# The Long-Run Supply and Demand for Venture

Capital Funds: Information and Endogenous Entry

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#### Abstract

The characteristics of entrepreneurs and venture capitalists are jointly determined by the set of real investment opportunities. An improvement in these opportunities has two effects: 1) it encourages marginal, low-quality entrepreneurs to pool with high-quality entrepreneurs, and 2) it induces the entry of venture capitalists that are less capable at screening new investments. The market for venture capital financing is therefore plagued by a double-sided asymmetric information problem. The severity of both frictions is increasing but concave in market heat, and this is shown to imply that supply and demand are more elastic in cold markets. The paradigm makes predictions regarding the evolution of venture capital partnership quality.

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### 1 Introduction

The venture capital market is characterized by dramatic fluctuations in the level of activity. Major research agendas have been to empirically decompose these cycles into changes driven by demand-side factors and those driven by supply-side factors<sup>1</sup> and to characterize the time-varying connections between returns and investment levels.<sup>2</sup>

A theme of this literature is that the supply of venture capital funds is "sticky" in the short-run. Private equity investing requires considerable skill and experience — attributes which are largely fixed in the short-run. During hot markets, general partners in established funds become the scarce resource in the market and earn significant economic rents (Gompers and Lerner, 2004; Ljungqvist and Richardson, 2003). As emphasized by recent theoretical literature, these temporary supply-demand imbalances alter the relative bargaining power of VCs and entrepreneurs.<sup>3</sup>

<sup>&</sup>lt;sup>1</sup>The pioneering approach is Gompers and Lerner (1998). For estimations emphasizing the impact of regulation, see Armour and Cumming (2006) and Cumming and MacIntosh (2006).

<sup>&</sup>lt;sup>2</sup>See Gompers and Lerner (2000), Gompers, Kovner, Lerner and Sharfstein (2005) and Kaplan and Schoar (2005).

<sup>&</sup>lt;sup>3</sup>See Inderst and Muller, 2004), Kannienen and Keuschnigg (2003), and Koskinen, Rebello and Wang (2008) for analysis of how these imbalances affect effort choices, portfolio size and security design, respectively.

One might conjecture that there is less economic content in the long-run adjustment story. In particular, suppose that entering VCs are identical to incumbents. Then, a candidate null hypothesis is that, once supply and demand again equilibrate, all of the important moments — average returns, risk, division of surplus between VCs and entrepreneurs, etc. — should be equal to that of their pre-shock state.

As this paper demonstrates, this candidate story is incomplete. When instead VCs are heterogeneous, the story of VC entry becomes a much richer one. Heterogeneity begs the following questions. Which *potential* VCs and entrepreneurs are likely to enter the market under which circumstances? How should we interpret a venture capitalist's decision to operate (or not) given a set of economic conditions, and what can be inferred from her survival?

This paper takes a first step towards such a model using the paradigm of information asymmetry. As is well known, financing of startups entails potentially extreme adverse selection costs given the absent track record of the firms seeking capital, and given the risky nature of the industries in which many of them operate. Exacerbating the problem, this scenario often involves an innovator who has extensive technical knowledge (Denis, 2004) but lacks the reputation capital necessary to convey this information credibly.

In such an environment of asymmetric information, perhaps the most nat-

ural definition of venture capitalist "ability" is effectiveness in screening new investments.<sup>4</sup> VCs go through an extensive process of due diligence before investing (Kaplan and Stromberg, 2000) and the vast majority of funding proposals are rejected. Viewed more broadly, skill in screening projects has enormous social welfare consequences because it determines how efficiently capital is directed.

The quantity of funds demanded depends upon economic conditions. In the model developed in this paper, agents choose to either start entrepreneurial firms (with VC financing) or continue in their current employment. In equilibrium, better quality agents are more likely to become entrepreneurs: their residual claim from doing so is relatively more valuable. Yet there is overinvestment, as some entrepreneurs with negative NPV projects pool with better entrepreneurs in an attempt to obtain the benefits of mispricing.

The capital supply side also has latent agents, i.e. information producers

<sup>&</sup>lt;sup>4</sup> "Extensive due diligence in the private equity market is needed because little, if any, information about issuers is publicly available and in most cases the partnership has had no relationship with the issuer. Thus, the partnership must rely heavily on information that it is able to produce de novo. Moreover, managers of the issuing firm typically know more than outsiders about many aspects of their business [but] the problem of adverse selection is mitigated by the extensive amount of due diligence conducted."

<sup>-</sup> Fenn, Liang and Prowse (1995, pg. 30)

who can choose to enter the market as venture capitalists if market conditions warrant doing so. Obviously, this entry decision hinges in part upon the agents' privately-known information production ability. The decision must also take into consideration the distribution of entrepreneurial quality, because the heterogeneity therein dictates the severity of the information problems being faced. In turn, entrepreneurial entry decisions are affected by the equilibrium skill level of VCs. This is because the capital suppliers' required returns depend upon their costs of information production. In addition, entrepreneurs face differential probabilities of being screened out (depending on their true quality), and these probabilities depend upon the skill level of VCs. All of these interdependencies underscore the importance of modeling these supply-side and demand-side frictions simultaneously.

These latent agents generate a supply curve of information production. In states of low capital demand (i.e., poor investment opportunities) only very efficient information producers are able to operate profitably. An improvement in real investment opportunities causes the market to "step up" the supply curve to progressively less efficient information production agents. Therefore, the average intrinsic quality of supply-side agents is countercyclical. This finding is in contrast to the null hypothesis adjustment story mentioned earlier, in which (by assumption) post-shock VCs are identical to pre-shock VCs in all

respects.

