CoCo Design, Risk Shifting Incentives and Financial Fragility[1]

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Abstract

Contingent convertible capital (CoCo) is debt that converts to equity or is written off if the issuing bank fails to meet a distress threshold. Conversion increases loss-absorption capacity, but leads to wealth transfers between CoCo holders and shareholders, which affect risk-shifting incentives of shareholders. We show that for writedown CoCos and insufficiently dilutive equity-converting CoCos, the risk-shifting incentives of shareholders always increase. Although CoCos are accepted as Additional Tier 1 Capital, we show that most CoCos lead to more risk taking than issuing subordinated debt instead. If there are social costs to bankruptcy, their use calls for higher capital requirements.

JEL classification: G01, G13, G21, G28, G32

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1 Introduction

This paper aims to show the risk-shifting incentives that arise from letting banks issue contingent convertible capital (CoCo) in order to fulfill capital requirements set by regulators. CoCos are hybrid instruments that are issued as debt but convert to equity or written off if the issuing bank fails to meet a distress threshold. The threshold may be contractual, as when the bank fails to meet a preset equity ratio, or discretionary, as when regulators deem the bank to be close to the point of nonviability. CoCos are designed this way in order to relieve the issuer of the burden of raising capital in situations of financial distress (Flannery (2005)). As a result, CoCos have become favored by regulators because of their enhanced loss absorption capacity relative to subordinated debt.

While CoCo conversion increases the loss absorption capacity of banks, it also potentially changes the order of seniority. If CoCos are written off, CoCo holders absorbs the first losses, instead of the original shareholders. This implies that at the moment of conversion, there is a wealth transfer in favor of the shareholders. If CoCos are converted to equity, CoCo holders absorb the losses together with the existing shareholders. In this case, the wealth transfer may be in favor of either the CoCo holder or the existing shareholder, depending on the terms of the conversion. The wealth transfers are defined relative to when the bank has issued subordinated debt in place of the CoCos, and always from the point of view of the original shareholder. Because of these wealth transfers, the bank may find it beneficial to engage in risk-shifting by choosing a riskier class of assets.

Our contribution to the literature is to provide a simple theoretical model of risk-shifting in the presence of CoCos, when the conversion is based on a breach of a preset equity ratio. The simplicity buys us a complete analytical solution, without much loss of generality. Using a call options framework, we show that risk-shifting incentives arise from two forces: an increase in the conversion probability of a given CoCo, and a decrease in the wealth transfer relative to issuing subordinated debt.

We define wealth transfers from the existing (i.e. prior to conversion) shareholders' point of view - that is, as the change in residual equity that results from a conversion-induced reduction in leverage. Within the call options framework, subordinated debt and unconverted CoCos of the same amount are equivalent because both are senior to equity. In the same way, there is no difference between equity and converted CoCos of the same amount, at least to the extent that the newly created equity value accrues to the old equity holders. This fact enables us to write the ex ante residual value of a CoCo-issuing bank as a weighted average of the respective residual values with subordinated debt, and with additional equity, with the conversion probability as the weight on the latter, and one minus that probability as the weight on the former. This approach allowed us to decompose this value as the residual value with subordinated debt, plus an expected wealth transfer term. The expected wealth transfer is the product of the conversion probability and the wealth transfer term. Our analysis differs from the existing literature in that we pay explicit attention to that probability of conversion, rather than treating it as a given term.

We apply our framework to the full range of CoCos issued so far: principal writedown (PWD) CoCos, which are not well-covered in the academic literature but widely issued, and convert-to-equity (CE) CoCos with dilutive and nondilutive conversion ratios. We show that for equal loss absorption capacity, all PWD and nondilutive CE CoCos each have substantially worse risk-shifting incentives than requiring additional equity would lead to. Moreover, we show that all PWD CoCos and nondilutive CE CoCos have worse risk-shifting incentives compared to the same amount of subordinated debt. This is because the wealth transfer is always away from the CoCo holders towards the existing shareholders.

But when the CoCos are of the dilutive CE variety, we show that the risk-shifting incentive turns negative. This is because the wealth transfer itself becomes negative - while shareholders in aggregate obtain a higher residual equity upon conversion, the old shareholders must share the total residual value (i.e. old and new claims) with the new shareholders created upon conversion. The sharing of residual equity, while not strictly skin in the game ex ante, is a credible threat such that the shareholders can be expected to choose risk levels that make the conversion probability smaller. As a result, the risk level chosen under dilutive CE CoCos will be lower than the risk level chosen under the same amount of subordinated debt.

Therefore, the risk-shifting incentives arising from the expected wealth transfers can be viewed as a wedge that affects a bank's optimal risk choices relative to when the bank has issued subordinated debt in place of CoCos. While there is no question about the superiority of additional equity over subordinated debt, the wedge brought about by the risk-shifting incentives matters in determining whether CoCos are superior to subordinated debt. We find that PWD and nondilutive CE CoCos encourage banks to take riskier choices relative to subordinated debt, while dilutive CE CoCos discourage them. However, as 60% of the CoCos issued to date are of the PWD kind, it is important to recognize the possibility that CoCos might contribute to, rather than mitigate the buildup of risk in the banking system.

Recent regulation has encouraged the use of CoCos in order to meet regulatory capital or loss absorption capacity requirements. However, regulation neither distinguishes between these two CoCo designs for the purpose of meeting capital requirements, nor considers the interaction of CoCo issuance with existing frameworks. We show that even though CoCos and equity provide equal loss absorption capacity ex post, replacing subordinated debt with CoCos changes the interaction of the regulator and the bank ex ante, because of the risk-shifting incentive wedge. The regulatory bodies would seem to be well advised to pay more attention to the risk incentives brought about by the design of CoCos.

The remainder of this paper is structured as follows. Section 2 provides a short primer on CoCos, including some statistics regarding their issuance. Section 3 discusses the related literature. Sections 4, 5 and 6 present the model and the analysis. Section 7 considers the implications of issuing CoCos in the context of existing financial regulation. Section 8 concludes. The mathematical foundations of this paper are presented in Appendix A, while the proofs that are not in the text are presented in Appendix B.

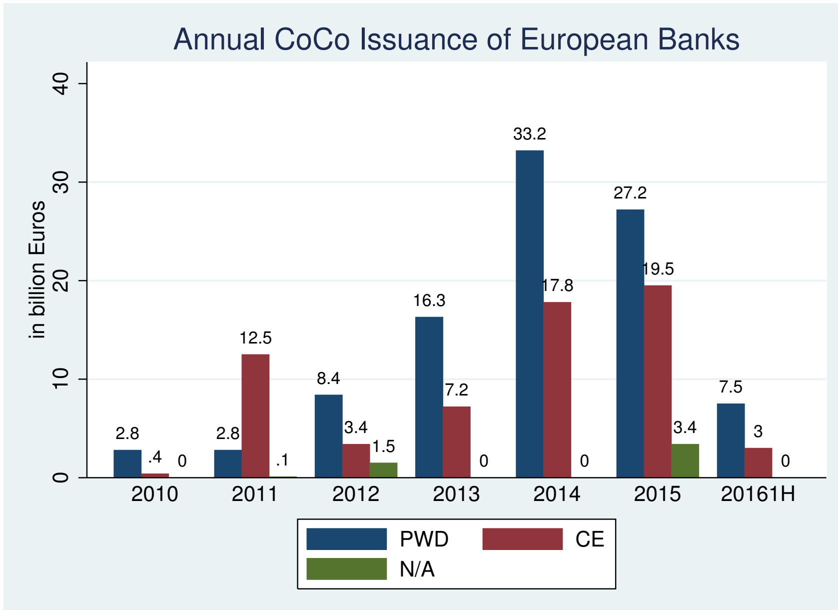
2 A Short Primer on CoCos

CoCos are hybrid instruments that are designed to improve the loss absorption capacity of the issuer without involving transfusions from new equity or taxpayer bailouts. These instruments were proposed by Flannery as early as 2005, but were thrust into the limelight after the financial crisis of 2008. Banks generally issue CoCos, though the insurance sector has already started looking into them as well. CoCos are issued as debt, but convert when the issuer encounters a trigger event. CoCos can be classified according to the type of trigger event and design.

There are two type of trigger events: automatic and discretionary. Automatic trigger events occur when the bank's equity ratio falls below a preset amount. The calculation may be based on either market or book values, although all of the issued CoCos have calculations based on book value. Discretionary trigger events occur when the regulator deems the bank to be near or at the point of nonviability (PONV). Because of the nature of the trigger event, CoCos have also been known as reverse convertible bonds. To qualify as part of regulatory capital under Basel III, CoCos must have at least the discretionary trigger. Because of this, most of the issued CoCos possess both types of triggers.

There are generally two types of CoCos based on design: principal writedown (PWD) CoCos are partially or fully written off the balance sheet, while convert-to-equity (CE) CoCos are converted to shares at a preset price.[1] Figure 1 presents the issuance of CoCos by design.

Figure 1: Annual CoCo Issuance of European Banks



Source: Dealogic (through the Association for Financial Markets in Europe)

It is notable that PWD CoCo issuances have overtaken CE CoCo issuances since 2012. By the first half of 2016, PWD CoCos amounted to 59% of total European issuances, while 38% were CE CoCos, and 3% were of an unspecified type. [2] Most of the CoCo issuance is by European and Asian banks. US banks have not participated in the wave of CoCo issuances because CoCos are treated as equity under US GAAP and as such, do not have tax benefits.

Because of their loss absorption capacity, CoCos have made their way into formal regulation. In June 2011, the Basel Committee on Banking Supervision released the final version of Basel III, [3] which addresses additional measures to ensure the stability of the banking system. One notable change from Basel II[4] is the strengthening of the capital base by enforcing stronger requirements for regulatory capital: loss absorption capacity is now a necessary quality for instruments to be included as part of Additional Tier 1 (going concern) capital and Tier 2 (gone concern) capital. Existing instruments that no longer qualify as

[1]Under Directive 2013/36/EU of the European Commission, the provisions governing the conversion of CE CoCos must specify either the rate of conversion and a limit on the permitted amount of conversion, or a range within which the instruments will convert into Common Equity Tier 1 instruments.

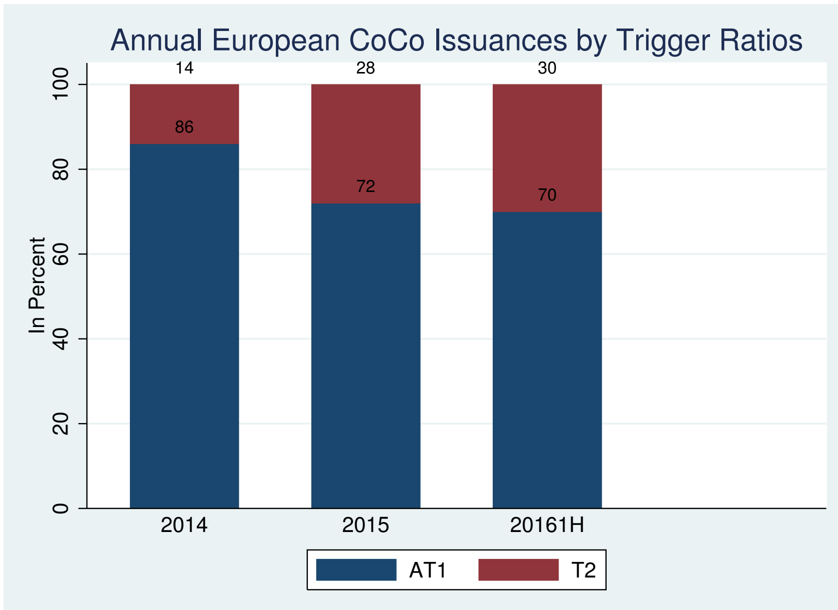
[2]In January 2011, RaboBank issued 2 billion Euros worth of PWD CoCos which had a cash payout to the CoCo holders in case of a trigger event. They have been redeemed by Rabobank in July 2016 . In 2016, ING launched a bond that contains an option allowing it to be transferred to ING's holding company, because of regulatory uncertainty over loss absorption.

