Pricing of Longevity Risk: The Case of China

Abstract: In this paper we use the Lee-Carter model to quantify longevity risk and to investigate the effect of longevity risk on pension and insurance pricing and liabilities in the context of China. We calculate the expected present value of life annuities for retired Chinese males and females, taking into account stochastic mortality development, revealing a significant impact of longevity risk on annuity pricing.

Keywords: Longevity risk, China, Lee-Carter model

1. Introduction

Like in most of the western world, the population of China has experienced a rapid aging over the past half century due to advances in public health, improved sanitation and personal hygiene, and general improvement in living standards (Lee, 2003; IMF, 2004). For example, the proportion of population aged 65 or older was only 4.41% in 1953 and 4.91% in 1982, respectively, but increased to 8.3% in 2008,[1] and by 2030 it will be more than doubled to 22% (James, 2002). Although age-specific death rates at all ages have declined exponentially at a constant rate in most developed countries (Tuljapurkar, Li, and Boe, 2000), it is the dramatically increased life expectancy at old age, along with lower fertility rates, that contributes to an increasing share of elderly people in the total population at a rapid rate in both OECD countries and emerging economies, most notably in China (Visco and d'Italia, 2006). In 1981, for example, a

[1] Data source: China Population and Employment Statistics Yearbook (2009).

60-year-old Chinese female individual had a life expectancy of 17.90 years, whereas in 2000 a 60-year-old female had a life expectancy of 19.62 years, representing an increase of 1.09 months per year (Zheng, 2005), i.e., more than five minutes per hour. The trends in improving mortality among the elderly are significantly challenging public pension plans as well as private pension funds and life insurers. In the U.K. and the U.S alone these institutions' exposure to longevity amounts to 400 billion USD in 2007 (Loeys, Panigirtzoglou, and Ribeiro, 2007). One more year of life expectancy at age 65 is estimated to add at least 3% to the present value of the pension liabilities in the U.K. (Biffis and Blake, 2009). In developing countries, including China, where pension systems are underdeveloped, these trends also significantly affect personal saving to fund retirement consumption. For example, consider a fairly-priced annuity with annual payoff ￥1 at the real interest rate of 3% in China. Then in 1981 the annuity price for a 60-year female should have been ￥14.11, but could have increased by 7% to ￥15.11 in 2000. Without other retirement income, this means that the 60-year-old females in 2000 should have saved 7% more to finance their retirement consumption than in 1981.[1]

Though the views regarding the outlook for human longevity are still controversial (Antolin and Blommestein, 2007),[2] the general opinion from the experts tends to be the presence of upward trends in longevity. However, there is a large degree of uncertainty concerning the improvement magnitude, especially at older ages.

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1 The result is based on author's own calculation.

2 For example, Olshansky et al. (2005) believe that there are natural limits to life expectancy, and suggest that the increase in life expectancy will slow down if not to stop; On the other hand, Oeppen and Vaupel (2002) argue that there are no limits to life expectancy and conclude from historical trends and age trajectories that longevity would keep increasing in the next decades.

From 1970 to 2000, the average increase in life expectancy of a 65-year-old male was 1.12 years/decade in the U.S. and 1.23 years/decade in the U.K., respectively, but the corresponding increase had only been 0.15 years over the previous decade in the U.S. and 0.17 years/decade over the previous century in the U.K. (Cocco and Gomes, 2008). China also experiences this uncertainty. According to Zhang (2005), in the 1980s the average increase in life expectancy of a 60-year-old Chinese male was 0.06 years per year, but increased to 0.09 years per year in the 1990s. Therefore, the major challenge faced by policy-makers, pension/insurance institutions, and individuals is not the trend in longevity itself, but rather be the uncertainty around the trend in life expectancy in the future (De Waegenaere, Melenberg, and Stevens, 2010). When future life expectancy outcomes and mortality improvement turn out to be different from anticipated, longevity risk occurs.

Cocco and Gomes (2008) find that, when individuals use official period life tables, -which do not allow for future life expectancy improvement-, to make their retirement finance decision, the effect of longevity improvement on individual welfare can be significant. Moreover, the importance of longevity risk for the liabilities of private pension funds and annuity providers is that increasing portfolio size can only mitigate but cannot eliminate this risk. Therefore, several innovative solutions to longevity risk through the financial system, namely, reinsurance (Richards and Jones, 2004), natural hedging (Cox and Lin, 2007), or securitization (Cowley and Cummins, 2005), are being discussed. But all these solutions require better understanding of future mortality development.

In this paper we use the Lee-Carter model and its alternative approach to quantify longevity risk and to investigate the effect of longevity risk on pension and insurance pricing and liabilities in the context of China. On the one hand, as the largest country in terms of population, China has been experiencing a faster decline in mortality among the elderly since the 1964-82 periods than the now low-mortality countries at comparable levels of overall mortality (Banister and Hill, 2004) and with its increasing prosperity these trends might be expected to continue. On the other hand, since most populations in developed countries (at least) is covered by the public pension systems and the systematic pension forms in these countries reach maturity, the focus in is more concentrated on the *distribution stage*; namely, depending on pension laws, people at retirement age receive their pension benefit either as lump sum, programmed withdrawal or as an annuity. On the contrary, in China the systematic pension reforms have entailed a significant downsizing of public pension pillars and an expansion of private provision in the form of individual accounts in defined-contribution plans. Since the overall pension coverage rate in China is still rather low, for example, in 2008 among its 302 million urban employees and 473 rural employees, only 55% and 12% were covered by the public pension system, respectively (Oksanen, 2010), the attention is almost exclusively paid on the *accumulation stage*, namely extending the pension coverage rate. Given the low (public) pension coverage and paramount importance of ensuring the safety and efficiency of the accumulated stage, the longevity risk is generally ignored by the public and policy makers. These facts characterize the severity of longevity risk in

China. Since most existing literature has scrutinized longevity risk in developed countries with fewer having sought to understand it in the context of the developing world, our first contribution is to fill in this gap. By quantifying this risk and assessing its impact on annuity pricing, we attempt to highlight the importance of longevity risk, to increase the public's awareness and understanding of longevity risk in the developing world, and to contribute to the current public pension reforms and product design in China. Second, following De Waegenaere, Melenberg, and Stevens (2010), and Hari, De Waegenaere, and Melenberg (2008), in this paper we distinguish diversifiable *individual mortality risk* and non-diversifiable *longevity risk*, investigating the impact of both risks on pension funds and annuity providers. Unlike its counterparts in the developed countries, the current public pension plans in China are decentralized to the local governments.[1] The relative small portfolio of each public pension plan means that these plans might face both risks. Third, since the Chinese statistical data on mortality are comparatively limited, the sampling inaccuracy might cause parameter risk, a special case of model risk arising due to the lack of knowledge regarding the true probability distribution of future mortality rates. Moreover, even though we might exactly know the true probability distribution of the future mortality rates, the uncertainty in the mortality trends still remains and may result in the process risk. We contribute to the existing literature on China's longevity risk by taking account of these risks. Finally, our research on the probability distribution of future mortality is important for China to respond to longevity risk

[1] It is reported by *China Business News* (15 August 2008) that the public pension plans in China now are organized by around 2,000 entities. Even though people are free to relocate, their pensions are not allowed to transfer freely, especially between provinces.

through other innovative channels such as securitization.

The remainder of this paper proceeds as follows. In section two, we introduce the source of longevity risk and its impact on annuity pricing. In section three, we present the data and the Lee-Carter model. In the next section, we show the estimation results, taking account of process risk and parameter risk. We analyze the impact of longevity on annuity pricing in section five. Finally, section six offers some concluding remarks.

