

Competing for Entrepreneurial Ideas: Matching and Contracting in the Venture Capital Market

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Abstract

We propose an equilibrium model of the venture capital (VC) market with two-sided matching between heterogeneous VCs and entrepreneurs in a principal-agent framework. Each VC matches endogenously with an entrepreneur, offering an equity share and investment that ensures the entrepreneur does not contract with another VC. There is positive assortative matching (PAM), and matching enhances investment and effort levels. Entry at the bottom of the ladder induces improvements across all ventures in the market (the *ripple effect*); but exit at the bottom of the ladder may trigger a wave of failure as it travels up the ladder (the *unraveling effect*).

Keywords: Entrepreneurship, venture capital, matching, contracts, ideas, expertise, moral hazard, incentives.

JEL classification: C78, D86, G24, L26, M13.

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1 Introduction

The venture capital (VC) market has long been recognized as a fundamental source for entrepreneurs to finance their ventures, and thus to bring their innovative ideas to market (Kaplan, Sensoy, and Strömberg (2009)). In addition to simply providing entrepreneurs with the required financial resources to start their own businesses, venture capitalists bring in specific expertise, which is particularly invaluable for entrepreneurs with little or no business-related skills. Specifically, venture capitalists monitor and are commonly represented on the boards of directors (Lerner (1995), Gompers and Lerner (1999)); provide technical and commercial advice (Bygrave and Tymmons (1992)); have valuable networks and meet with suppliers and customers (Gorman and Sahlman (1989), Hochberg, Ljungqvist, and Lu (2007, 2010)); help in designing strategy and human resource policies and have contacts with potential managers (Hellman and Puri (2002)); assist with the creation of strategic alliances (Lindsey (2008)); and frequently visit the firms they finance (Gorman and Sahlman (1989)). The expertise of venture capitalists can therefore be crucial for the success of new ventures and the exploration of innovative ideas with market potential. Given their specific expertise, the key challenge for venture capitalists is not only to identify entrepreneurs with suitable ideas, but also to attract them by offering mutually beneficial contracts.

This paper brings to light the importance of *heterogeneity* across entrepreneurs (in terms of their ideas) and venture capitalists (in terms of their expertise), as well as *matching* between entrepreneurs and venture capitalists. Venture capitalists (and the contracts they offer) are significantly heterogeneous in terms of their characteristics, skills, and reputation (Kaplan and Strömberg (2003, 2004), Hsu (2004), Kaplan and Schoar (2005), Kaplan, Sensoy, and Strömberg (2009)). Sørensen (2007) provides evidence highlighting the contributions of heterogeneity across VCs and matching between entrepreneurs and VCs in determining the eventual success or failure of start-ups. He shows that companies with more experienced VCs are more likely to go public, for two reasons: experienced VCs provide more value added and bring more companies public (which we label the "heterogeneity" effect); and experienced VCs select superior ventures to begin with (which we label the "matching" effect). He finds in an empirical two-

sided matching model that the matching effect is almost twice as important as the heterogeneity effect in explaining observed differences in IPO rates across VC investors.¹

Inspired by this evidence, we propose a two-sided matching model in a principal-agent framework with moral hazard consisting of a collection of venture capitalists that are heterogeneous in terms of their expertise and a collection of entrepreneurs that are heterogeneous with respect to the quality (or market potential) of their ideas. Each VC matches endogenously with an entrepreneur, whereby the expertise of the VC and the quality of the entrepreneur's idea jointly determine the *match quality* of the venture. The unobservable effort exerted by the entrepreneur determines the success or failure of the venture, while the match quality and investment supplied by the VC determine the profitability of the venture. Each VC offers its matched entrepreneur a contract that consists of an equity share and an investment. The contract must satisfy not only the usual incentive compatibility constraint associated with the moral hazard problem, but also a condition that ensures the entrepreneur does not desire to contract with another VC. In this sense, because VCs hold the bargaining power in the entrepreneur-VC relationship, VCs are competing for entrepreneurs, which plays a key role in the design of optimal contracts and the impact of entry into the VC market. The matching condition implies that the reservation utility of an entrepreneur is endogenous, as it is determined by the payoff he would receive upon contracting with the next best VC; thus, the manner in which changes in the VC market's environment affect contracting operates via an entrepreneur's reservation utility.

While the analysis of isolated entrepreneur-VC relationships in a contractual context undoubtedly provides important insights, a main implication of our study is that matching in the VC market significantly affects the structure and properties of VC contracts. We first derive the conditions under which we obtain *positive assortative matching* (PAM), the situation in which VCs with high expertise are matched to entrepreneurs with high quality ideas. PAM arises if there is complementarity between the entrepreneur's idea quality and the VC's expertise, complementarity between the match quality and the investment by the VC, and diminishing returns to the VC's investment. We refer to the collection of entrepreneur-VC matches as a *ladder*, wherein higher points correspond to higher quality matches. Endogenous matching affects the terms of the contract in each entrepreneur-VC pair. Since the outside option of every entrepreneur is endogenously determined by competition, each VC transfers more util-

¹Matching also matters in general principal-agent relationships (Akerberg and Botticini (2002), Serfes (2005, 2008)).

ity to its entrepreneur than would occur in the absence of matching considerations (as in an isolated principal-agent model). This is achieved through a greater investment in the venture and a larger equity share for the entrepreneur, which in turn enhance entrepreneurial effort and the venture's likelihood of success. Nevertheless, even taking into account matching considerations, the investment by the VC and effort exerted by the entrepreneur are below their socially efficient levels. An implication is that any managerial or policy implications should not be based on an isolated analysis of VC contracts; rather, it is indispensable to consider the two-sided VC market as a whole in order to account for the effects of competition.

The effects of entry and exit are particularly interesting. We begin by examining *endogenous entry*, under which fixed costs of entry determine the equilibrium number of entrepreneurs and VCs that enter the market. A threshold entrepreneurial idea determines the quality beyond which ventures are active. We find that entry at the bottom of the ladder (as brought about by a decrease in entry costs, for example) induces an increase in investment, effort, and probability of success across all ventures in the market. This occurs because, due to competition among VCs engendered by the matching process, enhancing the outside option of an entrepreneur (the source of which is the value of contracting with the VC that has the next lower expertise) must be accompanied by a corresponding increase in how favorable are the terms being offered the entrepreneur. We refer to this mechanism as the *ripple effect* since it propagates up the ladder by inductive reasoning. Because an entrepreneur-VC pair that enters the market does not internalize this positive externality, the equilibrium number of ventures is inefficiently low; thus, government policy should aim towards spurring entrepreneurial entry.

The ripple effect in the context of entry can have an *unraveling effect* in the context of exit. Exit at the bottom of the ladder potentially triggers a wave of failure in the VC market as it travels up the ladder. Ventures at the bottom of the ladder have the lowest probability of success. Upon failing and subsequently exiting the market, by reversing the inductive logic behind the ripple effect, we infer that all matched pairs of superior quality respond by diminishing their investments and levels of effort, which in turn raises their probability of failure. Thus, from a policy standpoint, low quality ventures may be "too small to fail." Paradoxically, it may be better for the VC market to experience failures at the top of the ladder since such failures do not exert a negative externality on entrepreneur-VC pairs below.

We next examine *exogenous entry* into the VC market. Entry can improve the match quality if the entrants are of relatively high quality, but it can also negatively affect the outside option of an entrepreneur since it depends on the maximum willingness to pay for that entrepreneur among VCs who are matched with lower quality entrepreneurs. If the venture capitalists' current profit improves, which can happen if entry enhances match qualities, then their willingness to pay for higher quality entrepreneurs diminishes. This lowers their outside option, thereby reducing their investment and effort levels. Thus, an important policy implication is that entry does not necessarily improve efficiency.

We consider three forms of exogenous entry. Entry by an entrepreneur-VC pair reduces the investment, effort, and success probability of entrepreneur-VC pairs of superior quality, while leaving unchanged those of inferior quality. Next, entry by a VC pushes down the ladder the matches of VCs of inferior quality, while simultaneously raising the investment, effort, and success probability of all entrepreneur-VC pairs in the market. This is consistent with evidence on the impact of VC entry (Hochberg, Ljungqvist, and Lu (2010)). Finally, entry by an entrepreneur pushes down the ladder the matches of entrepreneurs of inferior quality, while simultaneously reducing the investment, effort, and success probability of entrepreneur-VC pairs of superior quality.

Our paper primarily contributes to the theoretical literature on the contractual relationship between entrepreneurs and venture capitalists.² Casamatta (2003) considers the managerial incentives of investors and entrepreneurs as a double-sided moral hazard problem. Kirilenko (2001) studies the entrepreneur-VC relationship when the entrepreneur derives non-pecuniary benefits from having some control over the firm. Repullo and Suarez (2004) study the design of optimal securities in an environment with multiple investment stages and double-sided moral hazard. Schmidt (2003) shows the optimality of convertible securities in an environment in which investment is sequential and observable but not contractible. Ueda (2004) examines the source of finance when there are weak property rights. Hellmann (1998) focuses on the allocation of control rights between venture capitalists and entrepreneurs. Our model does not account for different allocations of control rights, the source of finance, or dynamic interactions between entrepreneurs and venture capitalists; instead, we examine the allocation of equity, the magnitude of the investment by the venture capitalist, and the roles of entry and exit when matching considerations are taken into account.

²See Hart (2001) for a review of the literature on financial contracting up to that point.

Our paper is closest in spirit to Inderst and Müller (2004), who study an equilibrium model of the VC market. Our paper, however, differs in two important aspects. First, Inderst and Müller consider a search equilibrium, whereas we study the matching equilibrium in the VC market. The search equilibrium in their model is characterized by the ratio of the number of venture capitalists to the number of entrepreneurs, commonly referred to as "market thickness" in the search theory literature. Using a matching framework, however, we show that it is not just the market thickness that matters; the qualities of entrepreneurial ideas and the expertise of venture capitalists are also critical in the optimal design of VC contracts. Second, Inderst and Müller consider homogenous venture capitalists and entrepreneurs, whereas we explicitly account for different match qualities by allowing entrepreneurs to be heterogenous with respect to the market potential of their ideas, and venture capitalists with respect to their expertise.

The remainder of the paper is structured as follows. Section 2 introduces the features of the model. Section 3 derives the optimal contract of an entrepreneur-VC pair in the absence of matching considerations. Section 4 incorporates two-sided matching in the VC market and characterizes the optimal contracts. Section 5 investigates the roles of entry and exit in the VC market, and how they affect the contractual relationships between entrepreneurs and VCs. Section 6 summarizes our main results and concludes.

