Strategic Asset Allocation and the Role of Alternative Investments

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ABSTRACT

In this paper, we provide a realistic framework that investors can use for their strategic asset allocation with alternative investments (buyouts, commodities, hedge funds, REITs, and venture capital). Our approach is not based on a utility function, but on an easily quantifiable risk preference parameter, λ. We account for higher moments of the return distributions within our optimization framework and approximate best-fit distributions. Thus, we replace the empirical return distributions, which are often skewed and/or exhibit excess kurtosis, with two normal distributions. We then use the estimated return distributions in the strategic asset allocation. Our results show in out-of-sample analyses that our framework yields superior results compared to the Markowitz framework. It also features better abilities to manage regime switches, which tend to occur frequently during crises. Lastly, to test our results for stability, we use further robustness tests, which allow for time-varying correlation structures in the return distributions and weight restrictions for the asset classes.

JEL Classification: G2, G12, G31

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Alternative investments, which totalled U.S. $9 trillion as of the end of 2009, have become increasingly important in a portfolio context for institutional investors such as endowments and high net worth individuals and account for approximately 12% of the worldwide managed assets [Boston Consulting Group 2009]. These investors enjoy regulatory freedom, have sufficient capital to invest in alternative investments like private equity or hedge funds, and have sufficiently long investment horizons to hold illiquid investments. The share of alternative investments in the portfolios of high net worth individuals reached a level of about 10% in 2009 [for more details, see World Wealth Report 2009 of Capgemini and Merrill Lynch]. The leading investors for alternative investments in the US are endowments, with an average share of alternative investments in an endowment portfolio that increased considerably from 3% in 1996 to 39% in 2007 (33% excluding real estate). Endowments with more than U.S. $1 billion, like Harvard and Yale, have an allocation of about 61% [for more details, see 2009 NACUBO Endowment Study and annual reports of the endowments].

But what factors are driving this rush to alternative investments? We argue that there are two main reasons. First, investors are seeking diversification to avoid a repetition of the substantial losses they experienced during the recent stock and bond market downturns (e.g., the Asian crisis of 1997, the Russian crisis of 1998, the new economy bubble in 2000, the World Trade Center attacks in 2001, and the current financial market crisis). Alternative investments are practical during more volatile market phases, because their return drivers differ from those of the equity and bond markets [Schneeweis, Kazemi and Martin (2001)].

Second, the positive diversification properties of alternative investments do not necessarily have to reduce expected portfolio returns, but instead may enhance risk-adjusted performance. For example, the top U.S. university endowments (e.g., Harvard, Princeton, and Yale) reported realized annual returns of 10%-25% over the last three years, which highlights that alternative investments can enhance expected portfolio returns, too. Lerner, Schoar and Wang [2007] attribute part of this success to willingness to rely on alternative investments.

However, if investors want to build an exposure to alternative investments, they must determine which investments to include, as well as the strategic asset allocation. Because strategic asset allocation explains most of the portfolio's return variability, it is the major determinant of investment performance and the most critical decision in the investment process (Hoernemann, Junkans and Zarate [2005][4]).

Investors must also consider the risk-return characteristics, because they are the primary influence on the strategic asset allocation models. The model of choice must be flexible enough to incorporate the risk-return characteristics. If they are not captured properly, or if the strategic asset allocation model is not flexible enough, the obtained optimal portfolio may include alternative investments only and thereby omitting traditional asset classes (Terhaar, Staub and Singer [2003]).

The majority of studies in the literature only focuses on the effects of including one alternative investment in a mixed-asset portfolio. If more than just one is included, the risk-return profiles are often not captured adequately, or the chosen model is not flexible enough (e.g. Schneeweis, Karavas and Georgiev [2002] and Conner [2003]). Alternatively, the alternative investments may not be representative of the entire universe. For example, Huang and Zhong [2006] consider commodities, REITs, and TIPs; Hoecht, Ng, Wolf and Zagst [2008] integrate only Asian hedge funds and Asian REITs. These papers do not provide a strategic asset allocation for a broad sample of alternative investments.

[4] The authors present an alternative to the often-cited studies of Brinson, Hood and Beebower [1986] and Brinson, Hood and Beebower [1991]. They use a slightly different framework and cover a longer time horizon. They also include alternative assets and use synthetic portfolios.

To our knowledge, this paper is the first that 1) incorporates a variety of alternative investments (e.g., commodities, private equity, hedge funds, and real estate) and traditional investments (stocks and government bonds), 2) adjusts risk-return profiles to account for data biases, 3) uses a strategic asset allocation model that is flexible enough to capture the risk-return profile adequately, and 4) incorporates real investor preferences.

Before the optimization, the return time series of some alternative investments (private equity and hedge funds) are corrected for data biases such as appraisal smoothing and stale pricing. The optimization is thus flexible enough to incorporate any potential risk arising from higher moments (skewness and kurtosis) that would not be covered by the standard deviation. This is important, because the empirical return distributions of some alternative investments are generally not normally distributed. Thus every portfolio optimization in the mean-variance space will likely be suboptimal.

Consequently, we use the mixture of normal method to replace the empirical return distributions (which often exhibit skewness and positive excess kurtosis) with two normal distributions to approximate a best fit distribution. This approach ensures that the best fit return distributions exhibit the higher moments close to their empirical pendants. We then use the best fit distributions in the optimization procedure. To derive the strategic asset allocation, we apply a goal function so that we can examine real investor preferences for risk aversion. Thereafter we apply four robustness checks in order to test the validity of our strategic asset allocation approach. The first robustness check tests sensitivity of our results against the background of the financial crisis, the second captures the consequences resulting from weight restrictions, the third tests our results for time-varying correlations, and the final robustness check is an out-of-sample analysis to evaluate the performance of the presented asset allocation procedure against the Markowitz framework.

Our general findings are that stocks of large US firms, as part of the traditional asset classes, are considered only in defensive portfolios but bonds are of high importance and are included up to the maximum weight restriction in all portfolios and emerging markets gain in relevance with a decrease in risk aversion. For alternative investments the picture is as follows: REITs play a major role in the portfolios with a decrease in risk aversion. In contrast, commodities have comparably stable medium allocations in all portfolios. Hedge fund allocations are comparable to bond allocations since they are integrated virtually into all optimal portfolios with the maximum portfolio weight. In comparison, private equity plays a very important role especially in defensive portfolios.

The robustness check for the financial crisis reveals that the importance of alternative investments for risk diversification in defensive portfolios was underestimated. In spite of the financial crisis the results for alternative investments are even stronger. The weight restrictions also do not alter our results. The cumulative weights for alternative investments remain stable for different values of the risk aversion parameter. Furthermore, when allowing for time-varying correlations, we end up with nearly identical allocations. Finally, the out-of-sample analysis shows that our optimization procedure in most cases generates superior results compared to the Markowitz framework.

In conclusion, we find that alternative investments are important for the strategic asset allocation of institutional investors such as endowments, family offices, pension funds, and high net worth individuals who have sufficient time horizons and investment capital. However, not all alternative investment classes are of equal importance. Alternative investments are not appropriate as substitutes for traditional asset classes, and may better serve as complements for achieving the desired risk-return profiles.

The rest of this paper proceeds as follows: In section 1, the data set and the correction of data biases is described. Section 2 presents the optimization procedure and the results. Section 3 concludes with a summary, discussion, and implications for future research.

1. Data Set Description

It has been well-known since Markowitz's [1952] seminal paper on portfolio theory that diversification can increase expected portfolio returns while reducing volatility. However, investors should not blindly add another asset class to their portfolios without carefully considering its properties in the context of the portfolio. A naively chosen allocation to the newly added asset class may not improve the risk-return profile or may even worsen it. This raises the question of whether alternative investments really improve the (risk-adjusted) performance of a (mixed-asset) portfolio, and whether they should be included in the strategic asset allocation.

