Inferring Default Correlation from Equity Return Correlation

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Abstract

This paper proposes a new approach to estimate default correlation. It overcomes an empirical difficulty encountered in the structural model when estimating default correlation from the unobservable asset process. The unique feature of this approach is that it links default correlation to equity return correlation while preserving the fundamental relation between default correlation and asset return correlation embedded in the structural model. Empirical results show that our model considerably outperforms Zhou's (2001) model in predicting default correlation, especially for bonds with long maturity horizon and low credit rating. Results indicate that a little more careful specification of the underlying mechanisms in the structural model can significantly improve its performance. Our finding strongly suggests that structural models are a very useful tool for estimating default correlation.

Keywords: *Default correlation, equity return correlation, defaultable bonds, and structural model.*

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1. Introduction

Default correlation is an important piece of information for risk management of credit portfolios because portfolio managers must accurately estimate portfolio losses that depend on joint default events between obligors in a portfolio. Das, Fong and Geng (2001) find that default rates of debts in credit portfolios are significantly correlated and estimates of credit losses are substantially different if default correlation is ignored. The recent subprime mortgage crisis is an acute example that underscores the importance of understanding default correlation. Cowan and Cowan (2004) show that default correlations between subprime loans are substantially higher than those between commercial bonds and loans, and that default correlation increases as the rating of the lender declines. High default correlations among loans compound the current problem in the subprime mortgage market.

Though default correlation is important for risk management and credit analysis, this information is often unavailable because default correlation cannot be measured directly. In particular, it is difficult to uncover default correlation based on observed default data for high-grade bonds because default is a rare event. A number of models have been developed to estimate default risk and to explore the structure of default correlation. Majority of these models adopt either the structural or reduced-form approach.

The reduced-form approach models default as an intensity process that is determined by exogenously state variables (see, among others, Jarrow and Turnbull, 1995; Madam and Unal, 1998; Duffie and Singleton, 1999; and Das, Duffie, Kapadia, and Saita, 2007). This approach allows the default intensity process to be directly estimated from the credit risk premium without relying on parameters related to the firm's underlying unobserved asset value. Because of this advantage, the reduced-form approach has been widely used to explain credit spreads (see Duffie and Singleton, 1999). Notwithstanding this advantage, formulation of default intensity as an exogenous factor limits the application of the reduced-form model to prediction of default correlation between firms.

The structural approach offers an excellent alternative to model default risk and default correlation. A distinct advantage of this approach is that it can be used to determine default probability, debt and equity values simultaneously in a unified framework. The structural model is pioneered by Merton (1974) and further developed by others (see, for example, Ingersoll, 1977; Smith and Warner, 1979). Merton assumes that the evolution of firm asset value follows a geometric Brownian motion and the default boundary is the face value of debt. Equity represents a European call option on the firm's asset with the strike price equal to the debt face value. In Merton's model, default can only happen at debt maturity. To allow default before debt maturity, Black and Cox (1976) introduce the first-passage-time model that specifies default as an event of the first time that the firm's asset value hits the default boundary. The default boundary can be exogenously specified as a covenant to protect bondholders' interests (see Black and Cox, 1976; Longstaff and Schwartz, 1995), or can be determined endogenously as a threshold at which stockholders maximize the equity value at default (see Leland, 1994; Leland and Toft, 1996).

Given interdependence of two firms, the structural models can be used to derive default probability for each firm and to infer the default correlation between them (Hull and White, 2001; Löffler, 2003; Overbeck and Schmidt, 2005). Since major components in structural models are asset, debt, equity, and default boundary, any dependence of one component on another can generate default correlation. We can thus differentiate various default correlation models by their correlation channels. Earlier studies focus on the correlation between two firms' assets (Zhou, 2001; Frey, McNeil, and Nyfeler, 2001). Asset values are treated as a function of common factors and firm-specific factors (Finger, 1999; Frey, McNeil, and Nyfeler, 2001) where the common factors dictate the asset return correlation between firms. Subsequent studies (see Giesecke, 2003, 2006) model default correlation by introducing the correlation between firms' default boundaries in addition to that between firms' assets to account for the contagious effects of credit risk. Giesecke (2003, 2006) assumes that each time a firm defaults, the true level of its default boundary is revealed, and investors use this new information to update their beliefs about the default boundaries of other firms. To model the dependence of firms' default boundaries, Giesecke uses copulas to link the probability distributions of individual firm default boundaries to a joint distribution function of default boundaries.

While the structural approach provides a cohesive framework to relate asset return correlation to default correlation, implementation of this type of models is limited in practice because the asset value process and default boundary are unobservable. A number of studies propose different ways to overcome this problem. For instance, Zhou (2001) assumes that asset return correlation is equal to equity return correlation, and CreditMetrics, an industrial credit risk model, approximates asset return correlation by equity return correlation. Intuitively, replacing asset return correlation ρ by equity return correlation ρ_S is more suitable for firms with a low level of debt over a short horizon as indicated by Zhou (2001). However, a high-rated firm can have high leverage if its assets are considered safe whereas a firm with risky assets may still has a low rating even though its leverage is lowered. This suggests that the equity-asset relationship can be complicated not only by leverage but also by the risk in its asset (i.e., asset volatility). Having a low leverage is not necessarily a valid reason for using the approximation that $\rho = \rho_S$. Thus, to better capture default correlation, the naïve approximation of $\rho = \rho_S$ should be replaced with a more subtle relationship based on the structure of the firm. For instance, as firms' debt level and time horizon increase, asset and equity return correlations often diverge. Therefore, additional mechanisms should be introduced to capture this effect. Zeng and Zhang (2002) show that equity return correlation is not a perfect proxy for asset return correlation because the covariance between the assets of two firms is composed of the covariance between their equities and the covariance between their risk-free components. DeServigny and Renault (2002) examine whether default correlation can be efficiently extracted from equity return correlation. Their empirical results show that default correlation implied by equity return correlation is generally not a good proxy for empirical default correlation.

In this paper we propose a new method to infer default correlation from equity return correlation using a structural approach without approximating the asset return correlation with the equity return correlation or introducing additional correlated processes, such as dependent default boundaries (e.g., Giesecke, 2006), and cross holdings between two firms (e.g., Elsinger, 2007). A rationale of working with the structural model is that if the underlying linkage among key financial decision variables is more accurately modeled, default correlation could be more reliably inferred from equity data, which in turn would result in a better understanding of the channels of default correlation in the structural model. This also allows us to better assess how much of default correlation is due to the correlated asset process and whether additional factors are needed. In our method, we first establish the links between equity return correlation and asset return correlation, and between asset return correlation and default correlation, respectively. We then put the two links together to eliminate the requirement for the information of asset return correlation. In this way, we are able to infer default correlation from observed equity return correlation more accurately based on a theoretically sound contingent claims framework. Since firms' stocks are actively traded, we can estimate default correlation easily using stock returns.

We develop a hybrid structural model to relate equity return correlation to default correlation and to provide estimates of default correlation. The model combines an extended Leland-Toft (1996, hereafter LT) model and Zhou's (2001) model (see Figure 1). The extended LT model links equity return correlation to asset return correlation whereas Zhou's model links asset return correlation to default correlation. Combining these two structural models establishes a link between equity return and default correlations. It is straightforward to integrate Zhou's (2001) asset-default correlation model to the extended LT model because the former is also based on the first-passage-time framework. We use this integrated hybrid model to estimate default correlation from empirical data and to compare its performance with the well-known Merton model and Zhou's model.

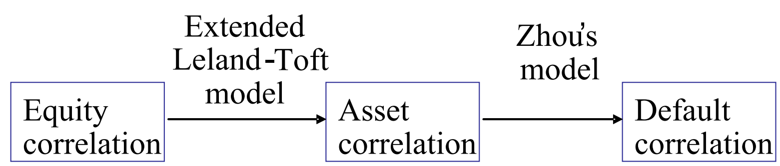

Figure 1. Hybrid model

Traditionally, the Merton-type model is used to estimate default probability and yield spread for each individual firm. It can be extended to the two-firm case to explore the default correlation between them. The extended Merton model has a closed-form solution and is easy to implement. Empirical results show that the model can capture the fact that as bond rating decreases, default correlation increases. However, it cannot capture the time horizon effect as shown by Lucas (1995).[1] The reason is that in the Merton-type model default can only happen at debt maturity. A firm having negative assets is permitted to continue its operation until debt maturity. As a result, default probability is reduced and so is the default correlation. In addition, the simple Merton-type model imposes restrictive assumptions, such as the equity value is a European call option on the firm's assets, no tax benefit from the use of debts, and default boundary is exogenously determined.

Zhou (2001) attempts to improve Merton's model by developing a first-passage-time model to estimate a joint default probability distribution and default correlation. This model avoids the restrictive assumption in Merton's model that default can only occur at the bond maturity date. The strength of this model is that it provides a theoretical framework that makes use of firm-specific information to determine default correlation among firms. Moreover, the model provides an analytical formula for calculating default

[1] Lucas estimates default correlations using the firm bankruptcy data in the period. Thus, his default correlations are based directly on the default events observed in the market.

correlations that can be easily implemented for a variety of applications. However, a potential drawback of this model is that it imposes a restrictive assumption that asset return correlation equals equity return correlation. In addition, Zhou (2001) uses an arbitrary value for the asset (equity) return correlation in his calculation for default correlation. Since one can always boost the equity return correlation coefficient to increase the size of default correlation, the performance of the model is called in question.

