# Trade Credit: The Interaction of Financing, Operations, Marketing and Risk Management *

Xiaojun Shi†‡ and Shunming Zhang§

March 14, 2011

**Abstract**

This paper builds a comprehensive model that treats trade credit as the interaction of financing, operations, marketing and default-risk management. Our model is featured by using a two-stage lottery method to describe default risk, in the context of which the incentive-compatible model is formulated. We find that the capital cost of the supplier is the most important factor determining the credit term. Default risk acts like a filtering criterion for selecting retailers eligible for credit. Empirical evidence supporting our theoretical considerations is obtained by estimating three panel econometric models, using a dataset of manufacturing companies drawn from COMPUSTAT database.

**Keywords**: Trade credit; Two-stage lottery; bilevel programming; default risk; economic ordering quantity (EOQ).

*We gratefully acknowledge support from National Natural Science Foundation of China (NSFC Grant Numbers: 70825003 and 70972001).

†Corresponding author.

‡School of Economics and Management, Beijing University of Aeronautics and Astronautics, Beijing, 100191, P.R.China. Phone and Fax: (86 10) 8233 9132 and E-mail: sxjstein@gmail.com.

§School of Finance, Renmin University of China, Beijing, 100872, P.R.China. Phone and Fax: (86 10) 82500626 and E-mail: szhang@ruc.edu.cn.

# 1 Introduction

The large literature of trade credit has focused mainly on the financial aspect. However, we argue that trade credit is far more complicated than what this unilateral theory suggests. In this paper, we build a comprehensive model that treats trade credit as the interaction of financing, operations, marketing and default-risk management. We model decision making on trade credit as an integrated process encompassing ordering, price discounting, cost sharing and risk shifting between the supplier and the retailer. This modeling framework factors in default risk, along with financial cost, market demanding, and inventory costs, which describe multiple aspects of trade credit.

The main feature of our approach is to propose a new two-stage lottery framework to describe default risk, in the context of which the incentive-compatible decision model is formulated. Two-stage lotteries were first introduced in seminal papers by Segal (1987 and 1990), in order to describe uncertainty. These lotteries are in nature very useful tools to model decisions under uncertainty. In recent years, they have attracted considerable attention in the financial economics, however, they have rarely been used in corporate financial decision modeling. To the best of our knowledge, this paper is the first attempt to apply two-stage lottery in the theoretical treatment of trade credit.

Our model and numerical experiments shed some fresh light on how multiple factors affect the trade credit term decision. We find that the financing capacity of the supplier is the most important determinant of the length of the credit term. This result corresponds with several prominent theories of trade credit, especially the financing motives theory of Schwartz (1974). In addition, the market demand factor is suggested to be positively related to the optimal term decision, and the holding cost of the retailer is negatively related. The empirical analysis also shows that default risk acts more as the criterion to screen default-prone retailers, than as a determinant of the length of credit term. Our approach bridges two previous categories of literature, focusing on the financial and inventory control aspects respectively. In this way, a multiple-perspective examination of the credit terms decision is made feasible. Supporting evidence is provided by a large-scale panel dataset of COMPUSTAT manufacturing companies.

# 2 The Model

In an economy, two agents, the supplier and the retailer, contract on trade credit. In this contracting, the supplier plays the principal role. She decides on trade credit, subject to financing capacity and inventory cost minimizing constraints. However, the principal could not make the decision on her own since her constraints are functions of the retailer's ordering and default risk. The retailer, as the agent, reacts on the supplier's term decision to determine her optimal ordering and default behavior, which in turn impacts on the supplier's cost. This interaction between the supplier and retailer approaches its equilibrium, which characterizes the incentive-compatible trade credit term as a function of financing capacity of the supplier, market demanding, inventory and default risk.

What the supplier produces is assumed to able to suffice the demand faced by the retailer, and the supplier replenishes its inventory instantaneously with a lot size that is an integer multiple of the retailer's ordering per cycle. This assumes that the retailer has no stochastic shortages or leadtime. However, the supplier faces possible default of the retailer, which introduces uncertainty into our modeling.

For the sake of mathematical tractability, we suppose the retailer operates in a non-stochastic marketing environment, facing a determinate demand rate. Moreover, the retailer agrees to pay back no later than the term granted by the supplier if it chooses to honor the debt. We argue that a stochastic demand is not requisite for the possible default of the retailer. Default could be endogenous. The retailer could choose to default even if she has the capacity to service the debt.

We summarize the notation as follows first, which are used through this paper.

$i$ = Subscript represents the player (the supplier or the retailer). More specifically, $i = s, r$ where $s$ = supplier, $r$ = retailer.

$A_i$ = Fixed unit setup cost of player $i$.

$h_i$ = Unit holding cost of player $i$ per unit time, only representing cost of capital, excluding storage cost.

$s_i$ = Unit storage cost of player $i$ per unit time, excluding cost of capital.

$c_i$ = Unit procurement cost of player $i$.

$D$ = Demand rate per unit time.

$m$ = The integer multiple of production lot-size of the supplier to the retailer's ordering quantity per cycle.

$k_i$ = The minimum rate of return of the investment required by player $i$.

$Q$ = Retailer's ordering quantity.

$t$ = Credit term offered by the supplier, or interest free period for permissible delay in payment.

### 2.1 The Supplier's Model

Incorporating the possible default of the retailer places the supplier in an uncertain environment. We model the supplier's decision in a *two-stage lotteries* framework to describe this uncertainty. From the perspective of the supplier, she don't know the possibility of the retailer's default on trade credit. Neither could she predict exactly how much loss the retailer's default would cause due to complicated interactions of production, inventory and marketing factors. The available methods such as simple or compound lotteries are unable to capture this two-layer uncertainty. We thus propose to use two-stage lottery to describe this case. We model the uncertainty of the possibility of default at the first stage and uncertainty of loss incurred at the second stage.

We define $A$ to be a two-stage lottery, if $A$ depends upon some simple lottery $X$ such that, for each realization $x$ of $X$, $A(x)$ is a simple lottery. In this case, $A(x)$ is the first level lottery for any realization $x$ of $X$ and $X$ is the second level lottery.