[3]Basel III: A global regulatory framework for more resilient banks and banking systems

[4]Basel II: Revised international capital framework

regulatory capital have been phased out beginning January 2013, and replaced by CoCos. The criteria for whether a CoCo falls under Additional Tier 1 or Tier 2 depends only on their trigger level: above 5.125% qualifies as Additional Tier 1, otherwise they qualify as Tier 2. Figure 2 shows the distribution of the European-issued CoCos by their trigger ratios.

Figure 2: European CoCo Issuances by Trigger Ratios



Source: Dealogic (through the Association for Financial Markets in Europe)

While Basel III itself has no legal bite, it was translated into EU law in 2013 by the issuance of Directive 2013/36/EU, also known as the Capital Requirements Regulation and Directive (CRR/CRD-IV). This means that for EU banks, at most 3.5% of the 8% regulatory capital requirement will be filled in by CoCos. Moreover, there is no upper bound to the amount of CoCos they can issue. In addition, in November 2015, the Financial Stability Board (FSB) has released its Total Loss Absorption Capacity (TLAC) Standard for globally systemic financial institutions. The TLAC Standard mandates that for these institutions, minimum loss absorption capacity must be raised to 16% of risk weighted assets by January 2019, and to 18% by January 2022. The TLAC Standard's description of the loss absorbing instruments fits CoCos. With this, one should see an increase in the CoCo issuances over the next few years.

As CoCos are new and not well-understood, steps have been taken to protect unwitting consumers. In October 2014, the U.K.'s Financial Conduct Authority has prohibited banks from issuing CoCos to ordinary retail investors. Moreover, the market has been shown to be sensitive to potential trigger events.

In February 2016, the price of CoCos issued by Deutsche Bank fell from fears that the bank would not be able to meet its coupon payment obligations. However, the prices of other CoCos followed suit, despite the absence of adverse news regarding their issuers.

3 Related Literature

There is a small but growing body of research on the impact of CoCos on the risk-shifting incentives of banks. Koziol and Lawrenz (2012) only consider CE CoCos, and argue that risk-shifting incentives always increase relative to ordinary bonds, as long as the old equity holder gets to keep *some* shares after conversion. This strong result depends critically on their assumption that the conversion trigger coincides with the default trigger: If asset values decline enough to trigger default at a particular leverage ratio, replacing some of the debt by CoCos will leave shareholders better off: with an equal decline in asset values they are left with some claims and default is staved off, while in the straight debt case they would have lost everything. Berg and Kaserer (2014) numerically simulate the value of equity given an exogenously set mixture of debt and equity converter CoCos for four specific conversion ratios as a function of asset return variance. They argue that risk-shifting rises as wealth transfers from CoCo holders to equity holders increase, and observe, like Chan and van Wijnbergen (2015), that the price at which conversion takes place has a direct impact on the magnitude and even sign of these wealth transfers. They also show that several of the existing CoCos such as those issued by Lloyds and Rabobank have prices that *fall* with changes in implied asset volatility, inferring that the market recognizes the risk taken by the banks This finding points at very clear risk-taking incentives inherent in the CoCo designs issued by those two banks. Hilscher and Raviv (2014) argue that risk-taking incentives of banks may be mitigated by choosing the conversion ratio properly. For a capital structure containing CoCos, they found conversion ratios such that the resulting equity vega[5] is equal to zero. This is akin to the suggestion of Calomiris and Herring (2013) on having CoCos which are sufficiently dilutive. On the other hand, Martynova and Perotti (2015) claim that both CE and PWD CoCos can mitigate risk-shifting if the trigger level is set properly. In their paper, risk-shifting takes the form of not exerting sufficient effort in monitoring the assets of the bank. However they do not consider the possibility that the bank's risk choice affects both wealth transfers and the probability of conversion. Accounting for the latter link is at the core of the analysis presented in this paper.

[5] Vega is the sensitivity of the option value with respect to the volatility of its underlying assets.

Chen et al. (2015) endogenize the conversion[6] in an asset pricing setup similar to Koziol and Lawrenz (2012) and like them, only consider equity conversion CoCos. Although they derive closed form solutions, they use numerical procedures to obtain their results, which necessarily depend on chosen parameter values. They chose parameter values such that at least some dilution of old shareholders is taking place. As a consequence, conversion in the cases they analyze always imply a loss to old shareholders. But of the more than 200 billion Euro face value CoCos issued as of the first half of 2016, substantially more than half are issued on terms that imply a wealth transfer towards equity holders once conversion takes place, a possibility that plays a substantial role in our paper. In their set up, banks need to continuously roll over debt. This gives rise to rollover costs whenever the market value of the issued debt is lower than the par value of the newly issued debt. The possibility of this happening leads to lower risk-shifting by banks, because higher risk increases rollover costs.

4 Revisiting the Call Options Approach to Residual Equity Valuation

Black and Scholes (1973) and Merton (1974) have noted that the shareholders of a firm effectively hold a call option on their company's assets. While it is true that the creditors of the firm have claim over the assets to the extent of the outstanding liability, the shareholders can obtain the full claim to the assets upon paying off all outstanding liabilities. Therefore, the residual claim held by the shareholders can be thought of as a call option on the firm's asset, with the outstanding liability as the strike price.

For a bank that has issued hybrid instruments such as CoCos, the valuation of its residual equity is slightly more involved. This is because the change in the hybrid's "state" necessarily changes the bank's capital structure. This implies a corresponding change in the valuation of the residual equity. Therefore, the valuation of residual equity involving hybrids must take the various "states" into account.

If the probability of conversion was exogenous, valuation is straightforward: the residual equity value of a CoCo-issuing bank can simply be expressed as a linear combination of the residual equity values before conversion (when the CoCo is treated as debt) and after conversion (when the CoCo is either written off or is converted to equity), with the conversion probability as the weighting factor. However, CoCos convert whenever the bank encounters either an automatic or a discretionary trigger. The bank's ability to choose risk levels affects the shape of the return distribution, which in turn affects the bank's ability to meet either

[6] In their continuous time framework, endogenizing conversion comes down to endogenously determining the timing of conversion.

type of trigger. Therefore, we cannot assume that the probability of conversion is exogenous.

By expressing the bank's residual equity as a call option, and by recognizing that the probability of CoCo conversion is affected by risk levels chosen by the bank, we are able to examine the risk-taking incentives of a CoCo-issuing bank. Moreover, using the method outlined above, we can examine each type of CoCo design and determine which of them provides the best and the worst incentives for risk-taking.

4.1 Setup

Issued CoCos have two kinds of trigger: an automatic one which occurs whenever the bank fails to meet a preset equity ratio, and a discretionary one which occurs whenever the regulator believes the bank has reached the point of non-viability. In this paper, we focus on the automatic type.

A model with CoCos must have at least three dates because the risk choice, the conversion itself, and the final payoffs happen at distinct dates. However, if one wants to determine the ex ante risk-shifting incentives induced by a CoCo, it is enough to know the impact of risk on the expected realizations of the asset value at the time of conversion. Therefore, while we refer to $t = 1$ and $t = 2$ events (for the sake of exposition), our analysis focuses only on the $t = 0$ actions.

Consider a CoCo-issuing bank. At $t = 0$, its capital structure is composed of D_d deposits, D_s CoCo, and E initial equity. We assume that the CoCo does not convert at $t = 0$. At this stage, the CoCo-issuing bank is indistinguishable from an ordinary bank with D_s subordinated debt in place of CoCos. We normalize the amounts such that $D_d + D_s + E = 1$. We take these amounts as given, because we are interested in seeing how banks choose risk for a given capital structure. Since banks face capital regulation, the bank is constrained in choosing its capital structure in the first place.

Upon obtaining these funds, the bank invests them in an asset that gives return R_t at $t > 0$. We assume that R_t follows a lognormal distribution with parameters $\left(\mu, \sigma^2\right)$ for the corresponding normal distribution of $\ln(R_t)$. The bank can choose the risk level σ of the assets at $t = 0$. However, once the bank has chosen σ, it cannot make changes at any further time. Because we analyze at $t = 0$, we assume that the bank only knows and works with expectations about future returns. In particular, the bank works with expected return $R = \mathbb{E}_0(R_1) = \mathbb{E}_0(\mathbb{E}_1(R_2))$. Also, to ensure that we analyze a pure risk effect not confounded with increases in wealth, we structure the increase in risk in such a way that $\mathbb{E}(R_t) = R$ stays unchanged (i.e. a mean-preserving spread in variance).

The setup described above allows us to write the equity holder's claim as a call option on the asset return, as in Black and Scholes (1973) and Merton (1974). For ease, we assume there is only one share, and the bank does not issue any new shares aside from those that may arise from CoCo conversion. Denote the value of the share at $t = 0$ as e_0. Thus, before conversion, the bank's residual equity may be expressed as

$$e_0 \quad = \quad C\left[R,\ D_d + D_s\right] \tag{1}$$

where $C[R,\ D]$ is a call option[7] on an asset with gross return R and strike price D. Henceforth, we use "liability", " leverage", and "strike price" interchangeably, to refer to a bank's outstanding liability. In all subsequent calculations, we use D to refer to a general strike price, but specify the actual level of debt (e.g. D_d or $D_d + D_s$) when appropriate. As the unconverted CoCo is indistinguishable from subordinated debt, we also refer to the amount e_0 as the bank's residual equity value with subordinated debt.

At $t = 1$, the asset return realization is observed to be R_1. Provided that R_1 exceeds the total liability $D_d + D_s$, the bank remains solvent, otherwise, the bank is in default. Of course it is possible for the realization R_1 to be low enough to cause default even at $t = 1$. In that case, the bank is assumed to be closed down. However, we only consider cases when conversion precedes default. Henceforth, we assume that the bank's $t = 0$ expectation about the $t = 1$ return is larger than $D_d + D_s$: $\mathbb{E}_0(R_1) > D_d + D_s$.

CoCos convert at $t = 1$ when R_1 is lower than what is consistent with a preset trigger equity ratio τ. At $t = 2$ (provided that the bank has survived $t = 1$ events) when R_2 materializes, the creditors of the bank are paid, and anything left accrues to the residual claimant, which is the equity holder of the bank. We assume there is no risk of depositor runs (for example because of deposit insurance) in order to focus entirely on the risk-shifting implications of various CoCo designs. [8]

4.2 The Endogenous Conversion Probability

We have shown in the previous subsection that it is straightforward to value residual equity when D_s is subordinated debt. When CoCos are involved, we need to consider both the change in the value of the residual equity arising from the change in the outstanding liability, as well as the probability that the CoCo converts. A number of papers (for instance, Martynova and Perotti (2015)) treat this probability as exogenous. However, since the bank's choice of risk affects the distribution of the asset returns, the probability

[7] Appendix A contains the mathematical foundations of the call options framework.

[8] In principle it is also possible to draw conclusions from those risk choices for run probabilities: for such an analysis in a global games framework, see Chan and van Wijnbergen (2015).

of CoCo conversion cannot be exogenous. In this section, we define this probability endogenously by using the concept of distance-to-default and modifying it accordingly.