Our hybrid model overcomes the shortcomings of the Merton-type model and Zhou's model. We adopt a first-passage-time model (e.g., the LT model) and extend it to a two-firm setting in which equity and default of these firms are intrinsically linked. We first establish the relation between asset return correlation and equity return correlation (see Figure 1) and then examine suitability of using the equity return correlation as a proxy for the asset return correlation. Simulations show that equity return correlation is generally not a good direct proxy for asset return correlation. In particular, as time horizon increases and bond ratings decrease, the gap between equity return correlation and asset return correlation widens increasingly. To overcome this problem, we employ the hybrid model that utilizes the fundamental relation between asset return correlation and default correlation implied by the structural model. Using this model, we abstract the asset return correlation from the observed equity return correlation, estimated from historical stock returns, using the calibrated LT model and then input it into Zhou's model to obtain default correlation.

We compare the performance of our model with that of the simple Merton-type model and Zhou's model using empirical data. Our results confirm the previous finding that equity return correlation is generally not a good proxy for asset return correlation (see DeServigny and Renault, 2002; Zeng and Zhang, 2002). Our paper contributes to the literature by proposing a new approach to resolve the problem of unobserved asset return correlation which is a key input in a default correlation model. Results show that our hybrid model based on this new approach performs better than other structural models in estimating default correlation of multiple firms. The greater estimation accuracy is achieved without having to impose more complicated structures such as correlated default boundary, networking, and cross holdings. Our results strongly suggest that the structural model is a useful tool for estimating default correlation.

The remainder of the paper is organized as follows. Section II develops a simple Merton-type model and a generalized first-time-passage (FTP) model to relate equity return correlation to default correlation. Section III discusses the FTP model implementation and Monte Carlo simulations. Section IV reports simulation results and presents empirical findings. Finally, Section V concludes the paper.

2. The Model

In this section, we first develop a simple Merton-type model to relate equity return correlation to default correlation. The simple model has a closed-form formula and is relatively easy to implement. However, the model imposes restrictive assumptions. To relax these restrictions, we extend the LT model (1996) to a two-firm setting to permit default before maturity. We then propose a hybrid model that integrates the extended LT model and Zhou's model to link default correlation directly to equity return correlation.

This model provides a convenient framework to estimate default correlation, which does not require the information for the unobserved asset return correlation.

2.1 The Simple Equity Return Correlation Model

2.1.1 Derivation of the simple model

Following Merton (1974), we assume a perfect and arbitrage-free capital market. The default-free interest rate, r, is constant and the money market account has value $B(t)=e^{rt}$ at time t. Denote V_1 and V_2 as total assets of firms 1 and 2, respectively. The dynamics of V_1 and V_2 are given by the following stochastic process:

$$d\ln V = \mu dt + \Sigma dw \tag{1}$$

where $\ln V = (\ln V_1, \ln V_2)'$, $\mu = (\mu_1, \mu_2)'$, and $w = (w_1, w_2)'$ are column vectors. μ_1 and μ_2 are instantaneous expected rates of return of firms per unit of time, and w_1 and w_2 are independent standard Brownian motions with volatilities σ_1 and σ_2, respectively. The returns can be determined as the change in $\ln V$ over a unit of time,[2] denoted as $\Delta\ln V_1$ and $\Delta\ln V_2$ for firms 1 and 2, respectively. Asset return correlation is therefore given by

$$\rho = \frac{\operatorname{cov}(\Delta\ln V_1, \Delta\ln V_2)}{\sqrt{\operatorname{var}(\Delta\ln V_1)\operatorname{var}(\Delta\ln V_2)}} \tag{2}$$

and the variance-covariance matrix is

$$\Sigma\cdot\Sigma' = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}.$$

The equity values S_1 and S_2 are the call options on firm assets with maturity T and strike price $B_{i,T}$. That is,

[2] For example, if the unit of time is month, then $\Delta\ln V$ represents monthly return. For Brownian motions, return correlation is independent of the frequency of the return data since the time factor cancels out in (2).

$$S_{i,T} = \left(V_{i,T} - B_{i,T}\right)^{+}, \quad i = 1, 2 \tag{3}$$

where $B_{i,T}$ is the promised payment (debt face value) of firm i to its debtholders at maturity T. If the firm's asset value (V_T) at maturity is greater than the face value of debt (B_T), the firm does not default and shareholders receive $V_{i,T} - B_{i,T}$. On the other hand, if $V_{i,T} < B_{i,T}$, the firm defaults on its debts, and debtholders take control of the firm.

We can derive the correlation coefficient of equity returns given the correlation coefficient of two firms' assets in (2). The correlation coefficient of equity returns at time T is

$$\rho_S = \frac{\operatorname{cov}\left(R_{1,T}, R_{2,T}\right)}{\sqrt{\operatorname{var}\left(R_{1,T}\right)\operatorname{var}\left(R_{2,T}\right)}}. \tag{4}$$

where the covariance of equity returns is given by

$$\begin{aligned}\operatorname{cov}(R_1, R_2) = {} & A_1 E\left(V_{1,T}V_{2,T}I_{\{V_{T,1}>B_{1,T}, V_{2,T}>B_{2,T}\}}\right) + A_2 E\left(V_{1,T}I_{\{V_{T,1}>B_{1,T}, V_{2,T}>B_{2,T}\}}\right) \\ & + A_3 E\left(V_{2,T}I_{\{V_{T,1}>B_{1,T}, V_{2,T}>B_{2,T}\}}\right) + A_4 EI_{\{V_{T,1}>B_{1,T}, V_{2,T}>B_{2,T}\}} \\ & + A_5 E\left(V_{1,T}I_{\{V_{1,T}>B_{1,T}, V_{2,T}\le B_{2,T}\}}\right) + A_6 EI_{\{V_{1,T}>B_{1,T}, V_{2,T}\le B_{2,T}\}} \\ & + A_7 E\left(V_{2,T}I_{\{V_{1,T}\le B_{1,T}, V_{2,T}>B_{2,T}\}}\right) + A_8 EI_{\{V_{1,T}\le B_{1,T}, V_{2,T}>B_{2,T}\}} + e^{2rT}I_{\{V_{1,T}\le B_{1,T}, V_{2,T}\le B_{2,T}\}},\end{aligned} \tag{5}$$

and the variance of equity returns for firm i is given by

$$\operatorname{var}\left(R_{i,T}\right) = D_{1,i}E\left(V_{i,T}^2 I_{\{V_{i,T}>B_{i,T}\}}\right) + D_{2,i}E\left(V_{i,T}I_{\{V_{i,T}>B_{i,T}\}}\right) + D_{3,i}I_{\{V_{i,T}>B_{i,T}\}} + e^{2rT}I_{\{V_{i,T}\le B_{i,T}\}}. \tag{6}$$

The definition of each term in (5) and (6) is shown in Appendix A, and the derivation of Eq. (5) and (6) is presented in Appendix B.

Next, default correlation is defined as

$$\rho_D = \frac{E\left(I_{\{V_{1,T}>B_{1,T}, V_{2,T}>B_{2,T}\}}\right) - E\left(I_{\{V_{2,T}>B_{2,T}\}}\right)E\left(I_{\{V_{1,T}>B_{1,T}\}}\right)}{\sqrt{Var\left(I_{\{V_{2,T}>B_{2,T}\}}\right)Var\left(I_{\{V_{1,T}>B_{1,T}\}}\right)}}, \tag{7}$$

where $I_{\{u\}}$ is an indicator function with a value equal to one if u is true, and zero otherwise and

$$Var\left(I_{\{V_{i,T}>B_{i,T}\}}\right)=E\left(I_{\{V_{i,T}>B_{i,T}\}}\right)\left[1-E\left(I_{\{V_{i,T}>B_{i,T}\}}\right)\right]. \tag{8}$$

The definition of each term in (7) is given in Appendix A.

Equations (4) and (7) indicate that both equity return correlation (ρ_S) and default correlation (ρ_D) are functions of asset return correlation. Using these two equations, we can determine the relationship between ρ_S and ρ_D directly without the knowledge of asset return correlation ρ . Asset return correlation can be determined from equity return correlation via (4) and default correlation can be obtained using asset return correlation as the input to (7). Thus, default correlation can be computed from equity return correlation without *a priori* knowledge of asset return correlation. In the following, we implement this procedure and present the simulation results. For ease of notations, we would often refer to asset (equity) return correlation as asset (equity) correlation henceforth.

2.1.2 Predictions of the simple model

To test the simple (Merton-type) model, we use the observed equity data to predict default correlation over the same period as Lucas (1995). Equity correlation and volatility are estimated using monthly stock return data from 1970 to 1993.[3] It is important to note that the Merton-type model underestimates default probability because it allows a bankrupt firm to continue to operate until debt maturity. To remedy this

[3] Section 4.2 gives the detailed description of estimation of equity correlation. To estimate equity volatility, we first calculate equal-weighted stock returns in a particular rating group for each month over the period of 1970 to 1993. Next, we calculate the standard deviations of the equal-weighted stock returns.

problem, we introduce a volatility multiplier (the only fudge factor in the model) in order for the model to generate default probabilities commensurate with historical default rates.