Like the supply-side, the demand-side informational friction is of procyclical severity. Consider a positive shock to the value of investment opportunities. With more profitable investments, competitive venture capitalists would be willing to hold smaller financial stakes in underlying firms, ceteris parabus. However, such an improvement in the terms of finance would induce the entry of marginal, lower-quality entrepreneurs. Thus the capital demand-side asymmetric information problem is of procyclical severity.

The model has the following properties. In all states of the world, VC information production is able to partially, but not fully, mitigate the demandside asymmetric information problem. In all states, within the set of active VCs, the lowest quality agents make zero profits. By contrast, high-quality VCs earn positive profits despite the fact that equilibrium profit-sharing rules are competitively determined. These rents are increasing in the value of investment opportunities. On both the supply-side and the demand-side, the average quality of agents is lower during good times, and the dispersion of quality across agents is higher during good times.

This paper's demand-side intuition has close parallels in the IPO literature. Khanna, Noe and Sonti (2008) and Yung, Colak and Wang (2008) develop theoretical models in which hot IPO markets are associated with lower-quality new issues on average. Their modeling of the supply side is very different from that of the current paper, however. In Khanna et. al the set of supply-side agents is fixed by assumption, and these agents are identical in skill. Underwriters are viewed as intermediaries who hire these skilled information production agents. The key step in the analysis lies in endogenizing these agents' wages. With fixed supply, wages rise in a hot market — capacity constraints become more important — and so underwriters economize by reducing their demand for screening labor. In Yung et. al the supply side is effectively unmodeled: there is unlimited capacity and no screening. The current model generalizes these approaches<sup>5</sup> to an environment with free entry on both the demand-side and the supply-side.

Endogenous entry in the model also permits examination of supply and demand elasticity; i.e. these elasticities are *derived* rather than *assumed* as in previous literature. When profitability is high, there are strong incentives to enter on both sides of the market. Consequently, marginal entry decisions are relatively insensitive to sharing rules, i.e., the division of the firm between 

5At the same time, both models make many predictions outside the scope of the current

model. Yung et. al (2008) tie together waves in IPO volume and underpricing. Khanna et al. identify the cyclical nature of screening intensity and the division of rents between intermediaries and specialized labor employed therein — considerations which are absent from the current model.

VCs and entrepreneurs. This implies that both supply and demand are more elastic in cold markets. Section 3.3 illustrates these tendencies graphically.

While the paper does not explicitly model the relationship between general partners (GPs) and limited partners (LPs), the findings give some reason to be skeptical of partnerships founded during hot markets. This viewpoint seems consistent with Gomper, Kovner, Lerner and Sharfstein (2005), who conclude that experienced VCs react more accurately to changing market conditions. A particularly promising scenario involves a partnership operating during hot markets but founded during cold markets. The decision to operate in a cold market suggests high innate ability (as does the ability to survive in tough conditions) whereas contemporaneous hot markets indicate attractive investment opportunities.

## 1.1 Comparison to the Agency Theoretic Approach

Separation between financing and real decision-making — a pillar of classical finance theory — breaks down in private markets. Venture capitalists have a myriad of non-financial roles, including the "professionalization" of the firm (Hellman and Puri, 2002), the decisions to liquidate or replace management (Gompers, 1995; Bergemann and Hege, 1997) and the form and timing of the exit strategy.

With both the entrepreneur and the VC having real inputs to firm value, the environment is one of double-sided moral hazard. This paradigm has important security design implications (Casamatta 2003; Repullo and Suarez, 1999; and Schmidt, 2003). Empirically, financial contracts cede to the VC a variety of state-dependent control rights (Kaplan and Stromberg, 2002), the terms of which are tailored to the circumstances of each individual firm. These contractual feature vary on a deal-by-deal, round-by-round basis (Kaplan and Stromberg, 2004).

By focusing on how these frictions vary with business conditions, two papers in the literature are particularly close to the spirit of the current model. Inderst and Muller (2004) model the process of search and bargaining between VCs and entrepreneurs. The relative supply and demand for funds determines the equilibrium stake held by VCs and the residual claim held by entrepreneurs, and in turn, the amount of effort chosen by each party. Total firm value can be nonmonotonic in the amount of capital market competition because at low (high) values of competition for funds the VC (entrepreneur) contributes too little effort.

In Inderst and Muller's analysis, shocks to investment conditions have a dynamic effect because the supply of venture capital is sticky in the short-run. Therefore, given a positive shock, VCs temporarily earn positive profits. In the long-run, these profits are driven away by the entry of new VCs. (The severity of frictions can either go up or down, depending on the pre-shock sharing rule.)

Kanniainen And Keuschnigg (2003) take an alternative approach to modeling the effect of changing business conditions. They emphasize that VC monitoring is a scarce resource (see also Sorenen, 2007) to be divided over multiple, simultaneously-held portfolio firms. They demonstrate that when industry returns are high, VCs hold a greater number of portfolio firms and hence contribute less effort to each firm. Hence, as in the current model, there is a sense in which the average supply-side "quality" deteriorates during good times, although the notion of quality is quite different from the one employed here.

A broader question is how these agency theoretic approaches might be empirically distinguished from asymmetric information models. In corporate finance more generally, drawing such distinctions is not always straightforward.<sup>6</sup> One difference is that in these agency models, VCs are homogeneous and all take the same actions. By contrast, an important property of the <sup>6</sup>Sorensen (2007) employs the density of experienced investors in geographical submarkets as a quasi-instrumental variable in order to disentangle "sorting" effects (i.e., better VCs tend to be paired with better firms) from direct effects of the VC's involvement. He concludes that the former effect is almost twice as strong.

current model is that the distribution of venture capitalist quality changes over time. As an implication, the model makes testable implications regarding cross-sectional dispersion in returns; see Section 4. This heterogeneity has some parallels in the literature, as Kaplan and Schoar (2005) conclude "we think the most likely explanation for [our] results is a model of underlying heterogeneity in the skills of GPs."

