Intangible Assets and Cross-Sectional Stock Returns: Evidence from Structural Estimation

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Abstract

The relation between a firm's stock return and its intangible investment ratio and asset tangibility is derived under the intangible-asset-augemnted (IAA) q-theory framework. Using firm level data and the Generalized Method of Moments (GMM), we estimate the model and three main results emerge. First, the IAA q-theory captures the value premium and the relation between R&D intensity and stock returns significantly better than the conventional q-theory. Two features of intangible assets, adjustment costs and investment-specific-technological-change, are crucial to the improved model performance. Second, the relation between R&D intensity and stock return is similar to the relation between tangible investment and stock return, which is different from what the previous literature documents. Third, the IAA q-theory gives a more reasonable estimate of adjustment costs of tangible investments than the conventional q-theory does. Moreover, the magnitude of adjustment costs of intangible investments is estimated to be larger than that of tangible investments on average, providing supporting evidence that intangible assets are more crucial for firms to sustain their comparative advantages and helping to explain the higher autocorrelation of R&D expenditures than that of capital investments observed in the data.

JEL Classification: G12, E21, D24, O31, O32

1 Introduction

Intangible assets have been widely recognized as the driving force of an economy's productivity growth and have become more and more crucial for a firm's survival and prosperity. Recent studies (Rauh and Sufi, 2010 and Rampini and Viswanathan, 2010) show that a firm's asset tangibility is an important determinant for corporate policies, such as capital structure. However, less attention has been paid to the impact of intangible assets on stock returns, with the exception of Chan, Lakonishock, and Sougiannis (2001) (henceforth CLS) among others. In this study, we explore the relation between intangible assets and stock returns, both theoretically and empirically, and quantify the characteristics of intangible assets based on the structural estimation of our theoretical model.

We build a q-theory model with both tangible and intangible assets where investments in both types of assets incur adjustment costs and the accumulation of intangible assets leads to increased productivity of intangible investment, the so called "investment-specific technological change (henceforth ISTC)" effect. Adjustment costs prevent firms from accumulating assets rapidly. The magnitude of adjustment costs hence determines the speed of capital growth and the persistence of industry incumbents' profitability. The ISTC effect of intangible assets has been widely used to explain the productivity growth of an economy at the aggregate level (Greenwood, Hercowitz, and Krusell, 1997). In this paper, we study the impacts of both the adjustment cost (henceforth AC) effect of intangible investment and the ISTC effect on stock returns. More importantly, we quantify the magnitudes of the adjustment costs of both tangible and intangible assets and that magnitude of the ISTC effect based on the estimation of the model.

The structural estimation is based on the relation between a firm's stock return and its observable characteristics: both tangible and intangible investment rates, asset tangibility, profitability, and leverage, derived from our theoretical model. By matching the model predicted stock returns with the realized returns, we estimate the model parameters and compare the performance of the intangible-asset-augmented q-theory (henceforth the IAA q-theory) with that of the conventional q-theory using three sets of testing portfolios. Due to the reason of data availability (Lev, 2001), we focus on one special type of intangible investments, research and development (R&D) expenditure, and construct the level of intangible assets based on the accumulation of past R&D expenditures. The three sets of testing portfolios are portfolios sorted by the book-to-market ratio, the R&D-to-intangible-asset ratio, and the R&D-to-market-equity ratio, respectively.

The main findings of the paper are as follows. First, the non-nested tests indicate that the IAA q-theory explains cross-sectional stock returns significantly better than the q-theory with only tangible asset across all three sets of testing portfolios. Moreover, both the AC effect and the ISTC effect of intangible assets are shown by the nested tests to be crucial to the improved explanatory power.

Second, the IAA q-theory implies a 19.88% adjustment-costs-to-investment ratio (henceforth the AC/I ratio) for capital investments, averaging across all three sets of testing portfolios, while the estimates of the conventional q-theory is 223%. The existing literature estimates the AC/I ratio either using simulation of calibrated models (Summers, 1981 and Cooper and Haltiwanger, 2006), or using reduced-form regression of investment data (Litchenberg, 1988). The estimates from the aforementioned three papers are 22.1%, 20%, and 33.09%, respectively. Therefore, using both stock return data and investment data, the structural estimation of the IAA q-theory model leads to a much closer estimate of the AC/I ratio to what the previous literature finds than the conventional q-theory does.

Third, the model implies a larger AC/I ratio for intangible investments than that for tangible investments. This finding provides empirical support for the conventional wisdom that intangible assets are more crucial for firms to sustain their comparative advantages than tangible assets because it is more costly to accumulate intangible assets rapidly. Therefore, it is beneficial for firms to consistently invest in intangible assets, which explains the higher persistence of R&D

expenditures than that of physical investments observed in the data: 0.81 vs. 0.45 if scaled by book assets, 0.87 vs. 0.43 if scaled by Property, Plant and Equipment (PP&E), 0.88 vs. 0.55 if scaled by sales. Going forward, we use "physical", "capital", and "tangible" interchangeably.

Fourth, we show that high R&D-intensive firms earn 10% higher stock returns per annum, with t-statistic being 4.03, than low R&D-intensive firms, using the model implied measure of R&D intensity, R&D-to-intangible-assets ratio. This finding overhauls the widely documented "puzzle" that high physical investment-intensive firms earn lower returns (Titman, Wei, and Xie, 2004, henceforth TWX), however high R&D-intensive firms earn higher returns (CLS). We show that this "puzzle" is due to the fact that R&D intensity used in CLS uses market value of equity as the scaler, while tangible investment intensity used in the literature use either PP&E (proxy for physical assets) or total assets as scaler.[1] Our theory implies that in the relation with stock returns, R&D-to-intangible-assets ratio plays a similar role as the investment-to-physical-assets ratio and should be the more suitable measure of R&D intensity. Our empirical evidence shows that the R&D-to-intangible-assets ratio is indeed negatively related to stock returns, the same as the relation between tangible investment ratio and stock returns documented in the literature.

Our paper contributes to several strands of literature. There is a large literature that attempts to capture the cross-sectional returns using the investment-based q-theory model, pioneered by Cochrane (1996). Our paper contributes to this literature by introducing intangible assets to the traditional q-theory model with only tangible assets. We use the methodology proposed by Liu, Whited, and Zhang (2009) (henceforth LWZ), who estimate a structural q-theory model with only tangible assets. We show that intangible assets play an important role in capturing the value premium and the R&D related cross-sectional return patterns.

Our paper contributes to the literature that studies the impact of R&D on stock returns. In addition to CLS, who document the positive relation between the R&D-to-market-equity ratio and

[1] TWX study abnormal capital investment growth, where capital investment is measured as investments scaled by PP&E. Copper, Gulen and Schill (2008) and Titman, Wei and Xie (2010) use total asset growth as a measure of investment intensity, of which investments-to-total-assets is a component.

stock returns, Li (2007) shows that such a positive relation mainly exists among R&D intensive firms and Hsu (2009) shows that technological innovations increase risk premium at the aggregate level using patent data and R&D data. Lin (2009) tries to explain this relation using a dynamic model with investment-specific technological change. Our paper emphasizes the importance of using the economically sensible measure of R&D intensity in studies on the relation between R&D and stock returns.

Our work is also related to the growing field that uses structural estimation to study corporate policies and characteristics of individual firms (Hennessy and Whited, 2007; Whited and Wu, 2006; Albuquerque and Schroth, 2010). Our work uses this framework to study firm's investments in intangible assets rather than external financing costs, financial constraints, or private benefits of control.

Last, but not least, our paper provides a new methodology to the literature that studies the distinctive features of intangible assets. The macroeconomics literature focuses on how the investment-specific technological change affects productivity growth at the aggregate level using model calibration (Greenwood, Hercowitz, and Krusell, 1997 and Huffman, 2007). The literature in organization science and evolutionary economics is devoted to understand how the accumulation process of intangible assets shapes the structure of an industry and the survival rate of new entrants into the industry (Knott, Bryce and Posen, 2003). Those studies use linear regression and firm-level investment data, which can be problematic as pointed out by Whited (1994). To our best knowledge, this is the first paper to quantify the magnitudes of the adjustment costs of (in)tangible investments and the ISTC effect using asset return data and using structural estimations.

The paper is organized as follows. Section 2 sets up the model and derives the investment return. Section 3 describes the three models that we estimate, explains the construction of the data set and the testing portfolios, and exhibit the empirical tests and the estimation results. Section 4 concludes. Appendix A shows the proof of Proposition 1 and Appendix B provides the definitions and sources of data items used in the estimation.

2 The Model Setup

Assume that a firm faces infinite horizon and the time is discrete. The firm's production requires both tangible and intangible capital/assets in addition to non-capital input. Let Y_{jt} denotes the revenue of firm j at time t

$$Y_{jt} = e^{X_t} \left[\left(K_{jt}^m\right)^{\gamma} \left(K_{j,t}^u\right)^{1-\gamma} \right]^{\alpha} \left(L_{jt}\right)^{1-\alpha} ,$$

where $K_{j,t}^m$ is the capital stock of tangible assets, $K_{j,t}^u$ is the capital stock of intangible assets, X_t is the exogenous productivity shock, α is the capital (including both tangible and intangible) share of total output, and γ is the elasticity of substitution between tangible and intangible assets. Without loss of generality, let L_{jt} be the composite non-capital factor input and assume that firm j is a price taker in the input market. We assume constant-return-to-scale Cobb-Douglas production function. The accumulations of tangible assets and intangible assets follow

$$K_{jt+1}^m = (1 - \delta_{m,jt})K_{jt}^m + I_{jt}^m , \tag{1}$$

$$K_{jt+1}^u = (1 - \delta_{u,jt})K_{jt}^u + \Theta(I_{jt}^u, K_{jt}^u) , \tag{2}$$

where I_{jt}^m and $I_{I,t}^u$ are the investments made by firm j at time t in tangible assets and intangible assets, respectively, and $\delta_{m,jt}$ and $\delta_{u,jt}$ are the corresponding depreciation rates. Both tangible and intangible investments are produced using final outputs. The production function of the new intangible assets, Θ, is defined as

$$\Theta_{jt} \equiv \Theta(I_t^u, K_t^u) = \left[a_1 \left(I_t^u\right)^{\xi} + a_2 \left(K_t^u\right)^{\xi} \right]^{1/\xi} \tag{3}$$

with positive constants ξ, a_1 and a_2 so that the amount of newly produced intangible assets increases with the levels of both intangible investments and existing intangible assets. Moreover, with $\xi < 1/2$, the productivity of the intangible investments increases with the level of the existing intangible assets of firm j, that is,

$$\frac{\partial^2 \Theta_{ji}}{\partial I^u_{jt} \partial K^u_{jt}} > 0 \,.$$

To better understand the economic intuition behind the production function of intangible assets, we rewrite Θ as

$$\Theta_{jt} = I^u_t \left[a_1 + a_2 \left(\frac{K^u_t}{I^u_t} \right)^{\xi} \right]^{1/\xi} \equiv I^u_t Q^u_t \,,$$

where Q^u_t is the amount of new intangible assets that can be produced from one dollar of intangible investment at time t, which can be written as

$$Q^u_t = \left[a_1 + a_2 \left(\frac{K^u_t}{I^u_t} \right)^{\xi} \right]^{1/\xi} \,.$$

The time series of Q^u_t represents the investment-specific technological changes (ISTC).[2] We can see that as a firm accumulates more intangible assets, intangible investments become more productive in generating new intangible assets, or equivalently, the dollar price of the new intangible assets, $1/Q^u_t$, decreases. Our formulation of the production of intangible assets captures the intuition that the accumulation of knowledge capital makes generating new knowledge less expensive.

[2]The ISTC effect in the macroeconomic literature pioneered by Greenwood, Hercowitz, and Krusell (1997) appears in the accumulation of quality-adjusted physical capital. In our estimation, we use the the book values of physical assets reported in firms' financial statements, which are not quality-adjusted. The capital accumulation law for physical assets in equation (1) has to be satisfied due to the way that those data items are constructed. To our best knowledge, there is no good data resource that provides the quality-adjusted price indices for all the categories of physical assets that firms use. To avoid measurement errors, we put the ISTC effect in the accumulation of intangible assets, which we believe captures the same economic intuition.

Both investments in tangible assets and in intangible assets incur adjustment costs

$$\Phi_{jt}^m \equiv \Phi^m \left(I_{jt}^m, K_{jt}^m\right) = \frac{a}{2}\left(\frac{I_{jt}^m}{K_{jt}^m}\right)^{\rho} K_{jt}^m \,, \tag{4}$$

$$\Phi_{jt}^u \equiv \Phi^u \left(I_{jt}^u, K_{jt}^u\right) = \frac{b}{2}\left(\frac{I_{jt}^u}{K_{jt}^u}\right)^{\psi} K_{jt}^u \,, \tag{5}$$

where a, b, ρ and ψ are positive constants, with the first two constants reflecting the magnitude of the adjustment costs and the latter two constants reflecting the curvature of the adjustment costs for tangible investments and intangible investments, respectively.

Firms are allowed to have both equity and debt financing. Assume that there are no external financing costs. Following Hennessy and Whited (2007) and LWZ, we assume that firms issue a one-period debt. The debt outstanding at the beginning of period t is B_{jt}, with the gross required return r_{jt}^B. At the end of period t, firm j issues new debt B_{jt+1}. The net cash flow accrued to the shareholders of firm j at period t is

$$D_{jt}^S = (1-\tau_{jt})\left(Y_{jt} - \varpi_t L_{jt} - \Phi_{jt}^m - \Phi_{jt}^u - I_{jt}^u\right) - I_{jt}^m + \tau_{jt}\delta_{m,jt}K_{jt}^m - \left[1 + \left(r_t^B - 1\right)(1-\tau_t)\right] B_{jt} + B_{jt+1} \,,$$

where ϖ_t is the price on non-capital input and τ_{jt} is the corporate tax rate on firm j at time t.