Table 1 reports the predictions of default correlations by the simple model using stock data. We find that the simple model generates a pattern of default correlation similar to Lucas (1995). For example, when the time horizon is five years,[4] the simple model predicts that default correlation increases as ratings of both firms decline. The correlation between two Aa firms is 1.2%, while that between two B firms increases to 13%. In addition, given the rating for one firm, default correlation generally increases as the rating for another firm declines. For example, default correlation between two A firms is 3.7%, and 5.2% between A and B firms. These results are consistent with the findings of Lucas (1995). Overall, default correlations estimated by the simple model tend to understate the default correlations of lower grade bonds estimated from historical data (see Lucas, 1995). Moreover, the default correlation estimates are insensitive to time horizon T.[5]

A potential caveat in the above analysis is that stock data are collected from firms that are going concerns. Thus, the analysis is subject to a survival bias when using the data of these firms. To address this issue, we match the survived firm's equity value process to the observed data and then back out the asset value process for the firm to predict the default correlation. The results are reported in the right panel of Table 1. As expected, the difference is hardly noticeable for investment grades. For junk bonds, the survival bias has a larger effect but this effect is not statistically significant except for the default correlation between two B-rated firms where correction of the survival bias leads

[4] We only report the results for the time horizon of five years for brevity. The results for the other time horizons are available upon request.
[5] Estimates of default correlations at different horizons are available upon request.

to an 18% default correlation compared to 13% without correction. However, compared to Lucas's result (29%), both numbers seem low, suggesting the survival bias is not a major issue here.

A possible cause for the underestimation of default correlation is that the Merton-type simple model allows a "bankrupt" firm's equity process to continue to evolve with a finite chance of becoming solvent again over the horizon before debt maturity. This setup could lead to an increasing divergence between equity correlation and asset correlation.

The above results show a substantial room for improvement of the model. For instance, if we could model default in a more realistic way, i.e., to allow firm default before debt maturity, it should increase the predictive ability of the model. In the following, we propose an alternative approach to estimate default correlation using the first-passage-time method to permit default before maturity.

2.2 The Hybrid Model

The hybrid model, as depicted in Figure 1, consists of two components. It first incorporates a model to back out the unobserved asset correlation ρ from the observed equity data. It then inputs the resultant asset correlation ρ into a first-passage-time (FPT) model to obtain default correlation ρ_D. In the former, we extend the LT model to a multiple-firm setting and use the equity correlation information to infer the asset correlation, while in the latter we employ the model of the first passage time suggested by Zhou (2001) to establish the link between asset correlation ρ and default correlation ρ_D. This procedure allows us to use a more accurate relationship between default and equity correlations rather than the restrictive substitution of $\rho = \rho_S$ in Zhou (2001). We summarize the joint default probabilities by Zhou (2001) in Appendix C. These joint default probabilities can be used along with (2) and (7) to calculate asset correlation ρ and default correlation ρ_D.

2.2.1 The Extended Leland-Toft model

In this section, we extend the LT model to the case with two firms. This extended model will be used later to combine Zhou's (2001) to yield the hybrid model. We relax the assumptions in the simple Merton-type model to incorporate bankruptcy costs and taxes, and allow the firm to go bankrupt as its asset value falls below a threshold (default boundary) for the first time. Assume that the asset value of an unlevered firm, V, has the following continuous diffusion process:[6]

$$\frac{dV}{V} = [\mu(V,t) - \delta]dt + \sigma\, dZ \,, \tag{9}$$

where $\mu(V,t)$ is the total expected rate of return on the firm's assets, δ is the total payout ratio, which is the proportion of the firm value paid to all security holders, Z is a standard Wiener process, and σ is the constant volatility of asset returns. The asset value V includes the net cash flows generated by the firm's activity.

Suppose there is an identical but levered firm issuing a risky debt d per unit time with t periods to maturity, and continuous constant coupon flow $c(t)$ and principal $p(t)$. The firm remains solvent until the asset value V hits a default boundary V_B, leading to bankruptcy. Upon bankruptcy, bondholders receive a fraction $\chi = (1-\beta)$ of the asset value V_B, where β is the bankruptcy cost ratio and βV_B is loss due to bankruptcy. Further we assume that r represents the continuous interest rate paid by a default-free

[6] A similar specification is used by Leland and Toft (1996), Merton (1974), Black and Cox (1976), and Brennan and Schwartz (1978).

asset and investors follow a buy-and-hold investment strategy. Under the risk-neutral valuation, it can be shown that the value of the debt, d, is given by

$$d\left(V,V_B,t\right)=\frac{c\left(t\right)}{r}+e^{-rt}\left[p\left(t\right)-\frac{c\left(t\right)}{r}\right]\left(1-F\left(t\right)\right)+\left[\chi V_B-\frac{c\left(t\right)}{r}\right]G\left(t\right), \tag{10}$$

where $F(t)$ and $G(t)$ are given in Leland and Toft (1996). Thus, the total outstanding debt D is the integration of the debt flow $d(V,V_B,t)$ over T (maturity of newly issued debt):

$$D(V,V_B,T)=\int_{t=0}^{T} d(V,V_B,t)dt \tag{11}$$

The integral can be carried out numerically. The tradeoff between the benefit of tax shields and bankruptcy cost suggests that there exists an endogenously determined bankruptcy threshold V_B that maximizes firm value. The equity value, as a function of V_B and asset value V, is given by,

$$E\left(V,V_B,T\right)=V+\tau_C\frac{C}{r}\left[1-\left(\frac{V_B}{V}\right)^{a+z}\right]-\beta V_B\left(\frac{V_B}{V}\right)^{a+z}-D\left(V,V_B,T\right), \tag{12}$$

where C is the annual coupon payment, τ_C is the corporate tax rate. Parameters a and z are functions of asset volatility σ and interest rate r.[7]

Equation (12) establishes a link between asset process V and equity process E. If asset value $V(t)$ follows a geometric Brownian motion, equity value $E(V)$ as a function of V will also exhibit a similar random process.

Consider two firms with asset values V_1 and V_2. The dynamics of V_1 and V_2 are specified by (1)-(4), where asset returns $\Delta\ln V_1$ and $\Delta\ln V_2$ are correlated with a coefficient ρ, and volatilities σ_1 and σ_2. Similar to (2), we define equity return correlation as

[7] Detailed derivations of (10) and (12) are given by Leland and Toft (1996).

$$\rho_S = \frac{\operatorname{cov}(\Delta \ln E(V_1), \Delta \ln E(V_2))}{\sqrt{\operatorname{var}(\Delta \ln E(V_1))\operatorname{var}(\Delta \ln E(V_1))}}. \tag{13}$$

The correlation between two asset processes, V_1 and V_2, will undoubtedly result in a correlation between two corresponding equity processes, $E(V_1)$ and $E(V_2)$. However, these two correlations can diverge significantly because as time evolves, both leverage ratio l and asset volatility σ may change.

3. Model Implementations and Monte Carlo Simulation

We choose interest rate r = 8% and payout ratio $\delta = 6\%$. These figures are in line with Huang and Huang (2003). Corporate tax rate τ_C is set at 35%. Bankruptcy ratio β is set at 20% based on estimates in Andrade and Kaplan (1995).[8]

To implement the structural model properly, a calibration is necessary. The objective of calibration is to choose equity premium and asset volatility σ such that the model generates a default probability consistent with the observed default rates for each rating given in Table 2A. The equity premiums for ratings Aa to B are given in Table 2B. When dealing with heterogeneous time horizons, from one to ten years in the present case, we carry out the model calibration by choosing an asset return volatility σ that minimizes the aggregate squared difference between the implied and observed default probabilities,

$$\sigma = \arg\min_{\varsigma \geq 0} \sum_{i=1}^{10} \left[P_i(\sigma) - \overline{P_i} \right]^2, \tag{14}$$

[8] Personal tax rate is set to zero in this exercise. We have also tested various personal tax rates. For the issues discussed in this study, a non-zero personal tax rate does not qualitatively change our results. For simplicity and clarity, we abstract away from personal tax influence. Nevertheless, personal taxes exhibit interesting effects which will be a subject in a sequel to this study.

where P_i is the model-implied default probability by year *i*, and $\overline{P_i}$ is the corresponding observed default rate. Since we input equity premiums into the model, these probabilities are physical probabilities. Table 2C reports the model-implied asset return volatility σ for bonds with rating categories from Aa to B. To value debt and equity with the model, we return to the risk-neutral measure by retaining asset return volatility σ and forcing equity premium to zero.

In the Monte Carlo simulation, for each iteration we generate a time series sample path according to (9) with the starting asset value $V_i(0)$ normalized to 100, where i = 1, and 2 denoting the two firms. For each random movement in $V_i(0)$ at time t, we apply (11) and (12) to obtain debt $D_i(t)$ and equity $E_i(t)$. For the next random movement in $V_i(t + \Delta t)$ at time $t + \Delta t$, we again apply the model to obtain $D_i(t + \Delta t)$ and $E_i(t + \Delta t)$ while keeping the coupon, principal and default boundary unchanged. This is to recognize the fact that the stationary capital structure of the LT model rules out any debt restructuring after the optimization is done. The procedure is repeated until we reach the horizon at $t = T_H$. This allows us to map out one sample path. For a second iteration, the same procedure is repeated to generate another sample path of $V_i(t)$ (and thereby $D_i(t)$ and $E_i(t)$ as well) for each firm.