On the other hand, these agency models have insights and applications not shared by the current model. For example, they make predictions on the search time and bargaining power (in Inderst and Muller's case) and the evolution of the number of portfolio firms (in Kanniainen And Keuschnigg's case). As emphasized by Kaplan and Stromberg (2004), agency considerations play a particularly important role in the governance of venture capital partnerships. Undoubtedly, both imperfections merit further study.

## 2 The Demand for VC Finance

There exists a continuum of agents in the economy with nontransferable access to an entrepreneurial project. Undertaking the project requires that the entrepreneur forgo labor income worth V.

In this section, to provide a benchmark against which Section 3 will be compared, I assume there exist many identical agents who can become venture capitalists if doing so is profitable. Entrepreneurial projects require a single unit of VC labor to function. If this labor is provided, then payoffs to the project are 1 with probability  $\pi_i$  and zero otherwise. The subscript i indicates the entrepreneur's privately known type. VC effort costs K are privately borne. If no effort is provided, the payoff of the project is zero.

Venture capitalists also serve an information production role. They can evaluate entrepreneurs, obtaining a signal s = G or s = B. This signal is associated with two costs. The VC incurs an information production cost  $C_{VC}$ , and the entrepreneur pays a nonpecuniary effort cost  $C_E$  (interpreted as the opportunity cost of meeting with VCs, preparing the pitch, etc.).

The relevant distinctions between the aforementioned costs are the following. While the entrepreneur bears  $C_E$  directly, he bears  $C_{VC}$  and K only indirectly through its effect on the venture capitalist's required stake. Finally, the opportunity costs V is borne only if the project is undertaken.

Assume that

$$Prob \{s = G \mid \pi_i\} = \pi_i \tag{1}$$

Condition (1) implies that the signal is informative: higher quality firms are more likely to yield the signal s = G. To rule out the scenario in which purchase of these signals is socially suboptimal — the "uninformed finance" environment is not of interest here — assume a mass of entrepreneurs with

 $\pi_{MIN}=0$  who pool if the VC operates as an uninformed agent in equilibrium. For example, one could assume that they have zero opportunity costs and a small but positive private benefit to running the firm. This assumption captures the spirit of Kaplan and Schoar's (2005) intuition that if VC did no due diligence, the candidate pool would be flooded with bad projects. Given assumption (1), however, such entrepreneurs can never secure funding: they always obtain the signal s=B. Accordingly, these entrepreneurs do not affect the comparative statics of the equilibria; rather, their presence is used to rule out "uninformed" finance scenarios.

The sum of the aforementioned opportunity costs must be less than one (i.e.,  $K + V + C_E + C_{VC} < 1$ ) otherwise no entrepreneurs are worth funding. Assume that success probabilities  $\pi_i$  are uniformly distributed on [0, 1]. Hence, some entrepreneurs have positive NPV projects while others have negative NPV projects.

Some fraction of the potential entrepreneurs opt to remain in the labor pool rather than enter the VC market. Let  $\pi_{MIN}$  denote the lowest quality entrepreneur that tries to obtain VC financing. This cutoff value will be determined later. The interval  $[\pi_{MIN}, 1]$  then determines the number of active

<sup>&</sup>lt;sup>7</sup>Alternatively, their presence could be motivated by overconfidence; e.g., entrepreneurs who always attempt to pool regardless of the signal they observe. Overeagerness to obtain funding is a widely recognized problem in venture finance (Gompers, 1995).

entrepreneurs. It is assumed that entrepreneurs are evenly distributed across VCs so that each VC has n active entrepreneurs. Evenly distributing entrepreneurs across VCs in this way ignores an "integer" problem, i.e., the number of entrepreneurs is unlikely to be divisible by the number of VCs, so that each pool cannot literally be of exactly the same size. However, in equilibrium, VCs have no incentive to lure away entrepreneurs from others' pools: each evaluation is associated with zero expected profit.

Assuming that entrepreneurial quality  $\pi_i$  is drawn from a uniform distribution on [0,1] it can be shown that the unconditional average quality of active entrepreneurs, and the quality condition on passing the VC screen, respectively, are

$$\overline{\pi} = E(\pi) = \frac{\pi_{MIN} + 1}{2} \tag{2}$$

$$E\{\pi \mid s = G\} = \frac{2}{3} \left( \frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2} \right)$$
 (3)

Naturally, because the signal s is informative,  $E\{\pi \mid s=G\} > \overline{\pi}$ . Moreover, both the unconditional mean and the conditional mean are increasing in  $\pi_{MIN}$ . An exogenous improvement in the entrepreneurial pool implies a better expected quality conditional on passing the venture capitalist's screen.

### 2.1 The VC's Problem

Supply-side agents must determine if it is profitable to enter the market (i.e., become VCs) or not. This entry occurs until the supply of VC labor is sufficient to meet demand. The expected profit from picking an entrepreneur and evaluating him is

$$\underbrace{\left(\frac{\pi_{MIN} + 1}{2}\right)}_{Prob(s=G)} * \underbrace{\left[\frac{2\alpha}{3} \left(\frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2}\right) - K\right]}_{Expected\ Prof\ it\ when\ s=G},$$

where  $\alpha$  is the equity stake ceded by the entrepreneur. (Since payoffs are binary, it is without loss of generality to describe the securities as equity.) If this value exceeds  $C_{VC}$  then the agent has strict incentives to enter the market. Moreover if the first entrepreneur tested is a failure (i.e., s=B) then by symmetry it is profitable to test another entrepreneur, repeating the process until a success is found.