As the name suggests, distance-to-default is a measure of the closeness of the asset return and the value of the outstanding liability. For lognormally distributed asset returns R and total face value of debt D, distance-to-default d_d at $t = 0$ can be written as

$$d_d = \frac{1}{\sigma}\left[\ln\frac{R}{D} + r - \frac{\sigma^2}{2}\right] \tag{2}$$

where r is the risk-free rate. [9] It is implicit from the use of this measure that the default event occurs when the equity ratio of the bank is 0. However, with CoCos, the relevant event is not default, but conversion. For CoCos with automatic conversion, the trigger event is when the bank's equity ratio falls short of the trigger level $\tau > 0$. We therefore introduce a measure similar to distance-to-default by incorporating the trigger level τ, and call it distance-to-conversion d_c.[10] Formally, automatic conversion occurs whenever

$$\frac{R-D}{R} \leq \tau \Leftrightarrow R(1-\tau) \leq D, \tag{3}$$

allowing us to write the distance-to-conversion d_c as

$$d_c = \frac{1}{\sigma}\left(\ln\frac{R\,(1-\tau)}{D} + r - \frac{\sigma^2}{2}\right). \tag{4}$$

With the assumption of lognormally distributed returns, the conversion probability is then simply

$$p^c = \Phi\left(-d_c\right) \tag{5}$$

where $\Phi\left(\cdot\right)$ is the cumulative standard normal distribution. With the conversion probability now well-defined, we are now able to value the equity of a bank that has issued CoCos within our framework, as a linear combination of values of residual equity with differing amounts of outstanding liability.

[9] The standard form for distance-to-default is $d_d = \left[\ln\frac{R}{D} + \left(r - \frac{1}{2}\sigma^2\right)T\right] / \left[\sigma\sqrt{T}\right]$, for T periods ahead. Since in our model, conversion only occurs at $t = 1$, T takes the value of 1. Moreover, since we are performing the analysis at $t = 0$, we use the expected asset return R at $t = 0$ instead of the actual realization at $t = 1$ which is R_1.

[10] A similar measure has been introduced by Sy and Chan-Lau (2006), in the context of an early warning system for bank regulators.

As d_c is a function of both τ and σ, the probability of conversion p^c must be as well. We have

$$\frac{\partial p^c}{\partial \tau} = -\phi\left(-d_c\right) \frac{\partial d_c}{\partial \tau} = \phi\left(-d_c\right) \times \left(\frac{1}{\sigma\left(1-\tau\right)}\right) > 0 \tag{6}$$

and

$$\frac{\partial p^c}{\partial \sigma} = -\phi\left(-d_c\right) \frac{\partial d_c}{\partial \sigma} = \phi\left(-d_c\right) \times \left(1 + \frac{d_c}{\sigma}\right) > 0 \tag{7}$$

where $\phi(\cdot)$ is the standard normal distribution. This leads to the following lemma:

Lemma 1. *The conversion probability is increasing in the risk σ taken, as well as in the trigger ratio τ that is given.*

The intuition behind this result lies in the distance-to-conversion expression. d_c is a standardized variable that is affected by the trigger ratio τ and the risk level σ. d_c falls in τ because ceteris paribus, the equity ratio of a bank is closer to a higher value of τ than to a lower one. On the other hand, an increase in σ always decreases the value of a variable that it standardizes. The fall in the distance-to-conversion induced by both of these factors, combined with the derivative of the cumulative standard normal distribution with respect to its parameter, deliver this lemma.

From Lemma 1, one can see that the trigger ratio τ and the risk level σ are substitutes to an extent, as they affect the conversion probability in the same direction. If one takes the cross partial derivative of (7) with respect to τ, one obtains

$$\frac{\partial^2 p^c}{\partial \tau \partial \sigma} = \frac{\phi\left(-d_c\right)\left(1-\tau\right)\left[\sigma d_c \frac{\partial d_c}{\partial \sigma} - 1\right]}{\sigma^2\left(1-\tau\right)^2} < 0, \tag{8}$$

which shows that the marginal conversion probability with respect to risk σ falls as the trigger ratio τ rises. By Young's theorem, the marginal conversion probability with respect to the trigger ratio τ also falls as the risk level σ rises. This leads to following corollary:

Corollary 2. *The risk level σ and the trigger ratio τ are substitutes in terms of their effect on the conversion probability.*

Corollary 2 suggests that if the bank has a target level of the probability of conversion, the bank can choose lower risk levels if the trigger ratio is high enough. Similarly, if the trigger ratio is low, the bank can achieve the target by choosing higher risk levels.

4.3 Residual Equity Valuation With CoCos In The Capital Structure

In this section, we consider the valuation of residual equity when CoCos are in the capital structure. The two states (pre- and post-conversion) must be considered in the valuation. To this end, we examine how conversion alters the issuing bank's residual equity.

There are two types of CoCos that have been issued to date: principal writedown (PWD) CoCos and convert-to-equity (CE) CoCos. PWD CoCos are written off by the fraction $(1 - \varphi) \in [0,\ 1]$ from the issuing bank's balance sheet whenever the bank encounters an automatic trigger event. That is, provided that a bank has the capital structure described in Section 4.1, but with D_s PWD CoCos instead of subordinated debt, conversion would change the bank's residual equity from $C\,[R,\ D_d + D_s]$ to $C\,[R,\ D_d + \varphi D_s]$, where φ represents the fraction of the CoCos that are retained on the balance sheet. We henceforth refer to φ as the retention parameter.

On the other hand, CE CoCos convert to equity at some conversion rate ψ per unit of CoCo when the issuing bank encounters an automatic trigger event. [11] That is, provided that a bank has the capital structure described in Section 4.1, but with D_s CE CoCos instead of subordinated debt, conversion would change the bank's residual equity from $C\,[R,\ D_d + D_s]$ to $\frac{1}{1+\psi D_s}\left(C\,[R,\ D_d]\right)$.

Both the writeoff and the equity conversion features can be accomodated by the expression in (9) to represent a general CoCo-issuing bank's residual equity after conversion.

$$\frac{C\,[R,\ D_d + \varphi D_s]}{1 + \psi D_s} \tag{9}$$

PWD CoCos can be represented by setting $\psi = 0$ in (9) and keeping $\varphi \in [0,\ 1]$. A PWD that is fully written off has $\varphi = 0$. Similarly, CE CoCos can be represented by setting $\varphi = 0$ in (9) and keeping $\psi \in [0,\ \infty)$. A full PWD CoCo ($\varphi = 0$) is equivalent to a CE CoCo with zero dilution ($\psi = 0$). At the time of writing, there does not exist an issued CoCo which has both writedown and equity conversion features.

Denote by e_{coco} the value of a general CoCo-issuing bank's residual equity at $t = 0$. As previously mentioned, the value of residual equity of a bank with CoCos in the capital structure can be written as a linear combination of the pre-conversion state and the post-conversion state, with the probability of

[11] Some papers refer to the conversion price, which is the inverse of the conversion rate. That is, for conversion rate ψ, the conversion price is $1/\psi$.

conversion p^c as the weighting factor. With this, we may write the CoCo-issuing bank's residual equity as

$$\begin{aligned} e_{coco} &= p^c \frac{C\left[R,\ D_d+\varphi D_s\right]}{1+\psi D_s} + (1-p^c)\ C\left[R,\ D_d+D_s\right] \\ &= C\left[R,\ D_d+D_s\right] + p^c\left(\frac{C\left[R,\ D_d+\varphi D_s\right]}{1+\psi D_s} - C\left[R,\ D_d+D_s\right]\right) \\ &= e_0 + p^c W, \end{aligned} \tag{10}$$

where the wealth transfer is

$$W = \frac{C\left[R,\ D_d+\varphi D_s\right]}{1+\psi D_s} - C\left[R,\ D_d+D_s\right] \tag{11}$$

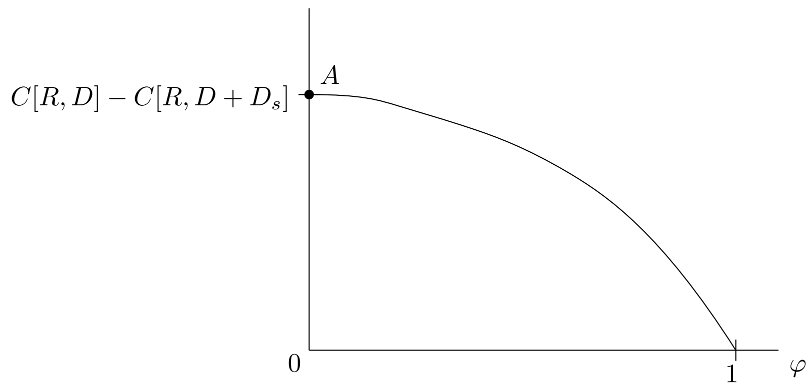
Thus, the ex ante value of residual equity of a CoCo-issuing bank can be expressed as the value of a bank's residual equity if it has issued subordinated debt e_0, plus an expected wealth transfer term $p^c W$.

The expected wealth transfer may be positive or negative, depending on the values of ψ and φ. A PWD CoCo's expected wealth transfer $p^c W_{pwd}$ is

$$p^c W_{pwd} = p^c\left(C\left[R,\ D_d+\varphi D_s\right] - C\left[R,\ D_d+D_s\right]\right), \tag{12}$$

which is always positive because the lower implied strike price after conversion ($D_d + \varphi D_s$) increases the value of the call option held by the bank's shareholder. Thus, the difference between $C\left[R,\ D_d+\varphi D_s\right] - C\left[R,\ D_d+D_s\right]$ is always larger than 0, and increases as φ moves from 1 to 0. Figure 3 illustrates the change in the wealth transfer from the point of view of the bank shareholder. At Point A in the Figure, when $\varphi = 0$, the wealth transfer from the CoCo holder to the existing shareholder is at its highest value. This is because nothing is left for the CoCo holder.

Figure 3: Wealth transfers from CoCo holders to equity holders for various levels of φ

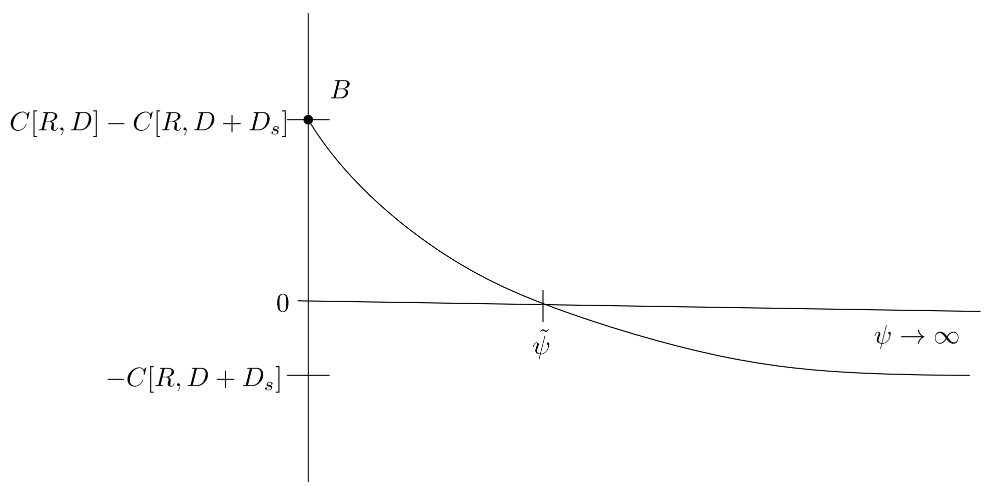


On the other hand, a CE CoCo's expected wealth transfer $p^c W_{ce}$ is

$$p^c W_{ce} = p^c \left(\frac{C\left[R,\ D_d\right]}{1 + \psi D_s} - C\left[R,\ D_d + D_s\right] \right), \tag{13}$$

which may be positive or negative over the the range of ψ, which is $\mathbb{R}^+$. Figure 4 illustrates the wealth transfer, again from the point of view of the original equity holder.