2. Introduction to Longevity Risk

The uncertain mortality development may cause two kinds of risk, namely *longevity risk* and *individual mortality risk*.[1] According to Dahl (2004), *longevity risk* results from changes in the underlying mortality density, whereas *individual mortality risk* results from the random individual deaths with a fixed mortality density. For better understanding of the distinction between the two risks, see also De Waegenaere, Melenberg, and Stevens (2010), we first introduce some scientific notation and terminologies of mortality.

2.1 Scientific Notations and Terminologies

The two basic building blocks of our projection of future life expectancy are the one-year death probability, denoted by $q_{x,t}^{(g)}$, and the central death rate, denoted by $m_{x,t}^{(g)}$. The one-year death probability, $q_{x,t}^{(g)}$, defines the probability that a x-year old person

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[1] In some literatures, the two kinds of risk are also named as systematic longevity risk and unsystematic longevity risk, respectively. In order to highlight the non-diversification of the former, following De Waegenaere, Melenberg, and Stevens (2010), we use *longevity risk* to indicate systematic longevity risk and *individual mortality risk* to unsystematic longevity risk.

belonging to group g (female or male; rural or urban) will die within one year in year t.

The central death rate is defined by

$$m_{x,t}^{(g)} = \frac{D_{(x,t}^{(g)}}{E_{x,t}^{(g)}}, \tag{1}$$

where $D_{x,t}^{(g)}$ denotes the death number of people belonging to group g at age x in year t, while $E_{x,t}^{(g)}$, also called exposure, denotes the number of person years in group g at age x in year t.

Since both $D_{x,t}^{(g)}$ and $E_{x,t}^{(g)}$ can be obtained from the national statistics, we could obtain the one-year death probability, $q_{x,t}^{(g)}$, from the central death rate, $m_{x,t}^{(g)}$ (McCutcheon and Nesbitt, 1973). In the general case, this relationship is complicated, but can be simplified with appropriate additional assumptions. For example, under the assumption that the central death rate equals to the force of mortality,[1] we could establish the following relationship

$$q_{x,t}^{(g)} = 1 - \exp(-m_{x,t}^{(g)}), \tag{2}$$

With the one-year death probability, we could also obtain one-year survival probability, i.e., the probability that a x-year old individual belonging to group g survives at least another year in year t, by

$$p_{x,t}^{(g)} = 1 - q_{x,t}^{(g)}, \tag{3}$$

Under the assumption of *constant* time-independent mortality rates and one-year death probabilities over time, the one-year death (survival) probabilities

[1] The force of mortality, often referred to as the hazard function in other fields such as in reliability theory, is defined as $\mu_x = \lim_{\Delta x \to 0} \frac{P(x < X \le x + \Delta x \mid X > x)}{\Delta x}$ and specifies the instantaneous rate of death for x-year old people belonging to group g in year t, given that these individuals survive up to age x.

would be independent of time and thus the subscript t can be suppressed. In this case, we could calculate the probability that a x-year old individual belonging to group g survives at least τ years, ${}_{\tau}p_x^{(g)}$, and the corresponding remaining life expectancy for this individual, $e_x^{(g)}$, as follows.

$$ {}_{\tau}p_x^{(g)} = \prod_{i=0}^{\tau-1} p_{x+i}^{(g)} , \tag{4} $$

$$ e_x^{(g)} = \sum_{\tau \geq 1} {}_{\tau}p_x^{(g)} , \tag{5} $$

where ${}_1p_x = p_x$. From the time point of year t, this individual is expected to die in $e_x^{(g)} + t$ years at the age of $e_x^{(g)} + x$.

However, the results above, based on the assumption of *constant* one-year death probabilities and mortality rates over time, might not be correct: neither are they constant over time, nor do they change in the same direction and at the same magnitudes for different cohorts. Figure 1 presents the mortality rates of selected age groups for different time periods, normalized to one for the year 1981.

[Insert Figure 1 here]

At least over longer time horizon, both Chinese females and males in these selected age groups experience significant mortality improvement, reflecting the increase in longevity over time. On the one hand, these improvements are different in terms of gender, ages, and years. On the other hand, at least to some extent, these improvements seem to be random, reflecting the stochastic chrematistics of the death probabilities. Since the death probabilities are not *constant* over time but rather *stochastic*, it’s inappropriate to use (4) and (5) for calculating the remaining life expectancy of a x-year old individual belonging to group g in year t. With varying death probabilities, the survival probability of a x-year old individual belonging to group g for at least τ years in year t should follow

$$ {}_{\tau} p_{x}^{(g)} = \prod_{i=0}^{\tau-1} p_{x+i,t+i}^{(g)} , \tag{6} $$

The corresponding remaining life expectancy for this individual in year t should be calculated by

$$ e_{x}^{(g)} = \sum_{\tau \geq 1} {}_{\tau} p_{x,t}^{(g)} , \tag{7} $$

Both (6) and (7) need future death probabilities that are unobservable for the current period. Thus, when using current death probabilities rather than the projected ones, the expected life expectancy as well as the discounted value of pension liabilities might be underestimated. See, for example, Hari et al. (2008). Moreover, it is appropriate to assume the future death probabilities in a stochastic way instead of a deterministic way. With the assumption of stochastic future death probabilities, therefore, the *longevity risk* resulting from the long-term deviation from deterministic mortality improvement is unavoidable (De Waegenaere, Melenberg, and Stevens, 2010).

2.2 Significance of Longevity risk

Assuming a finite number of scenarios for the evolution of future mortality probabilities, many studies (Olivieri, 2001; Coppola, Di Lorenzo, and Sibillo, 2000, 2003a, and 2003b) find that even when the size of portfolio is increased, the *longevity risk* cannot be diversified and does not disappear, whereas the *individual mortality risk* is diversifiable. In most developing countries the pooling size of pensions is relatively small. Thus, unlike their counterpart in developed world, the pension systems in developing countries typically might face both risks.

In order to demonstrate the non-diversifiable characteristics and significance of *longevity risk* and its distinction from *individual mortality risk*, we consider a pension plan composed of N x-year old immediate lifetime annuitants belonging to group g in year t. For simplicity, we assume that each annuitant gets one Chinese Yuan per year after retirement conditional on his/her survival, with a constant risk-free interest rate r. Thus, in year $t+\tau$ ($\tau \geq 1$) the present value of the future payment to annuitant i should equal

$$Y_i = \sum_{\tau \geq 1} 1_{i,t+\tau} \frac{1}{(1+r)^{\tau}}, \tag{8}$$

where $1_{i,t+\tau}$ donates a dummy variable with value equal to one if annuitant i is still alive in year $t+\tau$.

We first only consider *individual mortality risk*, namely that the future mortality improvements are known with certainty. In year t, the expected present value of the future payment to annuitant i is thus given by

$$A_{x,t} = \sum_{\tau \geq 1} E(1_{i,t+\tau}) \frac{1}{(1+r)^{\tau}} = \sum_{\tau \geq 1} {}_{\tau} p_{x,t} \frac{1}{(1+r)^{\tau}}, \tag{9}$$

According to the pooling argument, $A_{x,t}$ should be the fair price of this annuity and the fair price of Y_i should be the same as the fair price of $\frac{1}{N}\sum_{i=1}^{N} Y_i$. Under the assumption of independent annuitants, we can get the following variance

$$Var\left(\frac{1}{N}\sum_{i=1}^{N} Y_i\right) = \frac{\sigma^2}{N}, \tag{10}$$

where we take $\sigma^2 = Var(Y_i)$ and $\mu = E(Y_i)$.