In a competitive market, these values satisfy

$$\underbrace{\left(\frac{\pi_{MIN} + 1}{2}\right)}_{Prob(s=G)} * \underbrace{\left[\frac{2\alpha}{3} \left(\frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2}\right) - K\right]}_{Expected Profit when s=G} = C_{VC}. \tag{4}$$

Note that the *expected* profit from evaluating an entrepreneur is zero. Still, the *realized* profit of VCs can either be positive or negative, depending on whether a success is found in fewer-than-expected iterations or not.

## 2.2 The Entrepreneur's Problem

Expected payoff from submitting to evaluation is

$$\pi_i * \underbrace{\left[\pi_i(1-\alpha) - C_E\right]}_{Profit\ given\ funding} + (1-\pi_i) * \underbrace{\left[V - C_E\right]}_{Profit\ otherwise}. \tag{5}$$

The bracketed terms indicate the expected payoff if the project is funded or unfunded, respectively, while the leading terms reflect the probability of each outcome. Note that because entrepreneurs know their type, from their point of view the venture capitalist's signal adds no value-relevant information. Rather, the signal only determines whether the funding proposal is accepted.

Entrepreneurs seek funding when (5) exceeds the opportunity cost V.<sup>8</sup> Since this expression is increasing in  $\pi_i$ , it follows that there is a cutoff quality  $\pi_{MIN}$ , i.e., an entrepreneur just indifferent between seeking funding and not doing so. In particular,

$$\pi_{MIN} = \frac{V + \sqrt{V^2 + 4(1 - \alpha)C_E}}{2(1 - \alpha)}.$$
 (6)

Putting these two entry conditions together yields the following result.

<sup>&</sup>lt;sup>8</sup>The results are robust to making opportunity costs V a function of quality as well, provided that (5) is increasing in  $\pi_i$  more slowly than is  $V(\pi_i)$ . That is, the required assumption is that higher quality entrepreneurs can better take advantage of "scaling up" than can low quality entrepreneurs.

Theorem 1 (Existence) A necessary and sufficient condition for equilibrium to exist is that the solution  $\{\pi_{MIN}, \alpha\}$  to the system  $\{(4), (6)\}$  satisfies the following inequality.

$$\frac{1}{3}\pi_{MIN}^2 - \frac{1}{6}\alpha\pi_{MIN} - \frac{1}{2}\pi_{MIN}K - C_{VC} + \frac{1}{2}K - \frac{1}{6}\alpha \ge 0$$
 (7)

Proof: See the appendix.

The inequality (7) is an incentive compatibility condition. Equations (4)-(6) assume the signal is purchased, whereas (7) verifies that this purchase is optimal at the equilibrium. It is easy to show that the left-side of (7) is positive when  $\pi_{MIN}$  and  $C_{VC}$  are low, and is negative when either  $\pi_{MIN}$  or  $C_{VC}$  are high.

Non-existence can be motivated heuristically as follows. Suppose that  $\pi_{MIN}$  is near one. By definition, VCs are screening in equilibrium and their diligence prevents all but a few – the very best – entrepreneurs from seeking funding. Yet with  $\pi_{MIN} \approx 1$  there is virtually no dispersion in the pool of active entrepreneurs. This lack of uncertainty negates any incentive to purchase the signal in the first place: VCs would unilaterally defect to non-purchase, which violates the assumptions employed in deriving  $\{(4), (6)\}$ .

To summarize, the solution to  $\{(4), (6)\}$  must involve a nontrivial amount of heterogeneity in entrepreneurs in order for VC screening to be incentive

compatible. On the other hand, the entire thought exercise contained in this paper is of little interest without significant dispersion in agent quality.

Corollary 1 (Effect of Costs) The equilibrium values  $\{\alpha, \pi_{MIN}\}$  governed by  $\{(4), (6)\}$  satisfy

a) (Effect of K)

$$\frac{\partial \pi_{MIN}}{\partial K} > 0 \quad \frac{\partial \alpha}{\partial K} > 0 \tag{8}$$

b) (Effect of V)

$$\frac{\partial \pi_{MIN}}{\partial V} > 0 \quad \frac{\partial \alpha}{\partial V} < 0 \tag{9}$$

c) (Effect of  $C_E$ )

$$\frac{\partial \pi_{MIN}}{\partial C_E} > 0 \quad \frac{\partial \alpha}{\partial C_E} < 0 \tag{10}$$

d) (Effect of  $C_{VC}$ )

$$\frac{\partial \pi_{MIN}}{\partial C_{VC}} > 0 \quad \frac{\partial \alpha}{\partial C_{VC}} > 0 \tag{11}$$

Proof: Follows from the implicit function theorem; see appendix.

This outcome has the comparative static properties motivated in the introduction. Equations (8)-(11) indicate that as investment opportunities improve, lower quality entrepreneurs are drawn into the market. That is, as V and K shrink relative to terminal payoffs of a successful project), the cutoff quality  $\pi_{MIN}$  drops. In this sense the demand-side asymmetric information problem is of procyclical severity.

This result is closely related to Yung, Colack and Wang's (2008) analysis of the market for initial public offerings. In particular, their analysis focuses on the dispersion of quality within the set of IPOs in a given economic state. This dispersion is predicted to be procyclical. Taking the view that true quality will be revealed in the long-run, this result leads the prediction that cross-sectional variation in long-run returns should be higher within cohorts of firms that were funded during hot markets. Furthermore, this hypothesized expansion is more specific to the left side of the distribution rather than the right side. Consequently, they argue that financing waves will be associated with more portfolio firms going bankrupt ex-post. Corollary 1 indicates that these results generalize to an environment with informed capital suppliers and endogenous information acquisition.