Figure 4: Wealth transfers from CoCo holders to equity holders for various levels of ψ



Point B of Figure 4 shows that wealth transfer is highest when $\psi = 0$. At this value of ψ, the CE CoCo is equivalent to a full PWD CoCo. However, as $\psi \to \infty$, the CoCo holder completely dilutes the original shareholder such that the claim of the original shareholder disappears. Hence, the wealth transfer is from the original shareholder to the CoCo holder. As the wealth transfer term W_{ce} is continuous in ψ, there exists a value of ψ that sets the wealth transfer of a CE CoCo exactly equal to 0, and it is found by setting

$W_{ce} = 0$. Call this value $\bar{\psi}$. We have that

$$\bar{\psi} = \frac{1}{D_s}\left(\frac{C\left[R,\ D_d\right]}{C\left[R,\ D_d + D_s\right]} - 1\right). \tag{14}$$

At $\bar{\psi}$, the number of new shares $\bar{\psi} D_s$ valued at the pre-conversion value of $C\left[R,\ D_d + D_s\right]$ is just equal to the difference in the values of residual equity pre- and post-conversion: $C\left[R,\ D_d\right] - C\left[R,\ D_d + D_s\right]$.[12] Because a wealth transfer from the CoCo holder to the shareholder is observationally equivalent to the dilution of the shareholder, we also refer to ψ as the dilution parameter. Any value of $\psi < \bar{\psi}$ leads to a wealth transfer from the CoCo holder to the shareholder (nondilutive CoCos). Any value of $\psi > \bar{\psi}$ leads to a wealth transfer from the shareholder to the CoCo holder (dilutive CoCos) Only at $\psi = \bar{\psi}$ is there a neutral conversion in the sense of not causing any wealth transfers in either direction.

5 The Risk-Shifting Incentives Induced by CoCos

In the previous section, we have shown that PWD CoCos always have positive wealth transfers upon conversion, but the direction of CE CoCo wealth transfers vary with the dilution parameter ψ. To examine the risk-shifting incentives of each type of CoCo, we take the derivative of the expected wealth transfers with respect to σ. This is because the expected wealth transfer measures the impact of replacing a given amount of subordinated debt with an equivalent amount of CoCos. In effect, we are looking at the differential effect of CoCos on a bank's risk-making decisions, with subordinated debt as the benchmark. As previously mentioned, we assume that changes in σ do not change the expected return R - that is, we assume a mean-preserving spread in variance, in order to abstract away from wealth effects that are not brought about by changes in σ.

If one uses an exogenous probability of conversion in the expected wealth transfers, then CoCo conversion necessarily leads to lower risk-shifting. This is because wealth transfers shrink as σ rises, ceteris paribus. However, we cannot ignore the impact of risk on the conversion probability, as we have shown in Lemma 1 that the probability of conversion increases in risk. In this section, we find conditions for which the conversion probability effect dominates the wealth transfer effect. As PWD and CE CoCos have differing mechanisms, we discuss them separately.

[12]Calomiris and Herring (2013) has a similar discussion and the recommendation to use a conversion price closely related to our definition of $\bar{\psi}$. Also, this price is critical according to Sundaresan and Wang (2015) if multiple equilibria are to be avoided in the case of market-based (share price) conversion triggers.

5.1 Risk-shifting Incentives for Given CoCo Design

5.1.1 PWD CoCos

The value of residual equity of a bank that has issued a PWD CoCo is

$$e_{pwd} \quad = \quad e_0 + p^c \left(C\left[R,\ D_d + \varphi D_s\right] - C\left[R,\ D_d + D_s\right]\right). \tag{15}$$

The differential effect of using a PWD CoCo in place of the same amount of subordinated debt is given by the expected wealth transfer term $p^c W_{pwd}$:

$$p^c W_{pwd} = e_{pwd} - e_0 = p^c \left(C\left[R,\ D_d + \varphi D_s\right] - C\left[R,\ D_d + D_s\right]\right). \tag{16}$$

Define now the risk-shifting incentive of such a bank as RSI_{pwd}. This term is the derivative of $p^c W_{pwd}$ with respect to σ, as shown in (17).

$$RSI_{pwd} \quad = \quad \underbrace{\frac{\partial p^c}{\partial \sigma} \left(C\left[R,\ D_d + \varphi D_s\right] - C\left[R,\ D_d + D_s\right]\right)}_{CF_{pwd}} + \underbrace{p^c \frac{\partial}{\partial \sigma} \left(C\left[R,\ D_d + \varphi D_s\right] - C\left[R,\ D_d + D_s\right]\right)}_{WF_{pwd}} \tag{17}$$

Two components of RSI_{pwd} arise from the differentiation: the conversion probability factor (CF_{pwd}) and the wealth transfer factor (WF_{pwd}). CF_{pwd} represents the increase in the probability of conversion as risk increases, holding the wealth transfer constant. On the other hand, WF_{pwd} represents the change in the wealth transfer as risk increases, holding the conversion probability constant.

Let us first consider the conversion probability factor CF_{pwd}, reproduced in (18):

$$CF_{pwd} = \frac{\partial p^c}{\partial \sigma} \left(C\left[R,\ D_d + \varphi D_s\right] - C\left[R,\ D_d + D_s\right]\right). \tag{18}$$

CF_{pwd} has two components, the derivative of the conversion probability with respect to σ, and the wealth transfer itself. From Lemma 1, we know that $\frac{\partial p^c}{\partial \sigma} > 0$. The sign of CF_{pwd} then depends on the sign of the wealth transfer: for the case of PWD CoCos, it is always positive. Therefore, an increase in risk raises the probability of conversion, makes it more likely for the wealth transfer to be obtained. Considering only an exogenous probability of conversion would ignore the impact arising from CF_{pwd}.

Consider now the wealth transfer factor WF_{pwd}, reproduced below as (19):

$$WF_{pwd} = p^c \frac{\partial}{\partial \sigma} \left(C\left[R,\ D_d + \varphi D_s\right] - C\left[R,\ D_d + D_s\right] \right). \tag{19}$$

WF_{pwd} represents the impact of the increase in the risk level on the value of the wealth transfer itself, holding the probability of conversion constant. While the wealth transfer itself is positive, it is decreasing in the risk taken. The intuition behind this is that a conversion increases a bank's skin in the game. Prior to conversion, the bank has less of its own capital. After conversion, the disappearance of $1 - \varphi$ of the CoCo implies that the bank has more of its own capital, making risk-shifting less attractive than in the previous case. To see this formally, note that (19) takes the derivative of the difference of two call option expressions with respect to σ. This can be written as the difference between the vegas[13] of two call options that differ only in the strike price. That is,

$$WF_{pwd} \quad = \quad p^c \left(V\left[R,\ D_d + \varphi D_s\right] - V\left[R,\ D_d + D_s\right] \right) \tag{20}$$

where $V[\cdot]$ is the call option vega. As $V[\cdot]$ is continuously differentiable, we may rewrite (20) using the mean value theorem. Denote by V_D the derivative of vega with respect to the strike price D. Then, (20) may be rewritten as

$$WF_{pwd} \quad = \quad -p^c \left((1-\varphi)\, D_s\, V_D\left[R,\ D'\right] \right) \tag{21}$$

where $D' \in [D_d + \varphi D_s,\ D_d + D_s]$.

WF_{pwd} is negative given any value of risk and leverage. However, it consists of $V_D[\cdot]$, which is positive whenever $\sigma^2 > 2\left(\ln \frac{R}{D} + r\right)$, and goes to zero as σ outpaces d_1, where $d_1 = \frac{1}{\sigma}\left(\ln \frac{R}{D} + r + \frac{\sigma^2}{2}\right)$. Let us call these as the high fragility conditions. The high fragility conditions captures the substitutability of risk and leverage for banks: given a high leverage ratio $\frac{D}{R}$, the bank needs a smaller level of risk σ to keep $V_D[\cdot]$ constant, as well as the diminishing marginal returns to risk: a higher level of σ leads to lower values of $V_D[\cdot]$. The effect is more pronounced as σ outpaces d_1. When the high fragility conditions are met, WF_{pwd} goes to zero as well while CF_{pwd} stays positive, such that CF_{pwd} dominates WF_{pwd}.

[13] Vega is the sensitivity of the option value with respect to the volatility of its underlying assets, represented by the derivative of a call option with respect to σ.

Proposition 3. *The risk-shifting incentive of a principal writedown CoCo is positive whenever the high fragility conditions hold.*

5.1.2 CE CoCos

Consider now the value of residual equity when a firm has issued a CE CoCo:

$$e_{ce} = e_0 + p^c \left(\frac{C[R,\ D_d]}{1+\psi D_s} - C[R,\ D_d + D_s] \right). \tag{22}$$

The differential effect of using a CE CoCo in place of the same amount of subordinated debt is given by the expected wealth transfer term $p^c W_{ce}$:

$$p^c W_{ce} = e_{ce} - e_0 = p^c \left(\frac{C[R,\ D_d]}{1+\psi D_s} - C[R,\ D_d + D_s] \right). \tag{23}$$

Define now the risk-shifting incentive of such a bank as RSI_{ce}. This term is the derivative of $p^c W_{ce}$ with respect to σ, as shown in (24):

$$RSI_{ce} = \underbrace{\frac{\partial p^c}{\partial \sigma} \left(\frac{C[R,\ D_d]}{1+\psi D_s} - C[R,\ D_d + D_s] \right)}_{CF_{ce}} + \underbrace{p^c \left(\frac{V[R,\ D_d]}{1+\psi D_s} - V[R,\ D_d + D_s] \right)}_{WF_{ce}}, \tag{24}$$

where we have used the vega notation to simplify matters. As with RSI_{pwd}, RSI_{ce} also has two components, the conversion probability factor (CF_{ce}) and the wealth transfer factor (WF_{ce}). However, the expressions for CE CoCos involve the dilution parameter ψ, which causes changes in the direction of the wealth transfer. Analyzing the risk-shifting incentives must take the size of ψ into consideration.

To begin, take the derivative of RSI_{ce} with respect to ψ. We have that

$$\frac{\partial RSI_{ce}}{\partial \psi} = -\frac{D_s}{(1+\psi D_s)^2} \left(\frac{\partial p^c}{\partial \sigma} C[R,\ D_d] + p^c V[R,\ D_d] \right), \tag{25}$$

so the risk-shifting incentives fall as the dilution parameter increases. When $\psi = 0$, the CE CoCo is equivalent to a full PWD CoCo. Therefore, the risk-shifting incentives for this type of CE CoCo is positive, from the results of the previous section. On the other hand, when $\psi \to \infty$, we would have, at the limit,

$$RSI_{ce}\left(\psi \to \infty\right) \quad = \quad \underbrace{\frac{\partial p^c}{\partial \sigma}\left(-C\left[R,\ D_d + D_s\right]\right)}_{CF_{ce}} + \underbrace{p^c\left(-V\left[R,\ D_d + D_s\right]\right)}_{WF_{ce}}. \tag{26}$$

Conversion then allows the CoCo holder to completely dilute the original shareholder. This causes the wealth transfer to be negative, leading to a negative CF_{ce} term. Similarly, a full dilution leads to a negative WF_{ce} term because the shareholder compares the marginal risk incentive from having no share after conversion (0) with the marginal risk incentive from holding a call option value of $C\left[R,\ D_d + D_s\right]$. Thus, $RSI_{ce}\left(\psi \to \infty\right)$ has negative risk-shifting incentives.

The above analysis implies that there is a value of ψ that just makes the CE CoCo deliver zero risk-shifting incentives. Since $RSI_{ce}(\psi = 0) > 0 > RSI_{ce}(\psi \to \infty)$, we get by continuity a crossing at zero for a positive ψ. Call this value $\tilde{\psi}$. We obtain this value by setting (24) to 0 and solving for ψ. [14] The resulting expression for $\tilde{\psi}$ is

$$\tilde{\psi} = \frac{1}{D_s}\left(\frac{\frac{\partial p^c}{\partial \sigma}C\left[R,\ D_d\right] + p^c V\left[R,\ D_d\right]}{\frac{\partial p^c}{\partial \sigma}C\left[R,\ D_d + D_s\right] + p^c V\left[R,\ D_d + D_s\right]} - 1\right), \tag{27}$$

which we show to be less than $\bar{\psi}$ in Appendix B.3. Thus, any $\psi \in [0,\ \tilde{\psi})$ will yield a positive risk-shifting incentive (i.e. worse than in the alternative capital structure with subordinated debt instead of CoCos). Any $\psi \in [\tilde{\psi},\ \infty)$ makes the risk-shifting incentives negative, regardless of whether the high fragility conditions discussed in the previous section are met. This result is stronger than the one obtained for the case of PWD CoCos, because it holds for a nonlimiting value of ψ.