For the further analysis we use two traditional asset classes (proxy indices are in parentheses): stocks (S&P 500 TR Index and MSCI Emerging Markets TR Index) and government bonds (JPM US Government Bonds - TR Index), and four alternative assets: private equity, subdivided in buyout (US Buyout) and venture capital (US Venture Capital)[5], commodities (S&P GSCI Commodity TR Index), hedge funds (HFRI Fund of Funds Composite), and real estate investment trusts (REITs) (FTSE EPRA/NAREIT - TR Index).[6] All the time series in our investigation are on a monthly basis (except private equity time series which are based on quarterly data) with a January 1999 inception date, because all the indices report data from this date on. The end date for the time series is December 2009 (in Appendix D we build a sub-sample to control for the influence of the financial crisis and use the time period from January 1999 to December 2006).

[5] Both indices are based on the Thomson Reuters VentureXpert-data base. We followed the approach by Cumming, Haß and Schweizer [2010] for the calculation of the indices.

[6] Table A-I in Appendix A gives detailed descriptions of the proxy indices.

Before we start introducing the descriptive statistics of the considered asset classes we have to discuss several potential biases which could e.g. for alternative investments distort the inherent risk-return profile. Sources of distortion are manifold: For instance, appraisal based private equity indices like the calculated ones based on Thomson Reuters VentureXpert-data base show smoothed returns resulting from the deformation, which could occur through appraisal smoothing (estimated-value-method for determination of NAVs of portfolio companies), quarterly data availability and/or stale pricing (prices are distorted due to illiquid and not daily evaluated positions) and statistically cause a positive autocorrelation (see Table 1). These relations are common amongst illiquid investments like private equity, individual hedge funds strategies (see Avramov et al. [2009] and Table 2). They arise typically due to 1) irregular price determination 2) long time periods between price determination and 3) the use of book value instead of market prices (see, for instance, Geltner [1991]; Gompers and Lerner [1997]). The resulting positive autocorrelation causes a significant underestimation of risk due to the smoothed returns when naively using the raw data.

To adjust for appraisal-smoothing, stale pricing and for illiquidity in order to obtain an unbiased data set, we „de-smoothe" the private equity and hedge funds time series by using the Getmansky, Lo, and Makarov [2004] method, which incorporates the whole autocorrelation structure of the return distribution (the intuition behind this method can be found in Appendix C).[7] Thereafter, we obtain „de-smoothed" hedge fund and private equity time series and re-scaled private equity return series from quarterly data into monthly (see Cumming, Haß and Schweizer [2010] for further details).

Furthermore, some researchers emphasized that hedge fund time series are subject to a considerable survivorship bias. These studies use varying sample periods, calculation methods, and databases, and the resulting survivorship bias ranges from 0.16% (Ackermann,

[7] This method improves on Geltner's [1991] approach because the entire lag structure is considered simultaneously. In addition, there is no need for a de-smoothing parameter (see Byrne and Lee [1995] for the problematic determination of the de-smoothing parameter).

McEnally and Ravenscraft [1999]) to 6.22% (Liang [2002]).[8] Since we use an investable fund of hedge funds index, the performance of this index is not affected by a survivor ship bias. For that reason we do not conduct any adjustments.

After adjusting for the abovementioned distortions of the risk-return profile, Table 3 gives the resulting descriptive statistics. Note that emerging markets have the highest mean return (1.21%) but only the third highest standard deviation (6.69%), followed by REITs with a mean return of 0.81% and the highest standard deviation of 7.30%.

The higher moments (skewness and kurtosis) are additional potential sources of risk. Hedge funds exhibit the lowest skewness of -0.519 (kurtosis 6.728) whereas REITs show the highest kurtosis of 13.162 (skewness -0.300) among all asset classes. Therefore, hedge funds and REITs show the most unfavourable higher moment properties since a negative skewness and a positive excess kurtosis indicate that the outliers are on the left side of the return distribution and occur more often than expected under the normal distribution (known as tail risk). The excess kurtosis for most asset classes is close to zero (except for venture capital).

Analyzing the higher moments of the return distribution for the asset classes shows that some return distributions do not follow a normal distribution (the Jarque-Bera test rejects the null hypothesis of a normally distributed return distribution for REITs and venture capital at the 1% level. Thus, relying on a simple mean-variance framework and ignoring the higher moments will not adequately capture the risk-return profile. Failure to consider higher moments increases the probability of maintaining biased and sub-optimal weight estimations as well as underestimating tail losses.

Table 4 provides insight into the diversification potential of each asset class. Hedge funds have a high diversification potential, because the correlation to all other asset classes is

[8] However, most researchers usually amount survivorship bias to 2% to 3%. See, for example, Anson [2006], Brown, Goetzmann and Ibbotson [1999], and Fung and Hsieh [2000].

statistically not different form zero (except for private equity). Similar diversification potential applies for government bonds which also have a correlation to all other asset classes statistically not different form zero (except for venture capital). Noteworthy, no significant negative correlation between asset classes can be shown.

After reviewing the descriptive statistics of the return distributions, we cannot determine a priori that one asset class is a substitute for another. Therefore, we must consider all the asset classes for the portfolio construction. In order to create optimal investor portfolios, our model must consider the characteristics of the asset classes adequately. We present our framework for optimal portfolio construction in the next section.

2. Methodology and Results

We have discussed the descriptive characteristics of the different alternative asset classes as well as potential biases. We also concentrated on correcting these biases from the raw return series and discussed their statistical properties. Some of the resulting return distributions are not normally distributed, and exhibit skewness and excess kurtosis. For that reason, and assuming that investors do not have quadratic utility functions (therefore ignoring higher moments of the return distribution), a simple Markowitz's [1952] mean-variance framework will likely end up with inefficient portfolio composition and an underestimation of tail risk.

To capture the higher moments, the literature offers a number of alternative distributions to the normal distribution. The multivariate Student t-distribution is well-suited for fat-tailed data, but it does not allow for asymmetry. The non-central multivariate t-distribution also has fat tails and is skewed. However, the skewness is linked directly to the location parameter, making it somewhat inflexible. The lognormal distribution has been used to model asset returns, but its skewness is a function of the mean and variance, not a separate parameter.

Thus, to capture higher moments of not normally distributed returns, we need a distribution that is flexible enough to fit the skewness and the kurtosis. We use a combination of two different geometric Brownian motions to generate a mixture of normal diffusions. The normal mixture distribution is an extension of the normal distribution, and has been successfully applied in many fields of finance literature.[9]

We choose the normal mixture distribution primarily for its flexibility and its tractability.[10] In particular, let $f_1(x,\mu_1,\sigma_1)$ denote the probability density function of the first normal distribution, with mean μ_1 and standard deviation σ_1, and let $f_2(x,\mu_2,\sigma_2)$ denote the probability density function of the second normal distribution. We can then approximate the empirical distribution of hedge fund returns by a new distribution with the following probability density function:

$$\begin{aligned} f(x,\mu_1,\sigma_1,\mu_2,\sigma_2) &= 0.2\cdot f_1(x,\mu_1,\sigma_1)+0.8\cdot f_2(x,\mu_2,\sigma_2) \\ &= 0.2\cdot\frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\left(-\frac{(x-\mu_1)^2}{\sigma_1^2}\right)+0.8\cdot\frac{1}{\sqrt{2\pi\sigma_2^2}}\exp\left(-\frac{(x-\mu_2)^2}{\sigma_2^2}\right) \end{aligned} \tag{1}$$

Our economic justification is as follows. Consider a regime-switching model with two economic states: the usual and the unusual. The usual state exists 80% of the time, when the hedge fund can achieve a return with the distribution given by the second normal density; the unusual state exists 20% of the time, when the return is given by the other normal distribution.[11]

Note that we do not specify whether the unusual return is better than the usual return in terms of having a higher mean and/or lower volatility. Indeed, the unusual return could be better, worse, or even the same. The latter case harks back to the classic assumption that

[9] For example, Alexander and Scourse [2003] and Buckley, Saunders, and Seco [2004] have used this distribution to model asset returns and study option pricing problems in this setting. Venkataraman [1997] applied this concept to risk management. And Venkatramanan [2005] as well as Kaiser, Schweizer and Wu [2010] also used the normal mixture distribution in an asset allocation problem.

[10] Our approach is similar to that of Popova et al. [2007].

[11] The assumed breakdown of 80% and 20% may seem restrictive, but we tested different pairs for robustness: 90% and 10%, 85% and 15%, and 75% and 25%. We found the results remained qualitatively stable and were not driven by the assumed breakdown. Tables and figures are available from the authors upon request.

returns are unconditionally normal. In general, our setting allows for conditionally normal returns, but unconditional returns need not be normal.