2 Preliminaries

2.1 The Structure and Timing of the Model

Consider a market of $n \geq 2$ risk-neutral entrepreneurs and $n \geq 2$ risk-neutral venture capitalists. All entrepreneurs are wealth constrained and their reservation utilities are normalized to zero.³

There are four dates:

1. Entrepreneurs are endowed with ideas for new ventures.

³Besley and Ghatak (2005) also consider contracting between risk-neutral agents and principals in a framework with limited liability. The standard approach in the literature that studies moral hazard with limited liability is due to Innes (1990). While we initially assume a zero outside option for every entrepreneur, a strictly positive reservation utility for entrepreneurs with high-quality projects arises endogenously in our matching framework.

2. Each venture capitalist matches with an entrepreneur and offers him a contract that consists of a share of the venture and an investment in the venture.
3. Entrepreneurs exert unobservable effort.
4. Profits of ventures are realized and payments are made.

At date 1, each entrepreneur i is endowed with an idea, denoted μ_i , $i \in E = \{1, \dots, n\}$, where a higher value of μ_i reflects an idea of superior quality. Ideas are ranked according to their quality, i.e., $\mu_n \geq \dots \geq \mu_2 \geq \mu_1$. To commercially exploit his idea, each entrepreneur relies on a capital investment K_i as well as on managerial and marketing expertise, both provided by a venture capitalist (VC). Let x_j denote the expertise of venture capitalist $j \in V = \{1, \dots, n\}$, wherein superior expertise is reflected by a higher x_j , such that the levels of expertise are ranked, i.e., $x_n \geq \dots \geq x_2 \geq x_1$. Each VC matches with exactly one entrepreneur at date 2, upon which venture capitalist j offers its entrepreneur i a contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ that specifies a share λ_{ij} of total profit for the entrepreneur as well as a capital investment K_{ij} .⁴ The risk-adjusted cost of capital faced by each venture capitalist is denoted $r > 0$. At date 3, after signing the contract, an entrepreneur $i = 1, \dots, n$ matched with a VC j exerts effort e_{ij} to turn his idea into a marketable product. Implementing effort imposes the cost $c(e_{ij}) = e_{ij}^2/2$. The entrepreneur's effort e_{ij} determines the likelihood of whether the venture succeeds ($Y_{ij} = 1$) or fails ($Y_{ij} = 0$). Specifically, we assume that $\text{Prob}[Y_{ij} = 1 | e_{ij}] = e_{ij}$. Finally, at date 4, the profit of each venture is realized, and payments according to the respective contracts are made.

A key property of our framework is the reliance of entrepreneurs on the expertise of venture capitalists to turn promising ideas into marketable products. Intuitively, the value of this expertise is closely related to the entrepreneur's particular idea. To capture this notion, let $\Omega_{ij} \equiv \Omega(\mu_i, x_j) > 0$ denote the *match quality* between an entrepreneur with idea μ_i and a venture capitalist with expertise x_j . The match quality Ω_{ij} is strictly increasing in both the entrepreneur's idea quality μ_i and the venture capitalist's expertise x_j . Furthermore, ideas and expertise are assumed to be complements, i.e., $\partial^2 \Omega(\mu_i, x_j) / \partial x_i \partial \mu_j > 0$. Match qualities and thereby ventures may thus also be ranked, such that a collection of ventures consisting of entrepreneurs matched to venture capitalists is referred to as a *ladder*.

⁴Alternatively, we may interpret λ_{ij} as the fraction of shares for the entrepreneurs, whereas the remaining shares, $1 - \lambda_{ij}$, are held by the venture capitalist in exchange for its capital provision K_{ij} .

Let $\Pi_{ij} \in \{0, \pi(K_{ij}, \Omega_{ij})\}$ denote the gross profit of the venture started by entrepreneur i and financed by venture capitalist j , where $\pi(K_{ij}, \Omega_{ij}) \geq 0$ denotes the gross profit of a *successful* venture (i.e., $Y_{ij} = 1$).⁵ Given the entrepreneur's effort e_{ij} , the VC's capital investment K_{ij} , and the match quality Ω_{ij} , the expected profit of entrepreneur i 's venture financed by venture capitalist j is

$$E[\Pi_{ij}|e_{ij}, K_{ij}, \Omega_{ij}] = \pi(K_{ij}, \Omega_{ij})e_{ij}. \quad (1)$$

The realization of Π_{ij} is uncertain from an *ex ante* perspective, implying the VC cannot perfectly infer from the realized profit Π_{ij} the effort level e_{ij} , leading to a typical moral hazard problem.

We now shed more light on the properties of the venture's expected profit. First, the following four conditions need to be satisfied in order to generate a strictly positive expected payoff from the venture:

- The entrepreneur implements effort ($e_{ij} > 0$).
- The venture capitalist provides capital ($K_{ij} > 0$).
- The entrepreneur has a marketable idea ($\mu_i > 0$).
- The venture capitalist is equipped with expertise ($x_j > 0$).

Technically,

$$\pi(0, \Omega(\mu_i, x_j)) = \pi(K_{ij}, \Omega(0, x_j)) = \pi(K_{ij}, \Omega(\mu_i, 0)) = 0.$$

Second, we assume that the expected gross profit of a *successful* venture $\pi(K_{ij}, \Omega_{ij})$ is strictly increasing in the VC's investment K_{ij} and the match quality Ω_{ij} . Furthermore, we make the following assumptions to ensure interior solutions:

$$\left. \frac{\partial \pi(K_{ij}, \Omega_{ij})}{\partial K_{ij}} \right|_{K_{ij}=0} = \infty, \quad \left. \frac{\partial \pi(K_{ij}, \Omega_{ij})}{\partial \Omega_{ij}} \right|_{K_{ij}=0} = \frac{\partial \pi(K_{ij}, \Omega_{ij})}{\partial K_{ij}} \bigg|_{\Omega_{ij}=0} = 0.$$

These assumptions imply that the first unit of capital K_{ij} is very productive, but only as long as the match quality in question is strictly positive. We further assume that the function $\pi(K_{ij}, \Omega_{ij})$ is concave

⁵The assumption of π being non-negative implies that potential losses are only incurred by the venture capitalist up to its investment K_{ij} , but not by the entrepreneur, which is consistent with his limited wealth.

in K_{ij} and Ω_{ij} : $\partial^2\pi(K_{ij}, \Omega_{ij})/\partial z^2 \leq 0$ with $z = K_{ij}, \Omega_{ij}$ and $\pi_{K_{ij}K_{ij}}\pi_{\Omega_{ij}\Omega_{ij}} \geq (\pi_{\Omega_{ij}K_{ij}})^2$, where the subscripts denote partial derivatives. Finally, a higher match quality makes every unit of capital more productive, $\partial^2\pi(K_{ij}, \Omega_{ij})/\partial K_{ij}\partial\Omega_{ij} \geq 0$.

2.2 The Efficient (First-Best) Contract

So as to infer policy implications, we derive the contract between entrepreneur i and venture capitalist j which maximizes total surplus. Suppose a social planner can dictate entrepreneur i 's effort level e_{ij} as well as the investment K_{ij} by venture capitalist j . The social planner chooses the (first-best) effort level e_{ij}^{fb} and (first-best) investment K_{ij}^{fb} that maximize each venture's total surplus S_{ij} . The problem of the social planner can thus be stated as follows for a venture with match quality Ω_{ij} :

$$\max_{\{e_{ij}, K_{ij}\}} S_{ij}(e_{ij}, K_{ij}) = \pi(K_{ij}, \Omega_{ij})e_{ij} - \frac{e_{ij}^2}{2} - rK_{ij}. \quad (2)$$

Notice that $S_{ij}(e_{ij}, K_{ij})$ is not affected by the entrepreneur's share λ_{ij} of the venture's profit $\pi(K_{ij}, \Omega_{ij})$ since λ_{ij} determines only the allocation of surplus. The following lemma derives the solution to this problem:⁶

Lemma 1 (First-Best Contract) *The socially efficient investment K_{ij}^{fb} by venture capitalist j is implicitly characterized by*

$$\frac{\partial\pi(K_{ij}^{fb}, \Omega_{ij})}{\partial K_{ij}} = \frac{r}{\pi(K_{ij}^{fb}, \Omega_{ij})}; \quad (3)$$

and the socially efficient effort level e_{ij}^{fb} of entrepreneur i is

$$e_{ij}^{fb} = \pi(K_{ij}^{fb}, \Omega_{ij}). \quad (4)$$

3 The Venture Capital Market in the Absence of Matching

To establish a benchmark, we first derive the optimal venture capital contract in the absence of any matching considerations. To do so, we proceed in two steps. First, in Section 3.1, we characterize the

⁶Unless otherwise stated, all proofs are contained in the Appendix.

entrepreneur's effort choice for a *given* venture capital contract. Second, in Section 3, by accounting for the entrepreneur's effort policy, we solve for the optimal venture capital contract, assuming that the entire bargaining power rests with the venture capitalist.

3.1 An Entrepreneur's Effort Policy

We begin our analysis by investigating entrepreneur i 's effort policy for a given contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ offered by venture capitalist j , with $i \in \{1, \dots, m\}$ and $j \in \{1, \dots, n\}$. For a given match quality Ω_{ij} , capital investment K_{ij} , and share of the venture's profit λ_{ij} , the entrepreneur chooses his effort e_{ij} to maximize his expected utility

$$\max_{\{e_{ij}\}} U_i(e_{ij}, \lambda_{ij}, K_{ij}, \Omega_{ij}) = \lambda_{ij}\pi(K_{ij}, \Omega_{ij})e_{ij} - \frac{e_{ij}^2}{2}. \quad (5)$$

The entrepreneur's effort policy is thus

$$e_{ij}^* = \lambda_{ij}\pi(K_{ij}, \Omega_{ij}). \quad (6)$$

We assume that $\pi(K_{ij}^*, \Omega_{ij}) < 1$ for the optimal capital investment K_{ij}^* and any possible match quality Ω_{ij} in order to ensure that $e_{ij}^* < 1$ and thereby $\text{Prob}[Y_{ij} = 1 | e_{ij}^*] < 1$.⁷ The entrepreneur's effort e_{ij}^* is increasing in his share λ_{ij} of the venture's profit, the capital investment K_{ij} , his idea quality μ_i , and the expertise of the venture capitalist x_i : a higher capital investment or a better match quality makes every unit of the entrepreneur's effort more productive.

The entrepreneur's participation constraint is always satisfied because potential losses (up to K_{ij}) are only incurred by the venture capitalist. His limited liability further implies that the entrepreneur extracts economic rent, which can be shown (using the Envelope Theorem) to be strictly increasing in his share of the venture's profit λ_{ij} , the VC's capital investment K_{ij} , and the venture's match quality Ω_{ij} .