Obviously, with increasing pooling size, the variance of $\frac{1}{N}\sum_{i=1}^{N} Y_i$ approaches zero, if known risk free, so its fair price equals its expected present value, without risk premium. With certain future death probabilities, pension plans, and insurance companies only face *individual mortality risk* that can be eliminated by pooling.

When the future death probabilities are uncertain, however, *longevity risk* becomes dominant. We continue with the pension plan composed of N x-year old immediate lifetime annuitants belonging to group g in year t, given the set of future death rates in year t by $f_t = \{q_{x,t+\tau}^{(g)} \mid \tau \geq 1\}$. We follow the assumption of independent annuitants in *individual mortality risk* above but have different mean and variance both depending on f_t, i.e. $\mu(f_t)$ and $\sigma^2(f_t)$. Thus, (10) should be replaced by

$$Var(\frac{1}{N}\sum_{i=1}^{N} Y_i) = E[Var(\frac{1}{N}\sum_{i=1}^{N} Y_i) \mid f_t] + Var[E(\frac{1}{N}\sum_{i=1}^{N} Y_i \mid f_t] = \frac{E[\sigma^2(f_t)]}{N} + Var[\mu(f_t)]. \tag{11}$$

With increasing pooling size, the first term on the right side of (11) can still be eliminated, but the second term continues to exist, independently of N. With the existence of *longevity risk*, the pooling argument cannot eliminate mortality risk any more and a risk premium should be included into the pricing of financial products whose payoffs depend on the future mortality development.

In the context of China, on the one hand, the underdeveloped pension systems and low coverage might mean that both *individual mortality risk* and *longevity risk* exist. On the other hand, the incomplete market and non-diversifiable characteristics make the pricing of longevity risk and risk management more difficult in China than in developed countries.

3. Lee-Carter Models and Data

In this paper, we only discuss purely statistical mortality models, without considering other exogenous demographic or epidemiological factors, because pension funds and annuity providers are much more interested in "all-cause" mortality (Hari, 2006).

Generally, the stochastic mortality models are more parsimonious,[1] trying to explain the death rates with unobserved latent factors. Actually, when looking at sequences of mortality curves over a relatively long horizon, we can easily find that they change unpredictably, not only from one period to another, but also over the long term, though they do exhibit a general trend. Thus, it is more accurately to model the mortality in a stochastic fashion. Among these stochastic models, the Lee-Carter model (1992) has become the "leading statistical model of mortality in the demographic literature" (Deaton and Paxson, 2004) and, along with its extensions, has been widely applied for many developed countries for its simplicity and robustness in the context of linear trends in age-specific death rates, for example, Japan (Wilmoth, 1993), G7 countries (Tuljapurkar, Li, and Boe, 2000), Australia (Booth, Maindonald, and Smith, 2002), England and Wales (Renshaw and Haberman, 2003), Belgium (Brouhns, Denuit, and Vermunt, 2002), and the Netherlands (Hari et al., 2008; De Waegenaere, Melenberg, and Stevens, 2010). However, the existing literature using the Lee-Carter model for developing countries including China is rather limited and incomplete, partly due to the unavailability of data. To our knowledge, Hou, Yu, and Chen (2000) are the first to apply the Lee-Carter model to Chinese population. Using

[1] There are two types of mortality models: deterministic model and stochastic model. Starting from De Moivre (1724)1, the deterministic approach (Gompertz, 1825; Makeham, 1860; Heligman and Pollard, 1980) typically only considers the age dimension, though recent models try to fit mortality rates in both of age and of time dimension. However, since this kind of approach usually does not take account of uncertainty and also the accurate in-sample fit is translated into only small prediction intervals, it has not seemed to be very realistic in practice.

the mortality data on the rural males during 1988-1994,[1] they project the mortality movement for 1995-2000.The comparisons between the predicted mortality rates and the realized ones indicate that the Lee-Carter model has a significant prediction power. Yin (2005) estimates the Lee-Carter model with the mortality rates of Chinese males and females for 1986-2002. Since the Chinese mortality data might follow different ARIMA process, she first tests the appropriate model specification with the estimated coefficients and then compares the projected life expectancy at birth with the official prediction. Zhu and Chen (2009) use the Lee-Carter extension model and 1989-2006[2] mortality data to project the mortality dynamics of the urban population. Nevertheless, these researches allow for neither parameter risk not process risk. Furthermore, they only focus on a specific population group, for example, urban population or rural population, or a specific gender group.

3.1 Lee-Carter Model

According to Lee and Carter (1992), the log central death rate of the x-year-old persons in year t, $\ln(m_{x,t})$, is determined by a common latent factor κ_t, with an age-specific level parameter, α_x, and an age-specific sensitivity parameter, β_x. Mathematically, the model can be expressed as follows:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \tag{12}$$

where the white noise error terms, $\varepsilon_{x,t}$, represent the transitory non-systematic shocks.

Obviously, the OLS method cannot be applied to the Lee-Carter model because none of the variables on the right hand of equation (12) are observable. In

1 The mortality data for 1991 are missing.
2 The mortality data for 1987-1988, 1991-1993, and 2000 are missing.

order for a unique solution, Lee and Carter first normalize the sum of β_x terms to unity and κ_t terms to zero, i.e., $\sum_x \beta_x = 1$ and $\sum_\kappa \kappa_t = 0$, and get the value of α_x since it becomes the average value of $\ln(m_{x,t})$ over time. Then they use a two-stage approach to solve this under-identification problem. The singular value decomposition (SVD) approach is used in the first stage for the matrix of $\ln(m_{x,t}) - \hat{\alpha}_x$ to get estimates of κ_t and β_x. In the second stage, given the value of $\hat{\alpha}_x$ and $\hat{\beta}_x$, $\hat{\kappa}_t$ is re-estimated by iteration until the implied death number equals the actual death number such that

$$\sum_x D_{x,t} = \sum_x \left\lfloor E_{x,t} \exp(\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t) \right\rfloor, \tag{13}$$

Nevertheless, in the first stage of this two-step procedure above a weighted singular value decomposition could also be used (Wilmoth, 1993). Moreover, Lee and Miller (2001) proposed using a matching on the basis of observed and modeled life expectancy rather than the matching according to (13). In addition, in order to avoid the violation of the assumption of constant α_x and β_x, Booth, Maindonald, and Smith (2002) suggest using statistical techniques to select an appropriate sample period.

Originally, Lee and Carter find that κ_t satisfies a random walk with drift process as:

$$\kappa_t = \kappa_{t-1} + c + \xi_t, \tag{14}$$

where the white noise term, ξ_t, representing permanent shocks, is assumed to be independent of $\varepsilon_{x,t}$ and to follow a normal distribution with mean zero and variance of σ_ζ^2. With standard statistical or econometric time-series techniques, the parameters in (14) can be estimated. However, the ARIMA process of κ_t for other countries might be different from (13). For example, Yin (2005) finds that the Chinese male process follows ARIMA (0,1,1), whereas the Chinese female process is a random walk. Thus, standard statistical procedures should be applied to find an appropriate ARIMA model for the time series of κ_t (Liu, 2008).