This specification offers many advantages. First, we have four free parameters, $\mu_1, \sigma_1, \mu_2, \sigma_2$, so we can match the first through fourth moments of the empirical distribution exactly. We can also capture the skewness and the excess kurtosis. Second, with the normal density function, the new approximating distribution is still tractable. And third, as noted earlier, this specification treats the traditional normal approximation as a special case. Table 1 provides a visualization of this method.

Because we cannot solve the approximating parameters $\mu_1, \sigma_1, \mu_2, \sigma_2$ analytically, we must solve for them numerically. In particular, we look for means and standard deviations for the two normal distributions that can approximate as closely as possible the first four moments of the empirical distribution. Mean, variance, skewness, and kurtosis generally have different dimensions, so we decide to minimize the weighted relative deviation rather than the absolute deviation.

Let w = (w_1,w_2,w_3,w_4) be a vector of strictly positive constants, which serve as weights for the four moments we want to match. Our objective function is then:

$$\begin{aligned} \min\ & w_1 \times \left| \frac{\text{theoretical mean-empirical mean}}{\text{empirical mean}} \right| + w_2 \times \left| \frac{\text{theoretical variance-empirical variance}}{\text{empirical variance}} \right| \\ & + w_3 \times \left| \frac{\text{theoretical skewness-empirical skewness}}{\text{empirical skewness}} \right| + w_4 \times \left| \frac{\text{theoretical kurtosis-empirical kurtosis}}{\text{empirical kurtosis}} \right| . \end{aligned} \qquad (2)$$

The objective function takes a value 0 if all four moments can be matched exactly, with positive values otherwise. In our investigation, we use equal weights for all moments, i.e. each moment has the same importance in the objective function.[12] The approximating parameters that we obtain for the hedge fund strategies are provided in Table 4. Table 5 shows the first four moments of the empirical return distributions, and compares them with the

[12] Hence, it is unlikely to obtain a perfect match since the moment dependencies are not linear.

moments obtained from the mixture of normal method. Obviously, the moments are close and thus the fit is very good.

Our next step is to construct a strategic asset allocation with the broad variety of asset classes. Because the mean-variance approach does not work, we must find an appropriate objective function. We note that real-world investors looking to incorporate alternative investments into their portfolios are typically family offices, corporations, pension funds, high net worth individuals, and large endowments. These investors generally seek higher expected returns than in a money market, but are risk-averse and therefore pay special attention to downside risk.

We can thus specify the objective function of our investor as follows: Let r denote the random return of our portfolio, and r_1 and r_2 are some benchmark returns, which could be constants or random variables. Our investor's objective is to maximize the following function:

$$\max \ \Pr(r > r_1) - \lambda \Pr(r < r_2) \tag{3}$$

In other words, our investor wants to maximize the probability of outperforming some benchmark return, while minimizing the probability of underperforming another benchmark. Thus the first benchmark could be some constant, e.g., 10% p.a., or a random return of some other indices such as the S&P 500 as the market return. The second benchmark is usually chosen as 0%, the risk-free rate, or a government bond yield. For our analysis, we define the first benchmark as a constant 8% p.a., and the second as 0%.[13]

The term λ is a positive constant and represents the trade-off between these two objectives. It is obvious that λ depends on investors' risk aversion. The higher λ, the less aggressive the investors (higher risk aversion), as they weight the second objective more highly and are more concerned about losses than gains. Similar to the relative risk aversion

[13] For reasons of robustness, we assumed two stochastic benchmarks instead: the T-bill rate and the Barclays Capital Aggregate Bond Index for the second benchmark. The results remained qualitatively stable. Tables and figures are available from the authors upon request.

coefficient in canonical utility functions, plausible values of λ lie between 1 and 10. We also consider two constraints when optimizing our portfolios numerically: We do not allow short-selling, and we restrict the maximum asset class weight (CAP) to 20% (in appendix E we discuss the influence on the optimal portfolios when raising the CAP to 25%).[14] Using these constraints and the objective function stated above, we numerically calculate the optimal hedge fund portfolio for different parameters for λ. The scenario in which the entire sample period is covered and a CAP of 20% is considered will be referred to as the base case.

For different λs, all asset classes are at least incorporated into one optimal portfolio, but of course the weights vary by strategy and they are not of equal importance (see Figure 2, D-I, E-I, and E-II). For reasons of robustness, we analyze the optimal weights for the entire sample period January 1999-December 2009, and include the impact from the financial crisis and compare the results to the shorter sub-period which also starts in January 1999, but ends in December 2006 (see appendix D). Furthermore, we relax the strict CAP restriction to 25% instead of 20% and discuss the effects for the alternative investment classes (see appendix E).

The first interesting result in the base case for the traditional asset classes and stocks of large US firms (S&P 500 as proxy) is that it is considered in the optimal portfolios only for defensive respectively risk concerned investor portfolios (λ=1). In comparison, stock investments in emerging markets gain in importance with a decrease in risk aversion up to the CAP of 20% for λs greater than 3.5. Remarkably, bonds are of high importance and are included up to the CAP of 20% in all portfolios. For REITs, the first analyzed alternative investment, the weight in the optimal portfolios increases with decrease in risk aversion up to 20%. It is not surprising that allocations to REITs are not very large in defensive portfolios since REITs showed the highest historical standard deviation and the most unfavourable

[14] This maximum weight restriction aims to avoid having the portfolio dominated by a single asset class. When imposing the minimum diversification restriction the results are not as prone as for optimizations without such a restriction, since optimal portfolio weights do not comparably rely on the past performance of the respective assets.

higher moment properties among all considered asset classes. In contrast, commodities have a comparably stable allocation between 6% and 15% in all portfolios. Hedge fund allocations are comparable to bond allocations since they are integrated into all optimal portfolios with 20% (except λ=1). Private equity plays a very important role especially in defensive portfolios and is allocated with the maximal portfolio weight of 40% (buyout and venture capital) until a λ equal to 4.5. From this point on, the weight decreases and for a risk parameter value of 2.5, venture capital drops out of the portfolio. When summing up the weights for alternative investments we find that they have a cumulative weight of about 60% in offensive and performance orientated portfolios (λ=1) and about 77% in defensive portfolios (λ=6).

However, to show the dominance of our approach over the standard Markowitz approach we need to examine the out-of-sample performance. Therefore we conduct an out-of-sample Monte Carlo analysis according to Jobson and Korkie [1981] and Ledoit and Wolf [2008]. More specifically we use historical returns from January 1999 through June 2004 to construct Markowitz's efficient portfolios and equal expected return portfolios using our method for λ = 1, 3, and 6, respectively. Subsequently we use historical returns from July 2004 through December 2009 to construct 1,000 time series of future returns using a bootstrap approach according to Efron and Tibshirani [1994]. We then use the future returns time series to calculate portfolio returns.

To assess how beneficial our optimization technique is compared to the standard Markowitz approach, we calculate risk-adjusted performance for every risk measure separately as follows: the Sharpe ratio (SR) for standard deviation, the Sortino ratio (SoR) for LPM, the return on conditional value-at-risk (RoCVaR) for CVaR, and the Sterling ratio (StR) for MaxDD.

We note from Table 7 that our optimization technique outperforms the Markowitz approach significantly for the Sharpe Ratio[15] and for the other risk-adjusted performance measures. It performs especially well when the risk measures capture the downside risk (e.g., MaxDD, CVaR) and for the lower levels of risk aversion. For the out-of-sample analysis, we find that Markowitz is outperformed by all the risk-adjusted performance measures we study here, regardless of the level of risk aversion (see Table 7).

To approve our above mentioned results we conducted a series of robustness checks. In our first robustness check we analyze the influence of the financial crisis on the optimal portfolio weights for alternative investments (see appendix D). Firstly, we find that the importance of alternative investments for the risk diversification in defensive portfolio was underestimated before the financial crisis since the cumulative weight was about 54% only which is clearly below 77% for the entire sample period. This can mainly be attributed to private equity which was underrepresented in defensive portfolios und did not suffer as much as other asset classes like equity markets from market overreactions during the financial crisis, since the interim changes in private equity portfolio values are driven by appraisal changes (see e.g. de Bond and Thaler [1985] and Chopra, Lakonishok and Ritter [1992]). In contrast, the cumulative portfolio weights for offensive portfolios are about 20% higher when ignoring the financial crisis.