⁷Alternatively, one could incorporate a parameter γ in the entrepreneur's effort cost function so that $c(e_{ij}) = \gamma e_{ij}^2/2$. Assuming a sufficiently high cost parameter γ would then ensure that $\text{Prob}[Y_{ij} = 1 | e_{ij}^*] < 1$.

3.2 A Venture Capitalist's Contract

In this section, we derive the optimal contract offered by a venture capitalist in the absence of any matching considerations. This contract is not only aimed at securing the venture capitalist an adequate return on its investment, but also at motivating the entrepreneur to implement sufficient effort in order to turn his idea into a profitable product. Clearly, to indirectly control the entrepreneur's effort choice, the venture capitalist can simply adjust his share λ_{ij} of the venture's profit. Less obvious, however, is the fact that the venture capitalist can also utilize its investment K_{ij} to influence the entrepreneur's effort. As discussed in Section 3.1, a higher capital investment makes every unit of the entrepreneur's effort more productive, which in turn induces him to exert higher effort.

Consider the contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ offered by venture capitalist j to entrepreneur i . The venture capitalist adjusts this contract to maximize its expected profit, taking the entrepreneur's effort policy e_i^* into account. The venture capitalist's expected profit is thus given by

$$\Pi_{ij}^{VC}(\lambda_{ij}, K_{ij}, e_{ij}^*, \Omega_{ij}) = (1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij})e_{ij}^* - rK_{ij}. \quad (7)$$

By accounting for e_{ij}^* as defined by (6), the expected profit of venture capitalist j becomes

$$\Pi_{ij}^{VC}(\lambda_{ij}, K_{ij}, \Omega_{ij}) = \lambda_{ij}(1 - \lambda_{ij})\pi^2(K_{ij}, \Omega_{ij}) - rK_{ij}. \quad (8)$$

Before deriving the optimal VC contract, it is insightful to unravel the relationship between the allocation of shares and the venture capitalist's expected profit. We start by briefly discussing the extreme cases: all shares are either allocated to the venture capitalist ($\lambda_{ij} = 0$) or the entrepreneur ($\lambda_{ij} = 1$). Due to the lack of any prospective compensation, it is obvious that the entrepreneur refuses to implement effort whenever the venture capitalist holds all shares (i.e., $\lambda_{ij} = 0$), which in turn implies zero profit for the venture capitalist, i.e., $\Pi_{ij}^{VC}(\lambda_{ij} = 0, K_{ij}, \Omega_{ij}) = 0$.⁸ On the other hand, if the entire profit of the venture accrues to the entrepreneur (i.e., $\lambda_{ij} = 1$), the venture capitalist clearly does not benefit from the new enterprise. We infer that financing the new venture can only be profitable for the venture capitalist as long as *both* parties hold some shares (i.e., $\lambda_{ij} \in (0, 1)$).

⁸Note that in this case it is optimal for the venture capitalist to refrain from investing in the venture, i.e., $K_{ij}^* = 0$.

The optimal combination of the entrepreneur's share λ_{ij}^* and the venture capitalist's investment K_{ij}^* is dictated by two objectives: (i) to provide the entrepreneur with sufficient effort incentive to turn his idea into a profitable product; and (ii) to equip the new venture with sufficient capital, not only aimed at ensuring its survival, but also at generating an adequate return on investment for the venture capitalist. The following proposition describes the VC contract that maximizes (8).

Proposition 1 (VC Contract without Matching) *Suppose there is no matching in the venture capital market. The optimal contract between entrepreneur i and venture capitalist j comprises the entrepreneurial share $\lambda_{ij}^* = 1/2$ and the investment K_{ij}^* by the venture capitalist implicitly defined by*

$$\pi(K_{ij}^*, \Omega_{ij}) \frac{\partial \pi(K_{ij}^*, \Omega_{ij})}{\partial K_{ij}} = 2r. \quad (9)$$

In the absence of any competition between entrepreneurs as well as between venture capitalists, it is optimal for financiers to equally split the shares of new ventures. The even split arises because both entrepreneurs and venture capitalists are risk neutral and due to the manner in which effort affects the performance of a venture. The split of shares is not even when we consider endogenous matching in the next section.

As one would expect, the optimal contract $\{\lambda_{ij}^*, K_{ij}^*\}$ entails under-provision of capital from a social perspective, i.e., $K_{ij}^* < K_{ij}^{fb}$, as well as sub-efficient effort, i.e., $e_{ij}^* < e_{ij}^{fb}$.

The expected utility of entrepreneur i matched with venture capitalist j is given by

$$U_{ij}^V(K_{ij}^*, \Omega_{ij}) = \frac{1}{8} \pi^2(K_{ij}^*, \Omega_{ij}), \quad (10)$$

where the superscript V indicates that the entire bargaining power in this relationship rests with the venture capitalist.⁹ Entrepreneur i 's expected utility $U_{ij}^V(K_{ij}^*, \Omega_{ij})$ constitutes his lowest possible expected utility level, which will play a fundamental role in our analysis of matching in the VC market.

We next investigate the manner in which the optimal contract $\Gamma_{ij}^* = \{\lambda_{ij}^*, K_{ij}^*\}$ varies with respect to the match quality Ω_{ij} :

⁹We derive in the Appendix the entrepreneur's expected utility in case he holds the entire bargaining power. Thus, using the superscript V will ease the distinction between both cases.

Lemma 2 (Properties of the VC Contract without Matching) *Suppose there is no matching in the venture capital market. The entrepreneur's share λ_{ij}^* is independent of the venture's match quality Ω_{ij} while the investment by the venture capitalist K_{ij}^* is increasing in Ω_{ij} .*

A better match quality—stemming either from a more promising entrepreneurial idea or superior VC expertise—enhances the payoff to the VC of supplying the entrepreneur with more capital K_{ij}^* . On the other hand, providing the entrepreneur with the share $\lambda_{ij}^* = 1/2$ already maximizes the VC's return on investment, and is thus independent of the specific match quality.

Applying the Envelope Theorem yields the following lemma, which derives the effect of the match quality Ω_{ij} on the expected utility of entrepreneur i and the expected profit of venture capitalist j . We show that both parties strictly benefit from a superior match quality, rooted either in a more promising entrepreneurial idea or in greater VC expertise. This observation will have important implications for the properties of the matching process in the market for venture capital.

Lemma 3 (Impact of Match Quality without Matching) *Suppose there is no matching in the venture capital market. An entrepreneur $i = 1, \dots, m$ and venture capitalist $j = 1, \dots, n$ strictly benefit from a superior match quality Ω_{ij} , i.e., $dU_{ij}^V/d\Omega_{ij} > 0$ and $d\Pi_{ij}^{V,C}/d\Omega_{ij} > 0$.*

4 The Matching of Entrepreneurs with Venture Capitalists

4.1 Matching in the Venture Capital Market

Having derived the optimal venture capital contract in the absence of any matching considerations, we are now well equipped to investigate the optimal contracts offered by venture capitalists when forming partnerships with entrepreneurs in a two-sided heterogeneous market. To do so, we proceed in two steps. In this section, we identify the general properties of the matching process in the VC market. In the subsequent section, we elaborate on the optimal adjustment of VC contracts when taking matching considerations into account.

In a traditional principal-agent model, an entrepreneur would be forced to accept the offer of a specific VC. However, if entrepreneurs are free to choose the VC with the most attractive offer, it is intuitively clear that optimal VC contracts must take into consideration the best alternative available to an

entrepreneur. Venture capitalist j thus designs the contract $\{\lambda_{ij}, K_{ij}\}$ that maximizes its expected profit subject to the entrepreneur receiving at least u_{ij} , which will be formally defined later. The constrained optimization problem of venture capitalist j can thus be expressed as follows:

$$\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij}) = \max_{\{\lambda_{ij}, K_{ij}\}} (1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij})e_{ij}^* - rK_{ij} \quad (11)$$

s.t.

$$\lambda_{ij}\pi(K_{ij}, \Omega_{ij})e_{ij}^* - \frac{(e_{ij}^*)^2}{2} \geq u_{ij}, \quad (12)$$

where $e_{ij}^* = \lambda_{ij}\pi(K_{ij}, \Omega_{ij})$. The maximized objective function $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ defines the bargaining frontier between venture capitalist j and entrepreneur i . Whether the individual rationality (IR) constraint (12) is binding hinges on the specific value of u_{ij} . We can infer from our previous analysis that U_{ij}^V , as defined by (10), constitutes the lower bound of u_{ij} . This is the utility level entrepreneur i receives when contracting with venture capitalist j in the absence of matching, assuming that the entire bargaining power rests with the venture capitalist. The maximum value of u_{ij} is denoted by U_{ij}^E , which constitutes entrepreneur i 's expected utility in case the entire bargaining power rests with him, and not with venture capitalist j as considered above.¹⁰

The next lemma proves that the bargaining frontier $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ is decreasing in the entrepreneur's reservation utility u_{ij} . Figure 1 is a graphical depiction of the result. The bold curve depicts the bargaining frontier, which consists of the Pareto-optimal utilities for the (i, j) pair.

Lemma 4 (Bargaining Frontier) *The bargaining frontier $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ is decreasing in the entrepreneur's reservation utility $u_{ij} \in [U_{ij}^V, U_{ij}^E]$.*

We next define the venture capital market equilibrium.

Definition 1 (VC Market Equilibrium) *An equilibrium in the venture capital market consists of a one-to-one matching function $m : E \rightarrow V$ and payoff allocations $\Pi^* : V \rightarrow \mathbb{R}$ and $u^* : E \rightarrow \mathbb{R}_+$ that satisfy the following two conditions:*

¹⁰As will become clear, our next analysis does not rely on the specific functional form of U_{ij}^E , which is derived in the Appendix.