In this way, the systematic path of the central mortality rate of the x-year-old persons in year t satisfies:

$$\hat{m}_{x,t} = \exp(\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t), \tag{15}$$

In order for the projection of future mortality, we firstly need to forecast the future values of $\tilde{\kappa}_{T+\tau}$ (T is the final year of the sample) and then the systematic path of future central mortality rate by

$$\hat{m}_{x,t+T} = \exp(\hat{\alpha}_x + \hat{\beta}_x \tilde{\kappa}_{T+\tau}), \tag{16}$$

In order to avoid a jump-off bias, Lee and Miller (2001) alternatively propose using the observed (raw) central death rate of the final year in the sample as a jump-off value to predict the future central death rates such that

$$\tilde{m}_{x,t+T} = m_{x,t} \exp(\hat{\beta}_x (\tilde{\kappa}_{T+\tau} - \hat{\kappa}_T)).\,, \tag{17}$$

With the assumption that the force of mortality does not change during a year, i.e., $m_{x+s,t+s} = m_{x,t} (0 \le s < 1)$, the survival probability of one more year for one x-year-old person at time t is calculated by (2) and (3). From (6) and (7), we can obtain the projection of life expectancy at different ages. Undoubtedly, there might be several risks in our projection resulting from the scholastic nature of $\tilde{\kappa}_{t+T}$. First, since neither the true value of $\tilde{\kappa}_{t+T}$ nor its distribution is known at time T, the *process* *risk* might arise. Second, limited sample size and measurement error might cause inaccurately estimated coefficients of α_x, β_x and κ_t, which generates *parameter risk*. In addition, without knowing exactly the true distribution of $\hat{\kappa}_{t+T}$, but having to model it, there might cause *model risk*. For methods quantifying these risks, see Koissi, Shapiro, and Hognas (2006), Renshaw and Haberman (2008).

3.2 Data

Our data include 15 yearly observations of age-specific death and population counts for both males and females in China during the period of 1994-2008, provided by the China Population Statistical Yearbooks and the China Statistical Yearbooks compiled by the National Bureau of Statistics of China. Thus, we can obtain the age-specific central death rates through (1). However, it should be mentioned that the statistical methods obtaining these count numbers are inconsistent for each sample year. For example, the death and population counts in 2000 are based on national population census, while in other years on random samples of the population or sample survey on population changes. Additionally, for each year the data are different in terms of the last age category, with most being age group 85 and over. Since we are mainly concerned with the impact of longevity risk on pension and annuities, namely the impact of the uncertain life expectancy after retirement on pension and annuities, the missing mortality data at older ages might cause inaccurate estimation. Therefore, we need to first use the available mortality data to estimate the central death rates at older ages.

The patterns of mortality at older ages have been well documented in many studies (Horiuchi and Wilmoth, 1998; Thatcher, Kannisto, and Vaupel, 1998; Zeng and Vaupel, 2003). For example, Thatcher, Kannisto, and Vaupel (1998) use the observed population size and number of deaths for age 80-98 to examine the force of mortality, i.e. central death rate, at the oldest-old ages in 13 developed countries across several recent decades. By extrapolating for ages beyond 98, they find the mortality patterns are modeled better with Logistic, Kannisto, and quadratic methods than with Gompertz, Weibull, and Heligman and Pollar models for all the countries and across all these decades. Zeng and Vauple (2003) use the same methods to investigate mortality pattern for the oldest-old Han Chinese people in the 1990 census and find similar results. Since the Kannisto model works better than other model in fitting mortality pattern at old ages (Thatcher, Kannisto, and Vaupel, 1998), in this paper we use the Kannisto model proposed by Kannisto et al. (1994) to fit our available data to extrapolate the mortality rates at older ages. The model is expressed as

$$m_x = \frac{\alpha \cdot \exp(\beta x)}{1 + \alpha \cdot \exp(\beta x)}, \tag{18}$$

where m_x is the observable central death rate at age x; α and β are the two parameters that need to be estimated.

Since the Kannisto model is not supposed to fit the mortality data on the whole age range, we first use the *life-table ageing rate* defined by Horiuchi and Cole (1990) to choose our fitting age range. This measure is given by

$$k_x = \frac{\ln(m_{x,x+5}) - \ln(m_{x-5,x})}{5}, \tag{19}$$

where $m_{x,x+5}$ and $m_{x-5,x}$ are central death rates for successive 5-year of age. To calculate the confidence intervals, the corresponding standard error could be given approximately by

$$\sigma_x = \frac{1}{5}\sqrt{\frac{1}{D_{x,x+5}} + \frac{1}{D_{x-5,x}}}, \tag{20}$$

where $D_{x,x+5}$ and $D_{x-5,x}$ are the death counts in the age intervals between x and $x+5$ and between x-5 and x, respectively.

[Inset Table 1 Here]

Table 1 presents the estimation results based on (19) and (20). For each gender group, the life table ageing rates and their corresponding confidence intervals from age 60 to 95 are reported for each sample year. Since the increase (decrease) in k_x implies an acceleration (deceleration) in the age pattern of mortality (Horiuchi and Wilmoth, 1998), the lower age limit of our fitting range should be the one at which the estimate of life table aging rate falls off. Generally, the life-table aging rate begins to decrease at ages varying from 70 to 90. As a result, we use the Kannisto model to fit the mortality data from the age of 70 to the maximum age available in each year and extrapolate the central death rates up to age 120 for each sample year during the period of 1994-2008. Following Roli (2008), we replace the observed death rates for all ages at or above $\bar{x}$, where $\bar{x}$ is the lowest age at which there are fewer death counts than 100 but should satisfy $80 \le \bar{x} \le 95$.

Now, our dataset covers the age-specific mortality rate from age 0 to 120 during the period of 1994-2008. Figure 2 shows the logarithm of the central death rates of Chinese males and females for ages zero to 120 for the sample period

1994-2008. Like in most countries, the mortality pattern for each year in China firstly starts rather high for newborn infants and goes down at around age 15, then increasing again with the accident hump at around age 20-25.

[Inset Figure 2 Here]

4. Estimation Results

Using the singular value decomposition (SVD) approach, we firstly estimate the values of α_x, β_x and κ_t, respectively.

[Inset Figure 3 Here]

Figure 3 plots the estimated α_x (left panel) and β_x (right panel), respectively. Since α_x is the average value of $\ln(m_{x,t})$ over time, it can be interpreted as the mean age profile of mortality. The estimated α_x shows the similar mortality pattern as figure 2. Moreover, for most ages the estimated α_x for females is smaller than or equal to that for males, explaining the fact that females, on average, face a longer life expectancy than males. As the loading factor, β_x measures the age-specific response to the changes in the latent factor, κ_t. For example, a low (high) value of β_x represents slowly (rapidly) decrease of mortality at specific age if κ_t declines over time. Our estimation results (see right panel), after being smoothed, show that, even though the Chinese females show an overall declining sensitivity to the mortality movement as ages, indicating a diminishing increase in life expectancy, they still react more sensitively than the males before the age of 60. On the contrary, the U-shaped β_x curve of the males shows that the young and older populations are more sensitive to the mortality movement, with the middle aged population the least. It seems that older males experience much bigger mortality improvement κ_t declines over time during the sample period. Moreover, the smoothed curves follow different patterns for males and females, with the hump at around 10-30 for females.

[Insert Figure 4 Here]

Figure 4 plots the estimated κ_t, which, according to our expectation, shows the decreasing trends over time generally. In order to determine the appropriate ARIMA model for the time series of κ_t, we use the Augmented Dickey-Fuller test and the Phillips-Perron Test to check stationarity. Panel A of table 2 shows the test results and confirms the existence of a unit root for the κ_t processes in level, but stationarity after first differencing. Thus, the κ_t processes for both Chinese males and females seem to be integrated of order one. Moreover, from panel B of table 2 we conclude that both the male and female κ_t processes follow a random walk, which differs from previous findings. For example, Yin (2005) finds that the male process is an ARIMA (0, 1, 1) process. We use the OLS method to estimate (14) and the estimation results of κ_t process for males and females are shown by (18) and (19), respectively.

$$\kappa_t^{(m)} = -2.5793 + \kappa_{t-1}^{(m)} + e_t^{(m)}, \text{ with } \hat{\sigma}_{\xi}^{(m)} = 3.3846 \tag{18}$$

$$\kappa_t^{(f)} = -5.1769 + \kappa_{t-1}^{(f)} + e_t^{(f)}, \text{ with } \hat{\sigma}_{\xi}^{(f)} = 2.8483 \tag{19}$$

[Insert Table 2 Here]

Based on the results above, we can use project future values of $\tilde{\kappa}_{t+T}$ with (18) and (19) for males and females, respectively, and then calculate projected one-year death probabilities and life expectancies at different ages according to the relevant equations above.