When conducting the second robustness check to study the effect of the maximum weight restriction, we find that the cumulative portfolio weights for alternative investments do not differ substantially for the less restrictive CAP of 25% compared to the stricter one (see appendix E). In detail, allocations to private equity as an asset class are reduced even when for some portfolios the weight of buyout reaches the higher CAP. Furthermore, hedge funds

[15] The test for statistical significance is applied for the Sharpe Ratio following Korkie [1981] and Ledoit and Wolf [2008] only since for the other risk adjusted performance measures no test statistic can be found in the literature. Admittedly, we would expect that it is most difficult to outperform, given our optimization procedure, the Sharpe Ratio, because it is directly linked to the Markowitz approach. Given that we outperform the Sharpe Ratio significantly and find more favorable risk adjusted performance measures compared to the Markowitz approach, we are confident that the results hold for the other risk measures, too.

have larger allocations (25%) in defensive portfolios and slightly lower ones in offensive portfolios when considering the entire sample period – the weight is constantly at 25%, regardless of the risk aversion parameter, when the financial crisis is ignored.

Our approach so far has been based on the assumption that the correlation structures between all these assets remain constant over time. In reality, however, correlations are time-varying and stochastic. They are difficult to include when planning portfolios, because their dynamic nature can render the numerical optimizations very complicated. Therefore, instead of integrating a correlation directly into our portfolio selection problem, we conduct a third robustness check to test whether our portfolio remains robust against time-varying correlations.

In order to do this, we draw from the Wishart distribution ten times,[16] and simulate the new correlation matrix. Then, we run the same optimization procedure as before to determine the new optimal portfolio for three risk aversion classes: λ = 1, 3, and 6. If the new portfolio does not deviate greatly from the initial portfolio, we can conclude that our initial portfolio will remain stable and robust against time-varying correlations.

As the results in Table 6 show, our initial portfolios are quite stable and are only slightly affected by changes in the correlation matrix, especially the high risk aversion portfolio. The principal components remain the same for all fifteen new portfolios. In some cases, the new portfolio is exactly the same; in others, some asset weights undergo minor changes. For investors with high risk aversion (λ = 6), four of the ten portfolios are identical to the initial one. For investors with low risk aversion (λ = 1), three of the ten portfolios are identical.

The fourth and last robustness check is an out-of-sample analysis according to Jobson and Korkie [1981] and Ledoit and Wolf [2008] when the financial crisis is not included. Therefore we use historical returns from January 1999 through December 2003 to construct

[16] For further details, see for instance, Zhang [2006].

the benchmark portfolios and historical returns from January 2004 through December 2006 to generate future return time series.

We find that Markowitz is outperformed by all the risk-adjusted performance measures we study here, regardless of the level of risk aversion, also when the financial crisis is not included (see Table 7). Hence, our approach is more suitable for capturing regime switches, which were particularly prevalent during the financial crisis (see Tables D - III).

3. Conclusion

Markowitz's [1952] classic mean-variance approach is widely used for tactical asset allocation. But it fails to include further risk factors such as skewness and kurtosis, which occur because the return distributions of different hedge fund strategies are usually not normally distributed. This can lead to non-optimal strategic weight suggestions.

This paper introduces a more flexible method, the mixture of normal method, to incorporate the higher moments of different hedge fund strategy return distributions individually. We use the obtained distributions to optimize hedge fund strategy allocations for investors with different degrees of risk aversion and preferences. We are also able to incorporate stochastic and static benchmarks.

In our method, investors choose one benchmark they wish to outperform while simultaneously choosing a second benchmark for minimum acceptable performance. After defining the goal function, we solve the optimization problem for a set of risk parameters, and obtain very stable portfolio weights, regardless of the level of λ. Finally, we perform four robustness checks on our obtained portfolios with respect to the financial crisis, the maximum weight restriction, time-varying correlations as well as out-of-sample tests, and found robust results.

In conclusion, our approach incorporates the heterogeneity of different asset classes and individual investor preferences to deliver robust results for institutional investors' strategic asset allocation. Our results are in most cases superior to Markowitz's [1952] classic mean-variance approach during times that markets face regime switches, such as during the recent financial crisis. At these times, a robust and reliable strategic asset allocation gains in importance.

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Appendix A

Table A-I
Data Descriptions

This table reports the proxy indices for each asset class. The frequency, inception dates, end date, and additional information sources are given for the proxy time series.

Asset Class	Proxy Index	Frequency	Inception Date	End Date	Additional Information
U.S. Stocks	S&P 500 Composite - Total Return Index	Monthly	Jan 99	Dec 09	http://www2.standardandpoors.com
Emerging Markets	MSCI Emerging Markets - Total Return Index	Monthly	Jan 99	Dec 09	http://www.datastream.com
U.S. Government Bonds	JPM United States Govt. Bond - Total Return Index	Monthly	Jan 99	Dec 09	http://www.datastream.com
Real Estate Investment Trusts	FTSE EPRA NAREIT - Total Return Index	Monthly	Jan 99	Dec 09	http://www.nareit.com
Commodities	S&P GSCI Commodity - Total Return Index	Monthly	Jan 99	Dec 09	http://www.datastream.com
Hedge Funds	HFRI Fund of Hedge Fund Composite Index	Monthly	Jan 99	Dec 09	http://www.hedgefundresearch.com
Buyout	Thomson ReutersVentureXpert	Quarterly	Jan 99	Dec 09	http://www.thomsonreuters.com
Venture Capital	Thomson ReutersVentureXpert	Quarterly	Jan 99	Dec 09	http://www.thomsonreuters.com

Appendix B: Re-Scaling of Moments

The moments of a monthly return distribution can be rescaled to an annual return distribution as follows. Let r_i denote the monthly return, i and R denote the annual return. It is obvious that

$$R = \sum_{i=1}^{12} r_i \ .$$

Assume r_is are iid. Let $E[r_i] = \bar{r}$, $Var(r_i) = \sigma_r^2$, $E[R_i] = \bar{R}$, and $Var(R_i) = \sigma_R^2$. It is well known that

$$\bar{R} = 12\bar{r}$$

$$\sigma_R = \sqrt{12}\sigma_r .$$

The skewness of the annual return is defined as

$$\begin{aligned} Skew(R) &= \frac{E\left(R-\bar{R}\right)^3}{\sigma_R^3} \\ &= \frac{E\left(\sum_{i=1}^{12} r_i - 12\bar{r}\right)^3}{12\sqrt{12}\sigma_r^3} \\ &= \frac{E\left[\sum_{i=1}^{12}(r_i-\bar{r})\right]^3}{12\sqrt{12}\sigma_r^3} \\ &= \frac{E\left[\sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{12}(r_i-\bar{r})(r_j-\bar{r})(r_k-\bar{r})\right]}{12\sqrt{12}\sigma_r^3} \\ &= \frac{\sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{12}E\left[(r_i-\bar{r})(r_j-\bar{r})(r_k-\bar{r})\right]}{12\sqrt{12}\sigma_r^3}. \end{aligned}$$

Now since

$$E\left[(r_i-\bar{r})(r_j-\bar{r})(r_k-\bar{r})\right] = \begin{cases} E\left[(r_i-\bar{r})^3\right] = Skew(r_i)\sigma_r^3, & \text{if } i = j = k; \\ 0 & \text{if } i, j, k \text{ are not the same.} \end{cases}$$

The equation above can be written as

$$
\begin{aligned}
Skew(R) &= \frac{\left(\sum_{i=1}^{12} Skew(r_i)\sigma_r^3\right)}{12\sqrt{12}\sigma_r^3} \\
&= \frac{12 Skew(r_i)\sigma_r^3}{12\sqrt{12}\sigma_r^3} \\
&= \frac{Skew(r_i)}{\sqrt{12}}.
\end{aligned}
$$