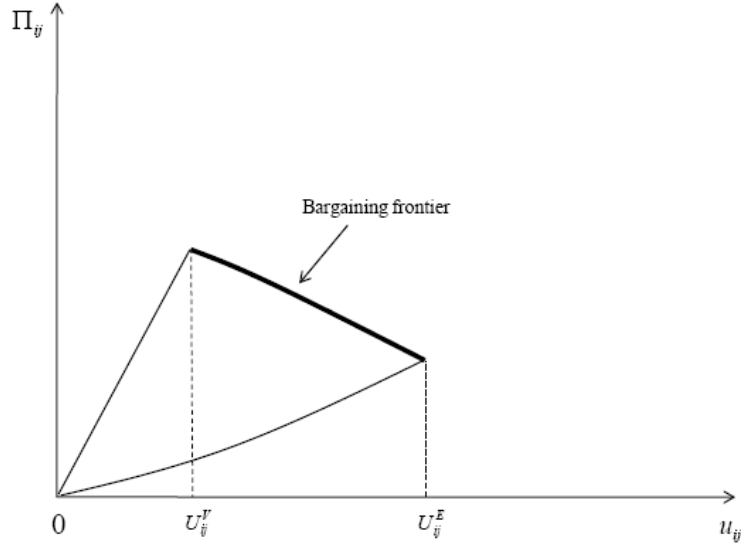


Figure 1: Bargaining Frontier

- *Feasibility of (Π^*, u^*) with respect to m : For all $i \in E$, $\{\Pi^*(m(i)), u^*(i)\}$ is on the bargaining frontier $\bar{\Pi}(u_{i,m(i)}, \Omega_{i,m(i)})$.*
- *Stability of m with respect to $\{\Pi^*, u^*\}$: There do not exist a pair $(i, j) \in E \times V$, where $m(i) \neq j$, and outside value $u > u^*(i)$ such that $\bar{\Pi}_{ij}(u, \Omega_{ij}) > \Pi^*(j)$.*

The two conditions guarantee the existence of a stable matching equilibrium in the venture capital market. Specifically, the first condition requires that the payoffs for venture capitalists and entrepreneurs must be attainable, which is guaranteed whenever the payoffs of any $(i, m(i))$ pair are on the bargaining frontier $\bar{\Pi}_{i,m(i)}(u_{i,m(i)}, \Omega_{i,m(i)})$. Moreover, the stability condition ensures that all matched venture capitalists and entrepreneurs cannot become strictly better off by breaking their current partnership (and match with a new venture capitalist or entrepreneur).

We obtain *positive assortative matching* (PAM) whenever entrepreneurs with high quality ideas are matched with venture capitalists enjoying high expertise. The opposite occurs with *negative assortative matching* (NAM). The next definition formalizes the characteristics of positive versus negative assortative matching.

Definition 2 (Assortative Matching in the VC Market) Consider two entrepreneurs i and i' with idea qualities $\mu_i > \mu_{i'}$, and suppose that entrepreneur i is matched with venture capitalist $j = m(i)$ and entrepreneur i' is matched with venture capitalist $j' = m(i')$. Then, the matching equilibrium is

- *positive assortative (PAM) if the venture capitalists' expertise satisfy $x_j > x_{j'}$;*
- *negative assortative (NAM) if $x_{j'} > x_j$.*

Applying the findings in Legros and Newman (2007) to our framework, we infer that the matching equilibrium is *positive assortative* if the cross-partial derivative of the bargaining frontier $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ with respect to the entrepreneur's idea quality μ_i and the venture capitalist's expertise x_j is positive, i.e., $\partial^2 \bar{\Pi}_{ij} / \partial \mu_i \partial x_j \geq 0$. At first glance, one would expect this to be the case when a high expertise venture capitalist benefits relatively more from contracting with a high quality entrepreneur; indeed, this would be the standard complementarity condition that guarantees positive assortative matching appearing in models with transferable utility, e.g., Shapley and Shubik (1971) and Becker (1973). However, as demonstrated by Legros and Newman, this is *not* a sufficient condition to guarantee PAM whenever utility is non-transferable, as in our framework. Rather, the sufficient condition for PAM is that $\partial^2 \bar{\Pi}_{ij} / \partial u_{ij} \partial x_j \geq 0$; that is, it must be relatively easier for a high (versus low) expertise venture capitalist to transfer surplus to an entrepreneur.

The next proposition derives sufficient conditions for PAM to arise in the VC market, i.e. for the condition $\partial^2 \bar{\Pi}_{ij} / \partial u_{ij} \partial x_j \geq 0$ to be satisfied. We henceforth assume these conditions are satisfied.

Proposition 2 (PAM in the VC Market) *The matching between venture capitalists and entrepreneurs is positive assortative (PAM) if the following two conditions are satisfied:*

- *The profit function $\pi(K_{ij}, \Omega_{ij})$ is increasing and concave in the capital investment K_{ij} and match quality Ω_{ij} , with complementarity between the two, i.e. $\partial^2 \pi(K_{ij}, \Omega_{ij}) / \partial K_{ij} \partial \Omega_{ij} \geq 0$.*
- *The match quality $\Omega_{ij} = \Omega(\mu_i, x_j)$ is increasing in the entrepreneur's idea quality μ_i and the venture capitalist's expertise x_j , with complementarity between the two, i.e. $\partial^2 \Omega(\mu_i, x_j) / \partial \mu_i \partial x_j \geq 0$.*

4.2 Matching and the Design of Venture Capital Contracts

Before we can draw any inferences about matching and the optimal design of VC contracts, it is essential to first derive conditions for a given venture capitalist-entrepreneur match being affected by an improved alternative for the entrepreneur as captured by a higher reservation utility u_{ij} . This is important because some matched entrepreneurs might be "out of reach" of other (competing) venture capitalists. For these partnerships, alternative matches are irrelevant, and the optimal venture capital contract is thus identical to the one derived in Section 3 in the absence of matching considerations.

Consider two adjacent entrepreneur-VC matches $(i, m(i))$ and $(i - 1, m(i - 1))$. Proposition 2 provides a useful characterization of the matching problem. Entrepreneur i is the most desirable entrepreneur among all venture capitalists that are ranked below venture capitalist $m(i)$. Nevertheless, it is venture capitalist $m(i - 1)$ who has the highest willingness to pay for entrepreneur i .¹¹ Therefore, venture capitalist $m(i)$ directly competes for entrepreneur i only with venture capitalist $m(i - 1)$, which allows us to explicitly define the outside option of entrepreneur i when he is matched with venture capitalist $m(i)$.

Let $u_{i,m(i-1)}$ denote the maximum utility that entrepreneur i can obtain from venture capitalist $m(i - 1)$, provided that the venture capitalist $m(i - 1)$ does not become worse off relative to his current match with entrepreneur $i - 1$:

$$u_{i,m(i-1)} = \max_{\{\lambda, K\}} \frac{1}{2} \lambda^2 \pi(K, \Omega_{i,m(i-1)})^2 \quad (13)$$

s.t.

$$\frac{1}{2} \lambda (1 - \lambda) \pi(K, \Omega_{i,m(i-1)})^2 - rK = \Pi^*(m(i - 1)). \quad (14)$$

Then, the equilibrium utility of entrepreneur i is $u^*(i) = \max \{u_{i,m(i-1)}, U_{i,m(i)}^V\}$.¹² Following from the properties of the bargaining frontier stated in Lemma 4, $u_{i,m(i-1)}$ is increasing in $\Omega_{i,m(i-1)}$ and decreasing in $\Pi^*(m(i - 1))$.

¹¹The positive cross-partial derivative of the bargaining frontier with respect to entrepreneurial quality and VC expertise and the fact that it is easier to transfer surplus to an entrepreneur as the quality of the venture capitalist increases ensure that the highest bid for entrepreneur i , among all venture capitalists below $m(i)$, originates from venture capitalist $m(i - 1)$.

¹²Note that $u_{i,m(i-1)}$ is the quasi-inverse of the bargaining frontier, $\bar{\Pi}(u_{i,m(i-1)}, \Omega_{i,m(i-1)})$, between entrepreneur i and venture capitalist $m(i - 1)$. See Legros and Newman (2007) for the formal definition of quasi-inverse.

Due to positive assortative matching, it is clear that the higher expertise venture capitalist $m(i)$ can always outbid the lower expertise venture capitalist $m(i-1)$. Nevertheless, the improved alternative for the higher idea quality entrepreneur i might require the higher expertise venture capitalist $m(i)$ to adjust its contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ in order to transfer more surplus to the entrepreneur. Technically, the higher expertise venture capitalist $m(i)$ needs to adjust the contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ whenever $U_{i,m(i)}^V < u_{i,m(i-1)}$.

When is the higher expertise venture capitalist $m(i)$ compelled to offer the higher idea quality entrepreneur i a contract that is less profitable to the VC? Put differently, when does the condition $U_{i,m(i)}^V < u_{i,m(i-1)}$ hold? To answer this question, we consider the following two cases.

Definition 3 (Case I: The VC Matters) *The expertise of the venture capitalist enhances the match quality, while the quality of the entrepreneurial idea does not:*

$$\Omega_{i,m(i)} > \Omega_{i,m(i-1)} = \Omega_{i-1,m(i-1)}. \quad (15)$$

Under Case I, the lower expertise venture capitalist $m(i-1)$ does not gain by contracting with the higher idea quality entrepreneur i . We can thus infer that $U_{i,m(i)}^V = u^*(i)$, which in turn implies that matching considerations are irrelevant for designing the contract between venture capitalist $m(i)$ and entrepreneur i . The optimal VC contract is therefore identical to the one derived in Section 3 in the absence of matching considerations, namely $\{\lambda_{i,m(i)}^*, K_{i,m(i)}^*\}$.

Definition 4 (Case II: The Entrepreneur Matters) *The quality of the entrepreneurial idea enhances the match quality, while the expertise of the venture capitalist does not:*

$$\Omega_{i,m(i)} = \Omega_{i,m(i-1)} > \Omega_{i-1,m(i-1)}. \quad (16)$$

Under Case II, it follows that $U_{i,m(i)}^V < u_{i,m(i-1)}$, implying venture capitalist $m(i)$ must offer the higher idea quality entrepreneur i a contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ that guarantees the entrepreneur a greater expected utility than $U_{i,m(i)}^V$. To see this, suppose for a moment that venture capitalist $m(i)$ offers the en-

trepreneur the contract $\{\lambda_{i,m(i)}^*, K_{i,m(i)}^*\}$ derived in Section 3 in the absence of matching considerations. Since the lower expertise venture capitalist $m(i-1)$ is clearly better off being matched with the higher idea quality entrepreneur i , it would offer a contract $\{\lambda_{i,m(i-1)}, K_{i,m(i-1)}\}$ aimed at guaranteeing entrepreneur i an expected utility level above $U_{i,m(i)}^V$. However, we can infer from Lemma 3 that the higher expertise venture capitalist $m(i)$ can profitably match $m(i-1)$'s contract offer $\{\lambda_{i,m(i-1)}, K_{i,m(i-1)}\}$, while also making the higher idea quality entrepreneur $m(i)$ strictly better off. To summarize, while the high idea quality entrepreneur-high expertise VC match $(i, m(i))$ is mutually beneficial, and—according to Proposition 2—constitutes the equilibrium outcome, competition for the higher idea quality entrepreneur nonetheless alters the equilibrium contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$.