We denote $L^1$ the set of random variables. Any element $X \in L^1$ is also called to be a lottery. A complex lottery relate to some simple lottery such that it is random variable for any realization of the simple lottery. We denote $L^2$ the set of two-stage lotteries. Any element $A \in L^2$ is a two-stage lottery. It is equivalent to that $A(x) \in L^1$ if there exists some lottery $X \in L^1$ such that $x$ is one of its realizations.

The cumulative distribution function (c.d.f.) of the two-stage lottery $A$ is defined by

$$F_A(z) = P\{A \leqslant z\} = \int_{(-\infty,+\infty)} P\{A(x) \leqslant z\} dF_X(x) = \int_{(-\infty,+\infty)} F_{A(x)}(z) dF_X(x). \tag{1}$$

The value of expected utility of the two-stage lottery $A$ is

$$EU(A) = \int_{(-\infty,+\infty)} U(z) dF_A(z) = \int_{(-\infty,+\infty)} U(z) d\left[\int_{(-\infty,+\infty)} F_{A(x)}(z) dF_X(x)\right]. \tag{2}$$

In our model, the possibility of default on trade credit by the retailer, as the first level lottery $A(x)$, follows a Logistic distribution $L(a, x)$ where $a > 0$ is a positive parameter. Modeling default probability using Logistic distribution is widely accepted in literature. Its density function is

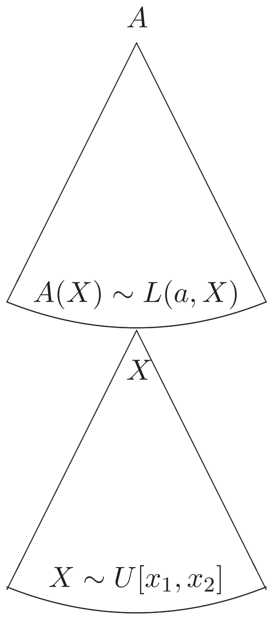
$$f_{A(x)}(z) = \frac{ae^{-(az-x)}}{\left[1 + e^{-(az-x)}\right]^2}, \qquad x \in (-\infty, +\infty). \tag{3}$$

Uncertainty of loss $X$, modeled as the second level lottery, is governed by a uniform distribution. Hence, $X \sim U[x_1, x_2]$ where $x_1$ is the what the supplier obtains in the worst case where retailer returns nothing; while $x_2$ is what the supplier has when the retailer services the trade credit obligation strictly according to the contract. Because the supplier has an information advantage and monitoring measures to recover the damages incurred by the default, at least to some degree (Rajan and Peterson, 1997; Giannetti, Burkart, and Ellingsen, 2008), the realized outcome for the supplier can lie at any point between $x_1$ and $x_2$. Thus, a simple and stable representation of this uncertainty is to assume that any point between $x_1$ and $x_2$ can appear with equal possibility. Its density function is

$$f_X(x) = \begin{cases} \dfrac{1}{x_2 - x_1}, & \text{if } \ x \in [x_1, x_2] \\ 0, & \text{if } \ x \notin [x_1, x_2]. \end{cases} \tag{4}$$

We thus have the two-stage lotteries representing the two-layer uncertainty faced by the supplier as Figure 1.

Figure 1. Two-Stage lottery for the Supplier under Default Risk




The c.d.f. of the two-stage lottery $A$ is

$$F_A(z) = \int_{[x_1, x_2]} F_{A(x)}(z) dF_X(x)$$

and its density function [1] is

$$\begin{aligned} f_A(z) &= \int_{[x_1,x_2]} f_{A(x)}(z)dF_X(x) = \int_{[x_1,x_2]} f_{A(x)}(z)f_X(x)dx \\ &= \int_{[x_1,x_2]} \frac{ae^{-(az-x)}}{\left[1+e^{-(az-x)}\right]^2}\frac{1}{x_2-x_1}dx \\ &= \frac{a}{x_2-x_1}\left[\frac{1}{1+e^{-(az-x_1)}}-\frac{1}{1+e^{-(az-x_2)}}\right]. \end{aligned} \tag{5}$$

We adopt CARA (Constant Absolute Risk Aversion) utility function

$$U(z) = -\frac{1}{\alpha}e^{-\alpha z}, \qquad z \in [0,+\infty). \tag{6}$$

where $\alpha > 0$ is the constant coefficient of absolute risk aversion. The value of von Neumann-Morgenstern expected utility of the two-stage lottery $A$ is [2]

$$\begin{aligned} EU(A) &= \int_{(-\infty,+\infty)} U(z)dF_A(z) = \int_{(-\infty,+\infty)} U(z)f_A(z)dz \\ &= \int_0^{+\infty} -\frac{1}{\alpha}e^{-\alpha z}\frac{a}{x_2-x_1}\left[\frac{1}{1+e^{-(az-x_1)}}-\frac{1}{1+e^{-(az-x_2)}}\right]dz \\ &= -\frac{1}{\alpha}\frac{a}{x_2-x_1}\int_0^{+\infty}\left[\frac{-e^{-\alpha z}}{1+e^{-(az-x_1)}}-\frac{-e^{-\alpha z}}{1+e^{-(az-x_2)}}\right]dz \\ &= -\frac{1}{\alpha}\frac{a}{x_2-x_1}\left\{\frac{1}{a}e^{-\frac{\alpha}{a}x_1}\ln\left[1+e^{x_1}\right]-\frac{1}{a}e^{-\frac{\alpha}{a}x_2}\ln\left[1+e^{x_2}\right]\right\} \\ &= \frac{1}{\alpha}\frac{1}{x_2-x_1}\left\{e^{-\frac{\alpha}{a}x_2}\ln\left[1+e^{x_2}\right]-e^{-\frac{\alpha}{a}x_1}\ln\left[1+e^{x_1}\right]\right\}. \end{aligned} \tag{7}$$

When the retailer pays back according to credit term agreement, the prize per unit time for the supplier is $x_1$ or $x_2$. $x_1$ is decomposed into fixed set-up cost, holding and storage cost of inventory, procurement cost, and opportunity capital cost of trade credit; $x_2$ is equal to $x_1$ plus sales value.