Equation (8)-(11) also indicate how VC contracts respond to changing market conditions. Unsurprisingly, as the required VC effort costs rise, VCs demand a larger stake. The result  $\frac{\partial \alpha}{\partial V} < 0$  has more subtle intuition. An increase in the value of outside opportunities (relative to VC finance) leads

<sup>&</sup>lt;sup>9</sup>The mean return is not predicted to change. Capital suppliers in the model are rational; i.e. offer prices fully account for changing demand-side conditions.

to improvement in the quality of the entrepreneurial pool. Because of this reduction in the severity of the asymmetric information problem, VCs are satisfied with a smaller stake. Putting these two results together, the model is ambiguous regarding how  $\alpha$  varies with the value of investment opportunities. If both V and K shrink relative to terminal payoffs, then there are competing effects on  $\alpha$ ; this issue is explored again in a numerical example in Section 3.3.

The two costs in the model have a symmetric effect on entrepreneurial entry: as either type of cost rises, marginal low quality entrepreneurs exit the market. This is true because both costs are ultimately borne by the entrepreneur. This symmetry is broken for  $\alpha$ . As venture capitalist costs  $C_{VC}$  rise, the obvious direct effect is that the VCs demand larger stakes as compensation. On the other hand, as the entrepreneurial costs rise, low-quality entrepreneurs exit the market. Because of this reduction in the severity of the asymmetric information problem, VCs are satisfied with a smaller stake.

# 3 The Supply of VC Finance

In Section 2, venture capitalists are identical and perfectly competitive, which drives their profits to zero. This modeling assumption precludes analysis of incentives to enter the VC market. To address this issue, Section 3 introduces heterogeneity in VC skill.

### 3.1 Two Venture Capitalist Types

Assume that some VCs have information production cost  $C_L$  and all others have cost  $C_H > C_L$ . Importantly, both types of venture capitalists obtain the same signal; it is only the costs of production that differ. Consequently, there is no post-financing difference between VCs and so, for a given contract, entrepreneurs have no strict preference regarding which VC supplies funding.

Further assume that there are some states in which the units of VC labor demanded exceeds the endowment of low-cost VCs. That is, low-cost VCs are in short supply, and this is a binding constraint. Thus, high-cost VCs must also be active to ensure that markets clear.<sup>10</sup>

The entry condition for venture capitalist i is

$$\underbrace{\left(\frac{\pi_{MIN} + 1}{2}\right)}_{Prob(s=G)} * \underbrace{\left[\frac{2\alpha}{3} \left(\frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2}\right) - K\right]}_{Expected\ Prof\ it\ when\ s=G} \ge C_i. \tag{12}$$

Competition among high-cost VCs leads to the following relations.

$$\underbrace{\left(\frac{\pi_{MIN} + 1}{2}\right)}_{Prob(s=G)} * \underbrace{\left[\frac{2\alpha}{3} \left(\frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2}\right) - K\right]}_{Expected\ Profit\ when\ s=G} = C_H > C_L$$
 (13)

<sup>&</sup>lt;sup>10</sup>There are also equilibria in which only the low-cost VC are active. In particular, these are states of low demand for capital. These outcomes are economically equivalent to those of Section 2 (substituting  $C_{VC} = C_H$ ) and so are not considered further.

This equilibrium has the following properties. High-cost VCs break even. Competition between them cannot reduce  $\alpha^*$  without violating the equality in equation (13). Low-cost VCs earn expected profits  $C_H - C_L > 0$  from their evaluations.

### 3.2 Continuum of VC Types

Section 2 demonstrates that venture capital labor demanded is a decreasing function of the cutoff quality  $\pi_{MIN}$ . When  $\pi_{MIN}$  is high, few entrepreneurs are active and therefore labor demand is low. More generally, denote this demand function  $D = D(\pi_{MIN})$  where D' < 0.11

Next suppose that each potential VC is a associated with an information production cost  $C_i$  drawn from some continuous distribution. Following the logic of Section 3.1, there is a cutoff  $C_{MAX}$  indicating the highest-cost VC active in equilibrium. The venture capitalist with  $C_i = C_{MAX}$  earns zero 11 Although unconditionally  $\pi_i \sim Unif[0,1]$ , so that active entrepreneurs are drawn from  $Unif[\pi_{MIN},1]$ , is not the case the demand is linear in  $\pi_{MIN}$ . As more (marginal) entrepreneurs are added to the pool during a hot market, the quantity of funded demanded goes up less than proportionally. The reason is that a smaller proportion of the added entrepreneurs receive the signal s=G. Hence, funds demanded are increasing and concave in market heat, or alternatively, decreasing and convex in  $\pi_{MIN}$ . This issue is developed numerically in Section 3.3.

profit. All those with cost exceeding  $C_{MAX}$  drop out out of the market, while those with cost less than  $C_{MAX}$  earn strictly positive profits from evaluating entrepreneurs. Therefore, in general, the measure of active agents generates a supply of venture capital finance  $S = S(C_{MAX})$  where S' > 0.

Equilibrium consists of the triple  $\{\pi_{MIN}, C_{MAX}, \alpha\}$  satisfying the following system of three equations.

(DEMAND) 
$$\pi_{MIN} [\pi_{MIN} (1 - \alpha) - C_E] + (1 - \pi_{MIN})(V - C_E) - V = 0$$

(SUPPLY) 
$$C_{MAX} - \left(\frac{\pi_{MIN} + 1}{2}\right) \left[\frac{2\alpha}{3} \left(\frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2}\right) - K\right] = 0$$

(MKT. CLEARING) 
$$D(\pi_{MIN}) - S(C_{MAX}) = 0$$

The demand equation comes from substituting  $\pi_i = \pi_{MIN}$  in expression (5). The left side of the equation is the profit of the worst entrepreneur active in equilibrium. By definition,  $\pi_{MIN}$  is the entrepreneurial quality associated with zero profit. The supply equation comes from substituting  $C_{VC} = C_{MAX}$  in equation (4). That is,  $C_{MAX}$  is the signal cost associated with zero profit. Finally, markets must clear, i.e., the aggregate supply must equal the aggregate demand.