Case I arises under two scenarios: first, the match quality (and, by extension, profitability of the venture) is significantly more sensitive to the expertise of a venture capitalist relative to the quality of an entrepreneurial idea; or, second, the distribution of expertise held by venture capitalists is considerably more heterogeneous than the distribution of entrepreneurial idea qualities. Case II arises under two scenarios: first, the match quality is significantly more sensitive to the quality of an entrepreneurial idea relative to the expertise of a venture capitalist; or, second, the distribution of entrepreneurial idea qualities is considerably more heterogeneous than the distribution of expertise held by venture capitalists. We argue that the second scenario of Case II is more plausible in light of empirical evidence suggesting that the matching process is relatively important (Sørensen (2007)).

Our discussion of Cases I and II in conjunction with Lemma 3 lead to the following result:

Lemma 5 (The Relevance of Matching Considerations) *Consider the entrepreneur-venture capitalist matches $(i, m(i))$ and $(i-1, m(i-1))$, for $i \geq 2$. There exists a threshold match quality $\bar{\Omega}_{i,m(i-1)} \in (\Omega_{i,m(i)}, \Omega_{i-1,m(i-1)})$ with the following property:*

- For $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$, matching considerations alter the contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ between entrepreneur i and venture capitalist $m(i)$, i.e., $U_{i,m(i)}^V < u_{i,m(i-1)}$;
- For $\Omega_{i,m(i-1)} \leq \bar{\Omega}_{i,m(i-1)}$, matching considerations do not alter the contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$, i.e., $U_{i,m(i)}^V \geq u_{i,m(i-1)}$.

Lemma 5 provides an important implication about the role of matching in the VC market. If the predominant force behind a match quality improvement is the idea quality μ_i of entrepreneur i (i.e., Case II arises), then matching considerations alter his contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ in his favor. Put differently, competition for high idea quality entrepreneurs forces venture capitalists to cede some of the surplus in order to attract entrepreneurs with suitable ideas. This in turn suggests that entrepreneurs generally benefit from the matching process in the VC market at the expense of venture capitalists. In other words, under Case II, the entrepreneur earns *matching rent*.

So far, we have examined whether venture capitalists are forced to adjust their contracts due to matching considerations. We now investigate in detail how matching alters the entrepreneurial share $\lambda_{i,m(i)}$ and capital investment $K_{i,m(i)}$. According to Lemma 5, we need to distinguish between the two cases defined above. Under Case I, entrepreneur i 's participation constraint (12) (see Section 4.1) is non-binding when offered the contract $\{\lambda_{i,m(i)}^*, K_{i,m(i)}^*\}$ derived in Section 3 in the absence of matching considerations. There is no need for the corresponding venture capitalist $m(i)$ to adjust its contract since entrepreneur i is better off accepting this contract rather than pursuing his best alternative. As a result, under Case I, the matching process does not alter the venture capital contracts derived in Proposition 1.

Under Case II, however, competition for entrepreneur i results in a new contract offer $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ from venture capitalist $m(i)$ which arises because, without such an adjustment, venture capitalist $m(i-1)$ would outbid venture capitalist $m(i)$. Technically, entrepreneur i 's participation constraint (12) is violated for the contract $\{\lambda_{i,m(i)}^*, K_{i,m(i)}^*\}$. Consequently, under Case II, matching has a significant impact on contractual arrangements between venture capitalists and entrepreneurs.

For the remainder of this paper, we exclusively focus on Case II, wherein venture capitalists need to take alternative matching outcomes into account when designing their contracts. The following proposition derives the optimal contract under this assumption.

Proposition 3 (VC Contracting with Matching) *Suppose Case II holds. The optimal contract between entrepreneur i and venture capitalist $m(i)$ comprises the entrepreneurial share*

$$\lambda_{i,m(i)}^M = \frac{\sqrt{2u^*(i)}}{\pi(K_{i,m(i)}^M, \Omega_{i,m(i)})}, \quad (17)$$

and the investment $K_{i,m(i)}^M$ by the venture capitalist implicitly defined by

$$\frac{\partial \pi(K_{i,m(i)}^M, \Omega_{i,m(i)})}{\partial K_{i,m(i)}} = \frac{r}{\sqrt{2u^*(i)}}. \quad (18)$$

The following lemma derives the manner in which higher quality entrepreneurial ideas impact the payoffs of venture capitalists and entrepreneurs:

Lemma 6 (Properties of the VC Contract with Matching) *Suppose Case II holds. The equilibrium expected profit of venture capitalists and the expected utility of entrepreneurs are increasing in the quality of entrepreneurial ideas, i.e., $\Pi^*(m(i)) \geq \Pi^*(m(i-1))$ and $u^*(i) > u^*(i-1)$ for $i \in E = \{1, \dots, n\}$.*

We established that competition for entrepreneurs via the matching process forces venture capitalists to offer them better terms. Matching considerations, and thus the competition for "talent" in the venture capital market, clearly benefits entrepreneurs at the expense of venture capitalists. The following proposition accordingly demonstrates that the matching process enhances the incentives of the venture capitalist to invest in the venture and the entrepreneur to exert effort. The proposition also proves that, despite the adjustments engendered by the matching process, the investment and effort levels remain sub-efficient.

Proposition 4 (VC Contracting with versus without Matching) *Suppose Case II holds. The optimal contract $\{\lambda_{i,m(i)}^M, K_{i,m(i)}^M\}$ between entrepreneur i and venture capitalist $m(i)$ with endogenous matching exhibits greater investment and effort compared to the optimal contract $\{\lambda_{i,m(i)}^*, K_{i,m(i)}^*\}$ in the absence of matching considerations, i.e., $K_{i,m(i)}^M > K_{i,m(i)}^*$ and $e_{i,m(i)}^M > e_{i,m(i)}^*$. However, the investment $K_{i,m(i)}^M$ and effort $e_{i,m(i)}^M$ with endogenous matching are still below their socially efficient levels, i.e., $K_{i,m(i)}^M < K_{i,m(i)}^{fb}$ and $e_{i,m(i)}^M < e_{i,m(i)}^{fb}$.*

5 Entry and Exit in the Venture Capital Market

5.1 Endogenous Entry into the Venture Capital Market

Suppose there are N venture capitalists and N entrepreneurs that are ready to enter the market, where N is a large integer. The fixed costs of entry are $F_{VC} > 0$ and $F_E > 0$ for the venture capitalist and entrepreneur, respectively. The highest quality idea and expertise are denoted by N and the lowest are denoted by 1. The following definition characterizes the equilibrium with endogenous entry, based on the premise that neither a venture capitalist nor an entrepreneur will enter the VC market if his expected payoff from doing so is negative or there is no available match.

Definition 5 (VC Market Equilibrium with Endogenous Entry) *An equilibrium with endogenous entry in the venture capital market consists of a one-to-one matching function m and payoff allocations Π^* and u^* that satisfy the conditions stipulated by Definition 1, along with a threshold entrepreneurial idea n^* that satisfies the following two conditions:*

- $\min\{\Pi^*(m(N - n^* + 1)) - F_{VC}, u^*(N - n^* + 1) - F_E\} \geq 0;$
- $\min\{\Pi^*(m(N - n^*)) - F_{VC}, u^*(N - n^*) - F_E\} < 0.$

The following proposition establishes the existence of a VC market equilibrium with endogenous entry.

Proposition 5 (Existence of a VC Market Equilibrium with Endogenous Entry) *Suppose Case II holds. An equilibrium with endogenous entry in the venture capital market exists.*

Entry into the venture capital market has a favorable *ripple effect* across the ladder of entrepreneur-VC matches. Consider a decrease in the fixed costs of entry, such that one more entrepreneur-VC pair enters at the bottom of the ladder. Suppose that, prior to the decline in entry costs, n^* entrepreneurs and venture capitalists had already entered the market; hence, the lowest qualities in the ladder are $N - n^* + 1$. Now suppose that the pair with qualities $N - n^*$ enters the market. This entry has an impact on the outside option of the entrepreneur $N - n^* + 1$ at the bottom of the ladder. Before entry, his outside option was

zero, but after entry it is positive.¹³ From (13), this obliges venture capitalist $N - n^* + 1$ to offer more utility to entrepreneur $N - n^* + 1$, which is achieved via greater investment, inducing the entrepreneur to exert higher effort. Hence, entry increases the probability of success of the venture that is right above the point in the ladder where entry occurred. By inductive reasoning, this has a ripple effect across the entire ladder. The profit of venture capitalist $N - n^* + 1$, namely $\Pi^*(N - n^* + 1)$, decreased and from (13) he is willing to pay more for entrepreneur $N - n^* + 2$, i.e., $u(N - n^* + 2, m(N - n^* + 1))$ increases. This raises the utility u^* of this entrepreneur, and as we mentioned above the venture capitalist will supply more capital which also leads to greater effort and higher probability of success. Following this logic, all matched pairs in the VC market will experience greater investment, effort, and probability of success. We summarize our inductive argument in the following proposition:

Proposition 6 (The Ripple Effect of Endogenous Entry into the VC Market) *Suppose Case II holds and consider an equilibrium with endogenous entry in the venture capital market. Entry at the bottom of the ladder brings about an increase in investment, effort, and probability of success across all ventures.*

Given that both a venture capitalist's investment and an entrepreneur's effort fall short of their socially efficient levels, we infer that entry at the bottom of the ladder generates positive externalities for the entire VC market. However, an entrepreneur-VC pair that enters the market does not internalize this positive externality and therefore the number of ventures is lower than what a social planner would prefer. Hence, our model suggests that government policy should aim towards spurring entrepreneurial entry, such as by lowering the costs of entry for both entrepreneurs and VCs and the cost of capital for VCs.

5.2 Exit from the Venture Capital Market

The ripple effect we identified in the context of entry can have an *unraveling effect* when considered in the context of exit. Exit at the bottom of the ladder potentially triggers a wave of failure as it travels up the ladder. Ventures at the bottom of the ladder have the lowest probability of success (i.e., level of effort) because the match quality and investment are all at the lowest levels in the VC market.¹⁴ Hence,

¹³In this discussion, it is implicitly assumed that the fixed cost of entry does not allow venture capitalist $N - n^*$ to contract with entrepreneur $N - n^* + 1$ and enjoy positive profit; thus, he is not a viable option for entrepreneur $N - n^* + 1$ before the venture capitalist enters the market.

¹⁴Failure can occur anywhere in the ladder, but it is more likely to happen at the lowest point.

if the lowest quality venture fails and subsequently exits the market, it will exert a negative externality on all matched pairs above it simply by reversing the logic behind entry outlined in the previous section. All ventures are now more likely to fail because there is less supply of capital and thereby effort. Thus, failures at the bottom of the ladder can trigger a crisis in the VC market; that is, low quality ventures may be "too small to fail."