The kurtosis of the annual return is defined as

$$
\begin{aligned}
Kurt(R) &= \frac{E\left(R-\overline{R}\right)^4}{\sigma_R^4} \\
&= \frac{E\left(\sum_{i=1}^{12} r_i - \overline{r}\right)^4}{144\sigma_r^4} \\
&= \frac{E\left[\sum_{i=1}^{12}(r_i - \overline{r})\right]^4}{144\sigma_r^4} \\
&= \frac{E\left[\sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{12}\sum_{l=1}^{12}(r_i-\overline{r})(r_j-\overline{r})(r_k-\overline{r})(r_l-\overline{r})\right]}{144\sigma_r^4} \\
&= \frac{\sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{12}\sum_{l=1}^{12}E\left[(r_i-\overline{r})(r_j-\overline{r})(r_k-\overline{r})(r_l-\overline{r})\right]}{144\sigma_r^4}.
\end{aligned}
$$

Now since

$$
E\left[(r_i-\overline{r})(r_j-\overline{r})(r_k-\overline{r})(r_l-\overline{r})\right] = \begin{cases} E\left[(r_i-\overline{r})^4\right] = Kurt(r_i)\sigma_r^4, & \text{if } i = j = k = l; \\ E\left[(r_i-\overline{r})^2(r_j-\overline{r})^2\right] = \sigma_r^4 & \text{if respective two of } i, j, k, l \text{ are the same;} \\ 0, & \text{otherwise.} \end{cases}
$$

The above equation can be rewritten as

$$\begin{aligned}Kurt(R) &= \frac{\left(\sum_{i=1}^{12} Kurt(r_i)\sigma_r^4\right) + \frac{12\cdot 11}{2}\cdot\frac{4\cdot 3}{2}\sigma_r^4}{144\sigma_r^4} \\ &= \frac{12Kurt(r_i)\sigma_r^4 + 396\sigma_r^4}{144\sigma_r^4} \\ &= \frac{Kurt(r_i)}{12} + \frac{11}{4}.\end{aligned}$$

Appendix C: Getmansky, Lo and Makarov's [2004] Method

The basis of the procedure of Getmansky, Lo and Makarov [2004] is the idea that the observable return does not equal the real return. The observable return R_t^o is rather composed of the real returns R_t of the previous periods. Therefore:

$$R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \cdots + \theta_k R_{t-k}.$$

$$\theta_k \in [0.1]. \quad j = 0. \ldots. k \quad \text{and}$$

$$1 = \theta_0 + \theta_1 + \cdots + \theta_k.$$

The observable return is therefore the weighted sum of real returns of the previous periods. It follows that the mean of the observable returns is equal to the mean of the real returns. However, the volatility of the observable returns is smaller than the volatility of the actual returns. More precisely, the following is valid for the volatility of the observable returns:

$$\text{Std}[R_t^o] = \frac{1}{\sqrt{\theta_0^2 + \theta_1^2 + \ldots + \theta_k^2}} \sigma \leq \sigma,$$

with σ being the volatility of the real returns.

In order to calculate the real returns, it is required at first to determine the weighting factors. Thereby we take advantage of the fact that the observable return can be written as Moving-Average Process, whereas the weighting factors stay the same. The weighting factors for this Moving-Average Process can be estimated via Maximum-Likelihood. Finally, the real returns can be calculated with the estimated weighting factors.

Appendix D: Robustness Check – Influence of the Financial Crisis

Table D-I
Descriptive Statistics of the Monthly Return Distributions of all Asset Classes

This table shows the mean, monthly standard deviation, skewness, kurtosis, minimum, maximum, median, 25% percentile, 75% percentile, square root of lower partial moment 2 with threshold 0 (LPM), Conditional Value at Risk (CVaR) at 95% confidence coefficient and Maximum Drawdown of the monthly return distributions of S&P 500, MSCI Emerging Markets, JPM US Government Bonds, FTSE EPRA/NAREIT, S&P GSCI Commodity, HFRI Fund of Funds, US Buyout, US Venture Capital, for the period from January 1999 to December 2006. Return time series with significant autocorrelation are considered after an autocorrelation (AC) adjustment (using Getmansky, Lo, and Makarov's [2004] method). All indices are total return indices or earnings are retained. All discrete returns were converted into logarithmical returns. Finally, the assumption of a normal return distribution is proved via Jarque-Bera-tests (see Jarque and Bera [1980]).

	S&P 500	MSCI Emerging Markets	JPM US Government Bonds	FTSE EPRA/ NAREIT	S&P GSCI Commodity	HFRI Fund of Funds	US Buyout	US Venture Capital
Mean	0.28%	1.47%	0.33%	1.36%	1.20%	0.59%	0.62%	0.79%
Standard Deviation	4.91%	6.61%	2.69%	4.46%	6.65%	3.03%	3.61%	6.11%
Kurtosis	2.928	2.723	3.779	4.848	2.594	6.237	2.582	5.621
Skewness	-0.248	-0.172	-0.641	-0.757	-0.032	0.930	-0.328	1.183
Minimum	-11.45%	-15.70%	-5.33%	-11.80%	-14.36%	-5.83%	-7.89%	-12.42%
Maximum	10.62%	15.08%	6.58%	14.11%	18.03%	9.22%	8.32%	23.01%
Median	0.59%	1.91%	0.48%	1.29%	1.47%	0.41%	0.96%	0.23%
Percentile 25%	-2.76%	-3.60%	-1.80%	-1.31%	-3.16%	-1.41%	-2.18%	-2.25%
Percentile 75%	3.19%	5.86%	2.19%	4.14%	6.02%	2.56%	3.47%	3.39%
LPM	1.80%	2.03%	0.96%	1.12%	2.10%	0.92%	1.18%	1.70%
CVaR	-9.95%	-12.60%	-4.58%	-7.37%	-11.85%	-5.11%	-7.16%	-9.33%
MaxDD	52.76%	48.68%	24.51%	23.19%	32.96%	24.12%	43.83%	69.85%
Jarque-Bera	1,004	0,782	9,006***	22,814***	0,675	55,760***	2,418	49,861***

***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively, based on monthly returns.

Table D-II
Correlation Matrix

This table shows the correlations between the asset classes from Table D-I. We use the S&P 500, MSCI Emerging Markets, JPM US Government Bonds, FTSE EPRA/NAREIT, S&P GSCI Commodity, HFRI Fund of Funds, US Buyout, and US Venture Capital for the period from January 1999 to December 2006. Values in boldface are significantly different from zero at the 5% level.

	S&P 500	MSCI Emerging Markets	JPM US Government Bonds	FTSE EPRA/ NAREIT	S&P GSCI Commodity	HFRI Fund of Funds	US Buyout	US Venture Capital
S&P 500	**1.000**							
MSCI Emerging Markets	0.173	**1.000**						
JPM US Government Bonds	-0.175	-0.092	**1.000**					
FTSE EPRA/ NAREIT	**0.262**	0.014	0.051	**1.000**				
S&P GSCI Commodity	0.060	-0.142	0.004	-0.155	**1.000**			
HFRI Fund of Funds	0.003	0.051	-0.149	-0.027	-0.062	**1.000**		
US Buyout	**0.291**	**0.471**	**-0.343**	0.029	0.012	0.188	**1.000**	
US Venture Capital	**0.242**	**0.507**	**-0.211**	-0.132	0.059	0.131	**0.715**	**1.000**

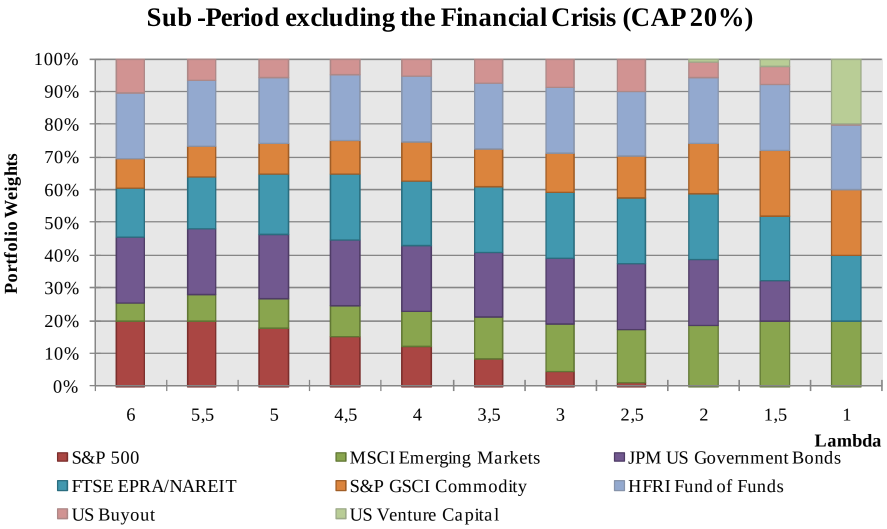
Table D-III
Robustness Check – Out-of-Sample Analyses