To permit correlation ρ between the returns of the two asset processes, we employ the following return dynamics:

$$\begin{cases} \Delta V_1 = V_1\left[\dfrac{\mu_1}{n} + \dfrac{\sigma_1}{\sqrt{n}} \times \Delta Z_1\right] \\ \Delta V_2 = V_2\left[\dfrac{\mu_2}{n} + \dfrac{\rho\sigma_2}{\sqrt{n}} \times \Delta Z_1 + \dfrac{\sqrt{1-\rho^2}\,\sigma_2}{\sqrt{n}} \times \Delta Z_2\right] \end{cases}, \tag{15}$$

where n denotes the number of time intervals partitioned for each year and μ_i is the net drift rate. The random variables ΔZ_1 and ΔZ_2 follow two independent standard normal distributions. The volatility for each period is $\sigma_i / \sqrt{n}$ where σ_i is the annualized return volatility for firm *i*. For example, when the time interval is month, n is set to 12 and $\sigma_i / \sqrt{12}$ is the monthly return volatility. In each simulation, t is represented by the number of time steps within the period [0, *t*]. The convergence of Monte Carlo simulation can be achieved by a large number of iterations but at the expense of computation time. For each rating pairs (e.g., Baa and Ba firms), we generate 10,000 sample paths. Return correlations of assets, equities and debts are calculated for each sample path and their averages are reported.

4. Simulation Results, Empirical Analysis and Discussions

In this section, we first demonstrate that equity correlation ρ_S and asset correlation ρ can be quite different and the difference grows as the horizon lengthens and the rating declines. This justifies our efforts to seek a more accurate relationship between ρ_S and ρ. Next, we establish the link between ρ_S and ρ through the extended LT model for different pairs of ratings. Equity return correlation ρ_S is estimated from stock return data. Combined with the link between asset correlation ρ and default correlation ρ_D given in Zhou (2001), we implement the hybrid model to estimate default correlation.

4.1. Deviation of equity from asset correlations – horizon and rating effects

Table 3 reports the simulation results for the relation between asset correlation and equity correlation. The condition that $\rho = \rho_S$ imposed by previous studies of default correlation (e.g., Zhou, 2001) is often violated as shown in Table 3 where we fix the correlation between the two asset processes at $\rho = 40\%$. The model-predicted equity correlations increasingly deviate from asset correlation $\rho = 40\%$ as horizon lengthens and ratings decline. This is due to the diffusion nature of the two asset value processes ($\ln V_1$ and $\ln V_2$) and the nonlinearity in the model-predicted asset-equity relationship. Taking the pair of Aa-B ratings for example, when time horizon $T = 1$ year, equity correlation ρ_S is about 38%, which is fairly close to the given asset correlation $\rho = 40\%$. But for $T = 10$ years, ρ_S drops to 22%, which is only about a half of ρ. In the case of B-B bonds, the discrepancy between ρ and ρ_S is even larger. Thus, the restriction of the equality of asset correlation and equity correlation is a very strong assumption in the previous default correlation studies.

4.2. Estimation of equity and asset correlations

In empirical investigation, we estimate equity correlation ρ_S from historical equity return data. We calibrate the model such that the implied equity correlation from the model matches the empirical equity correlation. We calculate equity correlation ρ_S for firms across ratings from stock returns.[9] The data of stock returns are retrieved from CRSP and the data of firm ratings are obtained from Compustat.[10] We use monthly returns to calculate the correlation for each year. Thus, if we have *n* firms with the same

[9] Lucas (1995) calculated historical default correlation based on default information provided by Moody's Investors Service from 1970 through 1993. To compare with his results, we use the equity information over the same period to calculate equity correlation.

[10] Compustat provides both short-term and long-term issuer credit ratings. In our analysis, we limit attention to those firms with assigned Standard & Poors Long Term Domestic Issuer Credit Rating.

rating, we will have $n(n-1)/2$ pairs of correlations between two firms within the rating in a given year. If we have n firms with one rating and m firms with another rating, we will have $n \times m$ pairs of correlations across the two ratings. We calculate pairwise equity correlations within and across ratings each year over the sample period of 1970 to 1993. We obtain an equally-weighted average of correlations for each pair of ratings and report them in Table 4. The summary statistics of the sample are given in Panel 1 and the mean and standard error are given in Panel 2.

The relationship between asset correlation ρ and equity correlation ρ_S can be determined by simulating the model to match empirical estimates of ρ_S in Table 4 (Panel 2). The estimates of the asset return correlation are reported in Panel A of Table 5.

4.3 Performance of the hybrid model

The results from the hybrid model are shown in Panel D of Table 5 for horizons T = 4, 6, 8 and 10 years, respectively. We estimated the results for ten horizons (1 to 10 years) but in the interest of brevity we only report the results for these four horizons here.[11] As shown, given the calibrated asset correlation ρ in Panel A, the hybrid model (Panel D) and Zhou's model (Panel C) generate quite different default correlations, which indicates that $\rho = \rho_S$ in Zhou (2001) is a poor assumption.[12] This evidence is consistent with previous findings that equity correlation is not a good proxy for asset correlation (e.g., see Zeng and Zhang, 2002; de Servigny and Renault, 2002). In addition, in accordance with de Servigny and Renault (2002), we find that asset correlation ρ

[11] The results for other horizons are consistent and available upon request.

[12] To compare with Zhou's results, we use the same values of Z_i used by Zhou (2001). Specifically, Z_i for AA, A, BBB, BB, and B bonds are 9.3, 8.06, 6.46, 3.73, and 2.1, respectively.

inferred from equity correlation ρ_S exhibits an increasing trend as time horizon lengthens (e.g., comparing different horizons in Panel A of Table 5).

First, for bonds with high rating, the hybrid model predicts higher default correlation than Zhou's model does as shown in Panels C and D. But the difference between the two models is not as big as that for bonds with low rating. When both ratings are investment-grade, Zhou's model and the hybrid model tend to overpredict default correlation for horizons greater than 6 years and underpredict for shorter horizons if we use Lucas's results in Panel B as a benchmark. However, the over- and underpredictions are small. For example, for A-A pair with $T = 8$ years, Lucas's estimate is 2% while the hybrid model and Zhou's model predicts 2.86% and 2.22%, respectively. It should be noted that Lucas's results may not be a reliable benchmark for high rating bonds. Lucas's results are essentially unchanged from 4 to 10 years considering possible rounding errors. This implies that no (additional) default is recorded between 4 and 10 years in his sample. Furthermore, estimating default correlation for safe firms can be extremely challenging in empirical analysis using historical data. This is because joint default is an even rarer event, which may result in very few or even no observations. For example, default correlation being 0 for Baa-Baa could mean no observed joint default in this category as explained in Lucas (1995). Also, default correlation is a ratio of two small numbers (i.e., functions of default probability; see (7) and (8)). This can make estimation of default correlation quite unreliable with even a small error in the default probability estimation.

Second, the hybrid model predicts much higher default correlations ρ_D than Zhou's approach does for bonds with lower ratings. These correlations are closer to Lucas's estimates. For example, for the B-B bond pair with 4-year horizon, the hybrid model predicts 18.21% for default correlation ρ_D, which is much higher than the default correlation obtained from Zhou's approach (12.96%). The improvement is even more significant for 10-year horizon – the hybrid model predicts 32.13% for B-B pair while Zhou's approach can only generate 13.68%. From the perspective of credit risk management, a more accurate default correlation estimate for lower-grade bonds is more important than that between high-grade bonds because when credit risk is high, joint default becomes a major concern. By contrast, for high-rated bonds, the low credit risk makes joint default a less likely event even with a high default correlation.

To the extent that there are more default and joint default events in low rating classes, Lucas's estimates should be more reliable. Indeed, his estimates show larger and more sensible variations as horizon changes for bonds with lower ratings. Therefore, Lucas's estimates may serve as a reasonable benchmark in this regime.[13] As shown, the hybrid model clearly outperforms Zhou's model by predicting much higher default correlations for bonds with lower ratings.

5. Conclusion

Default correlation information is important for credit analysis and risk management, but the scarcity of bankruptcy data makes it difficult to obtain this measure. Existing structural models rely on the asset return correlation to predict default correlation. Because asset value and asset return correlation are unobservable, equity value and equity return correlation are used as substitutes instead. Previous studies have

[13] We note that Zhou (2001) use Lucas's results as a benchmark across all rating classes and horizons.

indicated that these substitutions can result in poor estimates of default correlation. Our findings confirm this view.

In this paper, we establish a more accurate link between equity correlation and default correlation through their relations with asset correlation in a structural framework, which allows us to use the equity return data to estimate default correlation more reliably. We first develop a simple Merton-type model to directly link default correlation to equity correlation. We then develop a hybrid model to overcome the problem in Zhou's model where unobservable asset return correlation is approximated by equity return correlation. This hybrid model is an integration of the extended LT model in a two-firm setting and Zhou's model.

Empirical results show that the hybrid model performs substantially better than other structural models. The simple Merton-type model can capture the trends of historical default correlations but it cannot capture the time horizon effect exhibited in historical default correlations. By contrast, the hybrid model can predict the time horizon effect as well as the rating effect. Zhou's model predicts lower default correlations than the hybrid model particularly for low-grade bonds. Results show that the hybrid model performs much better than the Merton-type model and Zhou's model.

Our results show that with only one basic correlated stochastic driving force (asset process), a hybrid structural model can yield reasonable default correlation estimates without imposing more complex factors such as correlated default threshold, incomplete information, and cross holdings among firms. Results show that structural models are quite useful for estimating default correlations. The hybrid structural model developed in this paper can be used to enhance credit risk management. In particular, it makes dynamic management of credit risk possible because the required inputs to the model are equity data, which are readily available.

APPENDIX A

Relationship between Equity Correlation and Asset Correlation

In this appendix, we present the result needed to calculate equity correlation from asset correlation and we show the derivation of this result.