Corollary 4. *For any risk level σ and leverage D , the risk-shifting incentives of a convert-to-equity CoCo is negative if the dilution parameter ψ is larger than $\tilde{\psi}$, and positive otherwise.*

5.2 Effect of Other Design Features on Risk-shifting Incentives

Thus far, we had considered the risk-shifting incentives brought about by having CoCos in a bank's capital structure. These incentives were studied taking design parameters as given. However, certain aspects of CoCo design may mitigate the risk-shifting incentives. In the previous section, we have shown that the risk-shifting incentive for a CE CoCo falls when the dilution parameter increases. In this section, we examine

[14] The results are consistent with those of Hilscher and Raviv (2014), who find the conversion ratio that achieves zero vega. However, they only consider the wealth transfer and the leverage channels. Our calculations for the conversion ratio also takes the endogenous probability of conversion into account.

the impact of the retention parameter for a PWD CoCo, and the trigger ratio for both types of CoCo on the risk-shifting incentives. There are two channels where these operate: the probability of conversion, and the wealth transfer.

5.2.1 Risk Taking Incentives as a Function of the Retention Parameter φ

We have shown that the risk-shifting incentives for any PWD CoCo (RSI_{pwd}) are positive when the fragility condition is met, given the retention parameter φ. But the risk-shifting incentive changes with φ, because φ affects the size of the wealth transfer W_{pwd}, even though the probability of conversion is unaffected. We have

$$\frac{\partial RSI_{pwd}}{\partial \varphi} = \underbrace{-\frac{\partial p^c}{\partial \sigma} \exp(-r)\, \Phi(d_2^*)\, D_s}_{\partial CF_{pwd}/\partial \varphi} + \underbrace{p^c V_D^* \frac{D_s}{D_d + \varphi D_s}}_{\partial WF_{pwd}/\partial \varphi}, \tag{28}$$

where the notations V_D^* and d_2^* refer to V_D and d_2 evaluated at liabilities $D_d + \varphi D_s$.[15] The term $\partial CF_{pwd}/\partial \varphi$ is always negative: since φ is the fraction of the debt retained, a higher retention rate (smaller writedown) leads to lower risk-shifting incentives because the actual wealth transfer is also smaller.

Consider now the term $\partial WF_{pwd}/\partial \varphi$ in (28). While this expression is always positive, we show in Appendix A.4 that V_D tends to zero whenever the high fragility conditions hold, so $\partial CF_{pwd}/\partial \varphi$ dominates $\partial WF_{pwd}/\partial \varphi$. Thus the higher the writedown fraction, the higher the risk-shifting incentives become.

Corollary 5. *When the high fragility conditions hold, the risk-shifting incentive of a principal writedown CoCo is increasing in the fraction of the CoCo written off upon conversion.*

5.2.2 Impact of τ on the Risk-Shifting Incentives

In this section, we examine the impact of the trigger level τ on the risk-shifting incentives. The results from this section emanate from Lemma 1, which means the effect is solely through the probability of conversion, not the wealth transfer. To see this, we again use the residual equity of a bank that has issued a general CoCo, (9), introduced in Section 4.3 and reproduced here as (29):

[15] We have that $d_2 = \frac{1}{\sigma}\left[\ln \frac{R}{D} + r - \frac{1}{2}\sigma^2\right]$ for strike price D. d_2 is the same as distance-to-default measure introduced in Section 4.2.

$$e_{coco} = C\left[R,\ D_d + D_s\right] + p^c \left(\underbrace{\frac{C\left[R,\ D_d + \varphi D_s\right]}{1 + \psi D_s} - C\left[R,\ D_d + D_s\right]}_{wealth\ transfer} \right). \tag{29}$$

The trigger level τ does not appear in the wealth transfer component of (29), so we may use W to represent the wealth transfer without loss of information. As before, the risk-shifting incentive is calculated by taking the derivative of the expected wealth transfer $p^c W$ with respect to σ, as shown in (30).

$$RSI = \frac{\partial p^c W}{\partial \sigma} = \frac{\partial p^c}{\partial \sigma} W + p^c \frac{\partial W}{\partial \sigma} \tag{30}$$

Differentiating the risk-shifting incentive with respect to τ leads to the following expression:

$$\frac{\partial RSI}{\partial \tau} = \frac{\partial^2 p^c}{\partial \sigma \partial \tau} W + \frac{\partial p^c}{\partial \tau} \frac{\partial W}{\partial \sigma}. \tag{31}$$

Note that the effect of τ is solely through the probability of conversion. From Lemma 1, $\frac{\partial p^c}{\partial \tau} > 0$ while $\frac{\partial^2 p^c}{\partial \tau \partial \sigma} < 0$ follows from Corollary 2. The net effect must take the wealth transfers into consideration. For PWD and nondilutive CE CoCos, the wealth transfer is always positive, while the marginal effect of risk on the wealth transfer is negative. So raising the trigger level τ always reduces the risk-shifting incentives embedded in those CoCo designs.[16] This is a possible way of mitigating the ill effects of CoCos that were designed to favor the original shareholders. As for dilutive CE CoCos, the fact that $\frac{\partial^2 p^c}{\partial \sigma \partial \tau} < 0$ interacts with the negativity of the wealth transfer, such that the net effect is more ambiguous.

Corollary 6. *For PWD and nondilutive CE CoCos, the risk-shifting incentive is decreasing in the trigger ratio τ. For dilutive CE CoCos, the impact of τ depends on the size of the wealth transfer.*

This result supports the Basel III requirement of a trigger level of 5.125% or higher for a CoCo to qualify as Additional Tier 1 capital.

6 The Bank's Optimization Problem with CoCos

We have shown in the previous section that a bank's risk-shifting incentives are affected by CoCo design. These incentives are related to, but distinct from a bank's problem of maximizing the net value of residual

[16] Martynova and Perotti (2015) also find that increasing the trigger level induces the banks to exert more effort in order to stave off conversion. This is consistent with our result that risk-shifting incentives decline as the trigger level rises.

equity. In this section, we show how a bank would choose its risk levels when faced with a constrained optimization problem. To this end, we introduce expected costs of default, and show how a bank's risk decision changes for different roles of D_s: additional equity, subordinated debt, PWD CoCo and CE CoCo.

In the literature, imposing expected costs of default is usually associated with social objective functions, as in Kashyap and Stein (2004). In our model, it is necessary even for the private objective function. This is because while the call option function necessarily accounts for the probability of default by construction, it does not account for the costs associated with default other than the foregone asset returns. Moreover, without these expected costs, the bank's maximization problem would remain unbounded for the range of parameters that we are interested in.

The expected default costs we have in mind have two components: the actual costs of bankruptcy, and the probability of default. The bankruptcy costs may be reputational or legal in nature, and distinct from social costs such as contagion effects on other banks, or taxpayer-funded bailouts. We keep these costs exogenous to our analysis, as we use a partial equilibrium framework.

The probability of default is a function of both risk σ and leverage D. For analytical convenience we use the first order Taylor approximation of this probability function in σ^2 and in D. The probability of default is distinct from the probability of conversion, although a sufficiently low draw of R_1 at $t = 1$ would make both events coincide. The literature on CoCos has paid more attention to probability of default than on the probability of conversion, perhaps due to the emphasis on the loss-absorption capacity of CoCos. In Chen et al. (2015) and Hilscher and Raviv (2014), the probability of default is influenced by the asset value that leads to default, which is chosen endogenously by shareholders in their analysis. However, the interaction of risk choices with the bank's capital structure is not considered explicitly in these papers.

6.1 A bank's objective function for given leverage D

Let X represent the bank's private costs of default, and let p^d represent the bank's probability of default. As stated above, we let X be given, and we adopt a functional form for p^d which is a linear approximation of the probability of default that is obtained from the Merton model: that is,

$$p^d = \Phi\left(-d_d\right), \tag{32}$$

where d_d is the distance-to-default introduced in Section 3. We may write p^d as a linear approximation around values of σ^2 and D away from zero, say $\bar{\sigma^2}$ and $\bar{D}$. This can be done as we are interested in values of σ^2 and D for which the high fragility conditions hold:

$$\begin{aligned} p^d\left(\sigma^2,\ D\right) &\approx p^d\left(\bar{\sigma^2},\ \bar{D}\right) + \frac{\partial p^d}{\partial \sigma^2}\left(\bar{\sigma^2},\ \bar{D}\right)\sigma^2 + \frac{\partial p^d}{\partial D}\left(\bar{\sigma^2},\ \bar{D}\right)D \\ &= \frac{1}{2}\sigma^2 b + cD. \end{aligned} \tag{33}$$

The probability of default in (33) is then obtained by omitting the irrelevant constant term as well as the higher-order terms, and where b and c are positive constants. Thus, the expected costs of default of a given bank is

$$p^d\left(\sigma,\ D\right)X = \left(\frac{1}{2}\sigma^2 b + cD\right)X, \tag{34}$$

This parameterization reflects that a higher risk choice and a higher leverage level make default more likely.

The bank would like to maximize the value of its residual equity (represented by the call option function), subject to the expected default costs in (34). The objective function takes the following form for expected return R, given leverage D:

$$\max\ C\left[R,\ D\right] - p^d X = \max\ C\left[R,\ D\right] - \left[\left(\frac{1}{2}\sigma^2 b + cD\right)X\right]. \tag{35}$$

The bank maximizes (35) by choosing σ. Similar to Kashyap and Stein (2004), we assume that the bank's leverage D cannot be adjusted at the time of choosing σ. Therefore, in the maximization process, the leverage term D drops out. For a given D, the first-order conditions associated with (35) is

$$V\left[R,\ D\right]|\sigma^* = \sigma^* bX, \tag{36}$$

where the notation $V\left[R,\ D\right]|\sigma^*$ means that the function $V\left[R,\ D\right]$ is evaluated at $\sigma = \sigma^*$. The objective function in (35) is concave in σ when $\sigma^2 > 2\left(\ln\frac{R}{D} + r\right)$. Therefore, for this range of σ, we know that there exists a σ that solves first-order conditions of the form (36). Since we are determining how CoCos would be effective in a crisis, we assume throughout this section and the next that the bank is operating when $\sigma^2 > 2\left(\ln\frac{R}{D} + r\right)$ holds. The next subsections consider how the banks' optimal σ changes with the capital structure.

6.2 Subordinated Debt vs. Equity

Consider first the case where the bank's capital structure has D_d deposits, and $D_s + E$ initial equity at $t = 0$. Given this capital structure, the bank essentially holds a call option on the asset return R at a strike price of D_d, leading to an objective function of the form

$$\max C\left[R,\ D_d\right] - \left(\frac{1}{2}\sigma^2 b + cD_d\right) X \tag{37}$$

and the first-order condition

$$V\left[R,\ D_d\right] |\sigma_e^* = \sigma_e^* bX, \tag{38}$$

where σ_e^* represents the optimal risk level under the circumstances.

Consider now the case where the bank's capital structure at $t = 0$ consists of D_d deposits, D_s subordinated debt, and E initial equity. Valuation of the bank's residual equity in this case requires that the strike price be $D_d + D_s$, leading to the objective function

$$\max\ C\left[R,\ D_d + D_s\right] - \left(\frac{1}{2}\sigma^2 b + c\left(D_d + D_s\right)\right) X \tag{39}$$

and the first-order condition

$$V\left[R,\ D_d + D_s\right] |\sigma_s^* = \sigma_s^* bX, \tag{40}$$

where σ_s^* represents the optimal risk level with $D_d + D_s$ leverage.