This table shows the difference in risk-adjusted portfolio performance and expected return of the allocations for investor objective function maximization (λ=1, 3, 6) compared to benchmark allocations (determined by the Markowitz portfolio selection process where an efficient frontier portfolio with an equal return is selected) for a one year holding period. Calculations are based on a standard block bootstrap Monte Carlo simulation with 1,000 runs following Efron and Tibshirani [1994]. For the out-of-sample analysis, we use the January 1999-December 2002 period to construct the benchmark portfolio, and January 2003-December 2006 to construct the time series of future returns. We calculate a corresponding risk-adjusted performance measure for each risk measure. For the standard deviation, we calculate the Sharpe ratio (SR), for LPM 2 with threshold 0, we calculate the Sortino ratio (SoR), for VaR with a 95% confidence level, we calculate the return on value-at-risk (RoVaR), for CVaR with a 95% confidence level, we calculate the return on conditional value-at-risk (RoCVaR), and for MaxDD, we calculate the Sterling ratio (StR). All risk-adjusted performance measures are calculated using the same arithmetic equation: (portfolio return – risk-free return)/risk measure. For this analysis, the risk-free return is set to 3%. Results remain stable when using 0% or the historical risk-free return. ***, **, and * denote that the assumption of an equal risk-adjusted performance measure is rejected at the 1%, 5%, and 10% significance levels for the Sharpe Ratio. Equivalent test statistics for other risk measures are not available.

λ	Expected Return	Sharpe Ratio	SoR	RoVaR	RoCVaR	StR
1	1.67%	0.117***	0.302	0.437	0.610	0.271
3	1.86%	0.117***	0.286	0.366	0.507	0.310
6	0.87%	0.077***	0.190	0.247	0.327	0.175

Figure D-I
Optimal Portfolio Weights

This figure shows the relationship between the risk aversion factor λ and the corresponding optimal portfolio weights for the asset classes with a maximum weight restriction per asset class of 20%. The sample period is January 1999-December 2006.



Appendix E: Robustness Check – Influence of Weight Restrictions

Table E-I
Robustness Check – Out-of-Sample Analyses

This table shows the difference in risk-adjusted portfolio performance and expected return of the allocations for investor objective function maximization (λ=1, 3, 6) compared to benchmark allocations (determined by the Markowitz portfolio selection process where an efficient frontier portfolio with an equal return is selected) for a one year holding period. Calculations are based on a standard block bootstrap Monte Carlo simulation with 1,000 runs following Efron and Tibshirani [1994]. For the out-of-sample analysis, we use the January 1999-June 2004 period to construct the benchmark portfolio, and July 2004-December 2009 to construct the time series of future returns for the entire sample period. For the sub-period excluding the financial crisis, we use the January 1999-December 2002 period to construct the benchmark portfolio, and January 2003-December 2006 to construct the time series of future returns. We calculate a corresponding risk-adjusted performance measure for each risk measure. For the standard deviation, we calculate the Sharpe ratio (SR), for LPM 2 with threshold 0, we calculate the Sortino ratio (SoR), for VaR with a 95% confidence level, we calculate the return on value-at-risk (RoVaR), for CVaR with a 95% confidence level, we calculate the return on conditional value-at-risk (RoCVaR), and for MaxDD, we calculate the Sterling ratio (StR). All risk-adjusted performance measures are calculated using the same arithmetic equation: (portfolio return – risk-free return)/risk measure. For this analysis, the risk-free return is set to 3%. Results remain stable when using 0% or the historical risk-free return. ***, **, and * denote that the assumption of an equal risk-adjusted performance measure is rejected at the 1%, 5%, and 10% significance levels. Equivalent test statistics for other risk measures are not available.

a) Entire Sample Period / CAP 25%

λ	Expected Return	Sharpe Ratio	SoR	RoVaR	RoCVaR	StR
1	0.36%	0.034***	0.091	0.059	0.077	0.033
3	0.00%	0.001	0.003	0.007	0.013	0.002
6	0.26%	0.030***	0.077	0.065	0.078	0.028

a) Sub-Period excluding the Financial Crisis / CAP 25%

λ	Expected Return	Sharpe Ratio	SoR	RoVaR	RoCVaR	StR
1	1.57%	0.100***	0.230	0.340	0.325	0.173
3	2.44%	0.210***	0.526	0.698	0.866	0.462
6	0.14%	0.010	0.026	0.027	0.039	0.016

Figure E-I
Optimal Portfolio Weights

This figure shows the relationship between the risk aversion factor λ and the corresponding optimal portfolio weights for the asset classes with a maximum weight restriction per asset class of 25%. The sample period is January 1999-December 2009.

Entire Sample Period (CAP 25%)

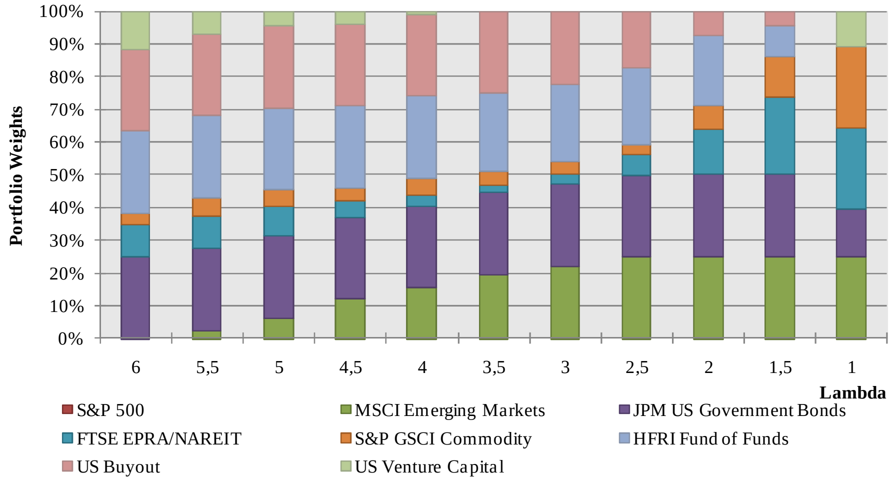


Figure E-II
Optimal Portfolio Weights

This figure shows the relationship between the risk aversion factor λ and the corresponding optimal portfolio weights for the asset classes with a maximum weight restriction per asset class of 20%. The sample period is January 1999-December 2006.

Sub -Period excluding the Financial Crisis (CAP 25%)

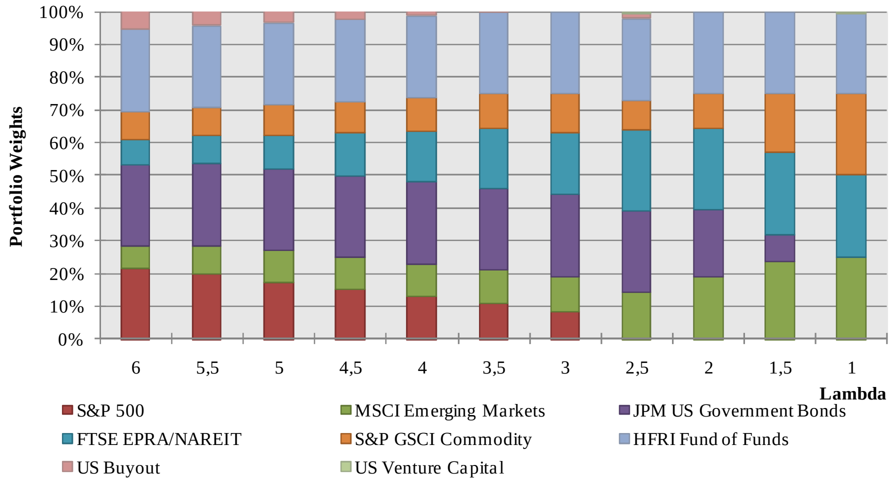


Table 1

Autocorrelation Structure of the Appraisal Value Based Private Equity Indices

This table shows the autocorrelation coefficient of the quarterly distribution of returns for the Appraisal Value Based Private Equity Indices (US Buyout and US Venture Capital) based on Thomson ReutersVentureXpert data base from January 1999 to December 2009 for the Lag 1 to 4. The bold formatting represents the significance at the level of 95%.