---

[1] We check that $f_A(z)$ is a density function as follows

$$\begin{aligned} \int_{-\infty}^{+\infty} f_A(z)dz &= \int_{-\infty}^{+\infty}\frac{a}{x_2-x_1}\left[\frac{1}{1+e^{-(az-x_1)}}-\frac{1}{1+e^{-(az-x_2)}}\right]dz = \int_{-\infty}^{+\infty}\frac{a}{x_2-x_1}\left[\frac{e^{-(az-x_2)}}{1+e^{-(az-x_2)}}-\frac{e^{-(az-x_1)}}{1+e^{-(az-x_1)}}\right]dz \\ &= -\frac{1}{x_2-x_1}\left\{\ln\left[1+e^{-(az-x_2)}\right]-\ln\left[1+e^{-(az-x_1)}\right]\right\}\Bigg|_{z=-\infty}^{z=+\infty} = -\frac{1}{x_2-x_1}\ln\frac{1+e^{-(az-x_2)}}{1+e^{-(az-x_1)}}\Bigg|_{z=-\infty}^{z=+\infty} \\ &= \frac{1}{x_2-x_1}\ln\lim_{z\to-\infty}\frac{1+e^{-(az-x_2)}}{1+e^{-(az-x_1)}} = \frac{1}{x_2-x_1}\ln\lim_{z\to-\infty}\frac{-ae^{-(az-x_2)}}{-ae^{-(az-x_1)}} = \frac{1}{x_2-x_1}\ln e^{x_2-x_1} = 1. \end{aligned}$$

[2] Set $t = e^{-(az-x)}$, then $z = \frac{1}{a}(x-\ln t)$, $e^{\alpha z} = te^{-\frac{\alpha}{a}x}$ and $dz = -\frac{dt}{at}$. Therefore

$$\int_0^{+\infty}\frac{-e^{-\alpha z}}{1+e^{-(az-x)}}dz = \int_{e^x}^{0}\frac{te^{-\frac{\alpha}{a}x}}{1+t}\left[-\frac{dt}{at}\right] = \frac{1}{a}e^{-\frac{\alpha}{a}x}\int_0^{e^x}\frac{dt}{1+t} = \frac{1}{a}e^{-\frac{\alpha}{a}x}\ln\left[1+t\right]\Big|_0^{e^x} = \frac{1}{a}e^{-\frac{\alpha}{a}x}\ln\left[1+e^x\right].$$

If $x_2 - x_1 \equiv \delta > 0$ is a positive constant, then

$$EU(A) = \frac{1}{\alpha\delta}\left\{e^{-\frac{\alpha}{a}(x_1+\delta)}\ln\left[1+e^{x_1+\delta}\right] - e^{-\frac{\alpha}{a}x_1}\ln\left[1+e^{x_1}\right]\right\}.$$

The expected utility maximization problem implies a unique solution $x_1^*\left(\frac{\alpha}{a}, \delta\right)$ dependent on $\frac{\alpha}{a}$ and $\delta$. That is to say, the unique $x_1^*\left(\frac{\alpha}{a}, \delta\right)$ solves $\max_{x_1} EU(A)$.

Using the method from Pan and Yang (2002), the average inventory level per unit of time for the supplier can be specified as

$$\frac{mQ\left[\frac{Q}{P} + (m-1)\frac{Q}{D}\right] - \frac{m^2Q^2}{2P} - [1+2+\cdots+(m-1)]\frac{Q^2}{D}}{\frac{mQ}{D}} = \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + 2\frac{D}{P}\right]. \quad (8)$$

where $P$ is the production rate of the retailer.

The opportunity capital cost of trade credit arises, as the supplier cannot receive payment on goods delivery and invest it immediately. In the case of permitted delay of payment, such an investment can only be made until the retailer pays at time $t$, and thus, the possible investment outcome in the time interval $[0, t]$ is lost. As the amount of investable money is $c_rQ$, the total investment outcome per unit of time is:

$$V_s(k_s, Q, t) = \frac{D}{mQ}\int_0^t k_s c_r Q d\xi = \frac{k_s c_r D t}{m}. \quad (9)$$

Therefore $x_1$ or $x_2$ can be specified as

$$x_1 = -\frac{A_s D}{mQ} - (h_s + s_s)\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + 2\frac{D}{P}\right] - \frac{k_s c_r D t}{m} \quad (10)$$

$$x_2 = \varphi_s(Q, t) = \frac{(c_r - c_s)D}{m} - \frac{A_s D}{mQ} - (h_s + s_s)\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + 2\frac{D}{P}\right] - \frac{k_s c_r D t}{m}. \quad (11)$$

## 2.2 The Retailer's Model

From the perspective of the retailer, fixed ordering cost $A_r$, inventory storage cost $I_r$ (excluding interest cost) and cost of capital for holding inventory $H_r$ during $t$ through $T_r$ occur during one ordering cycle $T_r$. Meanwhile, the retailer also reap the benefit of surplus funds $B_r$, as it can invest the credit surplus before it settles its payables (Haley and Higgins (1973)). This benefit reduces the retailer's inventory cost to some degree. Among the components analyzed above, ordering cost and inventory storage cost are fixed, while inventory holding cost and opportunity investment value vary with the credit term. Therefore, the net cost function of the retailer can be expressed as

$$\Phi_r(Q, t) = A_r + I_r + H_r(h_r, Q|t) - B_r(k_r, Q|t). \quad (12)$$

In the case of a determinate demanding rate $D$, the inventory storage cost $I_r$ for the retailer through one ordering cycle can be expressed as

$$I_r = s_r \int_0^{T_r} I(\xi)_r d\xi = \frac{s_r Q^2}{2D}, \tag{13}$$

where $I(\xi)_r$ is the inventory level of the retailer at time $\xi$, which equals $Q(1 - \frac{\xi}{T_r})$ in our case.

To specify $H_r$, we need to discuss in two cases, i.e., $0 < t \leqslant T_r$ and $T_r < t$, following Haley and Higgins (1973) and Goyal (1985). The cost of capital during $t$ through $T_r = \frac{Q}{D}$ when $0 < t \leqslant T_r$ is

$$H_r(h_r, Q \mid t) = h_r \int_t^{T_r} I(\xi)_r d\xi = \frac{h_r}{2} \frac{T_r - t}{T_r} Q(T_r - t) = \frac{h_r(Q - tD)^2}{2D}. \tag{14}$$

While $0 < T_r < t$, the cost of capital for the retailer is zero, $H_r(h_r, Q \mid t) = 0$. The retailer does not pay the supplier at all during the ordering cycle, but transfers the entire funds-holding cost of inventory to the supplier.