**Theorem 2** A necessary and sufficient condition for equilibrium to exist is

that the solution  $\{\pi_{MIN}, \alpha, C_{MAX}\}$  to the above three-equation system satisfies the following inequality.

$$\frac{1}{3}\pi_{MIN}^2 - \frac{1}{6}\alpha\pi_{MIN} - \frac{1}{2}\pi_{MIN}K - C_{MAX} + \frac{1}{2}K - \frac{1}{6}\alpha \ge 0$$
 (14)

Proof: See the appendix.

The existence condition is analogous to that following Theorem 1. It ensures that the marginal VC has an incentive to purchase the signal rather than supplying funding on an uninformed basis. Specifically, she earns zero profit as an "informed" agent but would earn negative profit as an uninformed agent. The same condition ensures that sidelined VCs would earn negative profits were they to enter as an uninformed source of finance.

Corollary 2 (Effect of Costs) The equilibrium triple  $\{\pi_{MIN}, C_{MAX}, \alpha\}$  governed by the system of equations on pg. (24) has the following comparative static properties.

#### a) (Effect of K)

$$\frac{\partial \pi_{MIN}}{\partial K} > 0 \quad \frac{\partial C_{MAX}}{\partial K} < 0 \quad \frac{\partial \alpha}{\partial K} > 0 \tag{15}$$

#### b) (Effect of V)

$$\frac{\partial \pi_{MIN}}{\partial V} > 0 \quad \frac{\partial C_{MAX}}{\partial V} < 0 \quad \frac{\partial \alpha}{\partial V} < 0 \tag{16}$$

#### b) (Effect of $C_E$ )

$$\frac{\partial \pi_{MIN}}{\partial C_E} > 0 \quad \frac{\partial C_{MAX}}{\partial C_E} < 0 \quad \frac{\partial \alpha}{\partial C_E} < 0 \tag{17}$$

The inequalities in (15)-(17) formalize the intuition laid out in the introduction, examining how the double-sided asymmetric information problem depends upon real investment opportunities. A positive shock to the economy (as indicated by a drop in K) has several implications. Most obviously, venture capitalists' required stakes fall because they are being asked to provide less costly effort while terminal payoffs are unchanged.

A key property of this asymmetric information paradigm is that  $\pi_{MIN}$  drops in response to a positive shock to investment opportunities. Financing frictions worsen because as the set of real opportunities improves, low-quality entrepreneurs enter the market. Analogously on the supply side,  $C_{MAX}$  deteriorates in response to this economic shock. Thus, hot markets induce the entry of low-quality agents on both sides of the market.

The model also makes a related observation regarding the rents to high-quality information producers. In Section 3.1, it was determined that the profit to high-quality VCs is zero when they are the only agents active (i.e., states of low demand) but their profit is  $C_H - C_L > 0$  in states of high demand. That is, high-quality agents' profits depend upon the difference between their own quality and that of the worst active agent. Analogously here, since the quality

of the worst agent is continuously decreasing in market heat, the profits of a fixed (active) VC are continuously increasing in market heat. As a result, total profits to the VC industry are also increasing in market heat.

Equations (16) and (17) have straightforward interpretations. As the relative costs to entrepreneurs of seeking VC finance increase, marginal entrepreneurs are forced out of the market – that is,  $\pi_{MIN}$  grows. Note that even though VCs do not bear the entrepreneurs' opportunity costs, there is a channel of indirect effects. Specifically, as these opportunity costs rise, weak entrepreneurs drop out. As the quality of pool improves, this enables VCs to demand smaller stakes.

The final indirect effect is that because demand falls, the equilibrium number of VCs operating must fall. Such a drop would be impossible if  $\alpha$  rose, because in that case VCs would make higher profits even for a fixed level of asymmetric information. These higher profits would encourage entry, not exit.

## 3.3 An Example

Fig. 2 displays the supply and demand curves governed by the system of equations on pg. 24 under the assumptions  $\{V = .2, K = .2, C_E = .1\}$ . It is further assumed that supply is proportional to  $C_{MAX}$ ; that is, information production costs are drawn from a uniform distribution.

The horizontal axis indicates  $\alpha$ , the share of the firm offered to the venture capitalist. The entrepreneur's residual claim is 1- $\alpha$ . The vertical axis indicates the quantity of labor. More specifically, it reflects the units of labor active in equilibrium. (Recall that some negative NPV projects are screened out by the VC whereas others self-select; the vertical axis here reflects the net effect of both types of removal.)

The demand curves in Fig. 1 have a downward slope. This is because a greater share  $\alpha$  of the firm ceded to VCs implies that fewer entrepreneurs find it profitable to seek funding. Likewise, the supply curves have an upward slope because when a greater share of the firm is ceded to VCs, venture capital profits are higher (all else equal) and therefore more VCs enter the market.

Next, consider a positive shock to the value of investment opportunities. The dotted lines show the supply and demand curves when V and K fall to from .2 to .1. This drop in V and K is equivalent to an increase in the terminal payoffs relative to other parameters.

The decrease in V shifts the demand curve up. As entrepreneurs' outside options become less valuable, they are more willing to enter the VC market. The drop in K has no direct effect on demand. Rather, entrepreneurs bear K only indirectly through the VCs' required stake.