	Lag 1	Lag 2	Lag 3	Lag 4
US Buyout	**0.3561**	**0.2945**	0.2178	0.1903
US Venture Capital	**0.6153**	**0.4988**	**0.3897**	0.0559

Table 2

Autocorrelation Structure of the Monthly Return Distribution of all Asset Classes

This table shows the autocorrelation co-efficient of the monthly return distributions of S&P 500, MSCI Emerging Markets, JPM US Government Bonds, FTSE EPRA/NAREIT, S&P GSCI Commodity, HFRI Fund of Funds, US Buyout, US Venture Capital, for the period from January 1999 to December 2009 for the monthly Lag 1 to 12. The bold formatting represents the significance at the level of 95%.

	S&P 500	MSCI Emerging Markets	JPM US Government Bonds	FTSE EPRA/ NAREIT	S&P GSCI Commodity	HFRI Fund of Funds
Lag 1	0.1008	0.2096	0.1236	0.0039	0.1762	0.0854
Lag 2	-0.0160	0.1845	0.0567	**-0.3224**	0.0963	0.1219
Lag 3	0.0195	0.0489	0.0858	0.1381	0.1258	0.0997
Lag 4	0.0260	-0.0176	-0.1206	**0.3031**	0.0171	-0.1228
Lag 6	0.0241	-0.0603	0.0542	-0.0707	0.0239	0.0451
Lag 7	-0.1282	-0.1060	-0.0681	**-0.2712**	-0.0079	0.0791
Lag 8	0.0900	0.0513	-0.0067	0.0636	-0.0608	0.0813
Lag 9	0.1304	0.0125	-0.1007	0.1748	-0.0189	**0.1839**
Lag 10	0.1732	0.0950	-0.0395	0.0012	-0.0385	**0.2078**
Lag 11	0.0184	0.0160	0.0989	**-0.2226**	0.0374	0.1185
Lag 12	-0.0435	-0.0097	0.0517	0.1047	0.1719	0.0352

Table 3
Descriptive Statistics of the Monthly Return Distribution of all Asset Classes

This table shows the mean, monthly standard deviation, skewness, kurtosis, minimum, maximum, median, 25% percentile, 75% percentile, square root of lower partial moment 2 with threshold 0 (LPM), Conditional Value at Risk (CVaR) at 95% confidence coefficient and Maximum Drawdown of the monthly return distributions of S&P 500, MSCI Emerging Markets, JPM US Government Bonds, FTSE EPRA/NAREIT, S&P GSCI Commodity, HFRI Fund of Funds, US Buyout, US Venture Capital, for the period from January 1999 to December 2009. Hedge fund and private equity (US Buyout and US Venture Capital) return time series with significant autocorrelation are considered after an autocorrelation (AC) adjustment (using Getmansky, Lo, and Makarov's [2004] method). All indices are total return indices or earnings are retained. All discrete returns were converted into logarithmical returns. Finally, the assumption of a normal return distribution is proved via Jarque-Bera-tests (see Jarque and Bera [1980]).

	S&P 500	MSCI Emerging Markets	JPM US Government Bonds	FTSE EPRA/ NAREIT	S&P GSCI Commodity	HFRI Fund of Funds	US Buyout	US Venture Capital
Mean	0.05%	1.21%	0.33%	0.81%	0.73%	0.33%	0.32%	0.43%
Standard Deviation	5.11%	6.96%	2.99%	7.30%	7.07%	3.14%	3.27%	5.37%
Kurtosis	4.478	2.976	4.791	13.162	4.252	6.728	2.834	7.183
Skewness	-0.462	-0.315	-0.001	-0.300	-0.510	-0.519	-0.135	1.441
Minimum	-14.14%	-19.53%	-8.24%	-32.87%	-22.66%	-10.74%	-7.89%	-12.42%
Maximum	10.62%	16.88%	9.46%	28.93%	18.03%	9.32%	8.32%	23.01%
Median	0.48%	1.83%	0.35%	1.37%	1.29%	0.28%	0.35%	-0.16%
Percentile 25%	-2.94%	-3.26%	-1.76%	-2.81%	-3.63%	-1.74%	-1.92%	-2.25%
Percentile 75%	3.15%	6.07%	2.10%	4.69%	5.79%	1.96%	2.49%	2.37%
LPM	1.96%	2.24%	1.01%	2.08%	2.44%	1.04%	1.14%	1.58%
CVaR	-11.03%	-14.06%	-5.48%	-17.96%	-14.94%	-6.02%	-6.77%	-8.78%
MaxDD	61.58%	56.08%	24.51%	69.36%	62.39%	24.18%	43.83%	69.85%
Jarque-Bera	16,707***	2,189	17,646***	569,887***	14,337***	82,390***	0,556	141,932***

***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively, based on monthly returns.

Table 3
Correlation Matrix

This table shows the correlations between the asset classes from Table 2. We use the S&P 500, MSCI Emerging Markets, JPM US Government Bonds, FTSE EPRA/NAREIT, S&P GSCI Commodity, HFRI Fund of Funds, US Buyout, US Venture Capital for the period from January 1999 to December 2009. Values in boldface are significantly different from zero at the 5% level.

	S&P 500	MSCI Emerging Markets	JPM US Government Bonds	FTSE EPRA/ NAREIT	S&P GSCI Commodity	HFRI Fund of Funds	US Buyout	US Venture Capital
S&P 500	**1.000**							
MSCI Emerging Markets	**0.275**	**1.000**						
JPM US Government Bonds	-0.183	-0.044	**1.000**					
FTSE EPRA/ NAREIT	**0.648**	0.153	-0.067	**1.000**				
S&P GSCI Commodity	**0.305**	-0.020	-0.102	0.189	**1.000**			
HFRI Fund of Funds	0.157	0.172	-0.176	0.161	0.184	**1.000**		
US Buyout	0.103	**0.292**	**-0.241**	-0.061	-0.082	0.088	**1.000**	
US Venture Capital	0.077	**0.337**	-0.144	-0.127	-0.043	0.049	**0.720**	**1.000**

Table 4

Moments of the Normally Distributed Auxiliary Distributions

This table shows the mean and the standard deviation of the two auxiliary distributions, as well as the weighting factor for S&P 500, MSCI Emerging Markets, JPM US Government Bonds, FTSE EPRA/NAREIT, S&P GSCI Commodity, HFRI Fund of Funds, US Buyout, and US Venture Capital for the period from January 1999 to December 2009. The values in the w-vector are all equal to 1.

	Distribution 1		Distribution 2	
Weighting Factor	0.2		0.8	
	Mean	Standard Deviation	Mean	Standard Deviation
S&P 500	0%	10%	1%	6%
MSCI Emerging Markets	1%	16%	18%	16%
JPM US Government Bonds	0%	9%	5%	11%
FTSE EPRA/NAREIT	5%	16%	11%	11%
S&P GSCI Commodity	0%	14%	11%	13%
HFRI Fund of Funds	3%	10%	5%	11%
US Buyout	1%	12%	5%	11%
US Venture Capital	0%	1%	7%	16%

Table 5

Comparison of the Moments of Empirical and Approximated Distributions

This table shows the first four moments (annualized) of the empirical and approximated distributions for the asset classes from Table 2 (see appendix B for the re-scaling from monthly to annual return distributions). The number on the left is the theoretical moment in the approximated distribution; the number in parentheses is the empirical moment.