Our analysis suggests that it may be better for the VC market to have failures and thereby exit at the top of the ladder rather than at the bottom. A failure at the top, for example, will have no impact on the rest of the matched entrepreneur-VC pairs. An implication is that policymakers should assist ventures at the lower points of the ladder because such interventions have the maximum impact on the entire VC market.

On the other hand, very high quality matches may be immune from turbulence at the bottom. Suppose there is a significant "break" in the ladder between two groups of entrepreneur-VC matches; for example, there is a collection of ventures that are of very high quality (e.g., the "outliers" that eventually become public), while there is another collection consisting of lower quality ventures. If the differences in quality between the two groups are sufficiently great, such that venture capitalists in one group are not competing for entrepreneurs in the other group, then failure in the bottom group may not affect success in the top group.

5.3 Exogenous Entry into the Venture Capital Market

We first consider entry by an entrepreneur-venture capitalist pair, then entry by a venture capitalist, and finally entry by an entrepreneur.

5.3.1 Exogenous Entry by an Entrepreneur-Venture Capitalist Pair

The following proposition demonstrates that entry by an entrepreneur-VC pair is detrimental to all matched pairs above the point on the ladder where entry occurred, whereas it has no effect below that point.¹⁵ An interesting implication is that improving the match quality between a subset of the matched pairs has a detrimental effect on capital, effort, and the probability of success in all pairs above them.

¹⁵Note, however, that if the pair enters at the bottom of the ladder, then $u^*(i)$ will increase, leading to higher capital provision.

Proposition 7 (Exogenous Entry by an Entrepreneur-VC Pair) *Suppose Case II holds. Suppose an entrepreneur and venture capitalist enter between $(i - 1, m(i - 1))$ and $(i, m(i))$. Matches do not change. Matched pairs at $(i, m(i))$ and above reduce their investment, effort, and probability of success, while matched pairs at $(i - 1, m(i - 1))$ and below are unaffected.*

5.3.2 Exogenous Entry by a Venture Capitalist

The following proposition proves that entry by a venture capitalist intensifies the extent of competition among venture capitalists, thereby increasing the payoffs of entrepreneurs, which in turn enhances investment and effort. Our prediction is consistent with evidence that incumbent VCs benefit from reduced VC entry by paying lower prices for their deals (Hochberg, Ljungqvist, and Lu (2010)).

Proposition 8 (Exogenous Entry by a VC) *Suppose Case II holds. Suppose venture capitalist j enters between venture capitalists $m(i-1)$ and $m(i)$. The matches of venture capitalists $m(i-1)$ and below are pushed down the ladder, while the matched pairs with venture capitalist $m(i)$ and above do not change. All matched pairs increase their investment, effort, and probability of success.*

5.3.3 Exogenous Entry by an Entrepreneur

The following proposition demonstrates that entry by an entrepreneur is detrimental to all matched pairs above the point on the ladder where entry occurred due to the decrease in the extent to which venture capitalists must compete for entrepreneurs.

Proposition 9 (Exogenous Entry by an Entrepreneur) *Suppose Case II holds. Suppose entrepreneur i' enters between entrepreneurs $i - 1$ and i . The matches of entrepreneur $i - 1$ and below are pushed down the ladder, while the matched pairs with entrepreneur i and above do not change. The matched pairs with entrepreneur i and above reduce their investment, effort, and probability of success, while the newly matched pair with entrepreneur i' increases its investment, effort, and probability of success relative to the previously matched pair with entrepreneur $i - 1$.*

6 Discussion and Conclusion

We consider a venture capital market that is comprised of VCs that are heterogeneous with respect to their expertise and entrepreneurs that are heterogeneous with respect to the quality of their ideas. The success of a venture depends crucially on the qualities of both sides. Within this market, we study how the two sides are matched endogenously. Our main interest is how matching affects the terms of the contracts and in particular the investments made by the VCs and equity shares offered the entrepreneurs. We show that, under reasonable conditions, matching is positive assortative (PAM): VCs with high expertise match with entrepreneurs that have high quality ideas. Each entrepreneur's outside option is endogenously determined by the fact that he can contract with an alternative VC. Because VCs have the bargaining power, they compete for high quality entrepreneurs; thus, matching increases the entrepreneurs' outside options relative to what entrepreneurs would receive in the absence of matching (as occurs in a traditional principal-agent model). Superior outside options imply that VCs must alter the terms of their contracts in order to transfer more utility to entrepreneurs. This is achieved via greater investments, which also boost effort levels and success probabilities. Matching thereby improves social efficiency since investment and effort levels are sub-efficient. Our analysis suggests that investment and effort levels depend not only on the quality of a particular match, but also on the entrepreneur's outside option. Entry into the VC market can reduce efficiency even if it leads to higher match qualities because it can lower the entrepreneurs' outside options. As a result, we derived a number of important policy implications.

The model yields a series of empirically testable predictions with regards to the roles of entry and exit in the VC market. We make specific predictions about the impact of different types of entry on entrepreneur-VC pairs above and below the match quality of the entrants; these predictions pertain to the magnitude of the investment and the likelihood of success (or survival). For such tests to be feasible, it is necessary to rank incumbent and entrant ventures according to their match quality so as to form the ladder of the VC market. Sørensen (2007) has already addressed the econometric issues associated with two-sided matching in such a framework, while Hochberg, Ljungqvist, and Lu (2010) have already examined entry and exit properties of the VC market. Thus, the required ingredients are available to implement the procedures we propose.

Appendix

Proof of Lemma 1.

The socially efficient effort level e_i^{fb} as well as the efficient capital investment K_{ij}^{fb} are characterized by the following two first-order conditions:

$$\pi(K_{ij}, \Omega(\mu_i, x_j)) = e_i \quad (19)$$

$$\frac{\partial \pi(\cdot)}{\partial K_{ij}} e_i = r. \quad (20)$$

Substituting (19) into (20) yields the lemma. \square

Proof of Proposition 1.

By accounting for the effort policy given by (6), the optimal contract components λ_{ij}^* and K_{ij}^* are implicitly characterized by the following two first-order conditions:

$$2\lambda_{ij}^*(1 - \lambda_{ij}^*)\pi(K_{ij}^*, \Omega_{ij}) \frac{\partial \pi(K_{ij}^*, \Omega_{ij})}{\partial K_{ij}} = r; \quad (21)$$

$$(1 - 2\lambda_{ij}^*)\pi^2(K_{ij}^*, \Omega_{ij}) = 0. \quad (22)$$

Solving (22) for λ_{ij}^* , and substituting the resulting expression in (21), yields the proposition. \square

Proof of Lemma 2.

Recall from Proposition 1 that $K_{ij}^*(\Omega_{ij})$ is implicitly characterized by

$$\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}} = 2r. \quad (23)$$

Implicit differentiating yields

$$\frac{dK_{ij}^*(\Omega_{ij})}{d\Omega_{ij}} = - \frac{\frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} \frac{\partial \pi(\cdot)}{\partial K_{ij}} + \pi(K_{ij}, \Omega_{ij}) \frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}}}{\frac{\partial}{\partial K_{ij}} \left[\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}} - 2r \right]} \quad (24)$$

Note first that the denominator is strictly negative due to the second-order condition. Moreover, recall that $\partial^2 \pi(\cdot) / (\partial K_{ij} \partial \Omega_{ij}) \geq 0$. Thus, $dK_{ij}^*(\Omega_{ij}) / d\Omega_{ij} > 0$. Finally, observe that $d\lambda_{ij}^* / d\Omega_{ij} = 0$. \square

Proof of Lemma 3.

Applying the Envelope Theorem to (8) yields the result for the VC. Next, the entrepreneur's expected utility under the optimal unconstrained contract $\Gamma_{ij}^*(\Omega_{ij}) = \{\lambda_{ij}^* = 1/2, K_{ij}^*(\Omega_{ij})\}$ is

$$U_i(e_{ij}^*, K_{ij}^*, \Omega_{ij}) = \frac{1}{2} \pi(K_{ij}^*, \Omega_{ij}) e_{ij}^* - \frac{(e_{ij}^*)^2}{2}. \quad (25)$$

Applying the Envelope Theorem yields

$$\frac{dU_i}{d\Omega_{ij}} = \frac{1}{2} \left[\frac{\partial \pi}{\partial K_{ij}^*} \frac{dK_{ij}^*}{d\Omega_{ij}} + \frac{\partial \pi}{\partial \Omega_{ij}} \right]. \quad (26)$$

Recall from Lemma 2 that $dK_{ij}^* / d\Omega_{ij} > 0$. Thus, $dU_i / d\Omega_{ij} > 0$. \square

Proof of Lemma 4.

The bargaining frontier is the solution to the following constrained maximization problem

$$\begin{aligned} \bar{\Pi}_{ij}(u_{ij}, \Omega_{ij}) &= \max_{\{\lambda, K\}} (1 - \lambda) \pi(K, \Omega_{ij}) e_i^* - rK \\ \text{subject to: } &\lambda \pi(K, \Omega_{ij}) e_i^* - \frac{(e_i^*)^2}{2} \geq u_{ij} \end{aligned}$$

where e^* is given by (6) and $u_{ij} \in [U_{ij}^V, U_{ij}^E]$.