We now show the longevity risk resulting from *process risk* and *parameter risk* through predicting the logarithm of the central death rate beginning from 2009, the first year after our sample period. Due to the random walk of the estimated κ_t, we can use the Girosi and King (2006)-variant of the Lee-Carter model to illustrate these risks (see appendix) because the T-asymptotic characteristics of the estimator based on this variant imply that making predictions as well as quantifying the longevity risk becomes a standard exercise in statistics or econometrics (De Waegenaere, Melenberg, and Stevens, 2010).

We show the observed and 30-year ahead prediction of the logarithm of the central death rate for 60-year old Chinese with parameter risk and process risk in figure 5 and figure 6, respectively. The prediction begins from 2009, the first year after our sample period. In figure 5, the two cases, i.e., only parameter risk and the combination of parameter risk and process risk, are taken into account, whereas figure 6 considers only process risk and the combination of process and parameter risk, using computing 95% confidence intervals. Both figures show clearly the downward trends not only in sample, but also out-of-sample, predicting mortality improvements in the future. For example, at the beginning of our sample (1994), the one-year death probability calculated based on (2) for one 60-year old male is 0.0141, but decreases to 0.074 in 2038, representing around 47% in just four decades. However, figure 5 and 6 also show the high uncertainty about future mortality movement in terms of direction and magnitude.

[Insert Figure 5 and 6 here]

5. Pricing of Longevity Risk

In this section we investigate the impact of longevity risk on public pension plans as well as private pension funds and life insurers by calculating the expected present value of a life annuity in different scenarios through simulation. We assume that each annuitant gets one Chinese yuan per year after retirement, conditional on his/her survival, with a constant risk-free interest rate r or under term structure of interest rate of government bond. Thus, in year $t+\tau$ ($\tau \geq 1$) the present value of the future payment should follow (8). In order to highlight the impact of scholastic death probabilities on the annuity price, we also calculate the expected present value of a life annuity under the assumption of constant one-year death probabilities based on (4) and (5).

[Insert Table 3 Here]

Table 3 presents the simulation results for the annuity price in different scenarios with corresponding 95% confidence intervals in parenthesis. Column (1) and (5) show the expected present value of a life annuity for 60-year old Chinese males and females in 2009 under constant death probabilities, respectively. Without accounting for the stochastic death probabilities and future mortality improvement, these results are unsurprisingly lower than those in the framework of stochastic death probabilities. For example, with interest rate of 3%, the life annuity price for 60-year old female in 2009 is only 15.17 based on the assumption of constant death probabilities; however, when stochastic mortality development is taken into account, the same person should pay, at least, 15.94 yuan or around 5% more for the same annuity product. Therefore, without taking into account longevity risk and its randomness when designing pension systems or products, the impact of longevity risk on risk management would be substantial.

6. Conclusions

In this paper we use the Lee-Carter model to quantify longevity risk and to investigate the effect of longevity risk on pension and insurance pricing and liabilities in the context of China. Unlike previous research, we find that both the latent factor process for Chinese males and females also follows a random walk. In addition to *process risk*, resulting from the unknown distribution of the latent factor in the future, we also take into account the impact of *parameter risk* on our projection of future mortality development. Though the future mortality development shows a strong downward trend, it also presents substantial uncertainties when *process risk* and *parameter risk* are involved. In order to investigate the impact of longevity risk on pension plans and insurance companies, we simulate the expected present value of life annuity for 60-year old Chinese males and female beginning from 2009. As comparison, we also calculate the corresponding annuity price under constant death probabilities for comparison. Our simulated results show that, without taking into account the stochastic mortality development in the future, the pricing of life annuity products would be underestimated, significantly challenging public pension plans as well as private pension funds and life insurers.

As the world’s largest country in terms of population, China has experienced a rapid aging over the past half-century and thus the Chinese government is reforming its pubic pension system to meet the urgent challenges of an ageing society. Since (public) pension coverage is still low in China, compared with other developed economies, much attention in China now is almost exclusively paid to the *accumulation stage*, with the ignorance of longevity risk by the public and policy makers. However, this paper reveals the significant impact of longevity risk on risk management and pension/annuity pricing. Thus, increasing awareness and understanding of longevity risk by the public, especially the policy makers, would contribute to the current public pension reforms and product design in China.

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Appendix: The Girosi and King (2006)-variant of the Lee-Carter model

First of all, let

$$l_t = \begin{pmatrix} \ln(m_{1,t}) \\ \vdots \\ \ln(m_{ma,t}) \end{pmatrix},$$

where ma stands for the maximum age.

Then, let

$$\alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{ma} \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_{ma} \end{pmatrix}, \text{ and } \quad \varepsilon_t = (\varepsilon_{1,t}, \cdots, \varepsilon_{ma,t})$$

Now, from

$$l_t = \alpha + \beta\kappa_t + \varepsilon_t \quad \text{and} \quad \varepsilon_t = \mu + \varepsilon_{t-1} + \delta_t$$

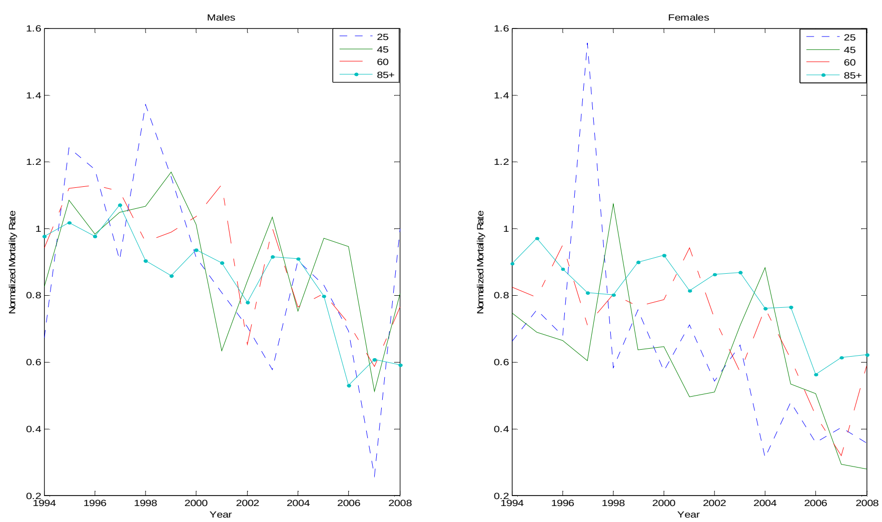
the Lee-Carter model can be rewritten as

$$l_t = \theta + l_{t-1} + \zeta_t$$

where $\theta = \beta\mu$ and $\zeta_t = \beta\delta_t + \varepsilon_t - \varepsilon_{t-1}$

Now, we can easily estimate the model, make prediction and quantify the longevity risk.