	Mean	Standard Deviation	Skewness	Kurtosis
S&P 500	0.80% (0.62%)	7.00% (17.70%)	-0.14 (-0.13)	3.44 (3.12)
MSCI Emerging Markets	14.60% (14.53%)	17.39% (24.12%)	-0.09 (-0.09)	3.01 (3.00)
JPM US Government Bonds	4.00% (4.02%)	10.81% (10.36%)	0.00 (0.00)	3.00 (3.15)
FTSE EPRA/ NAREIT	9.80% (9.74%)	12.40% (25.27%)	-0.09 (-0.09)	3.05 (3.85)
S&P GSCI Commodity	8.80% (8.82%)	13.92% (24.50%)	-0.15 (-0.15)	3.13 (3.10)
HFRI Fund of Funds	4.60% (4.00%)	10.84% (10.86%)	-0.16 (-0.15)	3.15 (3.31)
US Buyout	4.20% (3.82%)	11.32% (11.31%)	-0.03 (-0.04)	3.03 (3.00)
US Venture Capital	5.60% (5.16%)	14.59% (18.61%)	0.27 (0.42)	3.53 (3.35)

Table 6
Robustness Check – Time-Varying Correlation Matrix

This table shows the optimal portfolio weights when the variance of covariance is drawn from a Wishart distribution ten times. The number on the left is the average portfolio weight for our random sample. The number in parentheses is the portfolio weight for the original correlation matrix for the constructed indices for the January 1999-December 2009 period and a maximum weight restriction of 20%.

	λ=1	λ=3	λ=6
S&P 500	0.0 %(0.0%)	0.0 % (0.0%)	2.9% (3.0%)
MSCI Emerging Markets	20.0% (20.0%)	20.0% (20.0%)	0.0 % (0.0%)
JPM US Government Bonds	20.0% (20.0%)	20.0% (20.0%)	20.0% (20.0%)
FTSE EPRA/NAREIT	20.0% (20.0%)	6.9% (6.8%)	10.9% (10.7%)
S&P GSCI Commodity	16.7% (16.4%)	6.5% (6.3%)	6.2% (6.3%)
HFRI Fund of Funds	3.3% (3.6%)	20.0% (20.0%)	20.0% (20.0%)
US Buyout	0.0 % (0.0%)	20.0% (20.0%)	20.0% (20.0%)
US Venture Capital	20.0% (20.0%)	6.6% (7.0%)	20.0% (20.0%)

Table 7
Robustness Check – Out-of-Sample Analyses

This table shows the difference in risk-adjusted portfolio performance and expected return of the allocations for investor objective function maximization (λ=1, 3, 6) compared to benchmark allocations (determined by the Markowitz portfolio selection process where an efficient frontier portfolio with an equal return is selected) for a one year holding period. Calculations are based on a standard block bootstrap Monte Carlo simulation with 1,000 runs following Efron and Tibshirani [1994]. For the out-of-sample analysis, we use the January 1999-June 2004 period to construct the benchmark portfolio, and July 2004-December 2009 to construct the time series of future returns. We calculate a corresponding risk-adjusted performance measure for each risk measure. For the standard deviation, we calculate the Sharpe ratio (SR), for LPM 2 with threshold 0, we calculate the Sortino ratio (SoR), for VaR with a 95% confidence level, we calculate the return on value-at-risk (RoVaR), for CVaR with a 95% confidence level, we calculate the return on conditional value-at-risk (RoCVaR), and for MaxDD, we calculate the Sterling ratio (StR). All risk-adjusted performance measures are calculated using the same arithmetic equation: (portfolio return – risk-free return)/risk measure. For this analysis, the risk-free return is set to 3%. Results remain stable when using 0% or the historical risk-free return. ***, **, and * denote that the assumption of an equal risk-adjusted performance measure is rejected at the 1%, 5%, and 10% significance levels. Equivalent test statistics for other risk measures are not available.

λ	Expected Return	Sharpe Ratio	SoR	RoVaR	RoCVaR	StR
1	0.68%	0.095***	0.246	0.220	0.265	0.089
3	0.56%	0.063***	0.164	0.151	0.194	0.068
6	-0.08%	-0.006	-0.016	-0.015	-0.020	-0.007

Figure 1
Histograms and Fitted Distributions for all Asset Classes

The figure shows the monthly return histograms of the eight asset classes and the corresponding fitted return distribution for each strategy for the period from January 1999 to December 2009. The fitted return distribution is composed of two auxiliary distributions – distributions 1 and 2 – that are weighted with factors 0.2 and 0.8, respectively.

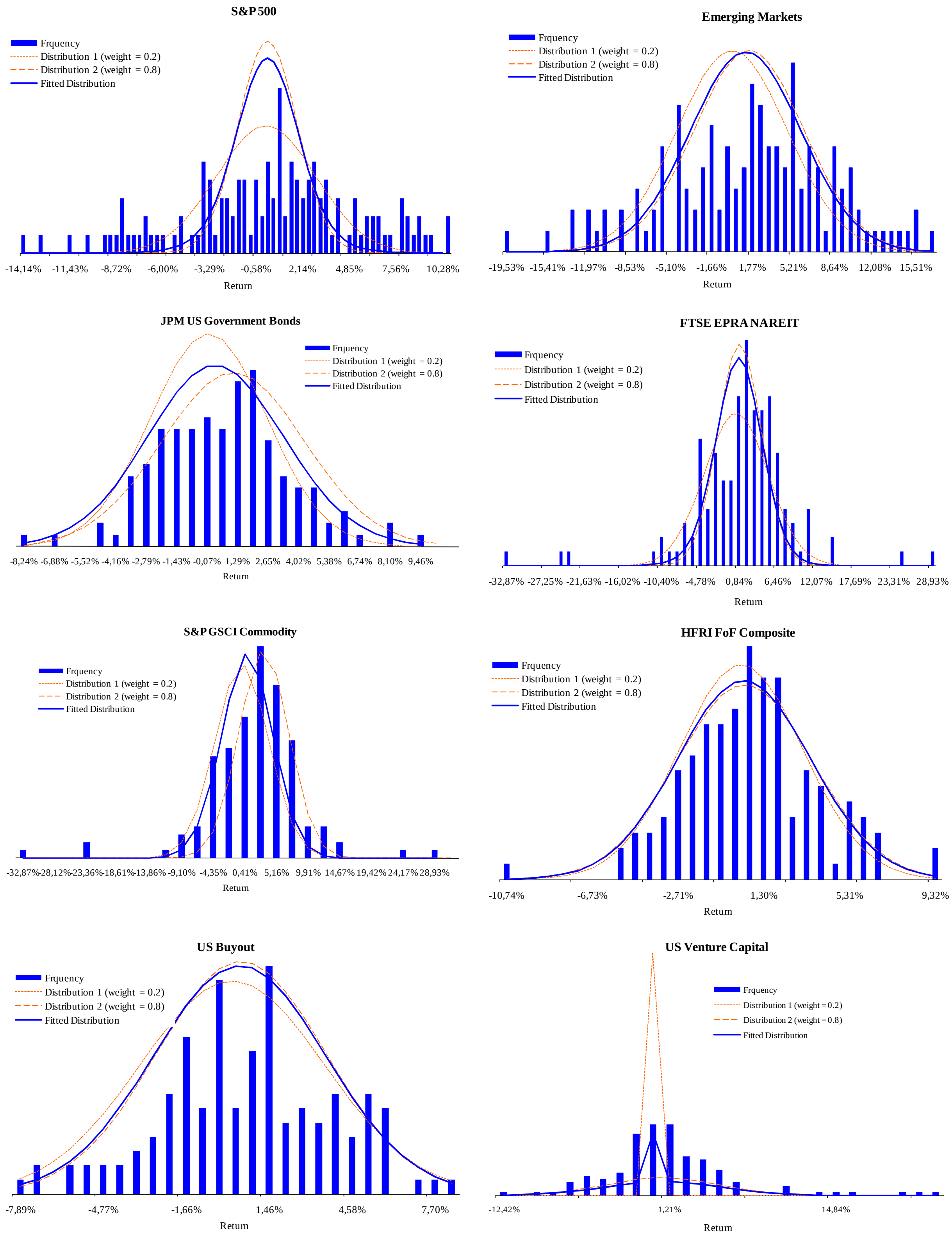


Figure 2
Optimal Portfolio Weights

This figure shows the relationship between the risk aversion factor λ and the corresponding optimal portfolio weights for the asset classes with a maximum weight restriction per asset class of 20%. The sample period is January 1999-December 2009.