Next, we specify $B_r$ in different cases.

Case 1. $0 < t < T_r$. Following Haley and Higgins (1973)[3], the surplus fund at any time $\xi \in (0, t)$ is $c_r[Q - I(\xi)_r]$ where $I(\xi)_r$ is the inventory level of the retailer at time $\xi$. This surplus can yield investment benefit $c_r[Q - I(\xi)_r]k_r d\xi$ in an infinitesimal time span $d\xi$. The term $k_r$ is the minimum rate of return required by the retailer. Thus, the overall investment benefits of the credit surplus over $[0, t]$ can be given as

$$B_r(k_r, Q \mid t) = \int_0^t c_r[Q - I(\xi)_r]k_r d\xi = \int_0^t c_r k_r D\xi d\xi = \frac{c_r k_r D}{2} t^2. \tag{15}$$

Case 2. $0 < T_r < t$. In this case, the investment amounts are different in two time intervals $[0, T_r)$ and $[T_r, t]$. They are $c_r[Q - I(\xi)_r]$ and $c_r Q$ respectively. Thus, the surplus investment benefits can be divided into two parts as

$$B_r(k_r, Q \mid t) = \int_0^{T_r} c_r[Q - I(\xi)_r]k_r d\xi + \int_{T_r}^t c_r k_r Q d\xi = c_r k_r Q t - \frac{c_r k_r Q^2}{2D}. \tag{16}$$

Accordingly, we specify the retailer's cost functions per unit time in different cases as

$$\varphi_r(Q, t) = \begin{cases} \frac{2A_r D + (h_r - c_r k_r) D^2 t^2}{2Q} + \frac{s_r + h_r}{2} Q - h_r D t, & if \quad 0 < t < T_r \\ \frac{A_r D}{Q} + \frac{c_r k_r}{2} Q + \frac{s_r Q - 2 c_r k_r D \tau}{2}, & if \quad 0 < T_r \leqslant t. \end{cases} \tag{17}$$

[3]However, if we suppose that the retailer has the required amount of cash to pay the supplier at the very beginning of the ordering cycle, then the retailer can invest this amount of money during the period of $[0, t]$. As the ordering quantity is $Q$ and its procurement price is $c_r$, the retailer can obtain investment revenue of $c_r Q(e^{k_r t} - 1)$.

However, in the second case with $T_r \leqslant t$, the unit cost of the retailer is simply a decreasing linear function with respect to $t$ in the interval $[T_r, \infty)$. A rational retailer should choose infinity as its optimal payment time. This choice implies that the retailer accepts that it is obligated to the supplier, but never actually pays back. Such a situation is intrinsically unfair and the supplier will not allow it to happen. Therefore, we only discuss the first case with $0 < t < T_r$ in the remaining parts.

## 2.3 The Trade Credit Model

In the case of $0 < t < T_r$, the incentive-compatible trade credit solves the bilevel programming as follows.

$$\begin{aligned} \max_{t} \quad & EU(A) = \frac{1}{\alpha}\frac{1}{x_2 - x_1}\left\{e^{-\frac{\alpha}{a}x_2}\ln\left[1+e^{x_2}\right] - e^{-\frac{\alpha}{a}x_1}\ln\left[1+e^{x_1}\right]\right\} \\ s.t. \quad & t \geqslant 0 \\ & \min_{Q}\ \varphi_r(Q,t) = \frac{2A_rD + (h_r - c_rk_r)D^2t^2}{2Q} + \frac{s_r + h_r}{2}Q - h_rDt \\ & \quad s.t.\ Q \geqslant 0 \end{aligned} \tag{18}$$

where $x_1$ and $x_2$ are given by Equations (10) and (11), respectively.

In the above programming (1), if it satisfies conditions $Q > 0$ and $h_r \geqslant c_rk_r$, then the lower level can be replaced by its First Order Condition equation, and the bilevel programming reduces to a non-linear programming.

One way to guarantee the validity of the FOA is to show that the retailer's unit cost function is convex with respect to $Q$ and $t$ under maximization. The Hessian matrix of the unit cost function in the lower level of (18) with respect to $Q$ and $t$ is

$$\begin{pmatrix} \dfrac{2A_rD + (h_r - c_rk_r)D^2t^2}{Q^3} & \dfrac{h_r - c_rk_rD^2t}{Q^2} \\ \dfrac{h_r - c_rk_rD^2t}{Q^2} & \dfrac{(h_r - c_rk_r)D^2}{Q} \end{pmatrix}.$$

The first order principal minor is $\dfrac{2A_rD + (h_r - c_rk_r)D^2t^2}{Q^3}$ which is obviously positive under the condition $Q > 0$ and $h_r - c_rk_r \geqslant 0$. The second order principal minor is $\dfrac{2A_rD(h_r - c_rk_r)D^2}{Q^4}$ which is also positive under the same conditions. The Hessian matrix is positive definite if we can prove $h_r - c_rk_r \geqslant 0$, as $Q > 0$ holds.

The inequality $h_r \geqslant c_rk_r$ can easily be proved. If the ordering quantity is supposed to be $q$, we multiply both sides of the inequality by $q$. The LHS becomes $h_rq$, which means pure holding cost of inventory with quantity of $q$. This term can be further interpreted as the lost investment return caused by the inventory of $q$ holding up $c_r q$ capital. If we denote the corresponding investment rate of return as $i_r$, we have $h_r q = i_r c_r q$. As $k_r$ is the minimum rate of return, it cannot be greater than $i_r$. Thus $i_r c_r q \geqslant k_r c_r q$ holds. Substituting $i_r c_r q$ by $h_r q$, we have $h_r \geqslant c_r k_r$.