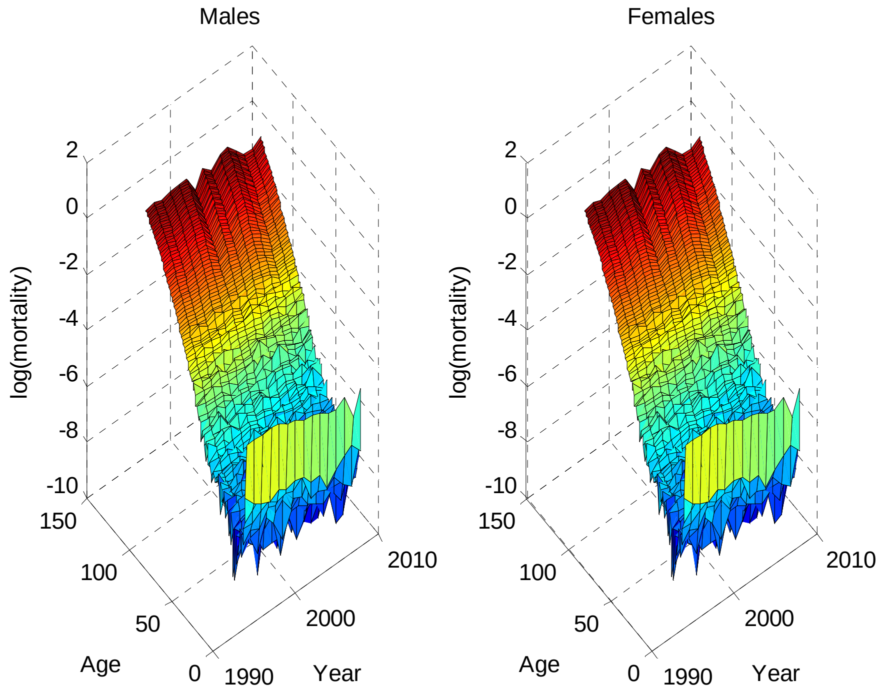
Figure 1: Normalized Death Rate for Selected Age Groups



This figure plots the observed death rates for Chinese males (left) and Chinese females (right), for selected age groups and for different time periods, normalized to one for year 1981. The data originates from the China Population Statistical

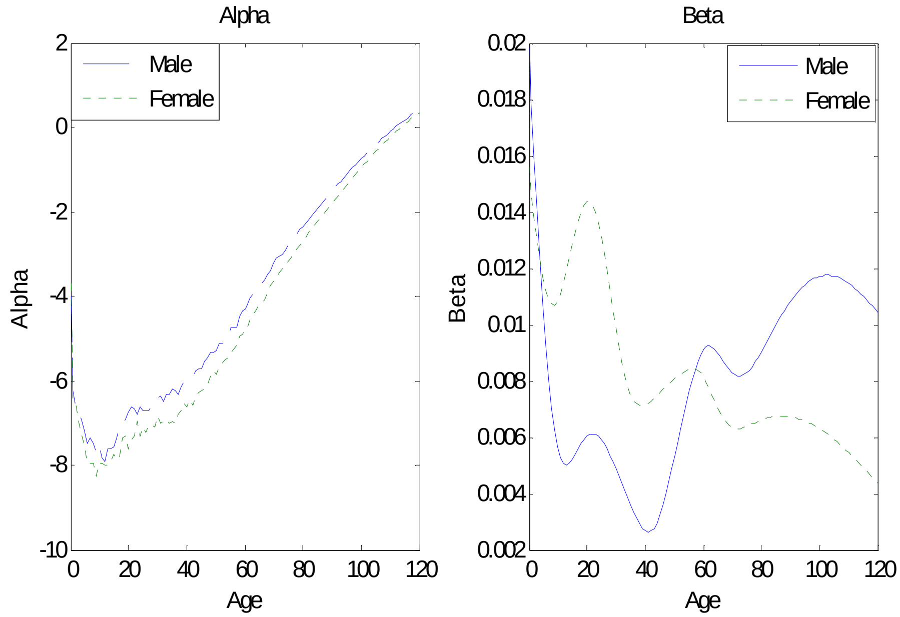
Yearbooks and the China Statistical Yearbook compiled by the National Bureau of Statistics of China.

Figure 2: Logarithm of Raw Central Death Rates in China, 1994-2008



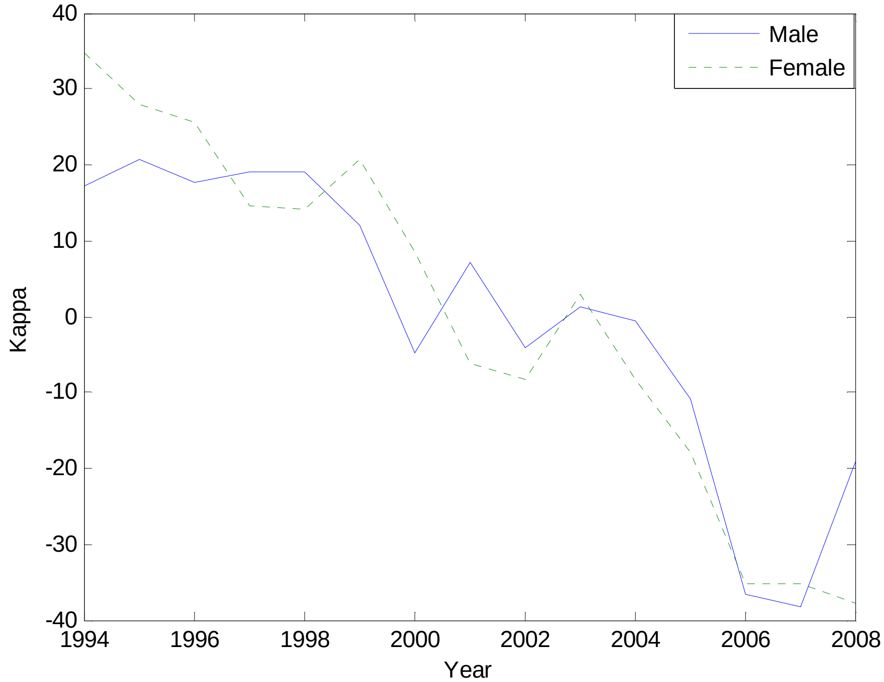
This figure plots the logarithm of dentral death rates during the 1994-2008 period for Chinese males (left) and Chinese females (right) from age 0, 1, 2, and up to age 120. The mortality data for age 0 to age 85+ originates from the China Population Statistical Yearbooks and the China Statistical Yearbooks, both of which are compiled by the National Bureau of Statistics of China. The mortality data at older ages are extrapolated by the Kannisto model.

Figure 3: Estimated α_x and β_x



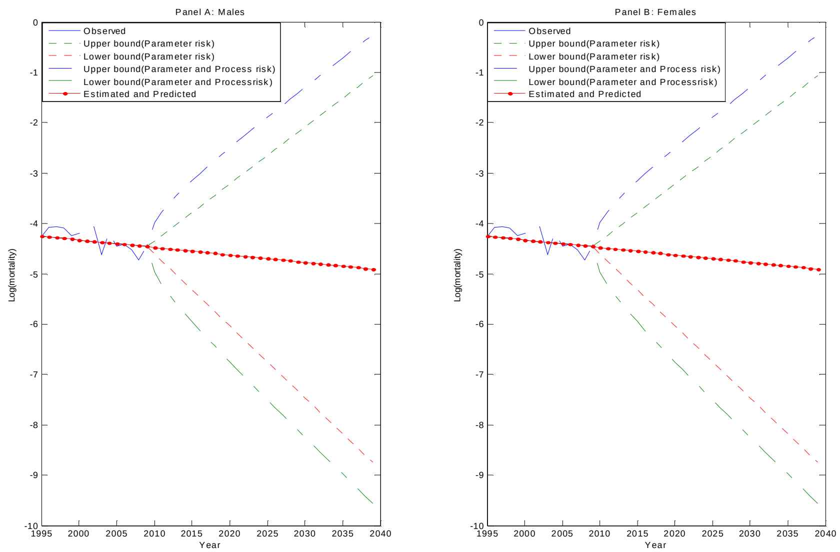
This figure presents the estimated α_x (left panel) and β_x (right panel, smoothed using cubic B-splines) for both Chinese males and females, from age 0, 1, 2, and up to 120.