We solve the maximization problem of a representative firm j and write its investment return as a function of firm's observable characteristics. To simplify the notation, we omit subscript j in all the equations where no ambiguity is present.

Proposition 1. *Firm's investment return* r^I_{t+1}*, defined as*

$$
\begin{aligned}
r^I_{t+1} \;=\; & \Bigg\{(1-\tau_{t+1})\frac{\alpha Y_{t+1}}{K^m_{t+1}} + \tau_{t+1}\delta_{m,t+1} - (1-\tau_{t+1})\Phi^m_{k,t+1} + (1-\delta_{m,t+1})\left[1+(1-\tau_{t+1})\Phi^m_{I,t+1}\right] \\
& + \left[\left[(1-\tau_{t+1})\left(1+\Phi^u_{i,t+1}\right)\right]\left(\frac{\Theta_{K,t+1}}{\Theta_{I,t+1}}\right) - (1-\tau_{t+1})\Phi^u_{K,t+1} + \frac{(1-\delta_{u,t+1})(1-\tau_{t+1})\left(1+\Phi^u_{i,t+1}\right)}{\Theta_{I,t+1}}\right] \\
& \times \left(\frac{K^u_{t+1}}{K^m_{t+1}}\right)\Bigg\} \Bigg/ \left\{\left[1+(1-\tau_t)\Phi^m_{I,t}\right] + \left[\frac{(1-\tau_t)\left(1+\Phi^u_{I,t}\right)}{\Theta_{I,t}}\right]\left(\frac{K^u_{t+1}}{K^m_{t+1}}\right)\right\}\,, \qquad (6)
\end{aligned}
$$

satisfies

$$\mathbb{E}_t\left[M_{t+1}\mathbb{I}_{t+1}r^I_{t+1}\right] = 1\,,$$

where M_{t+1} *is the stochastic discount factor from* t *to* $t+1$*.* r^I_{t+1} *is equal to the weighted average of the return on firm's equity and the after-tax return on its debt,*

$$r^I_{t+1} = (1-w_t)\, r^S_{t+1} + w_t\, r^{Ba}_{t+1}\,, \qquad (7)$$

where w_t *is the ratio of debt value to firm value at the end of period* t

$$w_t = \frac{B_{t+1}}{P_t - D^S_t + B_{t+1}}\,,$$

r^S_{t+1} *is stock return from period* t *to* $t+1$

$$r^S_{t+1} = \frac{P_{t+1}}{P_t - D^S_t}\,,$$

r^{Ba}_{t+1} *is the after-tax return on debt*

$$r^{Ba}_{t+1} = r^B_{t+1} - \tau_{t+1}\left(r^B_{t+1} - 1\right)\,,$$

and $\Phi^m_{*,t}$ *is the derivative of the adjustment cost function of tangible assets w.r.t. variable* $*$*. Similar* *definitions for* $\Phi^u_{*,t}$ *and* $\Theta_{*,t}$.

Proof. See Appendix A.

To understand the economics behind Proposition 1, we decompose firm's investment return into two components: the return on tangible assets $r^{I,m}_{t+1}$, defined as

$$r^{I,m}_{t+1} = \frac{(1-\tau_{t+1})\left[\alpha\frac{\gamma Y_{t+1}}{K^m_{t+1}} - \Phi^m_{K,t+1}\right] + \tau_{t+1}\delta_m + (1-\delta_m)\left[1+(1-\tau_{t+1})\Phi^m_{I,t+1}\right]}{1+(1-\tau_t)\Phi^m_{I,t}} \tag{8}$$

and the return on intangible assets $r^{I,u}_{t+1}$, defined as

$$\begin{aligned} r^{I,u}_{t+1} \;=\; & \left\{(1-\tau_{t+1})\left[\alpha\frac{(1-\gamma)Y_{t+1}}{K^u_{t+1}} - \Phi^u_{K,t+1}\right] + \frac{(1-\delta_u)(1-\tau_{t+1})\left(1+\Phi^u_{i,t+1}\right)}{\Theta_{I,t+1}}\right. \\ & \left. +(1-\tau_{t+1})\left(1+\Phi^u_{i,t+1}\right)\left(\frac{\Theta_{K,t+1}}{\Theta_{I,t+1}}\right)\right\} \Big/ \left[(1-\tau_t)\left(1+\Phi^u_{I,t}\right)/\Theta^u_{I,t}\right]. \end{aligned} \tag{9}$$

A firm's investment return should be a weighted average of its investment return on tangible assets and its investment return on intangible assets, with the weights being the ratio of the market value of tangible assets to total firm value and the ratio of the market value of intangible assets to total firm value, respectively.

For both tangible and intangible investment returns, the return from t to $t+1$ is a ratio of marginal benefit at $t+1$ of one more unit of investment made at t to its marginal costs at time t. The marginal benefit includes not only the marginal free cash flow at $t+1$, but also the marginal continuation value at time $t+1$. The marginal cost includes the price of one unit of investment, which is normalized to one for both types of investments, and the marginal adjustment costs of investment.

For tangible investments, the denominator in equation (8) is the marginal cost of tangible investment at time t, including the price of one unit of investment and the marginal adjustment cost,

$$\Phi_{I,t}^m \equiv \frac{\Phi_t}{I_t^m} = \frac{a\psi}{2}\left(\frac{I_t^m}{K_t^m}\right)^{\psi} .$$

Since the adjustment cost is categorized as part of the operating costs and it reduces a firm's taxable income, the net cost is given by $(1-\tau_t)\Phi_{I,t}^m$. Under the assumption that $\psi > 0$, the higher the investment ratio, the larger the marginal cost of investment.

The numerator in equation (8) is the marginal benefit of tangible investment made at time t, including both the immediate benefit at $t+1$ and the increased continuation value after $t+1$. Since one unit of tangible investment is transformed into one unit of tangible asset, the marginal benefit of investment made at time t is the same as the marginal benefit of capital at $t+1$. The first term in the numerator is the after-tax cash flow generated at time $t+1$ from one unit of increased tangible capital, including the increased sales, given by

$$\frac{Y_{t+1}}{K_{t+1}^m} = \alpha\gamma e^{X_{t+1}}\left(K_{t+1}^m\right)^{\alpha\gamma-1}\left(K_{t+1}^u\right)^{\alpha\gamma-1} L_{t+1}^{1-\alpha} = \alpha\frac{\gamma Y_{t+1}}{K_{t+1}^m} ,$$

minus the marginal increase in investment adjustment costs $\Phi_{K,t+1}$. The second term is the tax benefit from the deprecation of one unit of increased capital. The last term is the marginal continuation value (i.e., the market value at $t+1$ of one unit of increased capital after depreciation). Appendix A shows that the market price of capital at $t+1$ (i.e., the shadow price of capital) is given by

$$q_{t+1}^m = 1 + (1-\tau_{t+1})\Phi_{I,t+1}^m .$$

Hence, the marginal continuation value is given by

$$(1-\delta_m)q_{t+1}^m = (1-\delta_m)\left[1 + (1-\tau_{t+1})\Phi_{I,t+1}^m\right] .$$

Similarly, the return on intangible assets in equation (9) is the ratio of the marginal benefit of one more unit of intangible capital at time $t+1$ to the marginal cost of one more unit of investment made at time t. Compared to tangible investment, there are two major differences: (1) one unit of intangible investment generates more than one unit of intangible asset due to the ISTC effect and the productivity of the intangible investment depends on the parameter value of a_2; (2) intangible investment is expensed, instead of capitalized as tangible investment, and hence there is a corresponding tax deduction.

The denominator is the cost of producing the last unit of the new intangible asset. Due to the ISTC effect, one unit of intangible investment generates $\Theta_{I,t}$ units of intangible capital at time t. The marginal cost of intangible investment is similar to that of tangible investment, including the price of the one unit of intangible investment, which is normalized to one, and the corresponding marginal adjustment cost $\Phi^u_{I,t}$. Hence, the cost of producing the last unit of new intangible asset is given by

$$\frac{(1-\tau_t)\left(1+\Phi^u_{I,t}\right)}{\Theta_{I,t}}.$$

The numerator of equation (9) is the marginal benefit of one more unit of capital at time $t+1$, including the immediate benefit at time $t+1$ and the marginal continuation value (i.e., the present value of all the future benefits). The first term is the marginal after-tax cash flow generated at time $t+1$, equal to the marginal revenue from sales minus the marginal adjustment costs $\Phi^u_{K,t+1}$. Both the second term and the third term in the numerator are the marginal continuation value. The second term is the market value of one unit of intangible assets after depreciation, where the Appendix A gives the shadow price of the intangible asset at $t+1$:

$$q^u_{t+1} = \frac{(1-\tau_{t+1})\left(1+\Phi^u_{I,t+1}\right)}{\Theta_{I,t+1}}.$$

Moreover, due to the ISTC effect, for a given amount of intangible investment made at time $t+1$, the firm is able to produce $\Theta_{K,t+1}$ more units of intangible asset, which has a market value of $q^u_{t+1}\Theta_{K,t+1}$ and is the third term in the numerator.

Finally, a firm's investment return also depends on the relative value of the tangible assets and the intangible assets that the firm has. If a firm has more intangibles assets, its return on the intangible assets will have larger impact on the overall return of the firm and vice versa. The similar argument applies to the tangible assets. In general, the higher the ratio of intangible assets to tangible assets, the more important the return of intangible investment and *vice versa*.

Define the levered investment return as

$$r^{Iw}_{t+1} \equiv \frac{r^I_{t+1} - w_t r^{Ba}_{t+1}}{1 - w_t} \, .$$

Equation (7) implies that for any firm, at any period, and in any state of the world, its realized stock return equals the model predicted levered investment return, that is,

$$r^S_{t+1} = r^{Iw}_{t+1} \equiv \frac{r^I_{t+1} - w_t r^{Ba}_{t+1}}{1 - w_t} \, . \tag{10}$$

Through equation (6), we can relates a firm's characteristics with its stock returns, both of which are observable. Equation (10) is the equality that we use to construct the moment conditions for the structural estimation in Section 3.

Before we proceed to the empirical part of the paper, there are a couple of points that merit detailed discussion. First, our model is not a risk factor model and to derive equation (6), we do not need to specify the stochastic discount factor (henceforth SDF). The effect of the SDF is reflected implicitly on the firm's optimal corporate policies. Since we do not make any assumptions on the specific form of the SDF, the model is salient on the rationality of the investors. On the production side, the model assumes that the manager of the firm knows the SDF that her firm faces and makes the investment and financing decisions to maximize the shareholders' value.

Second, both returns and characteristics are endogenously determined by the exogenous factors (e.g., productivity shocks) and predetermined factors (e.g., the existing amounts of tangible and intangible assets that the firm has). Therefore, Proposition 1 gives us a relation, but not a causality, between a firm's realized stock return and its observable characteristics, such as profitability and investment rates.

3 Empirical Investigation of the Model

In this section, we take the model to the data and investigate the importance of intangible assets in capturing cross-sectional stock returns using structural estimations. Based on the parameter estimates of the models, we infer the magnitude of the adjustment costs of both tangible and intangible investments.

3.1 Test Design and Econometric Methodology

To investigate the importance of each feature of intangible assets in capturing the cross-sectional stock returns and to quantify the characteristics of the tangible and intangible investments, we construct and estimate four q-theory models: a q-theory model with only tangible assets (henceforth the Qm model), an IAA q-theory model with the ISTC effect (henceforth the Qu_ISTC model), an IAA q-theory model with the AC effect (henceforth the Qu_AC model), and an IAA q-theory model with both the ISTC effect and the AC effect (henceforth the Qu model). The parsimonious Qm model is the same as in LWZ and is used as the benchmark model. Because LWZ use quadratic adjustment costs for tangible investments, for comparison reason, we set ρ to be 2 for all four models. There are two parameters left to be estimated for the Qm model: the capital-to-output share, α, and the tangible investment adjustment cost parameter, a. For the ISTC effect, we set the curvature ξ to be 1/2 and normalize a_1 to one in order to focus on the magnitude parameter of the effect, a_2. Therefore, for the *Qu_ISTC* model, we add one more parameter: a_2. For the *Qu_AC* model, we add two parameters: b and ψ. Finally, there are 5 parameters to be estimated for the *Qu* model: α, a, a_2, b, and ψ.

Following Liu, Whited, and Zhang (2009), we test the ex-ante restriction implied by equation (7): expected stock returns equal expected levered investment return,

$$\mathbb{E}\left[r^S_{it+1} - r^{Iw}_{it+1}\right] = 0\,,$$

for testing portfolio i. Define the pricing error e_i from the above moment condition as

$$e_i = \mathbb{E}_T\left[r^S_{it+1} - r^{Iw}_{it+1}\right]\,, \tag{11}$$

where r^S_{it+1} is the observed stock return of portfolio i at $t+1$, r^{Iw}_{it+1} is the corresponding model-implied levered investment return, constructed from firm characteristics using equations (6) and (7), and $\mathbb{E}_T$ is the sample mean of the time series in the bracket. Both measurement errors and model specification errors contribute to the pricing error e_i, which is assumed to have a mean of zero.