Finally, the bilevel level programming (18) reduces to a nonlinear programming as follows:

$$\begin{aligned} \max_{t} \quad & EU(A) = \frac{1}{\alpha}\frac{1}{x_2 - x_1}\left\{e^{-\frac{\alpha}{a}x_2}\ln\left[1+e^{x_2}\right] - e^{-\frac{\alpha}{a}x_1}\ln\left[1+e^{x_1}\right]\right\} \\ s.t. \quad & -\frac{2A_rD + (h_r - c_rk_r)D^2t^2}{2Q^2} + \frac{s_r + h_r}{2} = 0 \\ & t \geqslant 0,\ Q \geqslant 0 \end{aligned} \tag{19}$$

Solving the second equation constraint in (19), we obtain that the optimal ordering quantity by the retailer of $Q^* = \sqrt{\frac{2A_rD + (h_r - c_rk_r)D^2t^2}{h_r + s_r}}$ under trade credit term $t$. This quantity is greater than $Q_0 = \sqrt{\frac{2A_rD}{h_r + s_r}}$ in the classic EOQ model with no permissible delay in payment as $h_r \geqslant c_rk_r$. Programming (19) reduces to a nonlinear programming when expressions of $Q^*$ and $x_2$ are inserted into the objective function. This yields

$$x_1^*\left(\frac{\alpha}{a}, \frac{(c_r - c_s)D}{m}\right) = -\frac{A_sD}{mQ^*(t)} - (h_s + s_s)\frac{Q^*(t)}{2}\left[m\left(1 - \frac{D}{P}\right) - 1 + 2\frac{D}{P}\right] - \frac{k_sc_rDt}{m} \tag{20}$$

Figure 2 to 5 show numerical analysis of how three broad kinds of factors affect the incentive compatible trade-credit decision. They are *financing factor*, *marketing factor*, and *inventory factor* represented by capital costs of the retailer $k_r$ and the supplier $k_s$, demand rate $D$ and holding cost of the retailer $h_r$, respectively.

[**Observation 1 (Financing Factor Effects)**] The supplier with greater financing capacity tends to grant a longer term, which yields increases in profits. On the other hand, if the retailer has a higher capital cost, which indicates that its financing is more constrained, the supplier must grant a longer term in this case, but suffers a decrease in profits.

On the part of the supplier, Figure 2 shows that the greater $k_s$ is less than $k_r$, the longer the term it is willing to grant. This situation implies that the supplier has greater financing capacity. Furthermore, offering a longer term in this case is actually a more efficient utilization of the supplier's financing capacity.

Regarding the retailer, if it is more financing-constrained with a higher capital cost $k_r$, the supplier must extend a longer term in order to bring the incentive-compatible scheme into effect. This is the case indicted in Figure 3. Such a compromised decision is made for the sake of incentive-compatible coordination.

[**Observation 2 (Marketing Factor Effects)**] A higher demand rate is associated with a longer credit term.

Figure 4 shows that the credit term is an increasing function of the demand rate. This result is quite understandable. A higher demand rate implies that the corresponding retailer is a more important client of the supplier, so that this retailer deserves a longer credit term.

[**Observation 3 (Inventory Cost Factor Effects)**] The retailer's holding cost basically determines the retailer's ordering behavior, in response to the supplier's credit term incentives. A higher holding cost leads to a shorter term.

This result is shown in Figure 5, which explains the cost-sharing mechanism of the credit term. Because the retailer's inventory cost increases with its holding cost, a longer term in the case of a higher holding cost, will shift a much larger share of inventory cost onto the supplier. Thus, the supplier prefers a shorter term.

## 3 Empirical Evidence

We propose a two-way approach to testing the main contents in Observations 1 through 3 and to testing the impact of default risk on the credit term decision. In reality, a firm acts as both a supplier and a retailer. When the sampled companies are viewed as suppliers, Observation 1 about the effects of financial cost and Observation 2 about the effects of marketing demand are tested. Conversely, the effects of default risk and Observation 3, on the effects of holding cost, are tested from the perspective of retailers.

Following the above reasoning, we need two dependent variables. For the supplier, we need the credit term it grants (the variable *logrt*). For the retailer, we need the credit term it receives (the variable *logdtp*). Because precise information on these two terms is very rarely available, we must find their proxies. It is well known that trade credit is represented by accounts receivable and accounts payable, when a firm is viewed as a supplier and a retailer respectively. We propose simply using the turnover days of accounts receivable as the term granted by the supplier, and the turnover days of accounts payable as the term received by the retailer. Based on the underlying logic of our models, it is more precise to use the term granted by the corresponding supplier of a retailer to represent the term received by the latter. However, it is almost impossible to identify who is the corresponding supplier of a given retailer.

Figure 6 shows the evolution of the logarithm of turnover days of accounts receivable and accounts payable over the 10 years of the sample period (1998 - 2007). The graph indicates that these two terms behaves quite differently to one another.