We show in Appendix A that the vega is decreasing in σ and increasing in D whenever $\sigma^2 > \ln\left(\frac{R}{D} + r\right)$. Therefore, since $D_d < D_d + D_s$, the graph of $V\left[R,\ D_d + D_s\right]$ should lie above that of $V\left[R,\ D_d\right]$ for any given σ. Figure 5 illustrates the case:

Figure 5: Optimal Risk Choice of Banks when D_s is Additional Equity/Subordinated Debt

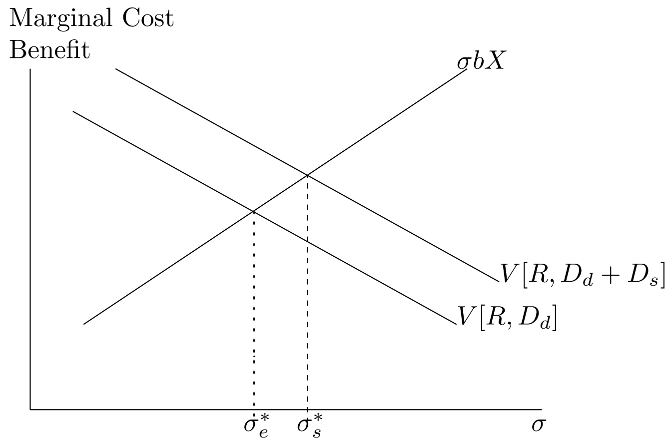


Figure 5 shows that the vega of a bank with D_s additional equity intersects the marginal cost line σbX at a smaller value of σ compared to the vega of a bank with D_s subordinated debt. That σ_s^* is higher than σ_e^* reflects the higher risk-shifting incentives from issuing D_s subordinated debt relative to issuing the same amount of additional equity. We may derive it more formally as follows: note that we may rewrite $V\left[R,\ D_d + D_s\right]|\sigma_s^*$ in terms of σ_e^* by using the mean value theorem, resulting in the following first-order approximation:

$$V\left[R,\ D_d + D_s\right]|\sigma_s^* = V\left[R,\ D_d\right]|\sigma_e^* + \left(V_\sigma|\sigma_e^*\right)\left(\sigma_s^* - \sigma_e^*\right) + \left(V_D|D_d\right)\ D_s, \tag{41}$$

where the notation $V_\alpha|\beta$ refers to the derivative of $V[\cdot]$ with respect to α, with α evaluated at β. By writing $\sigma_s^* bX$ as $\sigma_e^* bX + \left(\sigma_s^* - \sigma_e^*\right) bX$, we may rewrite (40) as

$$V\left[R,\ D_d\right]|\sigma_e^* + \left(V_\sigma|\sigma_e^*\right)\left(\sigma_s^* - \sigma_e^*\right) + \left(V_D|D_d\right)\ D_s = \sigma_e^* bX + \left(\sigma_s^* - \sigma_e^*\right) bX. \tag{42}$$

Subtracting (38) from (42) lets us obtain an expression showing that $\sigma_s^* > \sigma_e^*$.

$$\sigma_s^* = \sigma_e^* + \frac{\left(V_D|D_d\right)\ D_s}{bX - \left(V_\sigma|\sigma_e^*\right)} > \sigma_e^* \tag{43}$$

As $V_\sigma[\cdot]$ is always negative whenever $\sigma^2 > 2\left(\ln\frac{R}{D} + r\right)$, the denominator $bX - \left(V_\sigma|\sigma_e^*\right)$ is always positive as well.

Proposition 7. *The optimal amount of risk that a bank takes with D_s subordinated debt is higher than the* *optimal amount of risk if the bank has issued D_s additional equity.*

This result is intuitive: as the bank has more skin-in-the-game when it has issued more equity, it would choose lower risk levels as well.

6.3 Subordinated Debt vs. PWD and CE CoCos

When a bank issues D_s CoCos in place of the same amount of subordinated debt, the bank's objective function becomes

$$\max\ C\left[R,\ D_d + D_s\right] + p^c W - \left(\frac{1}{2}\sigma^2 b + c\left(D_d + D_s\right)\right) X \tag{44}$$

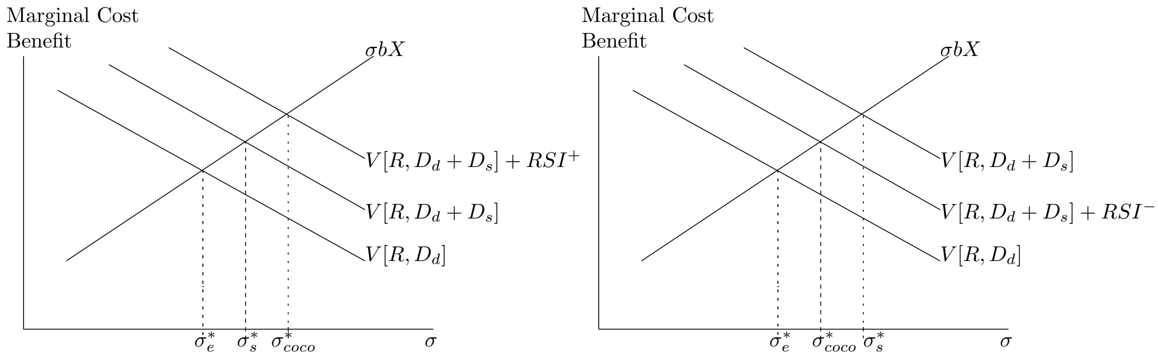
which is similar to (39) but with the expected wealth transfer term $p^c W$. The accompanying first order condition is

$$V\left[R,\ D_d + D_s\right] |\sigma^*_{coco} + RSI = \sigma^*_{coco} b X, \tag{45}$$

where RSI is the risk-shifting incentive arising from the expected wealth transfer $p^c W$. If RSI is zero, then (45) coincides with (40), because the strike price $(D_d + D_s)$ is the same regardless of whether D_s was issued as subordinated debt or as a CoCo. Therefore, the sign and magnitude of RSI determines how much the bank's behavior would change relative to the subordinated debt case.

We have shown in Section 5 that PWD CoCos and nondilutive CE CoCos have positive risk-shifting incentives, while dilutive CE CoCos have negative risk-shifting incentives. Therefore, for PWD CoCos and nondilutive CE CoCos, $V\left[R,\ D_d + D_s\right] + RSI$ must lie above that of $V\left[R,\ D_d + D_s\right]$ for any given σ provided that $\sigma^2 > 2\left(\ln \frac{R}{D} + r\right)$. Similarly, $V\left[R,\ D_d + D_s\right] + RSI$ must lie below $V\left[R,\ D_d + D_s\right]$ for dilutive CE CoCos. Figure 6 illustrates the first order conditions associated with D_s CoCos and D_s subordinated debt, for different RSI values.

Figure 6: Optimal Risk Choice of Banks when D_s is Subordinated Debt/CoCo



(a) PWD and Nondilutive CE CoCos (b) Dilutive CE CoCos

RSI^+ and RSI^- indicate that the risk shifting incentives are positive and negative, respectively.

As mentioned before, the forms in (44) and (45) accomodate both type of CoCos. We consider each type separately.

6.3.1 Optimal risk choices with PWD CoCos

To analyze PWD CoCos, we use (45) but use the subscript *pwd* to be more specific. Letting σ_{pwd}^* denote the solution to the bank's maximization problem, we may write the first-order condition as

$$V\left[R,\ D_d + D_s\right] |\sigma_{pwd}^* + RSI_{pwd} = \sigma_{pwd}^* bX. \tag{46}$$

Since (46) differs from (40) only by the risk-shifting incentive RSI_{pwd}, we can attribute the excess of σ_{pwd}^* over σ_s^* to the positive risk-shifting incentive brought about by the expected wealth transfer. Formally, we have

$$\sigma_{pwd}^* = \sigma_s^* + \frac{RSI_{pwd}}{bX - (V_\sigma | \sigma_s^*)} > \sigma_s^* \tag{47}$$

Proposition 8. *The optimal amount of risk that a bank takes with D_s principal writedown CoCos is higher than the optimal amount of risk if the bank has issued D_s subordinated debt.*

It is true that PWD CoCos improve loss absorption after conversion, and therefore meet the criteria for inclusion in Additional Tier 1 capital. However, as they elicit positive risk-shifting incentives before conversion, their use may make it more likely that the loss absorption capacity will be necessary in the future.

6.3.2 Optimal risk choices with CE CoCos

Similarly, to analyze CE CoCos, we use (45) but use the subscript *ce* to be more specific. Letting σ^*_{ce} denote the solution to the bank's maximization problem, we may write the first-order condition (up to a first-order approximation) as

$$V\left[R,\ D_d + D_s\right] |\sigma^*_{ce} + RSI_{ce} = \sigma^*_{ce} bX. \tag{48}$$

As with the PWD CoCos, we can express σ^*_{ce} in terms of σ^*_s in the following manner:

$$\sigma^*_{ce} = \sigma^*_s + \frac{RSI_{ce}}{bX - (V_\sigma|\sigma^*_s)} \tag{49}$$

The sign of RSI_{ce} determines whether σ^*_{ce} exceeds σ^*_s or not. We have shown in Section 5.1.2 that the dilution parameter ψ completely determines the sign of RSI_{ce}: a $\psi < \tilde{\psi}$ (nondilutive) leads to $RSI_{ce} > 0$, while $\psi > \tilde{\psi}$ (dilutive) leads to $RSI_{ce} < 0$.

Proposition 9. *The optimal amount of risk that a bank takes with D_s nondilutive CE CoCos is higher than the optimal amount of risk if the bank has issued D_s subordinated debt, but the opposite is true if the bank has issued the same amount of dilutive CE CoCos.*

It is then clear that dilutive CE CoCos induce better risk choices than the same amount of subordinated debt. As such, their inclusion as Additional Tier 1 capital is an improvement, but as they do not constitute skin in the game ex ante, they are still different from equity. Nonetheless, the threat of dilution effectively deters risk-shifting.

6.3.3 Dilutive CE CoCos vs. Equity

Thus far we have proven two sets of results, $\sigma^*_{ce} < \sigma^*_s$ with dilutive CE CoCos, and $\sigma^*_{ce} > \sigma^*_s$ otherwise. But can we determine how CE CoCos compare with straight equity in terms of risk choice? Post-conversion, dilutive CoCos and straight equity provide the same loss absorption capacity. But before conversion, it is the threat of a forthcoming dilution that leads to lower risk choices for dilutive CE CoCos. In contrast, it is higher skin in the game which leads to lower risk choices before conversion for the same amount of additional equity. It is worth examining whether there exists a dilution parameter that leads to better risk-shifting incentives for CE CoCos relative to additional equity.

Recall from (38) that when D_s is equity, the strike price is D_d, so the first order condition is $V[R,\ D_d]\,|\sigma_e^* = \sigma_e^* bX$. From (48), for the case when D_s is a convert-to-equity CoCo, the first order condition is $V[R,\ D_d + D_s]\,|\sigma_{ce}^* + RSI_{ce} = \sigma_{ce}^* bX$.