We use one-stage GMM with identity weighting matrix to estimate the aforementioned four models. It has been shown that the efficient two-stage GMM estimator has poorer small-sample properties than the estimator using one-stage GMM with identify matrix.[3] Because we use annual data on firm characteristics and our sample only starts in 1975 for reasons detailed next, we end up with a fairly small data sample and decide to use the more robust, albeit less efficient, one-stage GMM estimation. Consequently, the corresponding set of parameter estimates is chosen to minimize the equal-weighted pricing errors of each set of the testing portfolios. To be consistent

[3]See Hayashi (2000) page 215 for detailed discussions regarding the small-sample properties of GMM estimators.

with their economic meanings, the parameters are estimated within the following ranges: $0 < \alpha < 1$, $a \geq 0$, $a_2 \geq 0$, $b \geq 0$ and $\psi \geq 0$.

3.2 Data

We obtain the firm characteristics data from Standard and Poor's COMPUSTAT industrial files and the stock return data from the Center for Research in Security Prices (CRSP). The accounting treatment of the R&D expenditure only became standard after FASB issued SFAS No. 2 in 1974 to require the full expending of R&D outlays in financial reports of public firms. To reduce possible measurement errors, we choose our sample from 1975 to 2008. Following the literature, we exclude the financial firms (SIC code between 6000 and 6999) and regulated utilities (SIC code between 4900 and 4999) from the sample. As being explained in details later, because we construct the level of intangible assets from past R&D expenditures, only firm-year observations with positive R&D are included in our sample. Specific definitions of the data items we use can be found in Appendix B.

We use three sets of testing portfolios: ten book-to-market portfolios, ten R&D-to-intangible-asset portfolios, and ten R&D-to-market-equity portfolios. We choose portfolios that are likely to show a significant cross-portfolio spread of intangible assets because tests based on these portfolios are likely to be more powerful in identifying the effects of intangible assets in the IAA q-theory models.

Book-to-market portfolios are the natural choices for testing portfolios because the book-to-market ratio (henceforth B/M ratio) reflects not only the rent due to imperfect competition but also the value of intangible assets, with the later becoming more and more important in the last twenty years. Portfolios sorted on the R&D intensity ratio, by construction, have large spreads on $R\&D$, thus large spreads on the level of intangible assets, and are also used as our testing portfolios. We use R&D-to-market-equity ratios (henceforth I^u/ME ratio, where ME stands for market equity) as another sorting variable, following CLS.

The third sorting variable is the R&D-to-intangible-assets ratio (henceforth I^u/K_0^u ratio, where K_0^u is our proxy for intangible asset and will be defined later), which is our measure of R&D intensity. Our model implies that the R&D-to-intangible-asset ratio plays a similar role as the investment-to-tangible-asset ratio, commonly used as a measure of investment intensity, in the relation with stock return, especially when the ISTC effect is small. In the extreme case when the ISTC effect is absent, if we exchange the positions of the tangible investment and intangible investment and the positions of the tangible asset and intangible asset in equation (6), the equation stays the same. Based on this observation, the R&D-to-intangible-asset ratio should be the comparable measure of R&D intensity to the measure of investment intensity.

B/M portfolios. – The construction of the ten book-to-market (B/M) portfolios follows Fama and French (1993). In June of year t, we sort all the stocks into ten portfolios by their book-equity-to-market-equity ratios. Book value of equity is measured at the fiscal year ending in calendar year $t-1$ and market value of equity is measured in December of calendar year $t-1$. When forming portfolios, we select only firm-year observations that have positive total asset, positive sales, non-negative market value of debt, positive market value of asset, and positive capital stock at the most recent fiscal year end, and have been in Compustat for five years.[4] The breakpoints are based on the NYSE firms only. We hold the equal-weighted portfolios from July to next June and record the buy-and-hold annual returns.

I^u/ME portfolios. – The ten R&D-to-market-equity (I^u/ME) portfolios are constructed in a similar manner. In June of year t, we sort stocks into ten portfolios based on the I^u/ME ratio and hold the portfolios for a year. The numerator I^u is proxied by R&D expenditure, measured at the end of the fiscal year ending in calendar year $t-1$. The denominator ME is market level of equity, measured at the beginning of the same fiscal year. The portfolios are rebalanced every year.

[4] We need five years data to calculate the level of intangible assets.

I^u/K^u_0 portfolios. – To form the ten R&D-to-intangible-asset (I^u/K^u_0) portfolios, we need a proxy, labeled as K^u_0, for the level of intangible assets, which in theory depends on the magnitude of the ISTC effect that needs to be estimated and on the depreciation rate of intangible assets. To construct the proxy K^u_0, we ignore the ISTC effect, which leads to underestimation of intangible assets, and following CLS and Summers (1981), we use a depreciation rate of 20%. The proxy for intangible assets at the beginning of fiscal year $t-1$ is given by

$$K^u_{0,t-1} = R\&D_{t-2} + 0.8R\&D_{t-3} + 0.8^2R\&D_{t-4} + 0.8^3R\&D_{t-5} + 0.8^4R\&D_{t-6}\,.$$

The variable K^u_0 incorporates only the intangible investments made in the most recent five years. Lev and Sougiannis (1996) estimate the impact of the current and past R&D expenses on earnings. They show that the horizon of the impact varies across industries from five years to nine years. We take a low end of five years in order to keep as many observations as possible. This assumption also leads to underestimation of intangible assets.

Note that K^u_0 is different from the level of intangible assets, K^u, that is used in equation (6) to construct levered stock returns because K^u incorporates the ISTC effect. To construct K^u, we also uses the most recent five years' R&D expenditures and a depreciation rate of 20%. Specifically, the value of K^u_t is calculated by applying equation (2) recursively, using the R&D expenditure starting from time $t-5$ to t and assuming that K^u_{t-5} is zero. Because the value of a_2 varies across different models, the level of intangible assets for each firm is model-dependent.

The timing alignment between the accounting variables used in the L.H.S. of equation (6) and the return in the R.H.S. is the same as the one used in Liu, Whited, and Zhang (2009). In general, the flow variables reflecting the economic activities over one fiscal year are measured at the end of the fiscal year while the stock variables, such as K^m_{it} and K^u_{it}, are measured at the beginning of the fiscal year. The detailed description on the timing alignment can be found in Appendix C of Liu, Whited, and Zhang (2009).

Finally, since we need five years' data to construct K_0^u and K^u, our portfolio formation starts in June, 1980 and ends in June, 2007.

3.3 Summary statistics on portfolio returns

Table 2 reports summary statistics for the returns for all three groups of testing portfolios. The results for the B/M, I^u/K_0^u, and I^u/ME portfolios are shown in Panels A, B, and C, respectively. We report the means of the portfolio returns and the model errors (the intercepts) of the CAPM model and the Fama-French 3-factor model with their corresponding t-statistics.

B/M portfolios. – Consistent with previous studies, the annual return is monotonically increasing with the B/M ratio. The value premium (i.e., the annual buy-and-hold return spread between the firms with the highest B/M and the firms with the lowest B/M firms) is 15.37% per annum.[5] Neither the CAPM model nor the Fama-French 3-factor model can capture the value premium. The pricing errors of both models are significantly different from zero.

I^u/K_0^u portfolios. – The portfolio return decreases with the I^u/K_0^u ratio. As argued previously, our model implies that the I^u/K_0^u ratio is the measure of R&D intensity that is comparable to investment intensity. Our results show that similar to investment intensity, R&D intensity has a negative relation with stock returns, opposite to what the previous literature documents. The average annual return spread between the firms with the highest I^u/K_0^u ratio and the ones with the lowest I^u/K_0^u ratio is −10.18% per annum ($t = -4.03$). The CAPM alpha of the high-minus-low zero-investment portfolio is −9.46% per annum ($t = -3.39$) and the Fama-French alpha is −7.33% per annum ($t = -2.34$).

I^u/ME portfolios. – The portfolio return is increasing with the I^u/ME ratio, consistent with what CLS document. The average return of the high-minus-low zero-investment portfolio is 24.38% per annum ($t = 2.81$). The CAPM alpha is 21.34% per annum ($t = 2.23$) and the

[5]This magnitude is larger than the ones reported in other studies because we use buy-and-hold compound annual return, while most of the other studies report monthly return.

Fama-French model alpha is 31.20% per annum ($t = 3.98$). Previous literature concludes that intangible investment and tangible investment have opposite relations with stock returns based on this measure. However, with market value of equity as denominator, this measure of R&D intensity is likely to also reflect the value effect and the leverage effect and fail to provide a clear indication on the relation between R&D intensity and stock returns.

In summary, all three sets of testing portfolios have large cross-portfolio return spreads, which cannot be explained by either the CAPM model or the Fama-French 3-factor model. Both the book-to-market ratio and the R&D-to-market-equity ratio have positive relations with stock returns, while the R&D-to-intangible-assets ratio has a negative relation with stock returns. Going forward, we refer to the R&D-to-intangible-assets ratio (I^u/K_0^u) as R&D intensity.

3.4 Summary statistics on portfolio characteristics

Table 3 reports the summary statistics for the B/M, I^u/K_0^u, and I^u/ME portfolios in Panels A, B, and C, respectively, on the following portfolio characteristics: current and future investment-to-capital ratios (investment intensity), growth rate of investment intensity, current and future R&D-to-intangible-assets ratios (R&D intensity), growth rate of R&D intensity, sales-to-capital ratio, depreciation rate, market leverage, intangible-assets-to-capital ratio, and annual corporate bond return.

B/M portfolios. – Firms with higher book-to-market ratios (i.e., value firms) have lower values of investment intensity, R&D intensity, growth rate of R&D intensity, sales-to-assets ratio, and intangible-assets-to-tangible-assets ratio but higher values of leverage ratio, compared to firms with lower book-to-market ratios (i.e., growth firms). It suggests that value firms invest less in both tangible and intangible assets, grow less, have lower productivity, accumulate less intangible assets relative to tangible assets, and borrow more, relative to growth firms. All of the above differences are statistically significant. We do not find significant cross-portfolio differences in the

growth rates of investment intensity.

I^u/K_0^u portfolios. – Firms with high R&D intensity tend to have higher values of investment intensity, sales-to-assets ratios, and intangible-assets-to-tangible-assets ratio but lower values of growth rate of R&D intensity and leverage ratio, compared to firms with low R&D intensity. There is no clear pattern in the growth rate of investment intensity across the ten portfolios. The positive correlation between the physical investment intensity and R&D intensity suggests that both the tangible and intangible investment decisions might be driven by the same economic forces.

I^u/ME portfolios. – Different from the ten I^u/K_0^u portfolios, the I^u/ME portfolios do not show clear patterns in any firm characteristics except that the intangible-assets-to-tangible-assets ratio monotonically increases with the I^u/ME ratio. Even though firms with the highest I^u/ME ratios have lower values of growth rate of R&D intensity and higher values of investment intensity and sales-to-assets ratios, compared to firms with the lowest I^u/ME ratios, the differences in these characteristics are not monotonic across all ten portfolios. Across the ten portfolios, higher I^u/ME ratios are generally associated with lower R&D-to-intangible-assets ratios, which explains why their relations with stock returns are in the opposite directions.

To summarize, we observe significantly large spreads on intangible assets related portfolio characteristics across the B/M portfolios, which underscores the important role of intangible assets in capturing the value premium. Moreover, we find that firms' investment decisions on intangible assets are positively correlated with those on tangible assets. Next, we turn to the structural estimation of the aforementioned four q-theory models.

3.5 Parameter Estimates and Model Performance

We estimate each of the four models, Qm, Qu_ISTC, Qu_AC, and Qu, using all three groups of testing portfolios: the B/M, I^u/K_0^u, and I^u/ME portfolios. In addition to the parameter estimates, Table 4 also reports two measures of overall model performance: the average absolute pricing error (a.a.p.e.) across time and across portfolio, and the statistics of the χ^2 test. The economic meaning of pricing errors is analogous to the alphas in the factor model regressions, representing the part of portfolio returns unexplained by the model. The pricing errors of individual portfolios are reported in Table 5. The χ^2 test is the model overidentification test and constructed following Hansen (1982, Lemma 4.1.), with null hypothesis that the pricing errors are jointly zero.

We conduct two statistical tests to compare the model performance: the Wald test for the nested models:[6] Qu, Qu_ISTC, and Qu_AC, and the λ test developed by Singleton (1985) for the non-nested models: Qm and Qu. The null hypothesis of the Wald test is that the restrictions on a nested model are jointly satisfied. Applying the Wald test on the Qu_ISTC model and the Qu model, we jointly test the null hypothesis: $b = 0$ and $\psi = 0$ (i.e., the AC effect is not present in the data). Similarly, the Wald test between the Qu_AC model and the Qu model has the null hypothesis: $a_2 = 0$ (i.e., the ISTC effect is not present in the data). The p-values of the Wald test are reported in Table 4 Panels A, B, and C for the B/M, I^u/K_0^u, and I^u/ME models, respectively.