The decrease in K shifts the supply curve up. This is a direct effect: for a given  $\alpha$ , reducing the required effort contribution would increase VC profitability. Consequently, more VCs would enter, increasing the willingness to supply capital. By contrast, the decrease in V shifts the demand curve downward. This is an indirect effect. The adverse selection effect emphasized  $\overline{\phantom{a}}^{12}$ This point also emphasizes why Fig 1 uses the proportion of projects funded on the vertical axis rather than (numeraire-denominated) funds supplied. Otherwise, shocks to K

trivially change the amount of capital supplied and demanded.

in Section 2 implies that the quality of the entrepreneurial pool deteriorates. This deterioration indirectly reduces the profitability to capital suppliers, and the resulting exit of marginal VCs causes supply to fall. Over the relevant range in the graph, the direct effect dominates and leads to higher supply.

In this numerical example, the effect of the demand shock dominates that of the supply shock, leading to a higher equilibrium venture capital stake  $\alpha$ . This comparative static result is not general, however, as Corollary 2 suggests. That is, one can also construct shocks to  $\{V, K\}$  such that the supply effect dominates the demand effect and hence equilibrium  $\alpha$  is lower in hot markets.

It is also apparent in Fig 1 that both demand curves are less elastic (flatter) in hot markets (denoted by the dashed lines). This property is driven by V. To see why this effect occurs, consider the extreme case when V is very low compared to other parameters in the model. In that case, virtually every entrepreneur is active because no other choice is economically viable. Their collective entry decisions are therefore relatively invariant to  $\alpha$ . Hence the "hot" demand curve is flatter in  $\alpha$ .

A similar argument motivates why supply elasticity is a function of investment opportunities. In any economic state, increasing  $\alpha$  increases VC profits both directly (they keep a larger share of the firm) and indirectly (because higher  $\alpha$  improves the quality of the entrepreneurial pool). This indirect effect is most important in cold markets, however, when the incentives of all entrepreneurs to enter the market is weak. In that case, small changes in the sharing rule cause large changes in the average quality within the pool, and hence in VC profits.

Fig. 2 generalizes figure Fig. 1. The axis toward the front of the picture is  $\alpha$ , the share offered to VCs. The vertical axis is again the proportion of projects funded. The final axis, moving front to back in the picture, indicates entrepreneur's reservation utility V. Taking a slice of the figure along the V-axis one can see the supply and demand curves of the previous figure. Again it is clear that both supply and demand curves are more elastic when V is high. In this case, few entrepreneurs are active and so small changes rapidly alter the composition of the pool. Moving "backwards" in the picture toward lower V (interpreted as a positive shock to investment opportunities) causes an increase to both the equilibrium amount of capital supplied and to the stake of VCs.

### 4 Conclusions

The venture capital market is central to the financing of innovation but is subject to dramatic variation in the level of activity. Understanding the determinants of the underlying supply and demand curves is therefore an economic question of first-order importance.

This paper endogenizes this supply and demand in an environment of twosided information asymmetry. Entrepreneurs have heterogeneous, private information about the quality of their projects. Venture capitalists have hetergeneous ability levels. In effect, these agents generate a supply curve of information production. Thus, there are two distributions of agent quality, both of which vary continuously in the heat of the market.

Both types of variation have some support in the literature. Regarding the phenomenon of wider pooling during hot markets, Lerner (2002) comments

"[F]unds appear to be deployed much less effectively during the boom period. In particular, all too often these periods find venture capitalists funding firms that are too similar to one another."

Yet even when firms appear similar ex-ante, there is evidence — at least in the IPO market (Yung, et. al, 2008) — that firms funded during booms exhibit heightened variability in long-run returns. These findings together suggest that, while the firms may be difficult to distinguish ex-ante, there is actually significant heterogeneity revealed in the long-run.

The scientific study of returns in the VC market is a relatively young field, and is somewhat hindered by data availability issues. As Gompers, Lerner, Kovner and Sharfstein (2005) (hereafter, GLKS) point out that Venture Economics "does not collect valuation information for all the companies that were merged or acquired and it is possible that these outcomes are not as lucrative as those where the company exited with a public offering." Thus, for describing outcomes at the portfolio firm level, researchers have typically employed coarse measures such as whether a firm was liquidated, merged or went public.

Recent empirical papers circumvent the data problems alluded to by GLKS by using either proprietary databases (Ljungqvist and Richardson, 2003) or using fund-level cash flows (Jones and Rhodes-Kropf, 2004) rather than portfolio firm level cash flows. While this latter approach is useful for characterizing

the returns of the asset class, it does not permit analysis of the demand-side heterogeneity posited by the current model.

Similarly, the model's prediction of time-varying heterogeneity in the quality of supply side agents has not been subject to formal testing. Nervertheless, it has some parallels in Kaplan and Schoar's (2005) findings. They conclude that a large fraction of fund inflows during hot markets go to new entrants, and that these entrants appear to have poor ex-post performance. Among possible explanations for these findings, Kaplan and Schoar list a variety of disadvantages faced by new entrants, including lack of access to deal flow and better bargaining power vis-a-vis entrepreneur. Perhaps their strongest wording, however, is reminiscient of the current analysis: "we think the most likely explanation for [our] results is a model of underlying heterogeneity in the skills of GPs."

Hochberg, Ljungqvist and Vissing-Jorgensen (2008) also model an environment of GP heterogeneity. As funds season, LPs acquire a form of inside information vis-a-vis outside investors. For example, LPs may be able to distinguish luck from skill, thereby better forecasting future performance. As HLVJ show, the resulting informational hold-up leads to underadjustment of fees and performance persistence. There is also a possible efficiency gain, however, because LPs are sometimes willing to fund untested GPs owing to the

inherent option value in doing so. Thus, the informational hold-up does not necessarily serve as a friction but can also prevents a market failure.