At the bargaining frontier, the constraint must be binding. Using (6), the binding constraint becomes

$$\frac{\lambda^2 (\pi(K, \Omega_{ij}))^2}{2} = u_{ij}.$$

Hence, λ must satisfy

$$\lambda^* = \frac{\sqrt{2u_{ij}}}{\pi(K, \Omega_{ij})}. \quad (27)$$

Using (27) and (6) the venture capitalist's problem becomes

$$\max_{\{K\}} \sqrt{2u_{ij}} \pi(K, \Omega_{ij}) - 2u_{ij} - rK.$$

The first order condition is

$$\sqrt{2u_{ij}} \pi_K(K, \Omega_{ij}) - r = 0. \quad (28)$$

The solution is denoted by $K^*(u_{ij}, \Omega_{ij})$. Thus, the frontier contract $\{\lambda^*, K^*\}$ satisfies

$$\pi_K(K^*, \Omega_{ij}) = \frac{r}{\sqrt{2u_{ij}}} \quad \text{and} \quad \lambda^* = \frac{\sqrt{2u_{ij}}}{\pi(K^*, \Omega_{ij})}. \quad (29)$$

From (28) it follows that

$$\frac{dK^*}{du} = -\frac{\pi_K}{2u\pi_{KK}} > 0 \quad \text{and} \quad \frac{dK^*}{d\Omega} = -\frac{\pi_{K\Omega}}{\pi_{KK}} > 0. \quad (30)$$

Higher outside option or higher match quality imply higher capital investment. Using (27) and (30) we have

$$\frac{d\lambda^*}{du} = \frac{\pi + (\pi_K)^2 / \pi_{KK}}{\sqrt{2u}\pi^2}. \quad (31)$$

The maximized profit function $\bar{\Pi}(u_{ij}, \Omega_{ij}) = \sqrt{2u_{ij}} \pi(K^*(u_{ij}, \Omega_{ij}), \Omega_{ij}) - 2u_{ij} - rK^*(u_{ij}, \Omega_{ij})$ is the bargaining frontier. We differentiate the bargaining frontier with respect to u_{ij} , which yields

$$\frac{d\bar{\Pi}}{du_{ij}} = \frac{1}{\sqrt{2u_{ij}}} \pi + \sqrt{2u_{ij}} \pi_K \frac{dK^*}{du_{ij}} - 2 - r \frac{dK^*}{du_{ij}} \quad (32)$$

$$\text{using (28)} = \frac{1}{\sqrt{2u_{ij}}} \pi - 2. \quad (33)$$

When the above derivative is evaluated at the lowest possible u_{ij} which is U_{ij}^V , see (10), it is zero. Hence, any higher u_{ij} implies that the slope of the frontier, as expected, is negative. \square

Proof of Proposition 2.

We check whether the sufficient conditions for PAM are satisfied. We differentiate the venture capitalist's constrained profit function $\bar{\Pi} \equiv \sqrt{2u}\pi(K^*, \Omega) - 2u - rK^*$ with respect to x

$$\Pi_x = \sqrt{2u_{ij}}\pi_K \frac{\partial K^*}{\partial \Omega} \frac{\partial \Omega}{\partial x} + \sqrt{2u_{ij}}\pi_\Omega \frac{d\Omega}{dx} - r \frac{dK^*}{\partial \Omega} \frac{\partial \Omega}{\partial x} = \quad (34)$$

$$\text{using (28)} = \sqrt{2u_{ij}}\pi_\Omega \frac{\partial \Omega}{\partial x}. \quad (35)$$

Now we differentiate Π_x with respect to μ

$$\begin{aligned} \Pi_{x\mu} &= \sqrt{2u} \left(\pi_{\Omega\Omega} \frac{\partial \Omega}{\partial \mu} \frac{\partial \Omega}{\partial x} + \pi_{\Omega K} \frac{\partial K^*}{\partial \Omega} \frac{\partial \Omega}{\partial \mu} \frac{\partial \Omega}{\partial x} + \pi_\Omega \frac{\partial^2 \Omega}{\partial x \partial \mu} \right) \\ \text{using (30)} &= \sqrt{2u} \left(\pi_{\Omega\Omega} \frac{\partial \Omega}{\partial x} \frac{\partial \Omega}{\partial \mu} - \pi_{\Omega K} \frac{\pi_{K\Omega}}{\pi_{KK}} \frac{\partial \Omega}{\partial \mu} \frac{\partial \Omega}{\partial x} + \pi_\Omega \frac{\partial^2 \Omega}{\partial x \partial \mu} \right) \end{aligned}$$

The above is positive provided that

$$\sqrt{2u} \left(\pi_{\Omega\Omega} - \pi_{\Omega K} \frac{\pi_{K\Omega}}{\pi_{KK}} \right) \frac{\partial \Omega}{\partial \mu} \frac{\partial \Omega}{\partial x} \quad (36)$$

is positive. The positive sign follows from the assumption that $\pi(K, \Omega)$ is concave in both of its arguments and hence the Hessian determinant is positive, i.e., $\pi_{KK}\pi_{\Omega\Omega} - \pi_{\Omega K}\pi_{K\Omega} \geq 0$.

Next, we differentiate (35) with respect to u_{ij}

$$\Pi_{xv} = \frac{1}{\sqrt{2u_{ij}}}\pi_\Omega \frac{\partial \Omega}{\partial x} + \sqrt{2u_{ij}}\pi_{\Omega K} \frac{\partial K^*}{\partial u} \frac{\partial \Omega}{\partial x}. \quad (37)$$

Under our assumptions Π_{xv} is positive. □

Proof of Proposition 3.

The result follows directly from the proof of Lemma 4. □

Proof of Lemma 6.

By way of contradiction suppose that for some i , $\Pi^*(m(i)) < \Pi^*(m(i-1))$. Then, venture capitalist $m(i)$ can offer to entrepreneur $i-1$ exactly the same contract venture capitalist $m(i-1)$ offers to entrepreneur $i-1$. Because $x_{m(i)} > x_{m(i-1)}$ and $\Omega(i-1, m(i)) > \Omega(i-1, m(i-1))$, the utility of entrepreneur $i-1$ is greater than $u^*(i-1)$ and the profit of venture capitalist $m(i)$, is greater than $\Pi^*(m(i-1))$, which, given $\Pi^*(m(i-1)) > \Pi^*(m(i))$, implies that the matching m is not stable, contradiction. A similar argument can be used to show that $u^*(i)$ is increasing in i . \square

Proof of Proposition 4.

First, we can infer from (30) that $K_{ij}^M > K_{ij}^*$. Moreover, from (6), we have

$$\frac{de_i^*}{du_{ik}} = \frac{d\lambda_{ij}^*}{du_{ik}} \pi(\cdot) + \lambda_{ij}^* \frac{\partial \pi(\cdot)}{\partial K_{ij}} \frac{dK_{ij}^*}{du_{ik}}. \quad (38)$$

Using (27), (30), and (31), it follows that

$$\frac{de_i^*}{du_{ik}} = \frac{1}{\sqrt{2u_{ij}}} > 0. \quad (39)$$

Recall that if $U_i(\cdot) = u_{ik}$, K_{ij}^M , satisfies (29). This condition becomes equal to (3) only when $u_{ij} = \pi^2(K_{ij}, \Omega_{ij})/2$. However, on the bargaining frontier, we have

$$u_{ij} = \frac{1}{2} \lambda_{ij}^2 \pi^2(K_{ij}, \Omega_{ij}) < \frac{1}{2} \pi^2(K_{ij}, \Omega_{ij}), \quad (40)$$

because $\lambda_{ij} < 1$. This implies that $K_{ij}^M < K_{ij}^{fb}$, and hence, $e_i^M < e_i^{fb}$. From (30), higher outside option u_{ij} , due to the endogenous matching, implies higher capital investment. \square

Proof of Proposition 5.

A free entry equilibrium exists because, from Lemma 6, $\Pi^*(m(i))$ and $u^*(i)$ decrease as i decreases. Hence, if fixed costs of entry are high enough then there exists an $n^* < N$, meaning that only a subset of potential venture capitalists and entrepreneurs enter the market. \square

Proof of Proposition 7.

Let entrepreneur i' and venture capitalist j' enter between $(i - 1, m(i - 1))$ and $(i, m(i))$. The matchings below and above are not affected. From Lemma 6, we have $\Pi^*(m(i'^*(m(i - 1))))$, suggesting that $u^*(i)$ decreases after entry for all $i > i'$. This follows from (13) and the fact that nothing else, except Π^* , changes for $i > i'$. This implies, following Proposition 3, less capital provision for all matched pairs above where entry occurred and less effort. There will be no change in the pairs below i' . \square

Proof of Proposition 8.

Suppose venture capitalist j enters between venture capitalists $m(i' - 1)$ and $m(i')$. The matchings will change. In particular, venture capitalist j will match with entrepreneur $i' - 1$, venture capitalist $m(i' - 1)$ will match with entrepreneur $i - 2$ and so on. Venture capitalist 1 will be unmatched. We denote the new matching by m' . From Proposition 3 it follows that capital investment in the $(i, m(i))$ pair depends positively on the match quality $\Omega(i, m(i))$ and on the entrepreneur's outside option $u^*(i)$. Entry increases the match quality for all entrepreneur-venture capitalist pairs below the entrant. The key question is what happens to $u^*(i)$.

Let's examine (13), which is used to determine $u^*(i)$. Holding Π^* constant, a higher match quality $\Omega(i, m(i - 1))$ increases $u^*(i)$. Entry will increase Ω , i.e., $\Omega(i, m'(i - 1)) > \Omega(i, m(i - 1))$ for all $i < i'$, and hence it will have a positive impact on $u^*(i)$, holding Π^* fixed. But $\Pi^*(\cdot)$ also changes. If it increases, then $u^*(i)$ decreases, all else equal.

Naturally, then, we focus on the impact of entry on $\Pi^*(\cdot)$. There are two opposing effects on $\Pi^*(m'(i))$ relative to $\Pi^*(m(i))$, following the entry of venture capitalist j . First, because the new entrant pushes down all the venture capitalists, the match quality increases for entrepreneurs below the new entrant. This tends to increase the equilibrium profit of the VC all else equal, i.e., $\Pi^*(m'^*(m(i)))$, for each $i < i'$. This effect has a negative impact on $u^*(i)$, since a VC with a higher profit is willing to pay less to contract with the next higher quality entrepreneur. Second, $\Pi^*(m'(i))$ is also affected by $u^*(i - 1)$, which also changes. A higher entrepreneur outside option lowers the equilibrium profit of the VC, which reinforces a higher $u^*(i)$. If the VC equilibrium profit, $\Pi^*(m'(i))$, $i < i'$, decreases, then $u^*(i)$ will unambiguously increase. If not, then it is possible that entry leads to lower $u^*(i)$ and less capital.¹⁶

¹⁶The main point here is that improving the match quality does not necessarily ensure a higher capital provision. If the equilibrium profits of venture capitalists also increase, then they may be willing to pay less to contract with higher quality

We now show that, under Case II, $u^*(i)$ increases in the new matching m' , relative to the old matching m . In the new matching, m' , the utility of the first entrepreneur $u^*(1)$ will increase, because his outside option is venture capitalist 1, as opposed to zero before entry. Venture capitalist 2, with $m'(1) = 2$, provides more capital to entrepreneur 1, because $u^*(1)$ increases and $\Omega(1, m'(1))$ is higher than $\Omega(1, m(1))$. Moreover, $\Pi^*(m'^*(m(1)))$ under Case II, i.e., $\Omega(1, m(1)) \approx \Omega(1, m(2)) = \Omega(1, m'(1))$. Entrepreneur 1 is now matched with a higher quality venture capitalist, which boosts the profit of the VC relative to m (first effect), but at the same time entrepreneur 1 has a better outside option under m' than under m which lowers the VC's profit (second effect). Moreover, the second effect is much stronger. Under this assumption, venture capitalist 2 (who is now matched with entrepreneur 1) will bid more for entrepreneur 2 (than the bid of venture capitalist 1 under the old matching) and hence $u^*(2)$ increases and so on. \square

Proof of Proposition 9.