The Qm model and the Qu model are not nested because even under the restrictions: $a_2 = 0$, $b = 0$, and $\psi = 0$, the production function in the Qu model has intangible assets as an input, while the Qm model does not. Therefore, we apply the λ test developed by Singleton (1985) to compare the performance of the Qm model and the Qu model. For each set of testing portfolios, we calculate two statistics: $\lambda(Qm, Qu)$ and $\lambda(Qu, Qm)$.[7] If the Qm model is correctly specified, $\lambda(Qm, Qu)$ converges to a $\chi^2(1)$ distribution. On the other hand, if the Qu model is correctly specified, $\lambda(Qu, Qm)$ converges to a $\chi^2(1)$ distribution. The p-values of $\lambda(Qm, Qu)$ are reported

[6]We use the Wald test instead of the L test used in Whited and Wu (2006) for the nested models. The L test requires the weighting matrix to satisfy the efficiency condition. Because we use identity matrix as the weighting matrix in the GMM estimation, our estimator does not satisfy the efficiency condition. Hayashi (2000, page 223) provides detailed discussions on the differences between the Wald test and the test statistics by the LR principle, to which the L test belongs.

[7]Singleton (1985) Section 3 provides details on how to construct the λ statistic.

under the columns of Qm and the p-values of $\lambda(Qu, Qm)$ are reported under the columns of Qu in Table 4 Panels A, B, and C for the B/M, I^u/K_0^u, and I^u/ME models, respectively.

B/M portfolios. – The results from the Qm model are largely consistent with those reported in LWZ. Compared to the Qm model, the Qu model captures the value premium much better and reduces the a.a.p.e. from 3.88% to 1.36% per annum. As for individual portfolios, five out of ten portfolios have pricing errors less than 1% per annum and the largest pricing error is 3.30% under the Qu model. In contrast, the portfolio pricing errors from the Qm model range from -5.52% to 7.16%, as shown in Table 5 Panel A. The Qu model also gives the smallest pricing error for the high-minus-low portfolio among all four models. The χ^2 test cannot reject the hypothesis that the pricing errors of the ten B/M portfolios are jointly zero for none of the four q-theory models.

Figure **??** provides a visual representation of the model performance, plotting the predicted returns from each of the four models against the average realized returns for the ten B/M portfolios. The scatters from the Qm model and the Qu_ISTC model look almost identical, indicating little improvement by adding the ISTC effect of intangible assets to the Qm model. On the other hand, adding the AC effect of intangible assets greatly improves the performance of the Qm model. The scatters from the Qu_AC model are much more closely gathered around the 45-degree line. The scatters from the Qu model look almost identical to those from the Qu_AC model.

For the non-nested test, the p-value of $\lambda(Qm, Qu)$ approaches zero, rejecting the null hypothesis that Qm is the correct model specification at the 5% significance level. On the contrary, the p-value of $\lambda(Qu, Qm)$ is 0.98, failing to reject the null hypothesis that Qu is the correct model specification. We conclude that the Qu model fits the cross-sectional stock returns of the B/M portfolios significantly better than the Qm model.

The Wald test between the Qu_ISTC and the Qu model generates a p-value of 0.02, rejecting the null hypothesis that $b = 0$ and $\psi = 0$ at the 5% significance level. On the contrary, the p-value of the Wald test between the Qu_AC model and the Qu model approaches one, failing to reject the null hypothesis that $a_2 = 0$. The Wald tests indicate that the AC effect of intangible assets is crucial for the Qu model to capture the cross-sectional return spreads among the ten B/M portfolios, while the ISTC effect is not. Consistently, a_2 is estimated to be zero under the Qu model, which confirms the non-existence of the ISTC effect.

The Qu model gives a lower estimate of a than the Qm model does, 1.21 vs. 43.59, and a lower estimate of α, 0.40 vs. 0.77. Moreover, the Qu model estimates b to be 24.69, much larger than its estimate of a. The curvature of the adjustment costs of intangible investment ψ is 1.37. Note that the t-statistics of the parameter estimates are generally insignificant except for the capital-to-out ratio α. We suspect that the low statistical power is due to the small size of our data sample.

I^u/K_0^u *portfolios.* – The Qu model has an a.a.p.e. of 0.49, much smaller compared to the a.a.p.e. of 1.93 from the Qm model. Under the Qu model, the individual portfolio pricing errors are all below 1% per annum, while the pricing errors range from 0.44% to 6.12% under the Qm model as shown in Table 5, Panel B. Again, the χ^2 test cannot reject the hypothesis that the pricing errors of the ten I^u/K_0^u portfolios are jointly zero for none of the four q-theory models.

Figure **??** visualizes the performances of the four q-theory models. The scatters from both the Qu_ISTC model and the Qu_AC model are more concentrated around the 45-degree line than those of the Qm model, implying that both the ISTC and the AC effects improve the model performance. The scatters from the Qu model line up along the 45-degree line almost perfectly and exhibit the best model fit.

The results of the non-nested test for the I^u/K_0^u portfolios are identical to those for the B/M portfolios. The p-value of $\lambda(Qm, Qu)$ approaches zero, rejecting the null hypothesis that Qm is the correct model specification at the 5% significance level; the p-value of $\lambda(Qu, Qm)$ is 0.98, failing to reject the null hypothesis that Qu is the correct model specification. Therefore, the Qu model fits the cross-sectional stock returns of the I^u/K_0^u portfolios significantly better than the Qm model.

The Wald test between the Qu_ISTC model and the Qu model and the Wald test between the Qu_AC model and the Qu both have a p-value close to zero, indicating that both the ISTC effect and the AC effect are crucial to the improved model performance of the Qu model, compared to the Qm model.

The parameter estimates show similar patterns as what we see from the estimation of the B/M portfolios. The estimates of a and α from the Qu model are 7.05 and 0.28, respectively, smaller than the ones from the Qm model, 13.31 and 0.35. The Qu model again estimates a larger magnitude of b, 27.76, than its estimate of a. The curvature of adjustment costs ψ for intangible investments is estimated to be 1.64.

I^u/ME portfolios. – The Qu model has an a.a.p.e. of 0.78, much smaller than the a.a.p.e. of 3.19 from the Qm model. Panel C of Table 5 reports the individual portfolio pricing errors for the ten I^u/ME portfolios. The pricing errors under the Qu model range from 0.01% to 2.11% per annum, compared to the range of 0.21% to 7.05% under the Qm model. Moreover, the pricing error of the high-minus-low portfolio is 0.24%, the smallest among all four models. Same as what we find with the other two sets of testing portfolios, the χ^2 test cannot reject the hypothesis that the pricing errors of the ten I^u/ME portfolios are jointly zero for none of the four q-theory models.

The scatter plots in Figure **??** confirm that the Qu model gives the best fit among all. There is noticeable improvement in the model fit between the Qu_ISTC model and the Qu model and between the Qu_AC model and the Qu model, implying that both the ISTC and AC effects are important to the improved model performance of Qu.

For the λ test between the Qm and the Qu model, the p-value of $\lambda(Qm, Qu)$ is 0.05, rejecting the null hypothesis that Qm is the correct model specification at the 5% significance level; the p-value of $\lambda(Qu, Qm)$ approaches one, failing to reject the null hypothesis that Qu is the correct model specification. Therefore, the Qu model matches the return spreads of the I^u/ME portfolios significantly better than the Qm model. The Wald test between the Qu model and the Qu_ISTC and the one between the Qu and the Qu_AC model both have a p-value of zero, indicating that both the ISTC and the AC effects are crucial to the improved performance.

Consistent with what we find with the B/M portfolios and the I^u/K_0^u portfolios, the Qu model always gives smaller estimates of a and α than the Qm model, 2.71 vs. 69.34 for a and 0.14 vs. 1.00 for α. The magnitude of b estimated from the Qu model is 56.20, much larger than that of a. The curvature ψ is estimated to be 0.59.

In summary, there are several patterns that arise after comparing different models. First, all of the four q-theory models generally capture the cross-sectional stock returns pretty well. The χ^2 tests fail to reject that the pricing errors are jointly zero for neither model, using all three sets of testing portfolios. Second, the λ tests show that adding intangible assets to the conventional q-theory model significantly improves the model performance across all three sets of testing portfolios. Based on the Wald tests, both the ISTC and AC effects of intangible assets are crucial to the improved model performance. Third, the Qu model estimates a smaller value of a than the Qm model and the magnitude of b is larger than that of a, across all three sets of testing portfolios. Next, we calculate the average adjustment costs for both tangible and intangible investments based on the estimates of a, b, and ψ.

3.6 Magnitude of adjustment costs

It has been documented in the previous literature that the autocorrelation of R&D expenditure is much larger than that of the physical investment, e.g., Bloom (2007) among others. Similar patterns also show up in our sample. Table 6 shows that the autocorrelations of R&D scaled by total assets, PP&E, and sales are 0.81, 0.87, and 0.88, respectively, while they are 0.45, 0.43, and 0.55 for capital investments. The literature has been devoted to finding the economic reasons for the difference in persistence between tangible and intangible investments. In this subsection, we compare the magnitudes of adjustment costs of tangible and intangible investments implied from our model estimations, hoping to shed lights on this issue.

The AC/I ratio is commonly used as a measure of the magnitude of adjustment costs in the literature. Given the parameter estimates and the investment ratios, the average adjustment-cost-to-investment (AC/I) ratio across all testing portfolios can be calculated for the tangible investments as

$$\frac{AC^m}{I^m} = \frac{1}{10}\sum_{i=1}^{10}\frac{\hat{a}}{2}\overline{\left(\frac{I_i^m}{K_i^m}\right)},$$

and for intangible investments as

$$\frac{AC^u}{I^u} = \frac{1}{10}\sum_{i=1}^{10}\frac{\hat{b}}{2}\overline{\left(\frac{I_i^u}{K_i^u}\right)^{\psi-1}},$$

where i stands for portfolio i, $\hat{a}$ and $\hat{b}$ are the estimated values for a and b, and the variables with an overline are the time-series average of the corresponding variables. Table 7 reports both the AC^m/I^m ratios and the AC^u/I^u ratios implied by all three sets of testing portfolios. For the AC^m/I^m ratios, we report the values implied by both the Qm model and the Qu model.

Based on the estimates from the Qu model, the AC^m/I^m ratios are 6.43%, 39.05%, and 14.17% for the B/M, the I^u/K_0^u, and the I^u/ME portfolios, respectively, averaging to 19.88%. For the Qm model, those numbers are 232%, 73.7%, and 363%, averaging to 223%. The estimates of the AC^m/I^m ratio from Summers (1981), Litchenberg (1988), and Cooper and Haltiwanger (2006) are similar in magnitude, being 22.1%, 20%, and 33.09%, respectively.[8] The estimate from Hall (2004) is even smaller and close to zero. Therefore, the IAA q-theory gives a more reasonable estimate of adjustment costs than the conventional q-theory.

For intangible investments, the AC^u/I^u ratios are 839%, 589%, and 5,708% for the B/M portfolios, the I^u/K_0^u portfolios, and the I^u/ME portfolios, respectively, averaging to 2,379%. Across

[8]Cooper and Haltiwanger (2006) consider both the fixed and the quadratic parts of the adjustment costs. With only the quadratic adjustment costs, the AC^m/I^m ratio is 2.78%.

all testing portfolios, the AC^u/I^u ratio is consistently larger than the AC^m/I^m ratio , implying that it is more costly to rapidly accumulate intangible assets than tangible assets. This finding confirms the conventional wisdom that the comparative advantage due to intangible assets is easier to sustain than the one due to tangible assets. It is hence important for firms to consistently invest in intangible assets, which provides a possible explanation for the higher autocorrelation of R&D expenditures than that of capital investments observed in the data.

Even though it is consistent with the economic intuition and the aforementioned empirical facts to have a larger magnitude of adjustment costs for intangible investments than that for tangible investments, our model implied AC^u/I^u ratios seem fairly large. We suspect the following reasons that could lead to over-estimation of the adjustment costs of intangible investments: (1) the assumption that R&D expenditures older than 5 years do not contribute to the current intangible assets; (2) the omission of other types of intangible assets, especially human capital, which presumably can be quite large in magnitude. Both assumptions lead to under-estimation of the level of intangible assets, which gives a higher R&D-to-intangible-assets ratio and hence a higher AC^u/I^u ratio.

In summary, the IAA q-theory gives a more reasonable estimate of adjustment costs for tangible investments than the conventional q-theory does. The AC^m/I^m ratio from the Qu model is within the range of what previous studies find using different methodologies and data samples. Moreover, the AC^u/I^u ratio is estimated to be larger than the AC^m/I^m ratio.

Notice that the estimated model parameters and hence the average adjustment costs vary across different sets of testing portfolios, even though the portfolios are constructed using the same set of firms. In theory, every firm is different and the model parameters should vary across each individual firm. When grouping firms into portfolios and estimate the parameters at the portfolio level,[9] we treat each portfolio as a "representative" for firms in that portfolio and estimating the

[9]Using the current methodology, we are not be able to estimate the parameters at the firm level because many firms do not have long enough time series of data. Moreover, one motivation of the paper is to investigate the

model parameters that best describe the behaviors of the ten representative firms. With different sets of testing portfolios, we group firms differently and end up with representative firms with different characteristics, as shown in Table 3. Therefore, the parameter estimates based on those representative firms inevitably vary across the three sets of testing portfolios.

3.7 Comparative statics analysis

Equations (6) and (10) show the relation between a firm's realized stock returns and its observable characteristics as the results of shareholder value maximization. Even though we cannot argue causality between realized stock returns and firm characteristics based on this relation, it is interesting to see how important the cross-sectional variation of a specific characteristic, particularly the ones related to intangible assets, is for the model to match cross-sectional return spreads.