Figure 4: Estimated κ_t



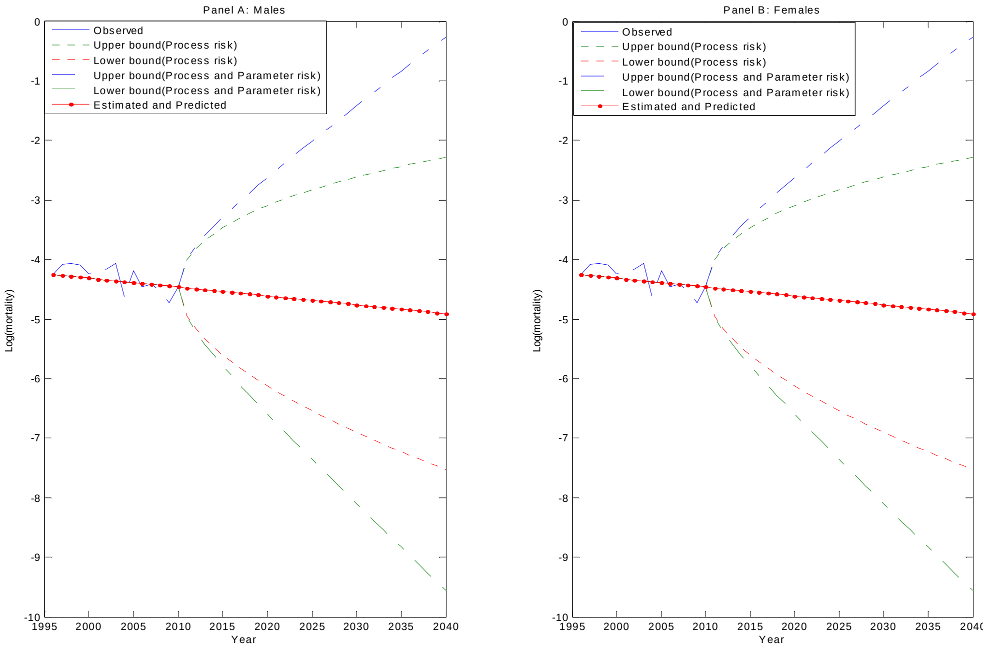
This figure plots the estimated κ_t for Chinese males and females for 1994-2008 period.

Figure 5: Prediction of Log Mortality Rate at 60 with Parameter Risk



This figure shows the (observed and predicated) logarithm of central death rates at the age of 60 for Chinese males (left) and Chinese females (right) with only parameter risk and the combination of parameter risk and process risk within 95% confidence intervals.

Figure 6: Prediction of Log Mortality Rate at 60 with Process Risk



This figure shows the (observed and predicated) logarithm of raw central death rates at the age of 60 for Chinese males (left) and Chinese females (right) with only process risk and the combination of parameter risk and process risk within 95% confidence intervals.

Table 1: Estimates of Life-Table Ageing Rate *(k_x)*, 1994-2008

Panel A: Males

	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
K_{60}	0.127	0.106	0.082	0.127	0.106	0.136	0.106	0.134	0.101	0.095	0.078	0.096	0.095	0.091	0.089
	(0.089,	(0.097,	(0.053,	(0.098,	(0.074,	(0.104,	(0.105,	(0.101,	(0.071,	(0.063,	(0.048,	(0.087,	(0.062,	(0.057,	(0.057,
	0.165)	0.115)	0.111)	0.156)	0.138)	0.168)	0.107)	0.167)	0.131)	0.126)	0.108)	0.104)	0.128)	0.126)	0.120)
K_{65}	0.074	0.095	0.097	0.061	0.121	0.083	0.1	0.094	0.075	0.080	0.078	0.101	0.101	0.143	0.107
	(0.042,	(0.087,	(0.071,	(0.036,	(0.096,	(0.057,	(0.099,	(0.068,	(0.050,	(0.052,	(0.051,	(0.094,	(0.072,	(0.113,	(0.078,
	0.106)	0.103)	0.122)	0.085)	0.147)	0.109)	0.101)	0.121)	0.101)	0.107)	0.106)	0.109)	0.131)	0.173)	0.136)
K_{70}	0.108	0.098	0.106	0.099	0.093	0.096	0.109	0.095	0.126	0.126	0.114	0.101	0.103	0.094	0.088
	(0.077,	(0.091,	(0.083,	(0.075,	(0.070,	(0.072,	(0.108,	(0.071,	(0.103,	(0.102,	(0.090,	(0.094,	(0.077,	(0.069,	(0.062,
	0.138)	0.106)	0.129)	0.123)	0.115)	0.120)	0.110)	0.118)	0.148)	0.149)	0.139)	0.107)	0.129)	0.119)	0.133)
K_{75}	0.086	0.085	0.085	0.083	0.095	0.098	0.09	0.090	0.071	0.102	0.086	0.095	0.094	0.090	0.092
	(0.055,	(0.077,	(0.061,	(0.059,	(0.072,	(0.074,	(0.089,	(0.066,	(0.049,	(0.079,	(0.063,	(0.089,	(0.070,	(0.067,	(0.069,
	0.118)	0.093)	0.108)	0.108)	0.118)	0.122)	0.090)	0.114)	0.093)	0.124)	0.109)	0.101)	0.118)	0.113)	0.116)
K_{80}	0.082	0.094	0.081	0.113	0.105	0.105	0.102	0.086	0.090	0.090	0.107	0.093	0.075	0.072	0.084
	(0.045,	(0.086,	(0.054,	(0.085,	(0.079,	(0.078,	(0.102,	(0.059,	(0.064,	(0.064,	(0.082,	(0.087,	(0.049,	(0.046,	(0.058,
	0.118)	0.103)	0.108)	0.141)	0.131)	0.132)	0.103)	0.114)	0.115)	0.115)	0.133)	0.100)	0.102)	0.098)	0.109)
K_{85}	0.083	0.075	--	0.091	0.050	0.056	0.07	0.104	0.070	0.083	0.096	0.087	0.052	0.075	0.073
	(0.032,	(0.062,		(0.053,	(0.013,	(0.020,	(0.068,	(0.067,	(0.035,	(0.050,	(0.062,	(0.078,	(0.015,	(0.041,	(0.040,
	0.133)	0.087)		0.129)	0.088)	0.093)	0.071)	0.140)	0.106)	0.117)	0.129)	0.096)	0.089)	0.108)	0.106)
K_{90}	--	0.080	--	--	--	--	0.071	--	--	--	--	0.074	0.082	--	--
		(0.058,					(0.069,					(0.060,	(0.026,		
		0.102)					0.073)					0.089)	0.138)		
K_{95}	--	0.070	--	--	--	--	-0.021	--	--	--	--	0.070	--	--	--

	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
		(0.016, 0.125)					(-0.026, -0.017)					(0.039, 0.101)			