Clearly, if an entrepreneur enters below entrepreneur 1, there will be no effect on the equilibrium. The simplest case is when entrepreneur i' enters between entrepreneurs 1 and 2. In the new matching m' we have $m'(i') = 1$, $m'(2) = 2$ and so on. The match quality in the $(i', m'(i'))$ pair, which is the lowest matched pair, increases. This implies a higher $\Pi^*(m'(i'))$ relative to $\Pi^*(m(1))$ and higher capital investment and effort. In turn, this suggests, using (13), that $u^*(2)$ decreases, leading to lower capital investment in the $(2, m'(2))$ pair. The same applies to all matched pairs above $(2, m'(2))$.

Now assume that entrepreneur i' enters between entrepreneur i and $i - 1$, with $i > 2$. In the new matching, $m'(2) = 1$, $m'(3) = 2$ and finally $m'(i) = i - 1$. Above the last pair there is no effect. Let's first compare $\Pi^*(m(2))$ with $\Pi^*(m'(2))$. In both cases, the entrepreneur is the same. A negative effect on Π^* , under matching m' , is that $\Omega(2, m(2)) > \Omega(2, m'(2))$, because $m'(2) = 1$, while $m(2) = 2$. A positive effect on Π^* under m' is that there is no venture capitalist below $m'(2)$, which implies a lower $u^*(2)$ under m' and hence a higher Π^* , implying less capital investment in the pair $(2, m'(2))$ relative to $(2, m(2))$. Under Case II, we have that $\Omega(2, m(2)) \approx \Omega(2, m'(2))$ and thus $\Pi^*(m'^*(m(2)))$. Hence, $u^*(3)$ is lower under m' . Next, let's look at the pair $(3, m'(3))$. Because $u^*(3)$ has decreased, capital investment decreases. For similar reasons as before, $\Pi^*(m'^*(m(3)))$, and $u^*(4)$ decreases. Moving up entrepreneurs. This lowers the outside option of entrepreneurs which, in the presence of endogenous matching, implies that venture capitalists can now afford to provide their entrepreneurs with less capital.

the ladder following the same argument, we can conclude that $u^*(i)$ decreases, which implies less capital investment in all pairs that lie above and consequently less effort. \square

Entrepreneurs' Preferences

Suppose that the entire bargaining power rests with the entrepreneur. When completely ignoring the policy of the venture capitalist, it is intuitively clear that the entrepreneur would prefer (i) to be the residual claimant of the entire profit of the new venture (i.e., $\lambda_{ij} = 1$); and (ii) an unlimited investment of capital (i.e., $K_{ij} \rightarrow \infty$). However, without any adequate revenues for its capital investment, the venture capitalist would never invest in the new venture. Technically, the participation constraint of the venture capitalist would be violated. This in turn implies that we need to account for the policies of the venture capitalist when considering the contractual preferences of the entrepreneur.

To better understand the competition of ideas in the market for venture capital and its effect on the corresponding contracts, consider the following scenario: The entrepreneur chooses his preferred share λ_{ij}^E on the venture's profit, while taking the corresponding investment $K_{ij}^*(\lambda_{ij}^E)$ by the venture capitalist into account. Let U_{ij}^E denote entrepreneur i 's expected utility, which constitutes his highest expected utility level when forming a partnership with venture capitalist j . Then, λ_{ij}^E is defined by

$$\lambda_{ij}^E \in \operatorname{argmax}_{\tilde{\lambda}_{ij}} U_i^E(e_i^*, \tilde{\lambda}_{ij}, K_{ij}^*(\tilde{\lambda}_{ij}), \Omega_{ij}) = \tilde{\lambda}_{ij} \pi(K_{ij}^*(\tilde{\lambda}_{ij}), \Omega_{ij}) e_i^* - \frac{(e_i^*)^2}{2}.$$

where $K_{ij}^*(\lambda_{ij})$ is implicitly characterized by (21) (ignoring condition (22)). Obviously, it is in the entrepreneur's best interest to leave some shares for the venture capitalist (i.e., $\lambda_{ij}^E < 1$). Otherwise, without any share on the generated revenue of the new venture (i.e., $\lambda_{ij} = 1$), the venture capitalist would not invest at all (i.e., $K_{ij}^*(1) = 0$), thus rendering the entrepreneur's pursuit of his business idea impossible.¹⁷

Drawing on the entrepreneur's contractual preference—as characterized above—eventually allows us to identify whether the entrepreneur favors a higher share on the venture's profit—compared to the unconstrained contract $\Gamma^*(\Omega_{ij})$ —at the expense of a higher capital investment, or vice versa. These insights are in turn critical for our subsequent analysis concerned with how contracts between venture

¹⁷In fact, one can show that there exists an inverse U-shaped relationship between the entrepreneur's expected utility $U_i^E(\cdot)$ and his share λ_{ij} on the venture's profit, similar to the relationship between the venture capital firm's expected profit $\Pi_{ij}(\cdot)$ and λ_{ij} .

capitalists and entrepreneurs respond to matching considerations in the market for venture capital. To unravel potential conflicts of interest between both involved parties with respect to the allocation of shares, the next proposition elaborates on the relationship between the entrepreneur's preferences and those of the venture capitalist, and how this relationship is affected by the specific match quality $\Omega(\mu_i, x_j)$.

Proposition 10 *The entrepreneur's preferred share $\lambda_{ij}^E(\Omega_{ij})$ on the venture's profit always lies above the share $\lambda_{ij}^* = 1/2$ preferred by the venture capitalist, even though this would concurrently imply a lower capital investment $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij})$ (i.e., $\lambda_{ij}^E(\Omega_{ij}) > \lambda_{ij}^* = 1/2$ and $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij}) < K^*(\lambda_{ij}^* = 1/2, \Omega_{ij})$). Moreover, the entrepreneur's preferred share $\lambda_{ij}^E(\Omega_{ij})$ is strictly increasing in the match quality $\Omega(\mu_i, x_j)$ (i.e., $d\lambda_{ij}^E(\Omega_{ij})/d\Omega_{ij} > 0$).*

Proof: By accounting for the entrepreneur's effort policy e_i^* , we get his expected utility as a function of λ_{ij} :

$$U_i(\lambda_{ij}, K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) = \frac{1}{2}\lambda_{ij}^2\pi^2(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}).$$

From the first-order condition, the share $\lambda_{ij}^E(\Omega_{ij})$ preferred by the entrepreneur is implicitly characterized by

$$\pi(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) + \lambda_{ij} \frac{\partial \pi(\cdot)}{\partial K_{ij}^*} \frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}} = 0. \quad (41)$$

The first-order condition implies that the optimal $\lambda_{ij}^E(\Omega_{ij})$ is where $dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$. Implicit differentiating $K_{ij}^*(\cdot)$ as defined by (21) yields

$$\frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}} = - \frac{2(1 - 2\lambda_{ij})\pi(\cdot) \frac{\partial \pi(\cdot)}{\partial K_{ij}^*}}{\frac{\partial}{\partial K_{ij}^*} \left[2\lambda_{ij}(1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}^*} - r \right]} \quad (42)$$

Note first that the denominator is strictly negative due to the second-order condition. Clearly, $dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$ requires that $\lambda_{ij} > 1/2$. Thus, $\lambda_{ij}^E(\Omega_{ij}) > \lambda_{ij}^* = 1/2$. Since $dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$ for $\lambda_{ij} > 1/2$, it follows that $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij}) < K^*(\lambda_{ij}^* = 1/2, \Omega_{ij})$.

Next, define

$$F \equiv \pi(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) + \lambda_{ij} \frac{\partial \pi(\cdot)}{\partial K_{ij}^*} \frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}}$$

$$G \equiv 2\lambda_{ij}(1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}} - r$$

Applying Cramer's rule gives

$$\frac{\partial \lambda_{ij}^E(\cdot)}{\partial \Omega_{ij}} = \frac{\det(A)}{\det(B)} \quad (43)$$

where

$$A = \begin{pmatrix} -\frac{\partial F}{\partial \Omega_{ij}} & \frac{\partial F}{\partial K_{ij}} \\ -\frac{\partial G}{\partial \Omega_{ij}} & \frac{\partial G}{\partial K_{ij}} \end{pmatrix} \quad B = \begin{pmatrix} \frac{\partial F}{\partial \lambda_{ij}} & \frac{\partial F}{\partial K_{ij}} \\ \frac{\partial G}{\partial \lambda_{ij}} & \frac{\partial G}{\partial K_{ij}} \end{pmatrix}$$

Note that

$$\det(A) = -\frac{\partial F}{\partial \Omega_{ij}} \frac{\partial G}{\partial K_{ij}} + \frac{\partial G}{\partial \Omega_{ij}} \frac{\partial F}{\partial K_{ij}}. \quad (44)$$

Clearly, $\partial G / \partial K_{ij} < 0$ due to the second-order condition. Moreover,

$$\frac{\partial F}{\partial \Omega_{ij}} = \frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} + \lambda_{ij} \frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}} \frac{dK_{ij}^*}{d\lambda_{ij}} > 0$$

$$\frac{\partial G}{\partial \Omega_{ij}} = 2\lambda_{ij}(1 - \lambda_{ij}) \left[\frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} \frac{\partial \pi(\cdot)}{\partial K_{ij}} + \pi(\cdot) \frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}} \right] > 0$$

$$\frac{\partial F}{\partial K_{ij}} = \frac{\partial \pi(\cdot)}{\partial K_{ij}} + \lambda_{ij} \underbrace{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2}}_{\leq 0} \underbrace{\frac{dK_{ij}^*}{d\lambda_{ij}}}_{< 0} > 0.$$

Thus, $\det(A) > 0$. Next, we have

$$\det(B) = \frac{\partial F}{\partial \lambda_{ij}} \frac{\partial G}{\partial K_{ij}} - \frac{\partial G}{\partial \lambda_{ij}} \frac{\partial F}{\partial K_{ij}}. \quad (45)$$

Notice that $\partial F/\partial \lambda_{ij}, \partial G/\partial K_{ij} < 0$ due to the respective second-order conditions. Moreover, recall that $\partial F/\partial K_{ij} > 0$. Furthermore,

$$\frac{\partial G}{\partial \lambda_{ij}} = 2(1 - 2\lambda_{ij})\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}} < 0. \quad (46)$$

Therefore, $\det(B) > 0$. Combining previous observations, we have $\partial \lambda_{ij}^E(\cdot)/\partial \Omega_{ij} > 0$. □

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