We conduct the following comparative static analysis. We reconstruct the predicted stock returns using the same parameter estimates from the Qu model, but make one change: for a given characteristic at a given year, we use its cross-sectional average at that year in equation (6) while keeping other characteristics unchanged. We then calculate the a.a.p.e. based on the reconstructed stock returns and measure the degree of performance deterioration based on the increase in a.a.p.e. relative to the corresponding one reported in Table 4. The more crucial a certain characteristic is to matching the cross-sectional return spreads, the greater the increase in a.a.p.e. we should observe. Table 8 reports the results of our comparative statics analysis on five characteristics: the tangible investment ratio I^m/K^m, the intangible investment ratio I^u/K^u, the intangible-to-tangible-assets ratio K^u/K, the sales-to-assets ratio Y/K^m and the leverage w, for all three sets of testing portfolios.

For the ten B/M portfolios, the most important characteristics are I^u/K^u and K^u/K^m. Taking away the cross-sectional variations of I^u/K^u and K^u/K^m increases the a.a.p.e. of the Qu model

ability of the IAA q-theory in capturing the value premium, which is a portfolio-level phenomena.

from 1.36% to 6.15% and 6.28%, respectively. This result confirms our previous estimation analysis that adding intangible assets to the conventional q-theory is important in capturing the value premium. Leverage w is the third important component. Without the variation in leverage, the a.a.p.e. increases to 3.31%. Y/K^m and I^m/K^m are less important compared to the other three characteristics, which explains why the conventional q-theory does a poor job in matching the value premium.

For the I^u/K_0^u portfolios, the most important characteristic is K^u/K^m, which leads to an increase in a.a.p.e. from 0.49% to 2.97%. I^u/K^u, I^m/K^m, and w are equally important, resulting in an increase of a.a.p.e. from 0.49% to 1.88%, 2.07%, and 1.83%, respectively. Y/K^m is the least important one, raising the a.a.p.e. to 1.58%.

For the I^u/ME portfolios, the most crucial characteristic is the leverage w. Without the variation in leverage, the a.a.p.e. of the Qu model increases from 0.78% to 5.85%. The importance of Y/K^m, K^u/K^m, and I^u/K^u follows that of leverage, raising the a.a.p.e. to 3.60%, 3.25%, and 3.24%, respectively. I^m/K^m is the least important one, raising the a.a.p.e. to 1.55%. The fact that leverage w, instead of the characteristics related to intangible assets, is the most important characteristic in matching the return spreads across the I^u/ME portfolios is consistent with our previous conjecture that using I^u/ME as the sorting variable may capture the leverage effect, in addition to the effect from R&D investment.

4 Conclusion

Intangible assets have become increasingly important for firm's survival and prosperity since the 1980s. The literature has emphasized two important features of intangible assets. One feature is the adjustment cost of intangible assets; the other feature is the investment-specifical technologic change. In this paper, we examine what the impacts of intangible assets are on asset returns and through which channels, based on structural estimations of four q-theory models. Moreover, we quantify the magnitude of the adjustment costs of both tangible and intangible investments.

The summary of our findings is as follows. First, the Qu model explains all three sets of testing portfolios: ten book-to-market portfolios, ten R&D-to-intangible-asset portfolios, and ten R&D-to-market-equity portfolios, significantly better than the Qm model. Second, both the AC effect and the ISTC effect are critical to the improved performance of the IAA q-theory. Third, incorporating intangible assets in a q-theory model generates a more reasonable magnitude of tangible investment adjustment costs than the conventional q-theory model. Fourth, the magnitude of adjustment costs of intangible investments is much larger than that of tangible investments. This finding provides empirical evidence for the conventional wisdom that intangible assets are more critical for a firm to sustain its comparative advantage and helps to explain the higher autocorrelation of R&D expenditures than that of capital investments observed in the data.

Last, but not least, we document that the R&D intensity, when measured as the I^u/K^u ratio, is negatively related to stock returns, which resembles the relation between stock returns and physical investment intensity, measured as the I/K ratio. This finding is opposite to the perception in the literature that the R&D intensity is positively related to stock returns. We argue that the R&D-to-market-equity ratio is not a good measure of R&D intensity because it likely reflects the value effect and the leverage effect.

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Appendix

A Proof of Proposition 1

Shareholder's maximization problem can be written as

$$P_t \equiv P(K_t^m, K_t^u, B_t, X_t) = \max_{\{I_t^m, I_t^u, K_{t+1}^m, K_{t+1}^u, L_t\}} \left\{ D_t^S + \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} P_{t+1} \right] \right\} , \tag{12}$$

where P_t is the cum-dividend equity value of the firm, D_t^S is the cash flow to shareholders at time t, M_{t+1} is the stochastic discount factor, and I_{t+1} is the default indicator, which equals 1 if the firm is solvent at time $t+1$ and 0 otherwise. If D_t^S is positive, the firm pays out dividends; if D_t^S is negative, the firm issues equity. D_t^S can be written as

$$D_t^S = (1-\tau_t)\left[Y_t - \varpi_t L_t - I_t^u - \Phi_t^m - \Phi_t^u\right] - I_t^m + \tau_t \delta_m K_t^m + B_{t+1} - \left[r_t^B - (r_t^B - 1)\tau_t\right] B_t \,,$$

and the maximization is subject to

$$q_t^u: \quad K_{t+1}^u = (1-\delta_u)K_t^u + I_t^u \tag{13}$$

$$q_t^m: \quad K_{t+1}^m = (1-\delta_m)K_t^m + \Theta\left(I_t^m, K_t^u\right) , \tag{14}$$

where r_t^B is the gross required return on debt, B_t is the-beginning-of-the-period debt outstanding at time t, and B_{t+1} is the-end-of-the-period debt outstanding at time t. The lagranian multipliers, q_t^u and q_t^m, can be interpreted as the shadow prices for tangible and intangible assets at time t, respectively. Since firm is a price taker in the input market, input price w_t is exogenously given.

Lemma 1. *Ex-dividend firm value V_t is given by*

$$V_t \equiv P_t - D_t^S + B_{t+1} = q_t^u K_{t+1}^u + q_t^m K_{t+1}^m \tag{15}$$

Proof: The first order conditions of shareholder's maximization problem are

$$I_t^u : \qquad \Theta_{I,t} q_t^u = (1-\tau_t)\left(1+\Phi_{I,t}^u\right) \tag{16}$$

$$I_t^m : \qquad q_t^m = 1 + (1-\tau_t)\Phi_{I,t}^m \tag{17}$$

$$L_t : \qquad \varpi_t = Y_{L,t}$$

$$K_{t+1}^u : \qquad q_t^u = \mathbb{E}_t\left[M_{t+1}\mathbb{I}_{t+1}V_{K^u,t+1}\right] \tag{18}$$

$$K_{t+1}^m : \qquad q_t^m = \mathbb{E}_t\left[M_{t+1}\mathbb{I}_{t+1}V_{K^m,t+1}\right]$$

$$B_{t+1} : \qquad 1 = \mathbb{E}_t\left[M_{t+1}\mathbb{I}_{t+1}\left[r_{t+1}^B - (r_{t+1}^B - 1)\tau_{t+1}\right]\right] ,$$

where $V_{K^u,t+1}$ is the derivative of the value function w.r.t. K_{t+1}^u and $V_{K^m,t+1}$ and $Y_{L,t}$ are defined similarly.

Equation (18) can be proved as follows. The continuation value of equity can be written as an integration over the possible realization of the productivity shock as

$$\mathbb{E}_t\left[M_{t+1}\mathbb{I}_{t+1}V_{t+1}\right] = \int_{\underline{X}_{t+1}}^{\infty} M_{t+1}V_{t+1}f(X_{t+1})dX_{t+1} ,$$

where $\underline{X}$ is the lower bound on the productivity shock, at which the firm defaults (i.e., $V_{t+1}(\underline{X}) = 0$). $\underline{X}$ is a function of the firm's state variables at $t+1$: $\{K_{t+1}^m, K_{t+1}^u, B_{t+1}, X_{t+1}\}$. The derivative of the continuation can be written as

$$
\begin{aligned}
\frac{\partial \mathbb{E}_t\left[M_{t+1}\mathbb{I}_{t+1}V_{t+1}\right]}{\partial K^u_{t+1}} &= \int_{\underline{X}}^{\infty} M_{t+1}V_{K^u,t+1}f(X_{t+1})dX_{t+1} + M_{t+1}(\underline{X})V_{t+1}(\underline{X})f(\underline{X})\left[\frac{\partial \underline{X}}{\partial K^u_{t+1}}\right] \\
&= \int_{\underline{X}}^{\infty} M_{t+1}V_{K,t+1}f(X_{t+1})dX_{t+1} \\
&= \mathbb{E}_t\left[M_{t+1}\mathbb{I}_{t+1}V_{K^u,t+1}\right].
\end{aligned}
$$

Similarly, we can prove the FOC's w.r.t K^m_{t+1} and B_{t+1}.

It's straightforward to show that the adjustment function, production function of new tangible assets, and the production function satisfy constant-return-to-scale, that is,

$$
\begin{aligned}
\Phi^m_t &= \Phi_{K^m,t}K^m_t + \Phi_{I,t}I^m_t \\
\Phi^u_t &= \Phi_{K^u,t}K^u_t + \Phi_{I,t}I^u_t \\
\Theta_t &= \Theta_{K^u,t}K^u_t + \Theta_{I,t}I^m_t \\
Y_t &= Y_{K^m,t}K^m_t + Y_{K^u,t}K^u_t + Y_{L,t}L_t\,.
\end{aligned}
$$

The derivatives of the investment adjustment costs w.r.t. the investments and the asset levels, both tangible and intangible, are given by

$$
\begin{aligned}
\Phi^m_{I,t} &= \frac{a\rho}{2}\left(\frac{I^m_t}{K^m_t}\right)^{\rho-1} \\
\Phi^m_{K,t} &= \frac{a(1-\rho)}{2}\left(\frac{I^m_t}{K^m_t}\right)^{\rho} \\
\Phi^u_{I,t} &= \frac{b\psi}{2}\left(\frac{I^u_t}{K^u_t}\right)^{\psi-1} \\
\Phi^u_{K,t} &= \frac{b(1-\psi)}{2}\left(\frac{I^u_t}{K^u_t}\right)^{\psi},
\end{aligned}
$$

and the partial derivatives of the capital production function Θ w.r.t. investment and asset level are given by

$$
\begin{aligned}
\Theta_{I,t} &= a_1 \left[a_1 + a_2 \left(\frac{I_t^u}{K_t^u} \right)^{-\xi} \right]^{\frac{1-\xi}{\xi}} \\
\Theta_{K,t} &= a_2 \left[a_1 \left(\frac{I_t^u}{K_t^u} \right)^{\xi} + a_2 \right]^{\frac{1-\xi}{\xi}} ,
\end{aligned}
$$

and the derivatives of the value function w.r.t. both tangible assets K_t^m and intangible assets K_t^u are given by

$$
\begin{aligned}
V_{K^m,t} &= (1-\tau_t) \left(Y_{K^m,t} - \Phi_{K,t+1}^m \right) + \tau_t \delta_m + q_t^m (1-\delta_m) \\
V_{K^u,t} &= (1-\tau_t) Y_{K^u,t} + q_t^u (1-\delta_u) + q_t^u \Theta_{K,t} \,.
\end{aligned}
$$

From the firs-order conditions, we can write the right hand side of equation (15) as

$$
q_t^u K_{t+1}^u + q_t^m K_{t+1}^m = \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} \left(V_{K^m,t+1} K_{t+1}^m + V_{K^u,t+1} K_{t+1}^u \right) \right] .
$$

Define a function Ω_{t+1} as

$$
\begin{aligned}
\Omega_{t+1} &= V_{K^m,t+1} K_{t+1}^m + V_{K^u,t+1} K_{t+1}^u \\
&= \left[(1-\tau_t) \left(Y_{K^m,t} - \Phi_{K,t+1}^m \right) + \tau_t \delta_m + q_t^m (1-\delta_m) \right] K_{t+1}^m \\
&\quad + \left[(1-\tau_t) Y_{K^u,t} + q_t^u (1-\delta_u) + q_t^u \Theta_{K,t} \right] K_{t+1}^u \,.
\end{aligned}
$$