Panel B: Females

	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
K_{60}	0.085	0.093	0.106	0.128	0.114	0.083	0.109	0.178	0.145	0.075	0.099	0.109	0.119	0.083	0.136
	(0.036, 0.134)	(0.081, 0.106)	(0.068, 0.144)	(0.088, 0.168)	(0.074, 0.154)	(0.043, 0.123)	(0.108, 0.110)	(0.134, 0.222)	(0.105, 0.186)	(0.032, 0.118)	(0.059, 0.139)	(0.097, 0.120)	(0.074, 0.165)	(0.040, 0.125)	(0.046, 0.178)
K_{65}	0.089	0.110	0.094	0.114	0.088	0.117	0.102	0.084	0.102	0.159	0.089	0.105	0.119	0.135	0.102
	(0.047, 0.132)	(0.099, 0.120)	(0.063, 0.126)	(0.083, 0.146)	(0.055, 0.121)	(0.084, 0.150)	(0.101, 0.103)	(0.051, 0.117)	(0.070, 0.133)	(0.123, 0.195)	(0.054, 0.124)	(0.095, 0.115)	(0.080, 0.158)	(0.097, 0.173)	(0.070, 0.139)
K_{70}	0.140	0.116	0.1	0.1	0.111	0.096	0.116	0.111	0.081	0.101	0.118	0.109	0.104	0.107	0.127
	(0.103, 0.177)	(0.107, 0.125)	(0.071, 0.128)	(0.072, 0.128)	(0.082, 0.140)	(0.067, 0.124)	(0.115, 0.117)	(0.082, 0.140)	(0.053, 0.108)	(0.073, 0.128)	(0.088, 0.147)	(0.101, 0.117)	(0.072, 0.136)	(0.076, 0.138)	(0.057, 0.158)
K_{75}	0.087	0.092	0.115	0.087	0.104	0.079	0.098	0.084	0.115	0.096	0.108	0.111	0.093	0.107	0.086
	(0.053, 0.120)	(0.083, 0.100)	(0.090, 0.141)	(0.061, 0.114)	(0.077, 0.130)	(0.051, 0.106)	(0.097, 0.099)	(0.058, 0.111)	(0.089, 0.141)	(0.070, 0.122)	(0.082, 0.134)	(0.104, 0.118)	(0.065, 0.122)	(0.080, 0.134)	(0.066, 0.113)
K_{80}	0.118	0.105	0.082	0.111	0.093	0.133	0.112	0.092	0.087	0.127	0.096	0.103	0.106	0.081	0.100
	(0.084, 0.151)	(0.096, 0.113)	(0.056, 0.108)	(0.084, 0.138)	(0.066, 0.120)	(0.105, 0.161)	(0.111, 0.113)	(0.064, 0.121)	(0.061, 0.133)	(0.101, 0.153)	(0.069, 0.122)	(0.096, 0.110)	(0.078, 0.135)	(0.054, 0.109)	(0.057, 0.127)
K_{85}	0.057	0.080	--	0.037	0.069	0.084	0.08	0.09	0.103	0.047	0.085	0.089	0.068	0.081	0.080
	(0.016, 0.098)	(0.069, 0.090)		(0.003, 0.072)	(0.035, 0.102)	(0.052, 0.116)	(0.079, 0.081)	(0.057, 0.123)	(0.072, 0.134)	(0.016, 0.078)	(0.054, 0.116)	(0.081, 0.097)	(0.035, 0.101)	(0.050, 0.113)	(0.042, 0.111)
K_{90}	--	0.095	--	--	--	--	0.093	--	--	--	--	0.096	0.111	--	--
		(0.080, 0.110)					(0.091, 0.094)					(0.085, 0.107)	(0.070, 0.152)		

K_{95}	--	0.027 (-0.006, 0.060)	--	--	--	--	0.051 (0.048, 0.053)	--	--	--	--	0.041 (0.021, 0.062)	--	--	--

Note: The corresponding confidence intervals are reported in the parentheses.

This table presents the estimates of life table aging rage and their corresponding confidence intervals for Chinese males and females during the period of 1994-2008 based on different age groups.

Table 2: Unit Root Test and Model Selection for κ_t

Panel A: Unit Root Test in Level and first Difference

	Male		Female	
Statistics	t-Stat	Prob.	t-Stat	Prob.
Augmented Dickey-Fuller Test in Level	-1.0225	0.7140	1.6843	0.9984
Phillips-Perron Test in Level	-0.9715	0.7324	0.2095	0.9625
Augmented Dickey-Fuller Test first Difference	-3.1315	0.0535	-3.9827	0.0140
Phillips-Perron Test in first Difference	-3.6908	0.0188	-5.5681	0.0008

Panel B: Autocorrelation and Partial Correlation of First Difference

	Male				Female			
	AC	PAC	Q-Stat	Prob.	AC	PAC	Q-Stat	Prob.
1	-0.108	-0.108	0.1996	0.655	-0.082	-0.082	0.1161	0.733
2	0.034	0.023	0.2217	0.895	-0.446	-0.456	3.8267	0.148
3	-0.002	0.004	0.2218	0.974	-0.013	-0.132	3.8302	0.280
4	-0.015	-0.016	0.2270	0.994	0.308	0.109	5.9514	0.203
5	-0.064	-0.068	0.3281	0.997	0.048	0.068	6.0085	0.305

Table 3: Simulation Results for Life Annuity Price, 60-year Male and Female in 2009

Panel A: Flat Rates

r	Male				Female			
	Constant Death Probabilities (1)	Process Risk (2)	Parameter Risk (3)	Process & Parameter Risk (4)	Constant Death Probabilities (5)	Process Risk (6)	Parameter Risk (7)	Process & Parameter Risk (8)
0.01	16.1499	17.0634	17.0692	17.0393	18.4207	19.5873	19.5933	19.5487
	(15.4562-16.8230)	(15.2385-18.6268)	(15.2834-15.6593)	(15.2418-18.6116)	(18.0970-18.7370)	(18.8202-20.2484)	(18.8335-20.2584)	(18.7609-20.2403)
0.02	14.7287	15.4900	15.4875	15.4627	16.6668	17.6302	17.6321	17.6043
	(14.1348-15.3001)	(13.9594-16.8147)	(13.9982-16.7869)	(13.9752-16.7864)	(16.3872-16.9318)	(16.9798-18.1824)	(17.0120-18.1826)	(16.9651-18.1654)
0.03	13.5066	14.1282	14.1372	14.1310	15.1659	15.9578	15.9660	15.9440
	(12.9968-14.0006)	(12.8373-15.2423)	(12.8533-15.2620)	(12.8629-15.2388)	(14.9342-15.3938)	(15.4330-16.4217)	(15.4361-16.4260)	(15.4036-16.4242)
0.04	12.4384	12.9832	12.9722	12.9592	13.8722	14.5351	14.5395	14.5161
	(11.9969-12.8631)	(11.9832-13.9337)	(11.8954-13.9082)	(11.8611-13.9187)	(13.6733-14.0678)	(14.0894-14.9274)	(14.1008-14.9306)	(14.0553-14.9237)
0.05	11.5180	11.9737	11.9694	11.9636	12.7542	13.3152	13.3085	13.2910
	(11.1309-11.8856)	(11.0747-12.7768)	(11.0666-12.7757)	(11.0427-12.7690)	(12.5797-12.9192)	(12.9426-13.6392)	(12.9243-13.6401)	(12.9010-13.6332)

Panel B: Term Structure

	Constant Death Probabilities (1)	Process Risk (2)	Parameter Risk (3)	Process & Parameter Risk (4)	Constant Death Probabilities (5)	Process Risk (6)	Parameter Risk (7)	Process & Parameter Risk (8)
Term	13.0671	13.6157	13.6158	13.5950	14.5559	15.2392	15.2362	15.2171
Structure	(12.6048-13.5097)	12.5002-14.6237)	(12.4963-14.5842)	(12.4556-14.5783)	(14.3468-14.7542)	(14.7850-15.6441)	(14.7758-15.6408)	(14.7459-15.6342)

This figure table presents the simulated annuity price for 60-year old Chinese males and females in 2009 under different scenarios. Panel A is based on the flat rates and Panel B on term structure of China's government bond 24th May, 2010.