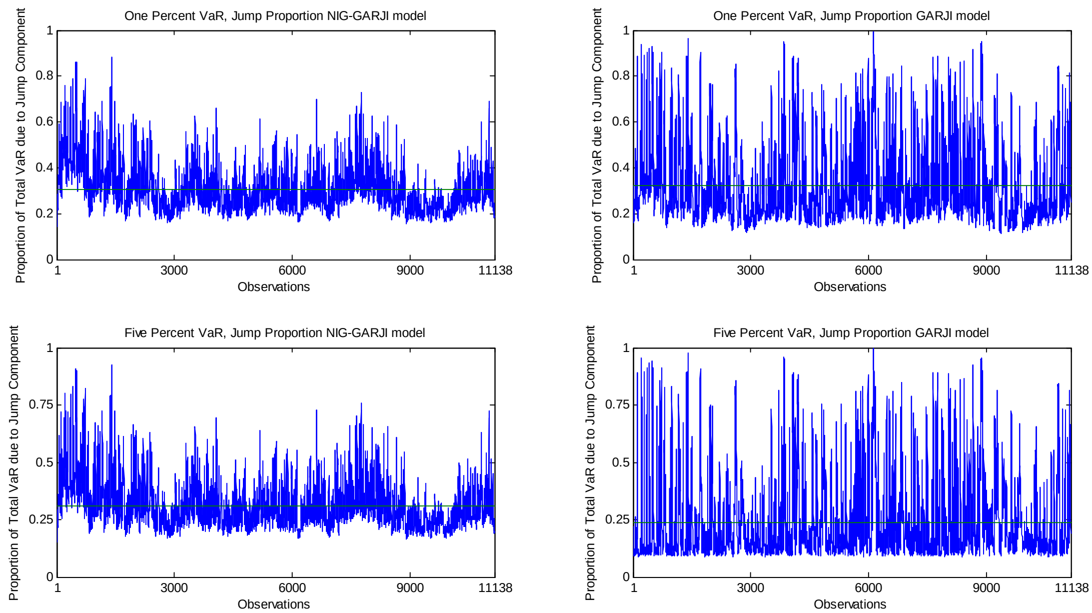
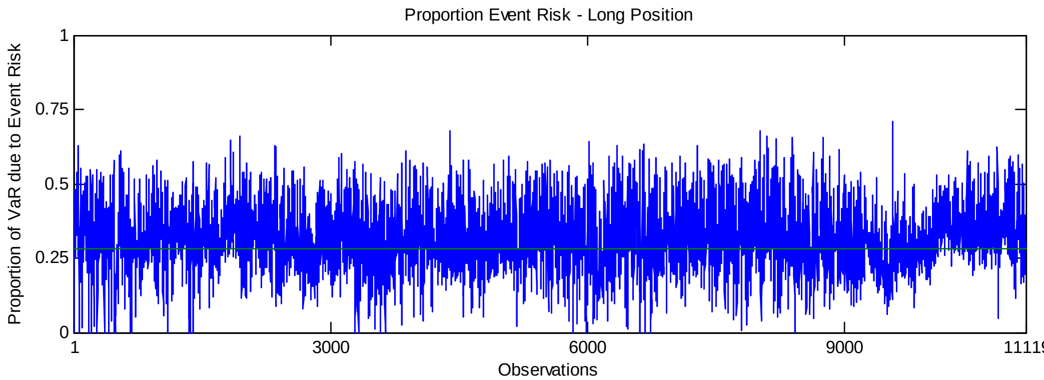


Figure 4

Event risk - Size1 (Smallest 30%)