Substituting the first-order conditions into the above equation and using the constant-return-to-scale property of the production function and the adjustment cost function, we get

$$\begin{aligned}
\Omega_{t+1} &= (1-\tau_{t+1})\left(Y_{K^m,t+1}K^m_{t+1} + Y_{K^u,t+1}K^u_{t+1} - \Phi^m_{K,t+1}K^m_{t+1} - \Phi^u_{K,t+1}K^u_{t+1}\right) + \tau_{t+1}\delta_m K^m_{t+1} \\
&\quad + q^m_{t+1}(1-\delta_m)K^m_{t+1} + q^u_{t+1}(1-\delta_u)K^u_{t+1} + q^m_{t+1}\Theta_{K,t+1}K^u_{t+1} \\
&= (1-\tau_{t+1})\left(Y_{K^m,t+1}K^m_{t+1} + Y_{K^u,t+1}K^u_{t+1} - \Phi^m_{K,t+1}K^m_{t+1} - \Phi^u_{K,t+1}K^u_{t+1}\right) + \tau_{t+1}\delta_m K^m_{t+1} \\
&\quad + q^m_{t+1}\left(K^m_{t+2} - i_{t+1}\right) + q^u_{t+1}\left(K^u_{t+2} - \Theta_{k,t+1}\right) + q^u_{t+1}\Theta_{K,t+1}K^u_{t+1} \\
&= (1-\tau_{t+1})\left(Y_{K^m,t+1}K^m_{t+1} + Y_{K^u,t+1}K^u_{t+1} - \Phi^m_{K,t+1}K^m_{t+1} - \Phi^u_{K,t+1}K^u_{t+1}\right) + \tau_{t+1}\delta_m K^m_{t+1} \\
&\quad - \left[1 + (1-\tau_{t+1})\Phi^m_{I,t+1}\right] I^m_{t+1} - \left[\frac{(1-\tau_{t+1})\left(1+\Phi^u_{I,t+1}\right)}{\Theta_{I,t+1}}\right]\left(\Theta_{t+1} - \Theta_{K,t+1}K^u_{t+1}\right) \\
&\quad + q^m_{t+1}K^m_{t+2} + q^u_{t+1}K^u_{t+2} \\
&= (1-\tau_{t+1})\left(Y_{K^m,t+1}K^m_{t+1} + Y_{K^u,t+1}K^u_{t+1} - \Phi^m_{K,t+1}K^m_{t+1} - \Phi^u_{K,t+1}K^u_{t+1} - \Phi^m_{I,t+1}I^m_{t+1} - \Phi^u_{I,t+1}I^u_{t+1}\right. \\
&\quad \left. - I^u_{t+1}\right) + \tau_{t+1}\delta_m K^m_{t+1} - I^m_{t+1} + q^m_{t+1}K^m_{t+2} + q^u_{t+1}K^u_{t+2} \\
&= (1-\tau_{t+1})\left(Y_{t+1} - w_{t+1}L_{t+1} - \Phi^m_{t+1} - \Phi^u_{t+1} - I^u_{t+1}\right) + \tau_{t+1}\delta_m K^m_{t+1} - I^m_{t+1} + q^m_{t+1}K^m_{t+2} + q^u_{t+1}K^u_{t+2} \\
&= D_{t+1} + q^m_{t+1}K^m_{t+2} + q^u_{t+1}K^u_{t+2}\,,
\end{aligned}$$

where D_{t+1} is the free cash flow of the firm at time $t+1$ and defined as

$$D_{t+1} = (1-\tau_{t+1})\left(Y_{t+1} - \varpi_{t+1}L_{t+1} - \Phi^m_{t+1} - \Phi^u_{t+1} - I^u_{t+1}\right) + \tau_{t+1}\delta_m K^m_{t+1} - I^m_{t+1}\,.$$

Therefore, the right hand side of equation (15) can be written as

$$\begin{aligned}
q^m_t K^m_{t+1} + q^u_t K^u_{t+1} &= \mathbb{E}_t\left[M_{t+1}\mathbb{I}_{t+1}\left[D_{t+1} + q^m_{t+1}K^m_{t+2} + q^u_{t+1}K^u_{t+2}\right]\right] \\
&= \mathbb{E}_t\left[\sum_{s=0}^{\infty} M_{t+s}\mathbb{I}_{t+s}D_{t+s}\right]. \qquad (19)
\end{aligned}$$

Firm value V_t is the sum of ex-dividend equity value and debt value, that is,

$$\begin{aligned} V_t &= P_t - D_t^S + B_{t+1} \\ &= \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} \left\{ D_{t+1}^S + P_{t+1} + \left[r_{t+1}^B - r_{t+1}^B (1 - \tau_{t+1}) \right] B_{t+1} \right\} \right] , \end{aligned}$$

where the second equation is derived from the first order condition on the optimal debt issuance

$$B_{t+1} = \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} \left[r_{t+1}^B - r_{t+1}^B (1 - \tau_{t+1}) \right] B_{t+1} \right] .$$

It's straightforward to show that

$$D_{t+1}^S + \left[r_{t+1}^B - r_{t+1}^B (1 - \tau_{t+1}) \right] B_{t+1} = D_{t+1} + B_{t+2} \,.$$

Therefore, we have

$$V_t = \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} \left(D_{t+1} + B_{t+2} + P_{t+1} \right) \right] = \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} \left(D_{t+1} + V_{t+1} \right) \right] .$$

Iterating the above equation, we get

$$V_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t+s} \mathbb{I}_{t+s} D_{t+s} \right] ,$$

which, combined with equation (19), implies that

$$V_t = q_t^m K_{t+1}^m + q_t^u K_{t+1}^u \,.$$

Q.E.D.

Lemma 2. *Define firm's investment return as*

$$r_{t+1}^I = \frac{D_{t+1} + q_{t+1}^m K_{t+2}^m + q_{t+1}^u K_{t+2}^u}{q_t^m K_{t+1}^m + q_t^u K_{t+1}^u} \,,$$

and r_{t+1}^I *satisfies the following equations:*

$$\mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} r_{t+1}^I \right] = 1 \tag{20}$$

$$r_{t+1}^I = \varpi_t \, r_{t+1}^S + (1 - \varpi_t) \, r_{t+1}^{Ba} \,. \tag{21}$$

Proof: Equation (20) is straightforward to prove. From Lemma 1, we know that

$$q_t^m K_{t+1}^m + q_t^u K_{t+1}^u = \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} \left(D_{t+1} + q_{t+1}^m K_{t+2}^m + q_{t+1}^u K_{t+2}^u \right) \right] \,.$$

Dividing the right-hand side of the equation by the left-hand side, we get

$$1 = \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} \left(\frac{D_{t+1} + q_{t+1}^m K_{t+2}^m + q_{t+1}^u K_{t+2}^u}{q_t^m K_{t+1}^m + q_t^u K_{t+1}^u} \right) \right] = \mathbb{E}_t \left[M_{t+1} \mathbb{I}_{t+1} r_{t+1}^I \right] \,.$$

We prove equation (21) by three steps:

Step 1: We show that given the level of both tangible and intangible assets and under the optimal choice of non-capital input L_t^*, for each period t,

$$Y_t - \varpi_t L_t^* = \gamma Y_t \,.$$

At any given level of tangible and intangible assets, the optimal non-capital input L_t^* is given by the following maximization problem

$$\max_{\{L_t\}} \quad Y_t - \varpi_t L_t \,,$$

subject to the revenue function

$$Y_t = e^{X_t} \left[(K_t^m)^\alpha (K_t^u)^{1-\alpha} \right]^\gamma (L_t)^{1-\gamma} .$$

The FOC w.r.t. L_t gives

$$(1-\gamma) e^{X_t} \left[(K_t^m)^\alpha (K_t^u)^{1-\alpha} \right]^\gamma (L_t^*)^{-\gamma} = \varpi_t .$$

If we substitute the above equation into the revenue function, the revenue after the input cost can be written

$$\begin{aligned} Y_t - \varpi_t L_t^* &= L_t^* \left(e^{X_t} \left[(K_t^m)^\alpha (K_t^u)^{1-\alpha} \right]^\gamma (L_t^*)^{-\gamma} - \varpi_t \right) \\ &= \gamma Y_t , \end{aligned} \tag{22}$$

when the optimal hiring of input is L_t^*. *Step* 2: We show that

$$(1-w_t)\, r_{t+1}^S + w_t\, r_{t+1}^{Ba} = \frac{D_{t+1} + q_{t+1}^m K_{t+2}^m + q_{t+1}^u K_{t+2}^u}{q_t^m K_{t+1}^m + q_t^u K_{t+1}^u} .$$

From Lemma 1, we get

$$\begin{aligned} \frac{D_{t+1} + q_{t+1}^m K_{t+2}^m + q_{t+1}^u K_{t+2}^u}{q_t^m K_{t+1}^m + q_t^u K_{t+1}^u} &= \frac{D_{t+1}^S + \left[r_{t+1}^B - \tau_{t+1} \left(r_{t+1}^B - 1 \right) \right] B_{t+1} - B_{t+2} + V_{t+1}}{V_t} \\ &= \frac{D_{t+1}^S + r_{t+1}^{Ba} B_{t+1} - B_{t+2} + P_{t+1} - D_{t+1}^S + B_{t+2}}{V_t} \\ &= \frac{P_{t+1} + r_{t+1}^{Ba} B_{t+1}}{V_t} \\ &= (1-w_t)\, r_{t+1}^S + w_t\, r_{t+1}^{Ba} . \end{aligned}$$

Step 3: Substitute equations (16), (17), (22), and the accumulation rules of both tangible and intangible assets into equation (21) and divide both the denominator and the numerator by K_{t+1}^m.

It's straightforward to get equation (6).

From Step 2 and Step 3, we conclude that

$$r^I_{t+1} = (1 - w_t)\, r^S_{t+1} + w_t\, r^{Ba}_{t+1}\,.$$

Lemma 3. *Firm's investment return* r^I_{t+1} *is a value-weighted average of its investment return on tangible assets* $r^{I,m}_{t+1}$ *and investment return on intangible assets* $r^{I,u}_{t+1}$*, where*

$$\begin{aligned} r^{I,m}_{t+1} &= \frac{(1-\tau_{t+1})\left[\alpha\frac{\gamma Y_{t+1}}{K^m_{t+1}} - \Phi^m_{K,t+1}\right] + \tau_{t+1}\delta_m + (1-\delta_m)\left[1 + (1-\tau_{t+1})\Phi^m_{I,t+1}\right]}{1 + (1-\tau_t)\Phi^m_{I,t}} \\ r^{I,u}_{t+1} &= \left\{(1-\tau_{t+1})\left[\alpha\frac{(1-\gamma)Y_{t+1}}{K^u_{t+1}} - \Phi^u_{K,t+1}\right] + \frac{(1-\delta_u)(1-\tau_{t+1})\left(1+\Phi^u_{I,t+1}\right)}{\Theta_{I,t+1}}\right. \\ &\quad \left. + (1-\tau_{t+1})\left(1+\Phi^u_{I,t+1}\right)\left(\frac{\Theta_{K,t+1}}{\Theta_{I,t+1}}\right)\right\} \Big/ \left[(1-\tau_t)\left(1+\Phi^u_{I,t}\right)/\Theta^u_{I,t}\right] \end{aligned}$$

and the weights are the market value of tangible assets and intangible assets, respectively, given by

$$w^m_t = \frac{q^m_t K^m_t}{V_t} \qquad w^u_t = \frac{q^u_t K^u_t}{V_t}\,.$$

Proof: Since q^m_t is the shadow price of one unit of tangible assets at time t, the market value of firm's tangible assets is $q^m_t K^m_{t+1}$. Similarly, the market value of firm's intangible assets is $q^m_t K^m_{t+1}$. From Lemma 1, we know that $V_t = q^m_t K^m_{t+1} + q^m_t K^m_{t+1}$. Hence, the weights w^m_t and w^m_t add up to 1. From the FOCs of the shareholder's value maximization in the proof of Lemma 1, we have

$$\begin{aligned} q^m_t &= 1 + (1-\tau_t)\Phi^m_{I,t} \\ q^u_t &= \frac{(1+\tau_t)\left(1+\Phi^u_{I,t}\right)}{\Theta_{I,t}}\,. \end{aligned}$$

Plug in the above equations and it is straightforward to show that

$$r_{t+1}^{I} = w_t^m r_{t+1}^{I,m} + w_t^u r_{t+1}^{I,u} .$$

Q.E.D.

B Definitions and Sources of Data Items

We list the Compustat item names for the variables used in the data construction.

Book Equity: Common Equity (CEQ) + Balance Sheet Deferred Tax (TXDB)

Total Asset: Total Asset (AT)

Market Value of Debt: Long-Term Debt (DLTT) + Short-Term Debt (DLC)

Market Value of Asset: Long-Term Debt (DLTT) + Short-Term Debt (DTC) + Share Outstanding (CSHO) × Stock Price - Annual Fiscal Year (PRCC_F)

Capital Stock: Gross Property, Plant, and Equipment (PPEGT)

R&D Expenditure: Research and Development Expense (XRD)

Physical Investment: Capital Expenditure (CAPX) − Sales of Property, Plant, and Equipment (SPPE)

Output: Sales (SALE)

Depreciation Rate of Tangible Assets: mean of the Depreciation (DP) to Gross Property, Plant, and Equipment (PPEGT) ratios over the entire time series

Market Leverage: Book Value of Debt / Market Equity

Table 1: **Descriptive Statistics of Testing Portfolio Returns**

For each testing portfolio i, we report in annualized percentage the average stock return, $\bar{r}_i^S$, the intercept from the CAPM regression, e_i^{CAPM}, and the intercept from the Fama-French 3-factor regression, e_i^{FF}. The H−L portfolio is long in the high portfolio and short in the low portfolio. The t-statistics for the model errors are reported in brackets beneath the corresponding errors. a.a.p.e. is the average of the absolute values of the errors for a given set of ten testing portfolios. Panel A reports results for ten B/M portfolios, Panel B for I^u/K_0^u portfolios and Panel C for I^u/ME portfolios.