The above story potentially interacts with the current analysis. Given a positive shock to investment opportunities, both incumbent GPs and their LPs earn economic rents on follow-up funds. In fact, the expected positive profits in hot markets accruing to limited partners — who effectively share residual claimancy with general partners in HLVJ — could subsidize their losses in cold markets. This effect would tend to smooth out the supply of capital over the market cycle relative to that predicted in this paper. The full range of implications (including time-varying incentives of GPs to signal quality) is left to future research.

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### 6 Proofs

**Proof of Theorem 1.** To prove sufficiency, note that equality in (4) and (5) ensure that VCs and the marginal entrepreneur earn zero profits. Thus, all agents have the incentive to enter the market and  $\alpha$  is competitively set. Profits from purchasing the signal are

$$\left(\frac{\pi_{MIN} + 1}{2}\right) * \left[\frac{2\alpha}{3} \left(\frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2}\right) - K\right] - C_{VC}$$
 (18)

and profits from funding all firms on an uninformed basis are

$$\left(\frac{\pi_{MIN} + 1}{2}\right)\alpha - K \tag{19}$$

The difference between these two values (after some simplification) is the expression IR in equation (7). Thus, VCs purchase the signal if and only if IR is positive. Necessity is similar.

**Proof of Corollary 1.** I derive the comparative static result  $\frac{\partial \pi_{MIN}}{\partial K} > 0$  here. The others are similar. From  $\{(4), (5)\}$ 

$$F := \alpha \frac{2}{3} \left( \frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2} \right) - K \tag{20}$$

$$G := \pi_{MIN}^{2}(1 - \alpha) - \pi_{MIN}V - C \tag{21}$$

are zero in equilibrium. Totally differentiating this system with respect to K, one obtains

$$\frac{\partial F}{\partial K} + \frac{\partial F}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K} + \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0$$
 (22)

$$\frac{\partial G}{\partial K} + \frac{\partial G}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K} + \frac{\partial G}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0$$
 (23)

The above is a system of two equations to be solved for two unknowns,  $\frac{\partial \pi_{MIN}}{\partial K}$  and  $\frac{\partial \alpha}{\partial K}$ . The solution for  $\frac{\partial \pi_{MIN}}{\partial K}$  is

$$\frac{\partial \pi_{MIN}}{\partial K} = \frac{\frac{\partial G}{\partial \alpha} \frac{\partial F}{\partial K} - \frac{\partial F}{\partial \alpha} \frac{\partial G}{\partial K}}{\frac{\partial F}{\partial \alpha} \frac{\partial G}{\partial \pi_{MIN}} - \frac{\partial G}{\partial \alpha} \frac{\partial F}{\partial \pi_{MIN}}}$$
(24)

Reporting only the signs of each of these partial derivatives, and noting that  $\frac{\partial G}{\partial K} = 0$ ,

$$sign\left(\frac{\partial \pi_{MIN}}{\partial K}\right) = \frac{\ominus \ominus - \oplus 0}{\oplus \ominus - \ominus \oplus} = \frac{\ominus}{\ominus} = \ominus$$
 (25)

**Proof of Theorem 2.** Similar to Theorem 1.

**Proof of Corollary 2.** I derive the comparative static result  $\frac{\partial \pi_{MIN}}{\partial K} > 0$  here. The others are similar. Denote H:=0, J:=0, M:=0 as the demand, supply and market clearing identities listed on page 24. Totally differentiating this system with respect to K, one obtains

$$\frac{\partial H}{\partial K} + \frac{\partial H}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K} + \frac{\partial H}{\partial C_{MAX}} \frac{\partial C_{MAX}}{\partial K} + \frac{\partial H}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0$$
 (26)

$$\frac{\partial J}{\partial K} + \frac{\partial J}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K} + \frac{\partial J}{\partial C_{MAX}} \frac{\partial C_{MAX}}{\partial K} + \frac{\partial J}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0$$
 (27)

$$\frac{\partial M}{\partial K} + \frac{\partial M}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K} + \frac{\partial M}{\partial C_{MAX}} \frac{\partial C_{MAX}}{\partial K} + \frac{\partial M}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0$$
 (28)

Note that the above is a system of three equations to be solved for three unknowns  $\frac{\partial \pi_{MIN}}{\partial K}$ ,  $\frac{\partial C_{MAX}}{\partial K}$  and  $\frac{\partial \alpha}{\partial K}$ . Solving the system, and using the fact that four of twelve leading terms are zero (e.g.  $\frac{\partial H}{\partial K} = 0$  by inspection on H), one obtains the solution

$$\frac{\partial \pi_{MIN}}{\partial K} = \frac{\frac{\partial H}{\partial \alpha} \frac{\partial M}{\partial C_{MAX}} \frac{\partial J}{\partial K}}{\frac{\partial M}{\partial C_{MAX}} \left[ \frac{\partial H}{\partial \pi_{MIN}} \frac{\partial J}{\partial \alpha} - \frac{\partial J}{\partial \pi_{MIN}} \frac{\partial H}{\partial \alpha} \right] + \frac{\partial J}{\partial C_{MAX}} \frac{\partial M}{\partial \pi_{MIN}} \frac{\partial H}{\partial \alpha}}$$
(29)

Reporting only the signs of each of these partial derivatives,

$$sign\left(\frac{\partial \pi_{MIN}}{\partial K}\right) = \frac{\ominus \ominus \ominus}{\ominus \left[\ominus \ominus - \ominus \ominus\right] + \ominus \ominus}$$
(30)

By inspection, both the numerator and denominator are positive and therefore the fraction is positive.