Result 1 Let Ψ(x,y) denote the bivariate normal distribution function, that is $\Psi(x,y) = P(X \le x, Y \le y)$ where X and Y are standard Brownian motions with means 0 and variances 1, and correlation coefficient $-1 \le \rho \le 1$ (we suppress the parameter ρ for simplicity). Then

$$E\left(V_{1,T}V_{2,T}I_{\{V_{1,T}>B_{1,T},V_{2,T}>B_{2,T}\}}\right)$$
$$= \exp\left(m_1 + m_2 + \rho\varsigma_1\varsigma_2 + \frac{\varsigma_1^2+\varsigma_2^2}{2}\right)\Psi\left(\varsigma_1 + \rho\varsigma_2 + \frac{m_1 - \ln B_1}{\varsigma_1}, \rho\varsigma_1 + \varsigma_2 + \frac{m_2 - \ln B_2}{\varsigma_2}\right), \tag{A1}$$

$$E\left(V_{1,T}I_{\{V_{1,T}>B_{1,T},V_{2,T}>B_{2,T}\}}\right)$$
$$= \exp\left(m_1 + \frac{\varsigma_1^2}{2}\right)\Psi\left(\frac{m_1 - \ln B_1 + \varsigma_1^2}{\varsigma_1}, \frac{m_2 - \ln B_2 + \rho\varsigma_1\varsigma_2}{\varsigma_2}\right), \tag{A2}$$

$$E\left(V_{2,T}I_{\{V_{1,T}>B_{1,T},V_{2,T}>B_{2,T}\}}\right)$$
$$= \exp\left[\frac{1}{2}\varsigma_2^2 + m_2\right]\Psi\left(\frac{m_1 - \ln B_1 + \rho\varsigma_1\varsigma_2}{\varsigma_1}, \frac{m_2 - \ln B_2 + \varsigma_2^2}{\varsigma_2}\right), \tag{A3}$$

$$E\left(I_{\{V_{1,T}>B_{1,T},V_{2,T}>B_{2,T}\}}\right) = \Psi\left(\frac{m_1 - \ln B_1}{\varsigma_1}, \frac{m_2 - \ln B_2}{\varsigma_2}\right), \tag{A4}$$

$$E\left(V_{1,T}I_{\{V_{1,T}>B_{1,T},V_{2,T}\le B_{2,T}\}}\right)$$
$$= \exp\left(m_1 + \frac{\varsigma_1^2}{2}\right)\left\{\Phi\left(\frac{m_1 - \ln B_1 + \varsigma_1^2}{\varsigma_1}\right) - \Psi\left(\frac{m_1 - \ln B_1 + \varsigma_1^2}{\varsigma_1}, \frac{m_2 - \ln B_2 + \rho\varsigma_1\varsigma_2}{\varsigma_2}\right)\right\}, \tag{A5}$$

$$E\left(V_{2,T}I_{\{V_{1,T}\le B_{1,T},V_{2,T}>B_{2,T}\}}\right)$$
$$=\exp\left[\frac{1}{2}\varsigma_2^2+m_2\right]\left\{\Phi\left(\frac{m_2-\ln B_2+\varsigma_2^2}{\varsigma_2}\right)-\Psi\left(\frac{m_1-\ln B_1+\rho\varsigma_1\varsigma_2}{\varsigma_1},\frac{m_2-\ln B_2+\varsigma_2^2}{\varsigma_2}\right)\right\}, \tag{A6}$$

$$E\left(I_{\{V_{1,T}>B_{1,T},V_{2,T}\le B_{2,T}\}}\right)=\Phi\left(\frac{m_1-\ln B_1}{\varsigma_1}\right)-\Psi\left(\frac{m_1-\ln B_1}{\varsigma_1},\frac{m_2-\ln B_2}{\varsigma_2}\right), \tag{A7}$$

$$E\left(I_{\{V_{1,T}\le B_{1,T},V_{2,T}>B_{2,T}\}}\right)=\Phi\left(\frac{m_2-\ln B_2}{\varsigma_2}\right)-\Psi\left(\frac{m_1-\ln B_1}{\varsigma_1},\frac{m_2-\ln B_2}{\varsigma_2}\right), \tag{A8}$$

where $\varsigma_i=\sigma_i^2T$ and m_i (i = 1, 2) is the expected value of $\ln V_i$ for firm i,

$$m_i=\ln V_{i,0}+\mu_iT-\frac{\varsigma_i^2}{2}$$

B_i is the debt value of firm i at time T, and Φ(x) is the cumulative normal distribution function, that is, $\Phi(x)=P(X\le x)$, and X is standard Brownian motions with means 0 and variances 1. $\ln V_T$ has a mean m_i and standard deviation ς_i :

$$\ln V_{i,T}=\Phi(m_i,\varsigma_i),$$

where Φ(x,y) is the cumulative normal distribution function with mean x and volatility y.

The expectations in (A9)-(A11) can be expressed in terms of Φ(x), that is,

$$E\left(V_i^2I_{\{V_i>B_i\}}\right)=\exp\left(2m_i+2\varsigma_i^2\right)\Phi\left(\frac{m_i-\ln B_i+2\varsigma_i^2}{\varsigma_i}\right) \tag{A9}$$

$$E\left(V_iI_{\{V_i>B_i\}}\right)=\exp\left(m_i+\frac{\varsigma_i^2}{2}\right)\Phi\left(\frac{m_i-\ln B_i+\varsigma_i^2}{\varsigma_i}\right) \tag{A10}$$

$$E\left(I_{\{V_i>B_i\}}\right)=\Phi\left(\frac{m_i-\ln B_i}{\varsigma_i}\right) \tag{A11}$$

Other terms in (6) are defined as below:

$$A_1 = \frac{1}{S_{1,0}S_{2,0}}, A_2 = -\frac{1}{S_{1,0}}\left(\frac{B_{2,T}}{S_{2,0}} + e^{rT}\right), A_3 = -\frac{1}{S_{2,0}}\left(\frac{B_{1,T}}{S_{1,0}} + e^{rT}\right),$$
$$A_4 = \frac{B_{1,T}B_{2,T}}{S_{1,0}S_{2,0}} + \left(\frac{B_{1,T}}{S_{1,0}} + \frac{B_{2,T}}{S_{2,0}}\right)e^{rT} + e^{2rT}, A_5 = -\frac{e^{rT}}{S_{1,0}},$$
$$A_6 = \frac{B_{1,T}}{S_{1,0}}e^{rT} + e^{2rT}, A_7 = -\frac{e^{rT}}{S_{2,0}}, A_8 = e^{rT}\frac{B_{2,T}}{S_{2,0}} + e^{2rT}$$

and

$$E\left(I_{\{V_{1,T} \le B_{1,T}, V_{2,T} \le B_{2,T}\}}\right)$$
$$= \Psi\left(-\frac{m_1 - \ln B_1}{\varsigma_1}, -\frac{m_2 - \ln B_2}{\varsigma_2}\right).$$

$$D_{1,i} = \frac{1}{S_{i,0}^2}, D_{2,i} = -2\left(\frac{B_{i,T}}{S_{i,0}^2} + \frac{e^{rT}}{S_{i,0}}\right), D_{3,i} = \left(\frac{B_{i,T}}{S_{i,0}} + e^{rT}\right)^2$$

and

$$I_{\{V_{i,T} \le B_{i,T}\}} = \Phi\left(-\frac{m_{i,T} - \ln B_{i,T}}{\varsigma_1}\right)$$

Derivation of Result 1

The left-hand-side of (A1) can be expressed as

$$E\left(V_1 V_2 I_{\{V_1 > B_1, V_2 > B_2\}}\right) = \int_{B_1}^{\infty}\int_{B_2}^{\infty} \exp\left(\mathbf{lnV}\begin{bmatrix}1\\1\end{bmatrix}\right)\psi(\mathbf{lnV})d\ln V_1 d\ln V_2 \tag{A12}$$

Suppose that $\sigma_1\sigma_2 \neq 0$ and $|\rho| < 1$. Then

$$\psi(\mathbf{lnV}) = \frac{1}{2\pi\sqrt{\det \boldsymbol{\Sigma}_\mathbf{T}}}\exp\left[-\frac{1}{2}\left((\mathbf{lnV - m})\boldsymbol{\Sigma}_\mathbf{T}^{-1}(\mathbf{lnV - m})'\right)\right] \tag{A13}$$

where

$$\mathbf{m} = (m_1, m_2)$$

$$\Sigma_{\mathrm{T}} \cdot \Sigma_{\mathrm{T}}^{'} = \begin{bmatrix} \varsigma_1^2 & \rho\varsigma_1\varsigma_2 \\ \rho\varsigma_1\varsigma_2 & \varsigma_2^2 \end{bmatrix}$$

By an affine transformation

$$\mathbf{U} = \mathbf{lnV} \cdot \mathbf{F} + \mathbf{v} \qquad \text{(A14)}$$

where

$$\mathrm{U} = (U_1, U_2)$$
$$\mathrm{F} = \begin{bmatrix} \dfrac{1}{\varsigma_1} & -\dfrac{\rho}{\varsigma_1\sqrt{1-\rho^2}} \\ 0 & \dfrac{1}{\varsigma_2\sqrt{1-\rho^2}} \end{bmatrix}$$
$$\mathrm{v} = (\nu_1, \nu_2) = -\mathrm{mF}$$

U_1 and U_2 are independent and identically distributed standard normal random variables and

$$\mathrm{lnV} = \mathrm{U} \cdot \mathrm{F}^{-1} + \mathrm{m}$$

The Jacobian of the transformation is

$$J(\mathbf{U}) = \left|\frac{\partial \mathrm{ln}\mathbf{V}}{\partial \mathbf{U}}\right| = \varsigma_1\varsigma_2\sqrt{1-\rho^2}$$