If we decompose $V[R,\ D_d + D_s]\,|\sigma_{ce}^*$ in terms of σ_e and $V[R,\ D_d]$, we can rewrite the first order condition of a CE CoCo as

$$\begin{aligned} V[R,\ D_d]\,|\sigma_{ce}^* + V_\sigma(\sigma_{ce}^* - \sigma_e^*) + (V_D|D_d)\ D_s + RSI_{ce} &= (\sigma_{ce}^* - \sigma_e^*)\,b + \sigma_e^* b \\ \sigma_{ce}^* &= \sigma_e^* + \frac{(V_D|D_d)\ D_s + RSI_{ce}}{bX - (V_\sigma|\sigma_e^*)} \end{aligned} \tag{50}$$

Thus, any ψ that sets $(V_D|D_d)\ D_s + RSI_{ce} \geq 0$ makes the risk-shifting incentive of D_s CE CoCo smaller than or equal to the risk-shifting incentive for D_s additional equity, for equal loss absorption capacity after conversion. In particular, it is

$$\psi \geq \psi_{eq} = \frac{1}{D_s}\left(\frac{p^c V[R,\ D_d] + \frac{\partial p^c}{\partial \sigma} C[R,\ D_d]}{p^c V[R,\ D_d + D_s] + \frac{\partial p^c}{\partial \sigma} C[R,\ D_d + D_s] - \left(\frac{R\phi(d_1)}{D}\right)\left(\frac{d_1}{\sigma}\right) D_s} - 1\right). \tag{51}$$

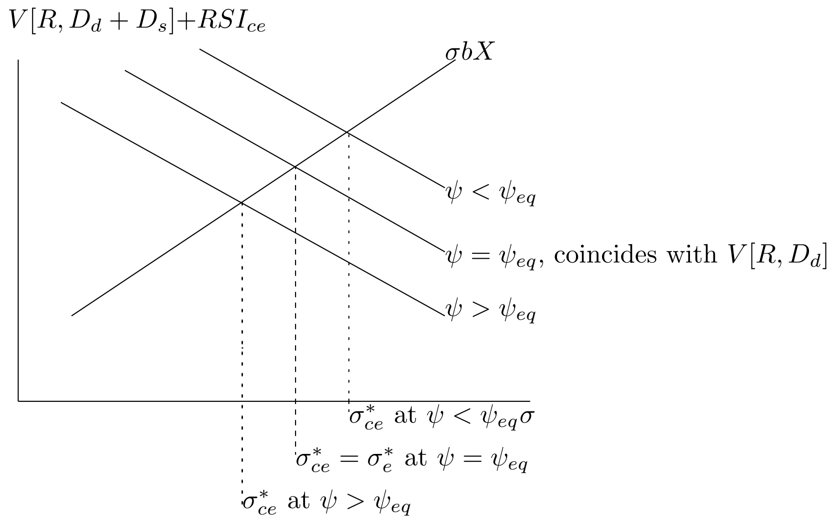
Note that ψ_{eq} resembles $\tilde{\psi}$ in (27). However, $\psi_{eq} > \tilde{\psi}$ because

$$\frac{\partial p^c}{\partial \sigma} C[R,\ D_d + D_s] + p^c V[R,\ D_d + D_s] > \frac{\partial p^c}{\partial \sigma} C[R,\ D_d + D_s] + p^c V[R,\ D_d + D_s] - \frac{R}{D}\frac{d_1}{\sigma}\phi(d_1)\,D_s$$

whenever $\sigma^2 > 2\left(\ln \frac{R}{D} + r\right)$. Also, $\psi_{eq} > \bar{\psi}$ because at $\bar{\psi}$, $RSI_{ce} = 0$ and since RSI_{ce} is decreasing in ψ, it must be that $\psi_{eq} > \bar{\psi}$.

This means that if the conversion ratio ψ of CE CoCos are superdilutive (i.e. when $\psi \in [\psi_{eq},\ \infty)$), they are better than straight equity in terms of risk-shifting incentives. Figure 7 illustrates the relationship between the risk-shifting line for equity and for CE CoCos with varying dilution parameters.

Figure 7: Optimal risk choices for additional equity/superdilutive CE CoCos



The following proposition holds:

Proposition 10. for $\psi \in \left[0,\ \tilde{\psi}\right]$, *we have* $\sigma^*_e < \sigma^*_s < \sigma^*_{ce}$ *For* $\psi \in \left[\tilde{\psi},\ \psi_{eq}\right]$ we have $\sigma^*_e < \sigma^*_{ce} < \sigma^*_s < \sigma^*_{pwd}$. Finally, for $\psi \in \left[\psi_{eq},\ \infty\right]$, we get a strong result: $\sigma^*_{ce} < \sigma^*_e < \sigma^*_s < \sigma^*_{pwd}$

So when the CoCo is superdilutive (i.e. $\psi > \psi_{eq}$), D_s CE CoCos provide lower risk-shifting incentive compared to straight equity, for equal loss absorption capacity. And even when they are not superdilutive but still provide at least a zero wealth transfer to the old shareholder, they still perform better than either subordinated debt or PWD CoCos, in that they provide less risk-shifting incentives for the same loss absorption capacity as subordinated debt would. But if the CoCos are not dilutive at all, they are worse than subordinated debt in that they provide even worse risk-shifting incentives for equal loss absorption capacity. In that case they clearly should not be part of Additional Tier 1 capital.

6.3.4 Interaction of τ with probability of default

In the previous sections, we have already seen that an increase in τ reduces the distance-to-conversion, thereby increasing the conversion probability. However, it does not play a role in the probability of default. To see this, consider again the first order condition for a general CoCo, as in (45) relative to the one for subordinated debt, as in (40). This results in the following optimal risk choice:

$$\sigma^*_{coco} = \sigma^*_s + \frac{RSI}{bX - (V_\sigma|\sigma^*_s)} > \sigma^*_s \tag{52}$$

τ only plays a role in RSI. Therefore, taking the derivative of σ^*_{coco} with respect to τ is equivalent to looking at the sign of RSI's derivative with respect to τ:

$$\frac{\partial \sigma^*_{coco}}{\partial \tau} = \frac{1}{bX - (V_\sigma | \sigma^*_s)} \frac{\partial RSI}{\partial \tau}. \tag{53}$$

We already know from Corollary 6 that $\frac{\partial RSI}{\partial \tau} < 0$ for PWD and nondilutive CE CoCos, while the sign is ambiguous for dilutive CE CoCos. Therefore, holding everything else constant, an increase in the trigger ratio causes a decrease in the risk taking incentives of a bank that has issued either PWD or nondilutive CE CoCos.

Corollary 11. *Taking the probability of default into consideration, a bank that has issued PWD or nondilutive CE CoCos will lower its risk-taking in response to a higher trigger ratio.*

7 Interaction of CoCos with pre-existing financial regulation

The goal of banking regulation is to protect the system from default externalities, and by extension, prevent the use of taxpayer money for bailout purposes. We consider the capital requirement aspect of banking regulation in this section. [17] There are two sides to capital requirements: a target probability of default, and the capital requirement itself. When the regulator sets a target probability of default, she does so taking the bank's leverage as an input, among other factors. The bank must choose a risk level which is compatible with its leverage, and complies with the target probability of default at the same time. When the regulator sets capital requirements, she forces the bank to change its capital structure in such a way that the bank's skin in the game increases. This increase leads to less risky behavior by banks. Both actions discourage banks from making risk choices that may adversely affect the financial system.

Recent regulatory changes pushed CoCos to the frontline. From Basel III, CoCos now form part of Additional Tier 1 and Tier 2 capital for bank. This means that CoCos will comprise at most 3.5% out of the 8.0% minimum total capital required based on risk-weighted assets. Moreover, in November 2015, the Financial Stability Board has mandated that an additional 8% of capital requirements (based on risk-weighted assets) be filled in by CoCos for globally systemic financial institutions. These regulations imply that CoCos will form a substantial portion of a bank's balance sheet in the near future, replacing subordinated debt to a

[17] See VanHoose (2007) for a very informative survey on bank behavior and capital regulation.

large extent. However, as we have seen in the previous section, the replacement of subordinated debt with CoCos have implications on a bank's risk choices because of the expected wealth transfers.

In this section, we examine how replacing subordinated debt with CoCos affects bank risk choices, given that the regulator has imposed both capital requirements and a target probability of default. [18] In order to do this, we build on the bank's maximization problem from the previous section. We have previously mentioned that the bank's expected costs of default are a function of both risk σ and leverage D, as in (34). This implies that for a target probability of default $\overline{p_d}$, there is a tradeoff between risk and leverage. Because the regulator is assured that the bank will comply with its mandates, we can model the situation as a Stackelberg game: the regulator sets the target probability level knowing the bank's objective function, letting the bank react to the requirements.

7.1 Setup

The expected costs of default were defined in (34), where the probability of default was

$$p^d = \frac{1}{2}\sigma^2 b + cD. \tag{54}$$

The regulator sets a target level of this probability to be a constant equal to $\overline{p^d}$, similar to what is set out under Basel II and III. [19] From (54), there is a tradeoff between risk σ and leverage D for a constant p^d. For a bank to comply with $\overline{p^d}$, any increase in σ must be compensated by a decrease in D and vice versa. By totally differentiating (54) and setting it to 0, we obtain the following negative slope:

$$\begin{aligned} 0 &= \sigma b X \, d\sigma + c \, dD \\ \left.\frac{d\sigma}{dD}\right|_{\overline{p^d}} &= -\frac{c}{\sigma b X}. \end{aligned} \tag{55}$$

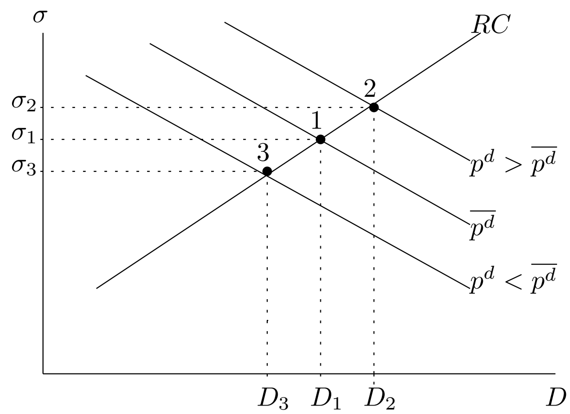
The downward sloping line labeled $\overline{p^d}$ in Figure 8 illustrates the tradeoff between risk and leverage that this choice of a given default probability implies. Given $\overline{p^d}$, a bank may choose higher σ if leverage D is lower. A higher (lower) target default probability corresponds to an upward (downward) shift in the downward

[18] The regulator's imposition of a target probability of default is a simple way of capturing bank-regulator interactions in the context of capital requirements, as in Nicole M. Boyson and Stulz (2014). This target may be derived from the regulator wanting to impose socially optimal risk levels, rather than privately optimal ones, as in Kashyap and Stein (2004).

[19] The internal ratings-based approach set forth in Basel II and III links capital requirements to the credit losses (and the probability that these losses occur) that regulators are willing to accept. This probability can be construed as the probability of default of a bank.

sloping line in Figure 8.

Figure 8: Bank's risk curve against regulator's chosen probability of default



We turn now to the bank's reaction function. In Section 6.1, we have shown that there is a positive relationship between a bank's leverage and choice of risk levels, because the bank's risk-shifting incentives increase with leverage. We can draw a reaction curve (*RC*) that shows the bank's best risk choice as leverage changes. *RC* can be interpreted as the reaction of the Stackelberg follower. As a benchmark, we first derive the bank's *RC* for a given leverage D . By totally differentiating the bank's first-order condition in (36), we obtain the condition that the bank must obey if it wants to maximize the value of its residual equity:

$$
\begin{aligned}
0 &= V_D\left[R,\ D\right]\ dD + \left(V_\sigma\left[R,\ D\right] - bX\right)\ d\sigma \\
\left.\frac{d\sigma}{dD}\right|_{RC} &= \frac{V_D\left[R,\ D\right]}{bX - V_\sigma\left[R,\ D\right]},
\end{aligned}
\tag{56}
$$

which is positive. *RC* is also illustrated in Figure 8. The representation is very much simplified: we draw the curves as linear, but it is only the slopes of the curves that are important.