Figure 2  Sensitivity Analysis of the Retailer's Capital Cost

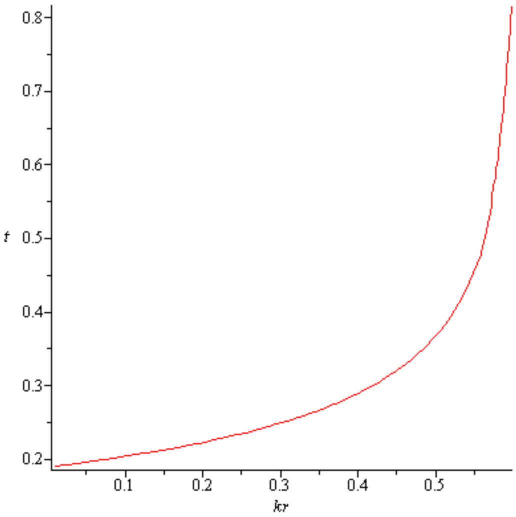



Figure 3  Sensitivity Analysis of the Supplier's Capital Cost

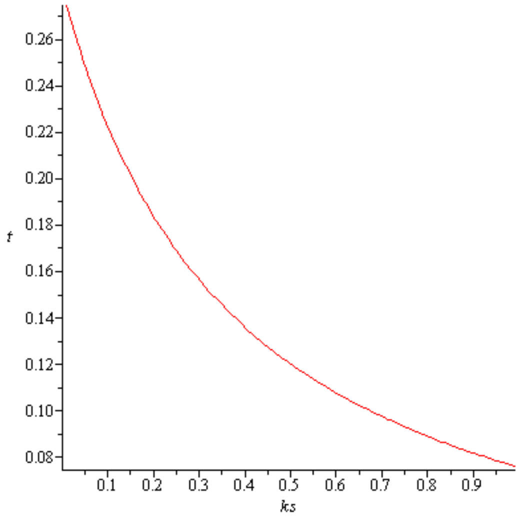



Figure 4  Sensitivity Analysis of the Retailer's Demand Rate

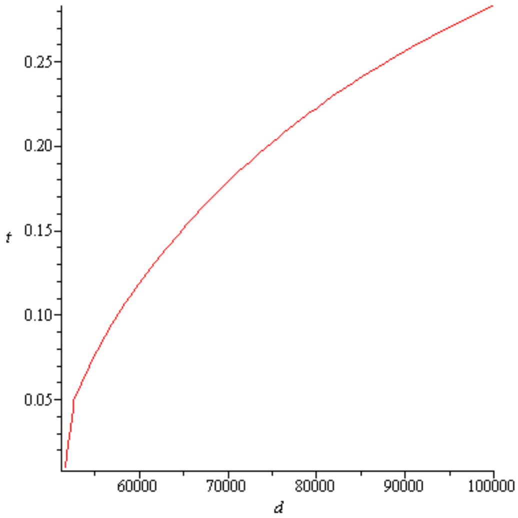



Figure 5  Sensitivity Analysis of the Retailer's Holding Cost

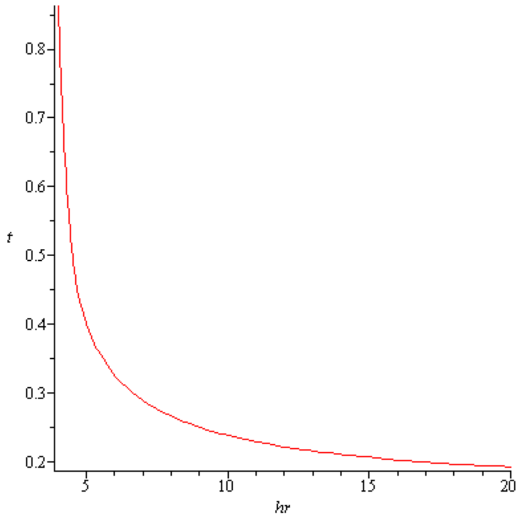
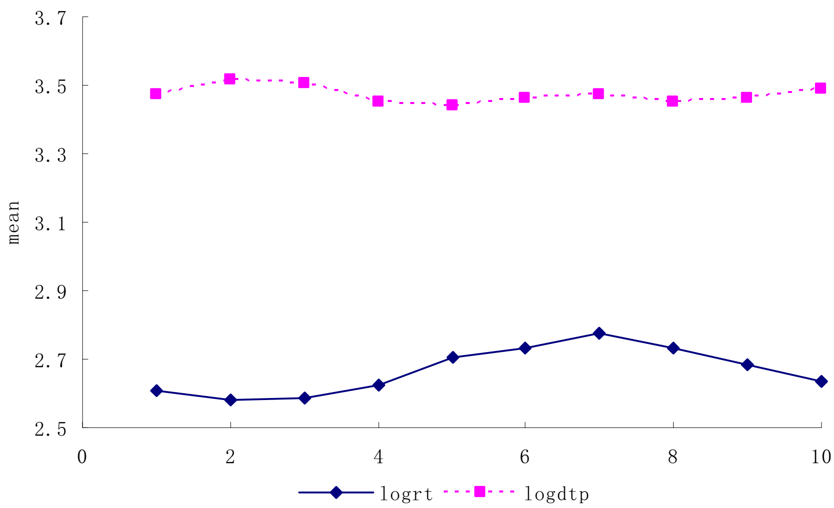



Figure 6  Two Kinds of Credit Terms During 1997 Through 2006



When we test Observations 1 and 2 through the perspective of the supplier, explanatory variables include financing cost (the variable *costdebt*) and market demanding (the variable *salesta*). We use *interest expense / (total current liability - accounts payable)* as the proxy of the financing cost, as accounts payable need not be covered by interest expense in practice. Regarding market demand, we use *sales / total assets* as its proxy. We expect a negative coefficient for the variable *costdebt* and a positive coefficient for the variable *salesta*.

When we test Observation 3 from the retailer perspective, the explanatory variable include holding cost (expressed in variable *hcost*). We use *inventory turnover days* $\times$ *financing cost in perspective of the retailer* as the proxy of holding cost. The logic behind this proxy is that, the holding cost per unit of inventory is its financing cost per unit of time. Thus for the entire inventory turnover period, the total holding cost for per unit inventory is simply the above expression. According to Observation 3, a negative coefficient accompanying the variable *hcost* must be expected.

We use a *Logistic* discrete choice econometric model to separately test the defaut risk effect. The dependent binary variable (*ngrant*) is specified as follows. For a company, if the term it receives is shorter than the term it grants, this implies that its supplier is rather strict, and the situation is quite similar to being not granted a trade credit at all, so that the dependent variable *ngrant* takes the value of 1. Otherwise, the variable ngrant assumes the value of 0, which implies that it is a net receiver of trade credit. Regarding the proxy of default risk, we choose Altman's Z-score, which is already available in COMPUSTAT database. As the Z-score is a decreasing function of default risk (Altman, 1968), we expect a negative accompanying coefficient.

Insert table 1 here.

Table 1 summarizes the dependent variables and indicators we choose as proxies for the four broad kinds of factors and the expected signs of their coefficients.

### 3.1 Data and Methodology

The data used in this study constitute an unbalanced panel, containing a total of 739 manufacturing firms for the period 1998-2007, drawn from COMPUSTAT database. The combination of a cross-section of information of N individuals (firms) with a time series for each present in the data sample, requires the panel data methodology to adequately express the non-observable heterogeneity of the individuals.