The region under the integration in (A13) is bounded in $(\ln V_1, \ln V_2)$ plane by

$$V_1 = B_1, V_2 = B_2$$

The image under the transformation in (U_1, U_2) plane is bounded by

$$U_1 = b_1$$
$$U_2 = -a_2 U_1 + b_2$$

$$b_1 = \frac{\ln B_1 - m_1}{\varsigma_1}, a_2 = \frac{\rho}{\sqrt{1-\rho^2}}, b_2 = \frac{\ln B_2 - m_2}{\varsigma_2\sqrt{1-\rho^2}}$$

The change of variable theorem gives

$$E\left(V_1 V_2 I_{\{V_1>B_1, V_2>B_2\}}\right)$$
$$= \int_{b_1}^{\infty} \int_{a_2 U_1 + b_2}^{\infty} \exp\left(\left(\mathrm{U} \cdot \mathrm{F}^{-1} + \mathrm{m}\right)\begin{bmatrix}1\\1\end{bmatrix}\right)\phi(U_1)\phi(U_2) J(\mathbf{U}) dU_1 dU_2 \tag{A15}$$

where ϕ is the standard normal probability density function. (A15) can be rewritten as

$$E\left(V_1 V_2 I_{\{V_1>B_1, V_2>B_2\}}\right)$$
$$= \gamma \int_{b_1}^{\infty} \phi\left(U_1 - (\varsigma_1 + \rho\varsigma_2)\right) dU_1 \int_{-a_2 U_1 + b_2}^{\infty} \phi\left(U_2 - \varsigma_2\sqrt{1-\rho^2}\right) dU_2 \tag{A16}$$

where

$$\gamma = \exp\left[\rho\varsigma_1\varsigma_2 + \frac{\varsigma_1^2 + \varsigma_2^2}{2} + m_1 + m_2\right]$$

Integrating (A16) with respective to U_2 yields

$$E\left(V_1 V_2 I_{\{V_1>B_1, V_2>B_2\}}\right)$$
$$= \gamma \int_{b_1}^{\infty} \phi\left(U_1 - (\varsigma_1 + \rho\varsigma_2)\right) \Phi\left(a_2 U_1 - b_2 + \varsigma_2\sqrt{1-\rho^2}\right) dU_1$$

The integral can be expressed in terms of the bivariate normal distribution function, Ψ, (Chuang, 1996)

$$E\left(V_1 V_2 I_{\{V_1>B_1, V_2>B_2\}}\right)$$
$$= \gamma \Psi\left(\varsigma_1 + \rho\varsigma_2 + \frac{m_1 - \ln B_1}{\varsigma_1}, \rho\varsigma_1 + \varsigma_2 + \frac{m_2 - \ln B_2}{\varsigma_2}\right) \tag{A17}$$

Using a similar transformation, we have (A2) to (A4). For (A5), it can be related to (A2) by

$$E\left(V_{1,T} I_{\{V_{1,T}>B_{1,T}, V_{2,T}\le B_{2,T}\}}\right) = E\left(V_{1,T} I_{\{V_{1,T}>B_{1,T}\}}\right) - E\left(V_{1,T} I_{\{V_{1,T}>B_{1,T}, V_{2,T}>B_{2,T}\}}\right) \tag{A18}$$

where

$$
\begin{aligned}
E\left(V_{1,T} I_{\{V_{1,T} > B_{1,T}\}}\right) &= \int_{b_1}^{\infty} \exp(\varsigma_1 U_1 + m_1)\phi(U_1) dU_1 \\
&= \exp\left(\frac{\varsigma_1^2}{2} + m_1\right)\Phi\left(\frac{m_1 - \ln B_1}{\varsigma_1} + \varsigma_1\right)
\end{aligned}
\tag{A19}
$$

Using (A19) and (A2), we obtain the result in (A5):

$$
\begin{aligned}
&E\left(V_{1,T} I_{\{V_{1,T} > B_{1,T}, V_{2,T} \le B_{2,T}\}}\right) \\
&= \exp\left(m_1 + \frac{\varsigma_1^2}{2}\right)\left\{\Phi\left(\frac{m_1 - \ln B_1}{\varsigma_1} + \varsigma_1\right) - \Psi\left(\frac{m_1 - \ln B_1}{\varsigma_1} + \varsigma_1, \frac{m_2 - \ln B_2}{\varsigma_2} + \rho\varsigma_1\right)\right\}
\end{aligned}
\tag{A20}
$$

By a similar transformation, we have (A6), (A7) and (A8).

Under the equivalent martingale measure, we have

$$
S_o = E\left(V_{1,T} I_{\{V_{1,T} > B_{1,T}\}}\right) - e^{-rT} B_T E\left(I_{\{V_{1,T} > B_{1,T}\}}\right)
$$

which is the Black-Scholes option formula. Since $V_0 = \exp\left(m + \frac{\varsigma^2}{2}\right)$, we have

$m_i = \ln V_{i,0} + rT - \frac{\varsigma_i^2}{2}$ under the equivalent martingale measure.

Appendix B

Derivation of Equity Correlation from Asset Correlation

Given the results in Appendix A, we can derive equity correlation from asset correlation. The equity value of firm *i* is the call value on the underlying firm asset process $V_{i,t}$. At time *T* the firm value is given by Eq. (5). The equity return for the period from 0 to *T* can be expressed as the call option with the strike equal to the firm's face value of debt

$$\frac{S_{i,T} - S_{i,0}}{S_{i,0}} = \frac{\left(V_{i,T} - B_{i,T}\right)^{+}}{S_{i,0}} - 1 \tag{B1}$$

We use the simple return rather than a continuous compounding in order to obtain explicit solutions.[14] For a constant interest rate and under the equivalent martingale measure, we have

$$E\frac{S_{1,T} - S_{1,0}}{S_{1,0}} = E\left[\frac{\left(V_{i,T} - B_{i,T}\right)^{+}}{S_{i,0}} - 1\right] = e^{rT} - 1 \tag{B2}$$

Since the volatility of a Brownian motion is invariant under different equivalent martingale measures, we can calculate the equity correlation between two firms under either the physical measure or the equivalent martingale measure. Thus, by definition the covariance between the equity returns of firms *i* and *j*, we have

[14] Given the joint probability distribution of the two assets, we may alternatively assume continuous return and compute the equity correlation directly by using the definition

$$Cov\left(\ln\frac{S_{1,T}}{S_{1,0}}\ln\frac{S_{1,T}}{S_{1,0}}\right)$$

However, this alternative does not provide much simplification.

$$
\begin{aligned}
\operatorname{cov}(R_1, R_2) &= \operatorname{cov}\left(\frac{S_{1,T} - S_{1,0}}{S_{1,0}}, \frac{S_{2,T} - S_{2,0}}{S_{2,0}}\right) \\
&= E\left(\frac{V_{1,T} - B_{1,T}}{S_{1,0}} - e^{rT}\right)\left(\frac{V_{2,T} - B_{2,T}}{S_{2,0}} - e^{rT}\right) I_{\{V_{,T1} > B_{1,T}, V_{2,T} > B_{2,T}\}} \\
&\quad - E\left(\frac{V_{1,T} - B_{1,T}}{S_{1,0}} - e^{rT}\right) e^{rT} I_{\{V_{,T1} > B_{1,T}, V_{2,T} \le B_{2,T}\}} \\
&\quad - E e^{rT}\left(\frac{V_{2,T} - B_{2,T}}{S_{2,0}} - e^{rT}\right) I_{\{V_{,T1} \le B_{1,T}, V_{2,T} > B_{2,T}\}} + E e^{2rT} I_{\{V_{,T1} \le B_{1,T}, V_{2,T} \le B_{2,T}\}}
\end{aligned}
\tag{B3}
$$

The first term on the right hand side of (B3) can be expanded and rewritten as

$$
\begin{aligned}
& E\left(\frac{V_{1,T} - B_{1,T}}{S_{1,0}} - e^{rT}\right)\left(\frac{V_{2,T} - B_{2,T}}{S_{2,0}} - e^{rT}\right) I_{\{V_{,T1} > B_{1,T}, V_{2,T} > B_{2,T}\}} \\
&= A_1 E\left(V_{1,T} V_{2,T} I_{\{V_{T,1} > B_{1,T}, V_{2,T} > B_{2,T}\}}\right) + A_2 E\left(V_{1,T} I_{\{V_{T,1} > B_{1,T}, V_{2,T} > B_{2,T}\}}\right) \\
&\quad + A_3 E\left(V_{2,T} I_{\{V_{T,1} > B_{1,T}, V_{2,T} > B_{2,T}\}}\right) + A_4 I_{\{V_{T,1} > B_{1,T}, V_{2,T} > B_{2,T}\}}
\end{aligned}
\tag{B4}
$$

where

$$
\begin{aligned}
& A_1 = \frac{1}{S_{1,0} S_{2,0}}, A_2 = -\frac{1}{S_{1,0}}\left(\frac{B_{2,T}}{S_{2,0}} + e^{rT}\right), A_3 = -\frac{1}{S_{2,0}}\left(\frac{B_{1,T}}{S_{1,0}} + e^{rT}\right), \\
& A_4 = \frac{B_{1,T} B_{2,T}}{S_{1,0} S_{2,0}} + \left(\frac{B_{1,T}}{S_{1,0}} + \frac{B_{2,T}}{S_{2,0}}\right) e^{rT} + e^{2rT}
\end{aligned}
\tag{B5}
$$