	$\bar{r}^S$	α^{CAPM}	α^{FF}	$\bar{r}^S$	α^{CAPM}	α^{FF}	$\bar{r}^S$	α^{CAPM}	α^{FF}
	Panel A: Ten B/M Portfolios			Panel B: Ten I^u/K_0^u Portfolios			Panel C: Ten I^u/ME Portfolios		
Low	9.35	-5.95	-2.84	23.37	6.44	7.48	9.12	-4.85	-7.92
		(-1.74)	(-1.31)		(1.05)	(2.13)		(-1.52)	(-3.05)
2	12.73	-0.79	0.33	21.98	8.06	7.50	12.66	-1.31	-2.26
		(-0.26)	(0.20)		(1.92)	(2.74)		(-0.54)	(-1.24)
3	14.11	-0.67	-1.19	20.54	6.33	4.40	14.29	0.53	-0.11
		(0.20)	(-0.66)		(1.91)	(2.08)		(0.19)	(-0.06)
4	16.91	3.59	0.19	17.64	4.34	3.70	15.01	1.22	0.62
		(1.11)	(0.11)		(1.47)	(2.15)		(0.37)	(0.32)
5	16.16	3.19	0.43	17.66	3.79	1.88	15.36	1.42	-0.05
		(1.03)	(0.23)		(1.15)	(0.88)		(0.44)	(-0.02)
6	18.17	4.97	0.63	18.82	4.77	4.21	19.73	5.01	7.54
		(1.54)	(0.36)		(1.57)	(2.09)		(1.17)	(3.09)
7	17.38	4.56	-0.09	19.27	4.88	5.80	19.69	5.33	7.45
		(1.24)	(-0.05)		(1.21)	(2.56)		(1.15)	(2.51)
8	19.63	7.42	2.47	17.54	2.91	5.34	20.12	3.91	5.04
		(2.22)	(1.26)		(0.67)	(2.21)		(0.77)	(1.53)
9	21.09	8.19	2.37	16.30	0.70	3.96	28.01	11.20	15.45
		(2.00)	(1.06)		(0.13)	(1.22)		(1.52)	(3.55)
High	24.72	10.73	3.45	13.19	-3.52	0.15	33.50	16.49	23.28
		(2.12)	(1.12)		(-0.56)	(0.05)		(1.63)	(3.62)
H-L	15.37	16.67	6.28	-10.18	-9.46	-7.33	24.38	21.34	31.20
		(3.61)	(1.81)		(-3.39)	(-2.34)		(2.23)	(3.98)
a.a.p.e		5.00	1.40		4.52	4.44		5.13	6.97

Table 2: **Descriptive Statistics of Testing Portfolio Returns**

For each testing portfolio i, we report in annualized percentage the average stock return, $\bar{r}_i^S$, the intercept from the CAPM regression, e_i^{CAPM}, and the intercept from the Fama-French 3-factor regression, e_i^{FF}. The H−L portfolio is long in the high portfolio and short in the low portfolio. The t-statistics for the model errors are reported in brackets beneath the corresponding errors. a.a.p.e. is the average of the absolute values of the errors for a given set of ten testing portfolios. Panel A reports results for ten B/M portfolios, Panel B for I^u/K_0^u portfolios and Panel C for I^u/ME portfolios.

	Low	2	3	4	5	6	7	8	9	High	H−L	a.a.p.e.
	Panel A: Ten B/M Portfolios											
$\bar{r}^S$	9.35	12.73	14.11	16.91	16.16	18.17	17.38	19.63	21.09	24.72	15.37	
α^{CAPM}	−5.95	−0.79	−0.67	3.59	3.19	4.97	4.56	7.42	8.19	10.73	16.67	5.00
$[t]$	[−1.74]	[−0.26]	[0.20]	[1.11]	[1.03]	[1.54]	[1.24]	[2.22]	[2.00]	[2.12]	[3.61]	
α^{FF}	−2.84	0.33	-1.19	0.19	0.43	0.63	−0.09	2.47	2.37	3.45	6.28	1.40
$[t]$	[−1.31]	[0.20]	[−0.66]	[0.11]	[0.23]	[0.36]	[−0.05]	[1.26]	[1.06]	[1.12]	[1.81]	
	Panel B: Ten I^u/K_0^u portfolios											
$\bar{r}^S$	23.37	21.98	20.54	17.64	17.66	18.82	19.27	17.54	16.30	13.19	−10.18	
α^{CAPM}	6.44	8.06	6.33	4.34	3.79	4.77	4.88	2.91	0.70	−3.52	−9.46	4.52
$[t]$	[1.05]	[1.92]	[1.91]	[1.47]	[1.15]	[1.57]	[1.21]	[0.67]	[0.13]	[−0.56]	[−3.39]	
α^{FF}	7.48	7.50	4.40	3.70	1.88	4.21	5.80	5.34	3.96	0.15	−7.33	4.44
$[t]$	[2.13]	[2.74]	[2.08]	[2.15]	[0.88]	[2.09]	[2.56]	[2.21]	[1.22]	[0.05]	[−2.34]	
	Panel C: Ten I^u/ME portfolios											
$\bar{r}^S$	9.12	12.66	14.29	15.01	15.36	19.73	19.69	20.12	28.01	33.50	24.38	
α^{CAPM}	−4.85	−1.31	0.53	1.22	1.42	5.01	5.33	3.91	11.20	16.49	21.34	5.13
$[t]$	[−1.52]	[−0.54]	[0.19]	[0.37]	[0.44]	[1.17]	[1.15]	[0.77]	[1.52]	[1.63]	[2.23]	
α^{FF}	−7.92	−2.26	−0.11	0.62	−0.05	7.54	7.45	5.04	15.45	23.28	31.20	6.97
$[t]$	[−3.05]	[−1.24]	[−0.06]	[0.32]	[−0.02]	[3.09]	[2.51]	[1.53]	[3.55]	[3.62]	[3.98]	

Table 3: **Summary Statistics of Portfolio Characteristics**

This table reports the averages of future investment-to-capital, I^m_{it+1}/K^m_{it+1}, current investment-to-capital, I^m_{it}/K^m_{it}, investment growth, $(I^m_{it+1}/K^m_{it+1})/(I^m_{it}/K^m_{it})$, future R&D-to-intangible-assets, $I^u_{it+1}/K^u_{0,t+1}$, R&D-to-intangible-assets, $I^u_{it}/K^u_{0,it}$, R&D growth, $(I^u_{it+1}/K^u_{0,it+1})/(I^u_{it}/K^u_{0,it})$, sales-to-capital, Y_{it+1}/K^m_{it+1}, the depreciation rate, δ_{it+1}, market leverage, w_{it}, intangible-assets-to-capital, $K^u_{0,it+1}/K^m_{it+1}$, and annual corporate bond returns in percentage, r^B_{it+1}, for all the testing portfolios. The column H−L reports the average differences between high and low portfolios and the column $[t_{\text{H}-\text{L}}]$ reports the t-statistics for the test that the differences equal zero. Panel A has results for ten B/M portfolios, Panel B for I^u/K^u_0 portfolios, and Panel C for I^u/ME portfolios.

	Low	2	3	4	5	6	7	8	9	High	H−L	$[t_{\text{H}-\text{L}}]$
	Panel A: Ten B/M portfolios											
I^m_{it+1}/K^m_{it+1}	0.15	0.12	0.11	0.11	0.11	0.10	0.10	0.09	0.09	0.09	−0.06	[−7.41]
I^m_{it}/K^m_{it}	0.16	0.13	0.12	0.12	0.11	0.10	0.10	0.09	0.10	0.09	−0.07	[−6.67]
$(I^m_{it+1}/K^m_{it+1})/(I^m_{it}/K^m_{it})$	0.97	0.98	0.98	0.97	1.01	1.01	0.95	1.01	0.97	0.98	0.01	[0.37]
$I^u_{it+1}/K^u_{0,it+1}$	0.43	0.38	0.36	0.36	0.34	0.34	0.33	0.34	0.33	0.31	−0.11	[−13.48]
$I^u_{it}/K^u_{0,it}$	0.42	0.38	0.37	0.36	0.35	0.34	0.34	0.33	0.34	0.34	−0.08	[−6.96]
$(I^u_{it+1}/K^u_{it+1})/(I^u_{it}/K^u_{it})$	1.01	1.00	1.00	1.01	0.99	0.99	0.97	1.02	0.98	0.95	−0.07	[−2.44]
Y_{it+1}/K^m_{it+1}	2.00	1.76	1.75	1.58	1.56	1.39	1.34	1.21	1.29	1.44	−0.57	[−7.32]
δ_{it+1}	0.09	0.09	0.08	0.08	0.08	0.08	0.07	0.07	0.08	0.08	−0.01	[−1.83]
w_{it}	0.10	0.18	0.26	0.25	0.26	0.31	0.34	0.42	0.48	0.48	0.37	[12.49]
$K^u_{0,it+1}/K^m_{it+1}$	0.25	0.16	0.12	0.11	0.10	0.07	0.07	0.05	0.06	0.06	−0.18	[−11.48]
r^B_{it+1}	10.99	10.63	10.55	10.65	10.75	11.02	11.12	11.14	11.24	11.52	0.53	[0.88]
	Panel B: Ten I^u/K^u_0 portfolios											
I^m_{it+1}/K^m_{it+1}	0.09	0.08	0.09	0.09	0.10	0.12	0.12	0.13	0.13	0.16	0.07	[5.91]
I^m_{it}/K^m_{it}	0.09	0.08	0.09	0.09	0.11	0.12	0.13	0.14	0.15	0.16	0.08	[7.15]
$(I^m_{it+1}/K^m_{it+1})/(I^m_{it}/K^m_{it})$	1.09	1.03	1.04	0.99	0.98	0.95	0.98	1.02	0.93	0.97	−0.12	[−1.36]
$I^u_{it+1}/K^u_{0,it+1}$	0.23	0.29	0.31	0.33	0.35	0.38	0.40	0.43	0.45	0.50	0.27	[16.60]
$I^u_{it}/K^u_{0,it}$	0.19	0.26	0.31	0.33	0.35	0.38	0.41	0.46	0.49	0.62	0.43	[14.68]
$(I^u_{it+1}/K^u_{0,it+1})/(I^u_{it}/K^u_{0,it})$	1.20	1.12	1.03	1.00	1.00	0.99	0.98	0.95	0.92	0.83	−0.37	[−9.30]
Y_{it+1}/K^m_{it+1}	1.61	1.45	1.51	1.61	1.61	1.86	1.99	1.95	2.34	2.76	1.15	[3.80]
δ_{it+1}	0.07	0.07	0.07	0.08	0.08	0.09	0.09	0.10	0.11	0.12	0.04	[5.92]
w_{it}	0.29	0.32	0.29	0.26	0.26	0.26	0.15	0.15	0.16	0.15	−0.14	[−10.12]
$K^u_{0,it+1}/K^m_{it+1}$	0.16	0.11	0.14	0.15	0.16	0.25	0.26	0.28	0.29	0.30	0.14	[3.87]
r^B_{it+1}	11.62	11.70	11.34	11.04	10.72	10.67	10.78	10.90	11.27	11.30	−0.32	[−1.41]
	Panel C: Ten I^u/ME portfolios											
I^m_{it+1}/K^m_{it+1}	0.09	0.09	0.10	0.11	0.10	0.10	0.10	0.11	0.12	0.11	0.02	[2.12]
I^m_{it}/K^m_{it}	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.10	0.12	0.11	0.01	[1.89]
$(I^m_{it+1}/K^m_{it+1})/(I^m_{it}/K^m_{it})$	0.97	0.98	1.02	0.99	0.98	0.97	0.97	1.03	1.02	1.04	0.07	[0.94]
$I^u_{it+1}/K^u_{0,it+1}$	0.39	0.39	0.41	0.40	0.37	0.36	0.36	0.36	0.35	0.31	−0.08	[−5.86]
$I^u_{it}/K^u_{0,it}$	0.39	0.39	0.41	0.40	0.38	0.37	0.37	0.36	0.35	0.32	−0.06	[−4.13]
$(I^u_{it+1}/K^u_{0,it+1})/(I^u_{it}/K^u_{0,it})$	1.02	1.01	1.02	1.01	0.99	1.00	0.98	0.98	1.01	0.95	−0.06	[−3.57]
Y_{it+1}/K^m_{it+1}	1.44	1.37	1.80	1.90	1.73	1.80	2.02	1.93	1.92	1.90	0.46	[5.81]
δ_{it+1}	0.07	0.07	0.08	0.08	0.08	0.09	0.09	0.10	0.11	0.11	0.05	[13.18]
w_{it}	0.22	0.23	0.21	0.15	0.16	0.18	0.22	0.25	0.43	0.55	0.33	[6.83]
$K^u_{0,it+1}/K^m_{it+1}$	0.02	0.07	0.16	0.21	0.23	0.30	0.35	0.38	0.35	0.38	0.36	[8.88]
r^B_{it+1}	10.64	10.55	10.76	10.99	11.09	11.06	11.16	11.60	11.78	11.92	1.28	[0.96]

Table 4: **Parameter Estimates and Tests for Model Performance Comparison**

Estimates and tests are from one-stage GMM with an identity weighting matrix. The moment conditions are $\mathbb{E}\left[r^S_{it+1} - r^{Iw}_{it+1}\right] = 0$. a is the adjustment cost parameter for tangible asset, b is the adjustment cost parameter for intangible asset, a_2 is the effect of intangible asset to tangible asset, ψ is the power parameter for the intangible asset adjustment cost function, α is capital's share. Their t-statistics, denoted as t, are reported in brackets beneath the estimates. a.a.p.e. is the average absolute value pricing error in annual percent. $p(\chi^2)$ is the p-value associated with the χ^2 statistic that tests the null hypothesis that the moment condition errors from one-stage GMM are jointly zero. $p(\lambda)$ is the p-value associated with the λ test defined in Singleton (1985), comparing the performance of the Qm model and the Qu model. $p(\lambda)$ under the column Qm tests the null hypothesis that Qm is the correct model while $p(\lambda)$ under the column Qu tests the null hypothesis that Qu is the correct model. $p(Wald)$ is the p-value associated with the Wald test. $p(Wald)$ under the column Qu_ISTC is for the Wald test between the Qu_ISTC model and the Qu model while $p(Wald)$ under the column Qu_AC is for the Wald test between the Qu_AC model and the Qu model. Panel A, B, and C report the results for the B/M portfolios, the I^u/K^u_0 portfolios, and the I^u/ME portfolios, respectively. For each panel, the results for four models are reported: the Qm model, the Qu_ISTC model, the Qu_AC model, and the Qu model.