Three panel models with random effects are specified, in order to test the main results of the theoretical analysis as follows:

$$logrt_{it} = \alpha + \beta_1 costdebt_{it} + \beta_2 salesta_{it} + \beta_3 acagr_{it} + \beta_4 ebt_{it} + \beta_5 ta_{it} + \alpha_i + c_{it} \tag{21}$$

$$logdtp_{it} = \gamma + \zeta_1 hcost_{it} + \zeta_2 acagr_{it} + \zeta_3 ebt_{it} + \zeta_4 ta_{it} + \gamma_i + u_{it} \tag{22}$$

$$logit(ngrant_{it}) = \lambda + \kappa_1 zsmb_{it} + \kappa_2 acagr_{it} + \kappa_3 ebt_{it} + \kappa_4 ta_{it} + \lambda_i + w_{it} \tag{23}$$

where equation (22) is a panel logistic model; *acagr*, *ebt*, and *ta* are all control variables representing the growth rate, earnings capacity, and size attributes and expressed as *3 year Compound Annual Growth Rate of total asset, EBIT to total asset* and *logarithm of total asset* respectively; $\alpha$, $\gamma$, and $\lambda$ are intercepts; $\alpha_i$, $\gamma_i$, and $\lambda_i$ are unobservable effects; $e$, $u$ and $w$ are residuals; and $\beta$, $\zeta$ and $\kappa$ are coefficients.

Endogeneity is one of the key issues, and should be adequately addressed in panel model estimations. Arellano and Bover (1995) propose using the first differences of the variables as instruments for the equations in levels, in addition to the instruments in levels for the equations in first differences. Thus, we adopt the Generalized Method of Moments (GMM) estimation method, that allows the use of all moments' restrictions in instrumental variables for the first differences. Another important attribute of trade credit is its *stochastic lagging*, as explored in Benishay (1968). We propose using a 1-lagged dynamic panel model to capture this attribute.

### 3.2 Estimation Results

Table 2 shows the two-step GMM estimations of Equation (18), which test the effects of the financing cost and demand rate. The first differences of *costdebt* and *salesta* have been used as instruments for addressing endogeneity. The Sargan test is based on a two-step estimator obtained by GMM. The value obtained from the test indicates that the validity of the instruments cannot be rejected. M1 and M2 yield P-values corresponding to the first and second order serial correlation tests, respectively. Generally speaking, they also support the consistency of the model specifications.

In Table 2, Column 5 shows that most parameters are significant. The statistical significance and negative sign of the *costdebt* variable reveal that the supplier with a lower financial cost is able to grant a longer credit term. This provides supporting evidence for Observation 1.

Likewise, statistical significance and the positive sign of the *salesta* variable provide supporting evidence for Observation 2. It is also worth noting that a 1 lagged *logrt* is also significant and has a negative coefficient, which validates the theoretical proposition with respect to stochastic lagging of accounts receivable.

Insert table 2 here.

Table 3 shows the two-step GMM estimations of Equation (19), which tests Observation 3 on the effect of holding cost. A one-lagged value of *hcost* is used as instrument. The Sargan test and P-values of M1 and M2 strongly support the consistency of the model specifications. In Table 3, Column 5 yields a 1-lagged value of the dependent variable *logdtp*, the explanatory variable *hcost* and its 1-lagged value are all statistically significant and with negative signs. This provides supporting evidence for Observation 3 and for the stochastic lagging theory of accounts payable.

Finally, Table 4 shows the estimations of Equation (20), testing the effect of default risk. The LR and Wald *Chi* test show that the model specifications are acceptable. The variable *zsmb* has a negative coefficient and is significant at the 99.9% level. This implies that a decrease in the retailer's default risk reduces the possibility of a non-granting of the credit term by its corresponding supplier. As indicated in Table 4, the default risk of the retailer impacts significantly on the grant or non-grant decision of the supplier. Thus, the default risk evaluation also acts as a filtering criterion for selecting retailers eligible for trade credit. This provides quite strong evidence in favor of the validity of the framework of the present paper, for building the models on the cornerstone of default risk.

Insert table 3 here.

Insert table 4 here.

# 4 Conclusions

We have examined the optimal credit term decision in an extended EOQ framework with the main feature of modeling default risk as a two-stage lottery. The element of default risk is, for the first time, incorporated explicitly into the coordination modeling of EOQ under a permitted delay in payment. We argue that a principal-agent-type modeling process is particularly suitable for credit term determination, as trade credit functions as an interactive incentive mechanism in the form of price discounting, ordering stimulating, risks shifting, costs sharing, and surplus profit partitioning between supplier and retailer. This is a typical case in which incentive-compatible coordination is required. Furthermore, we adopt a cost minimization paradigm subject to acceptable assumptions, so as to model the retailer's ordering decision. This leads to a tremendous improvement of mathematic tractability of the models, and without a loss of generality. Accordingly, we establish a bilevel programming process, in which the supplier acts as the leader in the first level programming, in making decisions on credit terms under uncertainty, by maximizing its expected utility. The retailer then makes decisions on ordering quantity, reacting to the credit term in the second level programming, by minimizing its net cost. The incentive-compatible credit term is obtained at the equilibrium of this principal-agent game. We provide a FOC procedure in order to optimize the bilevel programming of the principal-agent game.

The sensitivity analysis in the numerical experiments has direct implications for the formation of appropriate trade-credit term policy in practice. We find that the capital cost of the supplier plays the vital role in determining the credit term, which is consistent with the well-known financing motive theory on trade credit. Furthermore, the incentive-compatible credit term is a decreasing function of the retailer's holding cost, which corresponds to the inventory factor, while it is an increasing function of the demand rate, which corresponds to the marketing factor. Furthermore, we find that default risk acts more as a filtering criterion, than as a determinant of the length of credit term.

All the main results of our models are supported empirically by estimations of three panel models using a manufacturing company dataset drawn from COMPUSTAT database. To some degree, our approach takes a step in the direction of modeling, in a coherent whole, the financial aspects (capital cost and default risk), production aspect (inventory) and marketing aspect (demand) of trade credit. These aspects have typically been modeled separately in the literature.

There are at least two direct extensions of the present models that deserve further investigation. The first is to relax the determinate situation premise for the retailer, and design another two-stage lottery to describe its uncertain situation, possibly caused by stochastic demand, lead time or shortage. However, this will dramatically increase the mathematical complexity of the model. The second extension entails furthering the application of two-stage lotteries to incorporate the subjective dimension, by adding the distortion function component. This would enable to extend the analysis into the fascinating arena of behavioral financial decision making.