The second and the third terms on the right hand side of (B3) can be expanded and rewritten as

$$
\begin{aligned}
& E\left(\frac{V_{1,T} - B_{1,T}}{C_{1,0}} - e^{rT}\right) e^{rT} I_{\{V_{1,T} > B_{1,T}, V_{2,T} \le B_{2,T}\}} \\
&= -A_5 E\left(V_{1,T} I_{\{V_{1,T} > B_{1,T}, V_{2,T} \le B_{2,T}\}}\right) - A_6 e^{rT} I_{\{V_{1,T} > B_{1,T}, V_{2,T} \le B_{2,T}\}}
\end{aligned}
\tag{B6}
$$

$$
\begin{aligned}
& E e^{rT}\left(\frac{V_{2,T} - B_{2,T}}{C_{2,0}} - e^{rT}\right) I_{\{V_{1,T} \le B_{1,T}, V_{2,T} > B_{2,T}\}} \\
&= -A_7 E\left(V_{2,T} I_{\{V_{1,T} \le B_{1,T}, V_{2,T} > B_{2,T}\}}\right) - A_8 I_{\{V_{1,T} \le B_{1,T}, V_{2,T} > B_{2,T}\}}
\end{aligned}
\tag{B7}
$$

$$A_5 = -\frac{e^{rT}}{S_{1,0}}, A_6 = \frac{B_{1,T}}{S_{1,0}} e^{rT} + e^{2rT}, A_7 = -\frac{e^{rT}}{S_{2,0}}, A_8 = e^{rT} \frac{B_{2,T}}{S_{2,0}} + e^{2rT} \tag{B8}$$

Finally, the covariance of the equity returns of firms 1 and 2 can be expressed as

$$\begin{aligned} \operatorname{cov}(R_1, R_2) = & A_1 E\left(V_{1,T} V_{2,T} I_{\{V_{T,1} > B_{1,T}, V_{2,T} > B_{2,T}\}}\right) + A_2 E\left(V_{1,T} I_{\{V_{T,1} > B_{1,T}, V_{2,T} > B_{2,T}\}}\right) \\ & + A_3 E\left(V_{2,T} I_{\{V_{T,1} > B_{1,T}, V_{2,T} > B_{2,T}\}}\right) + A_4 E I_{\{V_{T,1} > B_{1,T}, V_{2,T} > B_{2,T}\}} \\ & + A_5 E\left(V_{1,T} I_{\{V_{1,T} > B_{1,T}, V_{2,T} \le B_{2,T}\}}\right) + A_6 E I_{\{V_{1,T} > B_{1,T}, V_{2,T} \le B_{2,T}\}} \\ & + A_7 E\left(V_{2,T} I_{\{V_{1,T} \le B_{1,T}, V_{2,T} > B_{2,T}\}}\right) + A_8 E I_{\{V_{1,T} \le B_{1,T}, V_{2,T} > B_{2,T}\}} + e^{2rT} I_{\{V_{1,T} \le B_{1,T}, V_{2,T} \le B_{2,T}\}} \end{aligned} \tag{B9}$$

where the expectations are given in Lemma 1. By definition, the variance of the equity returns is

$$\begin{aligned} \operatorname{var}(R_i) &= \operatorname{var}\left(\frac{S_{i,T} - S_{i,0}}{S_{i,0}}\right) = E\left[\frac{S_{i,T} - S_{i,0}}{S_{i,0}} - E\left(\frac{S_{i,T} - S_{i,0}}{S_{i,0}}\right)\right]^2 \\ &= E\left(\frac{V_{i,T} - B_{i,T}}{S_{i,0}} - e^{rT}\right)^2 I_{\{V_{i,T} > B_{i,T}\}} + e^{2rT} I_{\{V_{i,T} \le B_{i,T}\}} \end{aligned} \tag{B10}$$

The first term on the right hand side of (B10) can be expanded into

$$\begin{aligned} & E\left(\frac{V_{i,T} - B_{i,T}}{S_{i,0}} - e^{rT}\right)^2 I_{\{V_{i,T} > B_{i,T}\}} \\ & = D_{1,i} E\left(V_{i,T}^2 I_{\{V_{i,T} > B_{i,T}\}}\right) + D_{2,i} E\left(V_{i,T} I_{\{V_{i,T} > B_{i,T}\}}\right) + D_{3,i} I_{\{V_{i,T} > B_{i,T}\}} \end{aligned} \tag{B11}$$

where

$$D_{1,i} = \frac{1}{S_{i,0}^2}, D_{2,i} = -2\left(\frac{B_{i,T}}{S_{i,0}^2} + \frac{e^{rT}}{S_{i,0}}\right), D_{3,i} = \left(\frac{B_{i,T}}{S_{i,0}} + e^{rT}\right)^2 \tag{B12}$$

Combining (B10) and (B11) yields

$$\begin{aligned} \operatorname{var}(R_i) = & D_{1,i} E\left(V_{i,T}^2 I_{\{V_{i,T} > B_{i,T}\}}\right) + D_{2,i} E\left(V_{i,T} I_{\{V_{i,T} > B_{i,T}\}}\right) \\ & + D_{3,i} I_{\{V_{i,T} > B_{i,T}\}} + e^{2rT} E I_{\{V_{i,T} \le B_{i,T}\}} \end{aligned} \tag{B13}$$

Appendix C

Joint Default Probabilities

This appendix summarizes the joint default probabilities (see Zhou, 2001). When debt and equity have equal expected growth rates, i.e., leverage ratios l_1 and l_2 are constant, the expected default probability is

$$E[\omega_i(T)=1]=P[\omega_i(T)=1]=\Phi\left(-\frac{Z_i}{\sqrt{T}}-\frac{\mu_i}{\sigma_i}\sqrt{T}\right)+e^{-\frac{\mu_i}{\varsigma_i}Z_i}\Phi\left(-\frac{Z_i}{\sqrt{T}}-\frac{\mu_i}{\sigma_i}\sqrt{T}\right)$$

with properties

$$\operatorname{var}\left[\omega_i(T)\right]=P\left[\omega_i(T)=1\right]\left\{1-P\left[\omega_i(T)=1\right]\right\},\quad \text{and}\quad Z_i=\frac{\ln V_{i,0}-\ln V_{Bi}}{\sigma_i}$$

where ζ_i is the volatility of firm value. The joint default probability is

$$E[\omega_1(T)\cdot\omega_2(T)]=E[\omega_1(T)]+E[\omega_2(T)]-P[\omega_1(T)=1\text{ or }\omega_2(T)=1],$$

where the probability when at least one firm has defaulted is

$$P\left[\omega_1(T)=1\text{ or }\omega_2(T)=1\right]=1-\frac{1}{\alpha T}e^{\beta_1x_1+\beta_2x_2+\beta_tT}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi\theta_o}{\alpha}\right)e^{-\frac{r_o^2}{2t}}\int_0^{\alpha}\sin\left(\frac{n\pi\theta}{\alpha}\right)g_n(\theta)\,d\theta,$$

and the relevant parameters are given as follows,

$$\alpha=\begin{cases}\tan^{-1}\left(-\dfrac{\sqrt{1-\rho^2}}{\rho}\right) & if\quad \rho<0\\[2ex] \pi+\tan^{-1}\left(-\dfrac{\sqrt{1-\rho^2}}{\rho}\right) & otherwise,\end{cases}\qquad \theta_o=\begin{cases}\tan^{-1}\left(\dfrac{Z_2\sqrt{1-\rho^2}}{Z_1-\rho Z_2}\right) & if\quad \rho<0\\[2ex] \pi+\tan^{-1}\left(\dfrac{Z_2\sqrt{1-\rho^2}}{Z_1-\rho Z_2}\right) & otherwise,\end{cases}$$

$$g_n(\theta)=\int_0^{\infty}re^{-\frac{r^2}{2t}}e^{r\left[h_1\sin(\theta-\alpha)-h_2\cos(\theta-\alpha)\right]}I_{\frac{n\pi}{\alpha}}\left(\frac{rr_o}{t}\right)dr,\qquad r_o=\frac{Z_2}{\sin(\theta_o)},\qquad \beta_1=\frac{\mu_2\rho\sigma_1-\mu_1\sigma_2}{(1-\rho^2)\sigma_1^2\sigma_2},$$

$$\beta_2=\frac{\mu_1\rho\sigma_2-\mu_2\sigma_1}{(1-\rho^2)\sigma_1^2\sigma_2},\qquad \beta_t=\frac{\beta_1^2\sigma_1^2}{2}+\rho a_1a_2\sigma_1\sigma_2+\frac{\beta_2^2\sigma_2^2}{2}+\beta_1\mu_1+\beta_2\mu_2,$$

$$h_1=\beta_1\sigma_1+\rho\sigma_1\sigma_2,\qquad h_2=\beta_2\sigma_2\sqrt{1-\rho^2}.$$

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Table 1. Default correlation predicted by the Merton model

The simple model prediction uses inputs of equity correlation and volatility estimated from the historical stock returns between 1970 and 1993. The only undetermined factor is an equity volatility multiplier (a fudge factor) in order to generate reasonable magnitude of default correlation. The time horizon in the simulation is five years.

	Default correlation (%)					Default correlation (%) with survival bias				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	1.2					1.1				
A	1.8	3.7				1.8	3.8			
Baa	1.6	4.2	5.4			2.0	4.8	6.0		
Ba	2.4	5.0	6.4	9.4		2.3	6.2	7.9	12.6	
B	1.9	5.2	7.2	11.1	13.0	3.4	7.3	9.7	15.1	18.0

Table 2. Calibration parameters for the extended LT model

(A) Historical cumulative default rates (%), 1970-1993 – Source: Fons (1994).