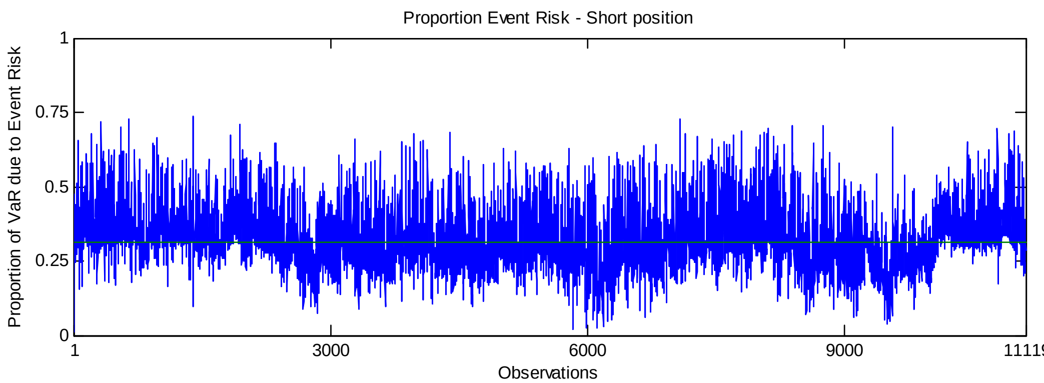


Figure 5

Event risk - Size2 (Middle 40%)

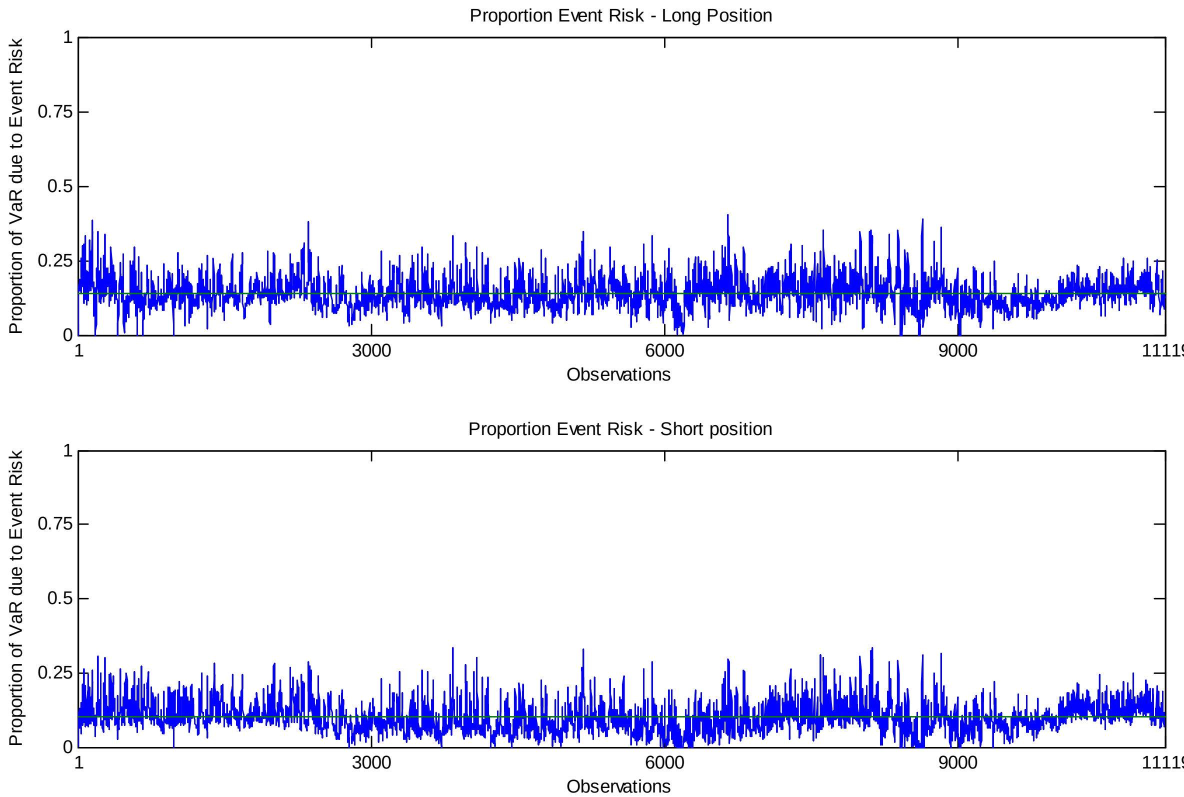


Figure 6

Event risk - Size3 (Largest 30%)