## References


[1] Abad, P.L. and C.K. Jaggi (2003) *A Joint Approach for Setting Unit Price and the Length of the Credit Period for a Seller When End Demand Is Price Sensitive.* International Journal of Production Economics 83, 115-122.

[2] Benishay, H. (1968) *Parameters and Relations of Stochastically Lagged and Disaggregative Time Series.* Econometrica 36, 155-171.

[3] Bias, B. and C. Gollier (1997) *Trade Credit and Credit Rationing.* Review of Financial Studies 10, 903-937.

[4] Brennan, M., V. Maksimovic and J. Zechner (1988) *Vendor Financing.* Journal of Finance 43,1127-1141.

[5] Chapman, C.B., D.F. Cooper, M.J. Page and S.C. Ward (1984) *Credit Policy and Inventory Control.* Journal of Operational Research Society 35, 1055-1065.

[6] Chung K-J. and J-J Liao (2009) *The Optimal Ordering Policy of the EOQ Model under Trade Credit Depending on the Ordering Quantity from the DCF Approach.* European Journal of Operational Research 196, 563-568.

[7] Haley, C.W. and R.C. Higgins (1973) *Inventory Policy and Trade Credit Financing.* Management Science 20, 464-471.

[8] Jaber, M.Y. and I.H. Osman (2006) *Coordinating a Two-Level Supply Chain with Delay in Payments and Profit Sharing.* Computers and Industrial Engineering 50, 385-400.

[9] Jaggi, C.K., S.K. Goyal and S.K. Goel (2008) *Retailer's Optimal Replenishment Decisions with Credit-Linked Demand under Permissible Delay in Payments.* European Journal of Operational Research 190, 130-135.

[10] Pan, J C-H. and Yang, J-S (2002) *A study of an integrated inventory with controllable lead time.* International Journal of Production Research 40, 1263 - 1273.

[11] Perterson, M.A. and R.G. Rajan (1997) *Trade Credit: Theories and Evidence.* The Review of Financial Studies 10, 661-691.

[12] Schwartz, R.A. (1974) *An Economic Model of Trade Credit.* Journal of Financial and Quantitative Analysis 9, 643-657.

[13] Segal, U. (1987) *The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach.* International Economic Review 28, 175-202.

[14] Segal, U. (1990) *Two-Stage Lotteries without the Reduction Axiom.* Econometrica 58, 349-377.

[15] Sohn S.Y. and H.S. Kim (2007) *Random Effects Logistic Regression Model for Default Prediction of Technology Credit Guarantee Fund*. European Journal of Operational Research 183, 472-478.

**Table 1. Dependent variables, indicators of influencing factors and their expected effects on term decisions**

| Dependent variables and factors | Proxy indicators | Expected impacts on term |
|---|---|---|
| The term granted: *logrt* | Logarithm of turnover days of accounts receivable | Dependent variable |
| The term received: *logdtp* | Logarithm of turnover days of accounts payable | Dependent variable |
| Not grant: *ngrant* | If *logdtp* is small it takes the value of 1, otherwise it takes 0. We define *logdtp* as small, if it is less than *logrt*. | Dependent variable |
| financing cost of the supplier: *costdebt* | Interest expense/(current liability-accounts payable) | − |
| default risk of the retailer: *zsmb* | Z-score by Altman (1968) | − |
| market demand of the retailer: *salesta* | Sales/assets, in view of the supplier | + |
| holding cost of the retailer: *hcost* | Inventory turnover days × financing cost | − |

**Table 2. GMM Estimations of the Equation (20) to Test Effects of Financing Cost and Demand in View of the Supplier**

| logrt | Coefficients | Std. Err. | $z$ | $P > \|z\|$ |
|---|---|---|---|---|
| logrt(1-lag) | - 0.107 | 0.024 | - 4.400 | 0.000 |
| salesta | 0.279 | 0.021 | 13.190 | 0.000 |
| salesta(1-lag) | - 0.032 | 0.015 | - 2.200 | 0.028 |
| costdebt | - 0.015 | 0.004 | - 4.160 | 0.000 |
| costdebt (1-lag) | 0.001 | 0.002 | 0.380 | 0.704 |
| acagr | 0.000 | 0.000 | 4.640 | 0.000 |
| ebt | - 0.067 | 0.051 | - 1.320 | 0.188 |
| ta | 0.000 | 0.000 | - 0.370 | 0.712 |
| constant | 2.485 | 0.096 | 25.950 | 0.000 |
| NO. observations | 3789 | | Sargan test | 0.741 |
| M1 | 0.000 | | M2 | 0.132 |

**Table 3. GMM Estimations of the Equation (21) to Test Effects of Holding Cost from the Retailer Perspective**

| logdtp | Coefficients | Std. Err. | $z$ | $P > \|z\|$ |
|---|---|---|---|---|
| logdtp(1-lag) | - 0.176 | 0.036 | - 4.910 | 0.000 |
| hcost | - 0.011 | 0.001 | - 17.740 | 0.000 |
| hcost(1-lag) | - 0.003 | 0.001 | - 4.390 | 0.000 |
| acagr | 0.000 | 0.000 | 0.390 | 0.000 |
| ebt | 0.016 | 0.058 | 0.280 | 0.782 |
| ta | 0.000 | 0.000 | 0.190 | 0.845 |
| constant | 4.185 | 0.136 | 30.670 | 0.000 |
| NO. observations | 2768 | | Sargan test | 0.543 |
| M1 | 0.000 | | M2 | 0.001 |

**Table 4. Estimations of the Equation (22) to Test Effects of Default Risk from the Retailer Perspective**

| ngrant | Coefficients | Std. Err. | $z$ | $P > \|z\|$ |
|---|---|---|---|---|
| zsmb | - 0.285 | 0.086 | - 3.330 | 0.001 |
| acagr | 0.000 | 0.000 | 0.760 | 0.446 |
| ebt | - 0.542 | 0.776 | - 0.700 | 0.485 |
| ta | 0.000 | 0.000 | 2.990 | 0.003 |
| constant | 6.117 | 0.358 | 17.080 | 0.000 |
| NO. observations | 3867 | | Wald Chi 2 (4) | 21.44 (0.000) |
| Likelihood | - 1030 | | LR Test | 0.000 |