Year	Aa	A	Baa	Ba	B
1	0.02	0.01	0.16	1.79	8.31
2	0.04	0.09	0.51	4.38	14.85
3	0.08	0.28	0.91	6.92	20.38
4	0.20	0.46	1.46	9.41	24.78
5	0.32	0.62	1.97	11.85	28.38
6	0.43	0.83	2.46	13.78	31.88
7	0.52	1.06	3.09	15.33	34.32
8	0.64	1.31	3.75	16.75	36.71
9	0.76	1.61	4.39	18.14	38.38
10	0.91	1.96	4.96	19.48	39.96

(B) Equity premium (%) – Source: Bhandari (1998)

Aa	A	Baa	Ba	B
5.6	5.99	6.55	7.3	8.76

(C) Implied asset volatility σ (%) from the calibrated model. We calibrate the extended LT model against the historical default rates. The resulting implied asset volatilities σ for various rating categories are reported below.

Aaa	Aa	A	Baa	Ba	B
3.260	3.432	3.811	9.507	23.252	42.892

Table 3. Model-implied equity correlation with fixed asset correlation ($\rho = 40\%$)
The numbers reported in the table are in percentages.

(1) T_H (years)	(2) **Aa - Aa**	(3) **Aa-A**	(4) **Aa-Baa**	(5) **Aa-Ba**
1	38.03	38.53	38.14	38.48
2	37.88	38.13	37.91	37.54
3	37.67	37.63	37.11	36.10
4	37.34	37.11	36.32	34.57
5	37.09	36.60	35.43	33.13
6	36.82	36.25	34.74	31.82
7	36.64	35.96	34.11	30.70
8	36.50	35.75	33.53	29.66
9	36.38	35.57	32.98	28.63
10	36.30	35.41	32.52	27.79
	Aa-B	**A-A**	**A-Baa**	**A-Ba**
1	37.95	38.38	38.40	38.14
2	35.95	37.81	37.87	37.23
3	33.76	36.95	36.82	35.54
4	31.58	36.33	35.84	33.98
5	29.55	35.81	34.90	32.54
6	27.78	35.39	34.14	31.23
7	26.22	35.10	33.47	30.03
8	24.82	34.81	32.87	28.99
9	23.52	34.59	32.31	27.98
10	22.41	34.40	31.82	27.11
	A-B	**Baa-Baa**	**Baa-Ba**	**Baa-B**
1	38.26	38.40	38.40	37.84
2	35.96	37.87	37.69	36.31
3	33.50	36.90	36.02	33.88
4	31.21	35.76	34.38	31.59
5	28.94	34.70	32.74	29.37
6	27.17	33.76	31.34	27.53
7	25.61	32.89	30.13	25.88
8	24.19	32.04	29.00	24.45
9	22.86	31.29	27.92	23.07
10	21.74	30.66	26.97	21.95
	Ba-Ba	**Ba-B**	**B-B**	
1	38.09	38.30	37.84	
2	36.94	36.00	35.02	
3	34.96	33.23	31.82	
4	32.94	30.63	28.91	
5	30.91	28.23	26.24	
6	29.17	26.28	24.08	
7	27.72	24.59	22.23	
8	26.34	23.10	20.62	
9	25.07	21.69	19.15	
10	24.00	20.48	17.96	

Table 4. Equity correlation ρ_S estimated with monthly return data
(Based on monthly equity return data 1970-1993; source: CRSP)

Panel 1: Summary statistics

Rating1	Rating2	Observations	min	max	Q1	Q3
Aa	Aa	21,348	-0.90	0.97	0.04	0.51
Aa	A	99,144	-0.99	0.99	0.03	0.50
Aa	Baa	133,420	-0.97	0.99	0.02	0.48
Aa	Ba	65,292	-0.89	0.98	-0.01	0.46
Aa	B	36,795	-0.96	0.98	-0.02	0.47
A	A	478,221	-0.98	0.99	0.04	0.50
A	Baa	646,946	-0.97	0.99	0.02	0.49
A	Ba	319,887	-0.98	1.00	0.00	0.47
A	B	182,209	-0.98	0.99	0.00	0.48
Baa	Baa	728,526	-0.99	1.00	0.01	0.48
Baa	Ba	365,471	-0.97	0.99	-0.01	0.46
Baa	B	205,840	-0.97	1.00	-0.01	0.47
Ba	Ba	176,547	-0.99	0.99	-0.01	0.46
Ba	B	100,141	-0.93	0.99	0.00	0.48
B	B	63,932	-0.90	0.98	0.01	0.48

Panel 2: Mean correlations* and standard errors

	Aa	A	Baa	Ba	B
Aa	25.83 (0.0022)				
A	25.02 (0.0010)	25.80 (0.0005)			
Baa	23.67 (0.0009)	24.28 (0.0004)	23.21 (0.0004)		
Ba	21.65 (0.0013)	22.80 (0.0006)	21.43 (0.0005)	21.93 (0.0008)	
B	21.34 (0.0017)	22.98 (0.0008)	21.61 (0.0007)	22.72 (0.0010)	23.14 (0.0013)

* Mean correlation are in percentage.

Table 5. Calibrated asset correlation ρ and default correlation ρ_D

Asset correlations are obtained by calibrating the hybrid model using monthly equity return data 1970-1993 from CRSP. Default correlations estimated by the hybrid model are compared with those estimated by Zhou's (2001) model and estimates by Lucas (1995). Reported are results for horizons $T = 4, 6, 8$ and 10 years.[15]

Panel A. Calibrated asset correlation from empirical equity correlation ρ_S

Calibrated asset correlation ρ in percentage is obtained by matching model-implied equity correlation ρ_S to our empirical estimates (in Table 4).

	$T = 4$ years					$T = 6$ years				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	27.38					27.86				
A	26.93	28.29				27.53	29.06			
Baa	25.89	26.98	25.77			27.12	28.39	27.40		
Ba	24.89	26.68	24.72	26.28		27.04	29.08	27.19	29.82	
B	26.74	27.23	27.16	29.51	31.81	30.51	31.31	31.23	34.46	38.22

	$T = 8$ years					$T = 10$ years				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	28.16					28.37				
A	27.93	29.56				28.21	29.93			
Baa	28.13	29.52	28.89			29.01	30.51	30.21		
Ba	29.05	31.36	29.43	33.05		29.92	33.57	31.65	36.34	
B	34.19	35.20	35.23	39.22	44.58	37.86	39.18	39.23	44.24	51.27

[15] We obtain results for $T = 1$ through 10 years. Results for other years are available upon request.

Panel B. Lucas's (1995) estimates of default correlation

This panel shows empirical estimates of default correlation ρ_D in percentage from Lucas (1995).

	$T = 4$ years					$T = 6$ years				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	0.0					1.0				
A	1.0	1.0				1.0	1.0			
Baa	1.0	1.0	0.0			1.0	1.0	0.0		
Ba	2.0	3.0	3.0	13.0		3.0	4.0	3.0	15.0	
B	2.0	4.0	5.0	22.0	27.0	4.0	7.0	7.0	25.0	29.0

	$T = 8$ years					$T = 10$ years				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	0.0					1.0				
A	1.0	2.0				2.0	2.0			
Baa	1.0	1.0	0.0			1.0	1.0	0.0		
Ba	3.0	5.0	2.0	10.0		3.0	4.0	2.0	8.0	
B	5.0	11.0	7.0	23.0	37.0	8.0	9.0	6.0	17.0	38.0

Panel C. Default correlation estimates from Zhou's model

Reported are default correlation ρ_D estimates in percentage using Zhou's (2001) approach (i.e., $\rho = \rho_S$ with ρ_S given in Table 4, Panel 2).

	$T = 4$ years					$T = 6$ years				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	0.03					0.31				
A	0.07	0.17				0.50	0.96			
Baa	0.12	0.35	0.88			0.72	1.49	2.66		
Ba	0.17	0.61	1.91	6.93		0.87	2.02	4.16	9.41	
B	0.14	0.56	1.99	8.92	12.96	0.73	1.81	4.06	10.80	13.60

	$T = 8$ years					$T = 10$ years				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	0.97					1.90				
A	1.35	2.22				2.44	3.62			
Baa	1.75	3.00	4.52			2.95	4.51	6.12		
Ba	1.90	3.60	5.99	10.79		3.01	5.01	7.35	11.62	
B	1.61	3.18	5.65	11.70	13.73	2.52	4.39	6.80	12.16	13.68

Panel D. Default correlation estimates from the hybrid model

Reported are default correlation ρ_D estimates in percentage from the hybrid model.

	$T = 4$ years					$T = 6$ years				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	0.04					0.39				
A	0.08	0.23				0.62	1.26			
Baa	0.15	0.45	1.09			0.94	2.00	3.50		
Ba	0.21	0.77	2.34	8.69		1.15	2.77	5.60	13.44	
B	0.18	0.66	2.53	11.86	18.21	1.01	2.43	5.87	16.74	23.12

	$T = 8$ years					$T = 10$ years				
	Aa	A	Baa	Ba	B	Aa	A	Baa	Ba	B
Aa	1.18					2.27				
A	1.68	2.86				3.01	4.63			
Baa	2.34	4.11	6.22			4.01	6.29	8.74		
Ba	2.70	5.26	8.68	17.06		4.33	7.77	11.42	20.19	
B	2.43	4.70	9.10	20.63	27.54	4.14	7.09	12.06	24.25	32.13