	Panel A: B/M					Panel B: I^u/K^u_0					Panel C: I^u/ME			
	Qm	Qu_ISTC	Qu_AC	Qu		Qm	Qu_ISTC	Qu_AC	Qu		Qm	Qu_ISTC	Qu_AC	Qu
a	43.59	16.43	1.21	1.21	a	13.31	18.70	5.79	7.05	a	69.34	46.78	1.73	2.71
$[t]$	[0.92]	[0.85]	[0.20]	[0.18]	$[t]$	[1.16]	[0.79]	[0.50]	[0.76]	$[t]$	[0.41]	[0.57]	[0.07]	[0.16]
b			24.69	24.69	b			3.26	27.76	b			67.47	56.20
$[t]$			[1.85]	[0.81]	$[t]$			[1.10]	[0.57]	$[t]$			[0.07]	[0.93]
a_2		0.00		0.00	a_2		1.03		0.30	a_2		1.35		0.65
$[t]$		[0.00]		[0.00]	$[t]$		[0.49]		[5.82]	$[t]$		[0.65]		[5.77]
ψ			1.37	1.37	ψ			4.12	1.64	ψ			10.00	0.59
$[t]$			[3.86]	[0.97]	$[t]$			[0.56]	[3.70]	$[t]$			[2.09]	[4.90]
α	0.77	0.44	0.40	0.40	α	0.35	0.33	0.31	0.28	α	1.00	0.56	0.27	0.14
$[t]$	[1.38]	[2.32]	[3.35]	[2.46]	$[t]$	[2.78]	[2.93]	[2.39]	[3.60]	$[t]$	[0.54]	[0.81]	[0.78]	[1.07]
$a.a.p.e.$	3.88	3.61	1.36	1.36	$a.a.p.e.$	1.93	1.82	0.87	0.49	$a.a.p.e.$	3.19	2.44	5.12	0.78
$p(\chi^2)$	0.79	0.73	0.68	0.56	$p(\chi^2)$	0.83	0.79	0.68	0.61	$p(\chi^2)$	0.81	0.73	0.68	0.77
$p(\lambda)$	0.00			0.98	$p(\lambda)$	0.00			0.98	$p(\lambda)$	0.05			1.00
$p(Wald)$	0.00	0.02	1.00		$p(Wald)$	0.00	0.00	0.00		$p(Wald)$	0.00	0.00	0.00	

Table 5: **Euler Equation Errors for Different Models**

Euler equation errors and t-statistics are from the from one-stage GMM estimation with an identity weighting matrix. The moment conditions are $E\left[r^S_{it+1} - r^{Iw}_{it+1}\right] = 0$. The mean errors are defined as $e_i \equiv E_T\left[r^S_{it+1} - r^{Iw}_{it+1}\right]$, in which $E_T[\cdot]$ is the sample mean of the series in brackets. In the last column, we report the difference in the mean errors in annual percent between the high and low portfolios, as well as their t-statistics. Panel A reports results for B/M portfolios, Panel B for I^u/K^u_0 portfolios and Panel C for I/K portfolios. e^{Qm} represents Euler equation error for the Qm model, e^{Qu_ISTC} for the Qu_ISTC model, and e^{Qu_AC} for the Qu_AC model. All the numbers are in percentage and per annum.

	Low	2	3	4	5	6	7	8	9	High	H−L
	Panel A: Euler equation errors for B/M Portfolios										
e_i^{Qm}	−5.52	−4.27	−6.23	2.77	−4.99	0.15	4.87	−0.86	7.16	2.02	7.54
$[t]$	[−1.68]	[−1.46]	[−1.87]	[1.01]	[−1.36]	[0.04]	[1.77]	[−0.25]	[1.71]	[0.41]	[1.20]
$e_i^{Qu\_ISTC}$	−5.78	−4.12	−6.38	1.77	−4.15	0.64	3.69	0.56	5.99	3.00	8.77
$[t]$	[−4.07]	[−2.63]	[−2.07]	[0.90]	[−1.51]	[0.21]	[2.81]	[0.21]	[4.00]	[0.81]	[2.77]
$e_i^{Qu\_AC}$	0.82	0.54	−3.05	1.09	−1.27	−0.19	−0.47	−2.35	0.54	3.30	2.48
$[t]$	[0.95]	[0.32]	[−1.68]	[0.97]	[−1.02]	[−0.12]	[−0.31]	[−1.43]	[0.48]	[1.71]	[1.91]
e_i^{Qu}	0.82	0.54	−3.05	1.09	−1.27	−0.19	−0.47	−2.35	0.54	3.30	2.48
$[t]$	[1.29]	[0.31]	[−2.05]	[0.65]	[−1.15]	[−0.12]	[−0.31]	[−1.70]	[0.50]	[2.61]	[2.04]
	Panel B: Euler equation errors for I^u/K^u_0 portfolios										
e_i^{Qm}	−1.65	1.71	−0.44	−1.13	0.96	3.39	1.76	0.55	1.59	−6.14	−4.48
$[t]$	[−0.51]	[0.92]	[−0.10]	[−0.39]	[0.46]	[1.32]	[0.71]	[0.14]	[0.67]	[−1.71]	[−0.97]
$e_i^{Qu\_ISTC}$	−1.39	2.64	−0.35	−0.73	1.20	2.67	1.01	−1.08	1.61	−5.54	−4.15
$[t]$	[−0.52]	[1.05]	[−0.07]	[−0.28]	[0.53]	[0.93]	[0.49]	[−0.46]	[0.56]	[−1.79]	[−1.16]
$e_i^{Qu\_AC}$	0.09	0.79	−0.19	−2.51	−0.59	1.46	1.01	1.16	−1.01	−0.10	−0.19
$[t]$	[0.03]	[0.46]	[−0.05]	[−0.99]	[−0.42]	[1.39]	[0.84]	[0.55]	[−0.53]	[−0.39]	[−0.06]
e_i^{Qu}	−0.12	0.33	0.77	−1.18	−0.28	0.54	0.57	−0.60	−0.27	0.20	0.32
$[t]$	[−0.05]	[0.29]	[0.24]	[−0.47]	[−0.23]	[0.33]	[0.78]	[−0.48]	[−0.12]	[0.22]	[0.21]
	Panel C: Euler equation errors for I^u/ME portfolios										
e_i^{Qm}	−5.54	0.21	−7.05	−3.97	−0.59	3.83	0.56	−3.08	3.05	4.02	9.56
$[t]$	[−1.41]	[0.03]	[−1.62]	[−1.89]	[−0.21]	[1.52]	[0.17]	[−0.57]	[0.50]	[1.08]	[1.76]
$e_i^{Qu\_ISTC}$	−0.43	3.71	−3.85	−2.70	−0.54	2.60	−1.21	−5.29	1.16	2.93	3.36
$[t]$	[−0.26]	[0.71]	[−1.00]	[−1.70]	[−0.22]	[1.39]	[−0.46]	[−0.92]	[0.27]	[0.80]	[0.70]
$e_i^{Qu\_AC}$	−11.67	−4.18	−6.93	−6.23	−3.12	1.81	−1.10	0.70	3.60	11.91	23.58
$[t]$	[−6.36]	[−2.56]	[−4.54]	[−3.38]	[−1.88]	[0.76]	[−0.85]	[0.19]	[5.39]	[5.30]	[9.85]
e_i^{Qu}	−0.23	2.11	0.07	−0.81	−1.64	1.63	−0.11	−0.86	0.32	0.01	0.24
$[t]$	[−0.39]	[3.03]	[0.07]	[−0.84]	[−1.29]	[1.02]	[−0.19]	[−0.82]	[0.45]	[0.03]	[0.39]

Table 6: **Autocorrelations of Tangible and Intangible Investment Rates**

This table reports the autocorrelations of both tangible and intangible investment rates. Tangible investment rate is measured by physical investment scaled by the beginning of the year total assets, by the beginning of the year Property, Plant, and Equipment (PP&E), and by sales over the same year. Intangible investment rate is measured by R&D expenditure scaled by the beginning of the year total assets, by the beginning of the year Property, Plant, and Equipment (PP&E), and by sales over the same year. The data sample is all Compustat firms with positive R&D expenditure from 1980 to 2008. We winsorize all the tangible investment rate variables at 99 and 1 percentiles and all the intangible investment rate variables at 99 percentile.

	Scaled by total assets	Scaled by PP&E	Scaled by sales
R&D	0.81	0.87	0.88
Investment	0.45	0.43	0.55

Table 7: **The Adjustment-Costs-to-Investment (AC/I) Ratio**

This table reports the adjustment-costs-to-investment (AC/I) ratios estimated using the ten B/M portfolios, the ten I^u/K^u_0 portfolios, and the ten I^u/ME portfolios, respectively, for both tangible investments and intangible investments, based on the parameter estimate in Table 4. For tangible investments, we report the estimates based on both the Qm model and the Qu model. For intangible investments, we report the estimate based on the Qu model only.

	B/M portfolios	I^u/K^u_0 portfolios	I^u/ME portfolios	Average
AC^m/I^m (Qm)	232%	73.7%	362 %	223%
AC^m/I^m (Qu)	6.43%	39.05%	14.17%	19.88%
AC^u/I^u (Qu)	839%	589%	5,708%	2,379%

Table 8: **Expected Return Errors from Comparative Static Experiments**

This table reports the results from comparative static experiments. For rows denoted by $\overline{I_{it+1}/K_{it+1}}, \overline{I_{it}/K_{it}}$, we set I_{it+1}/K_{it+1} for a given portfolio, denoted by i, to its cross sectional average value at time $t+1$ and we set I_{it}/K_{it} for a given portfolio to its cross sectional average value at time t. We use parameters reported in Table 4 for the Qu model to reconstruct the expected return. The difference between these reconstructed expected return and the realized return for each portfolio, the high-minus-low portfolio and the average absolute pricing errors are then reported. The results for others rows are designed analogously.

	Low	2	3	4	5	6	7	8	9	High	H−L	a.a.p.e.
Panel A: Ten B/M portfolios												
$\overline{I^m_{it+1}/K^m_{it+1}}, \overline{I^m_{it}/K^m_{it}}$	1.97	0.73	−2.97	1.64	−1.49	0.07	−0.92	−1.90	−0.27	2.76	0.79	1.47
$\overline{I^u_{it+1}/K^u_{it+1}}, \overline{I^u_{it}/K^u_{it}}$	−9.95	−4.54	−4.80	1.95	−0.23	4.92	5.51	9.14	9.78	10.70	20.65	6.15
$\overline{K^u_{it+1}/K^m_{it+1}}$	−10.48	−4.74	−4.84	1.82	−0.07	5.06	5.37	9.47	9.72	11.25	21.73	6.28
$\overline{Y_{it+1}/K^m_{it+1}}$	−3.44	−0.88	−0.38	2.73	0.92	2.45	1.65	2.48	4.03	9.21	12.65	2.82
$\overline{w}$	−4.93	−1.96	−0.99	1.93	1.12	3.06	2.21	4.43	6.23	10.21	15.14	3.71
Panel B: Ten I^u/K^u_0 portfolios												
$\overline{I^m_{it+1}/K^m_{it+1}}, \overline{I^m_{it}/K^m_{it}}$	4.13	3.41	1.87	−0.71	0.63	2.41	2.15	0.47	0.21	−4.71	−8.84	2.07
$\overline{I^u_{it+1}/K^u_{it+1}}, \overline{I^u_{it}/K^u_{it}}$	2.41	3.30	1.63	−0.60	0.28	1.85	2.08	0.43	−0.06	−6.14	−8.54	1.88
$\overline{K^u_{it+1}/K^m_{it+1}}$	5.85	6.10	3.44	0.25	0.91	0.76	1.41	0.12	−2.88	−7.97	−13.82	2.97
$\overline{Y_{it+1}/K^m_{it+1}}$	3.95	2.39	0.19	−1.97	−1.40	0.42	2.24	0.88	0.28	−2.08	−6.03	1.58
$\overline{w}$	4.83	3.44	1.89	-0.98	−0.77	1.08	1.49	0.11	−0.51	−3.17	−8.00	1.83
Panel C: Ten I^u/ME portfolios												
$\overline{I^m_{it+1}/K^m_{it+1}}, \overline{I^m_{it}/K^m_{it}}$	−0.45	2.00	−1.01	−1.63	−1.68	1.78	0.30	−0.31	1.18	5.12	5.57	1.55
$\overline{I^u_{it+1}/K^u_{it+1}}, \overline{I^u_{it}/Ku_{it}}$	−8.75	−4.61	−3.75	−2.52	−1.99	2.11	0.87	0.32	1.84	5.69	14.44	3.24
$\overline{K^u_{it+1}/K^m_{it+1}}$	−8.85	−4.73	−3.62	−2.51	−2.00	2.01	0.85	0.49	1.92	5.56	14.41	3.25
$\overline{Y_{it+1}/K^m_{it+1}}$	−9.48	−5.94	−3.85	−2.29	−2.11	1.97	1.34	0.80	2.56	5.66	15.14	3.60
$\overline{w}$	−10.22	−6.62	−4.79	−3.88	−3.54	0.97	1.13	1.80	9.95	15.61	25.83	5.85