The regulator can also set capital requirements (leverage) D in addition to $\overline{p^d}$, which when combined with the bank's reaction curve, forces a bank to choose a particular level of σ. At issue then is how the Stackelberg leader (regulator) picks the right point off that curve by imposing capital requirements or equivalently in our set up, the maximum amount of leverage D. To a regulator, there is a tradeoff between risk and leverage. Imposing a maximum leverage D_3 will allow the regulator to accept leverage of at most σ_2, if the target is $\overline{p^d}$. However, to a bank, risk and leverage reinforce each other, as reflected in the slope of the reaction curve. Therefore, it will choose a low level of risk, say σ_3, meaning that the bank takes too little risk relative to that which is considered optimal by the regulator, as Point 3 lies on $p^d < \overline{p^d}$. Similarly, if the regulator imposes a maximum leverage D_2, the optimal risk from her viewpoint is σ_3. The bank's reaction curve implies that it will choose σ_2, which is now too much risk compared to what the regulator deems optimal, as Point 2 lies on $p^d < \overline{p^d}$. Only if the regulator imposes leverage D_1 will the bank choose a risk level σ_1 that is compatible with the $\overline{p^d}$ specified by the regulator, at the intersection of the $\overline{p^d}$ and *RC* lines: Point 1 is the equilibrium solution to the Stackelberg game between the regulator and the bank. This example shows that the regulator must keep a bank's reaction curve in mind when setting capital requirements.

7.2 Replacing subordinated debt with CoCos

While a bank is always able to meet a leverage requirement with both deposits D_d and subordinated debt D_s, the regulator can only force a bank to choose her desired risk level when D_s cannot be bailed in, written down, or converted to equity. This is because the ability to eliminate all or part of D_s changes a bank's reaction curve, meriting further attention. Consider now what happens when, possibly in response to the recent change in capital standards, subordinated debt is replaced by CoCos. In Section 6.2, we have shown that CoCos have risk-shifting incentives which differ from subordinated debt, because of the expected wealth transfers. Therefore, a CoCo-issuing bank's first order condition for a given debt D should take the risk-shifting incentives into account, as in (57):

$$V[R,\ D] + RSI = \sigma b X. \tag{57}$$

This means that replacing subordinated debt by CoCos necessarily alters the reaction curve of a bank, because of the additional *RSI* term, which involves both σ and D as well. If we totally differentiate *RSI* with respect to both parameters, we obtain

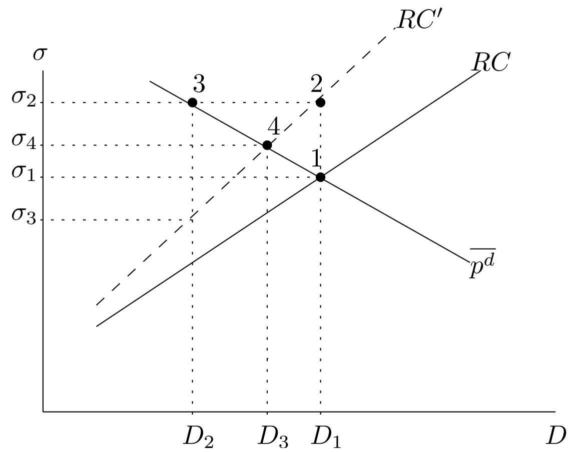
$$\begin{aligned} 0 &= \frac{\partial RSI}{\partial \sigma}\, d\sigma + \frac{\partial RSI}{\partial D}\, dD \\ \frac{d\sigma}{dD} &= -\frac{\frac{\partial RSI}{\partial D}}{\frac{\partial RSI}{\partial \sigma}}. \end{aligned} \tag{58}$$

For a CoCo with positive *RSI* (such as PWD and nondilutive CE CoCos), (58) is positive, because the risk-shifting incentive is increasing in leverage (less skin in the game implies higher gambling incentives) and decreasing in risk (diminishing marginal returns). Of course, for a CoCo with negative *RSI* (dilutive CE

CoCos), (58) is negative.

Consider first PWD and nondilutive CE CoCos. Let RC' denote the reaction curve drawn using (57). Since the risk-shifting incentive is positive, the reaction curve RC' must lie above that of RC. Figure 9 represents the change simply as an upward twist in the slope.

Figure 9: Upward twist in the risk curve due to replacing subordinated debt by risk-inducing CoCos



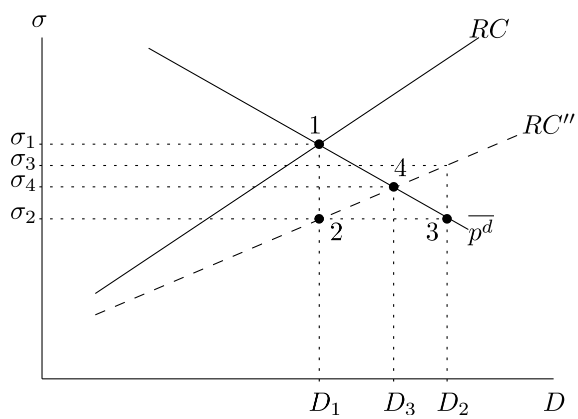
So suppose that the regulator has chosen the probability of default $\overline{p^d}$ and has imposed leverage D_1 on the banks, i.e. Point 1 in Figure 9, as in the benchmark case. Then, suppose for the sake of increasing loss absorption capacity, D_s subordinated debt is completely replaced with either a PWD or a nondilutive CE CoCo. This change causes the reaction curve to twist up from RC to RC'. As the bank did not change its leverage ratio, it still has D_1 leverage, but because of the potential wealth transfer brought about by the change from subordinated debt to equity, the risk incentives are higher: the bank's position is now at Point 2, where leverage is at D_1 but risk choice is at $\sigma_2 > \sigma_1$. What should the regulator do in this situation? At Point 2, the risk level σ_2 and leverage D_1 combination implies a probability of default which is higher than $\overline{p^d}$. To get back at $\overline{p^d}$ for risk level σ_2, she should impose higher capital requirements (lower leverage) D_2, as indicated in Figure 9 . But raising capital requirements by an additional $D_1 - D_2$ in turn leads to a lower risk choice of σ_3, which now implies a probability of default below $\overline{p^d}$, and so on. The new set of equilibrium values is at Point 4, with a higher risk choice than at Point 1 but a correspondingly larger loss absorption capacity because of the associated higher capital requirement.

Proposition 12. *When PWD and nondilutive CE CoCos are used by banks in their capital structure in place of subordinated debt, regulators should increase capital requirements if they want banks to choose risk levels that are consistent with the regulators' own preference.*

So given that subordinated debt only qualifies as Tier 2 capital under Basel III, it is arguable that PWD CoCos should not have been included as Additional Tier 1 equity regardless of the trigger level, because PWD CoCos lead to higher risk-shifting incentives. As conversion of a writedown CoCo wipes out a junior creditor, it allows the shareholder/manager to jump the seniority ladder. Therefore, they will not act in a safer manner even when compared with the case where these instruments are subordinated debt instead. Much of the CoCos issued between 2013 to 2015 have done just that, replace expiring subordinated debt.[20]

The situation is better when dilutive CE CoCos are considered, because the movement of the expected wealth transfer is away from the shareholder to the CoCo holder. Relative to subordinated debt, the same amount of CoCos have an additional term, *RSI*. The *RSI* for CE CoCos fall as the dilution parameter ψ increases, and are negative for $\psi < \tilde{\psi}$. Therefore, combining (57) and (58) for a negative value of *RSI*, the *RC* twists downwards to some RC'' instead of upwards. Figure 10 shows this other case.

Figure 10: Downward shift of the risk curve due to replacing subordinated debt by dilutive CoCos



As with the other case, suppose that the regulator has chosen the probability of default $\overline{p^d}$ and has imposed leverage D_1 on the banks, i.e. Point 1 in Figure 10, as in the benchmark case. Then, suppose for the sake of increasing loss absorption capacity, D_s subordinated debt is completely replaced with a dilutive CoCo. This change causes the reaction curve to twist down from *RC* to RC''. The fall in the reaction curve for a given leverage D_1 actually causes the bank's risk choice to fall from σ_1 to σ_2, in contrast to if the reaction curve twists upwards. To reach Point 4 in Figure 10, the regulator actually has to lower capital requirements to induce banks to take the optimal level of risk given RC'' and $\overline{p^d}$, which is σ_4. Seen this way, dilutive CoCos are a legitimate component of Additional Tier 1 capital, because they induce banks to

[20]Thompson, C. (2013, Dec. 17). Big rise in subordinated debt issuance by EU banks. *Financial Times.* Retrieved from https://www.ft.com/content/ca18e0f4-6732-11e3-a5f9-00144feabdc0

choose lower risk levels for a given leverage D.

Proposition 13. *When dilutive CE CoCos are used by banks in their capital structure in place of subordinated debt, regulators may decrease capital requirements if they want banks to choose risk levels that are consistent with their own preference.*

8 Conclusion

CoCos have become popular among banks since the emergence of Basel III and the Total Loss Absorption Capacity (TLAC) Standard by the Financial Stability Board. The reason is that CoCo conversion enhances loss absorption capacity by reducing the bank's leverage. However, an unintended consequence of this feature is that a wealth transfer occurs between the CoCo holders and the original shareholders when the conversion takes place. The wealth transfers may encourage the issuing bank to make conversion more likely. In this paper, we have looked at the implications of these wealth transfers on the issuing bank's risk-shifting incentives, relative to the same amount of subordinated debt.

By writing the issuing bank's residual equity as a linear combination of the pre-and post-conversion states, with the probability of conversion as the weighting factor, we were able to express the residual equity as one of a bank that has issued subordinated debt, plus an expected wealth transfer. The expected wealth transfer is the product of the wealth transfer and the conversion probability. While the literature has paid attention to the wealth transfer, it has largely taken the conversion probability as exogenous. We have endogenized this probability, as we recognize that this is influenced by a bank's risk choices.

The expected wealth transfer is affected by risk in two conflicting ways. First, higher risk levels increase the probability of conversion, which increases the expected wealth transfer. Effectively, this allows the shareholder to make conversion more likely. Second, the gains from the wealth transfer decrease as risk increases. In short there is a diminishing marginal gain in wealth transfers as risk increases, as the bank's skin in the game rises upon conversion. Unfortunately, the positive first effect dominates the negative second effect when initial risk levels or leverage ratios are sufficiently high, which are the circumstances that should give regulators cause for concern.

We have shown that the strength of the risk-shifting incentives is strongly influenced by CoCo design. As PWD CoCos and nondilutive CE CoCos always transfer wealth to equity holders upon conversion, the risk-shifiting incentive is positive. On the other hand, dilutive CE CoCos transfer wealth from equity holders to CoCo holders. The threat of dilution results in negative risk-shifting incentives relative to subordinated debt. The risk-shifting incentives act as a wedge in a bank's optimization problem, such that the optimal risk choice is different from that under the same amount of subordinated debt. For PWD CoCos and nondilutive CE CoCos, the risk choices are higher than under the same amount of subordinated debt, while for dilutive CE CoCos, it is lower.

These results naturally lead to further questions concerning capital requirements. A corollary of our results is that the interaction between capital requirements and asset-side portfolio risk must be carefully considered whenever amendments are made to existing policies. If CoCos are to continue to play an important role in the capital structure of banks, the level of capital requirements should also depend on how they are met. In that vein we have shown that some of the disadvantages of nondilutive CoCos can be offset by raising the bar higher: if inappropriate CoCo design increases risk taking incentives, that effect can be counteracted by requiring more skin in the game, i.e. by setting the requirement ratios higher than they are set for the case of pure equity or sufficiently dilutive CoCos.

These results are important in setting regulations. Basel III and the TLAC Standard were written with the focus on increasing loss absorption capacity of the financial system. To a substantial extent, this loss absorption capacity is being filled by CoCos, in particular for meeting TLAC requirements. But to achieve a more robust financial system, it is not enough to only consider loss absorption capacity. We must also consider regulation that prevents banks from choosing excessively risky actions in the first place, as the designers of Basel II fully realized when introducing risk weights. Capital regulation is meant to force banks to put more skin in the game in order to reduce risk-shifting incentives, and not just to increase loss absorption capacity for given risk levels. While CoCos are hybrids of debt and equity, it doesn't always mean that the risk levels they induce will be between those induced by debt and equity. As we have shown, not all CoCos are created equal - some have higher risk-shifting incentives than others. At the very least, the type of CoCo that is allowed to fill in Additional Tier 1 capital requirements should be restricted to equity converters, and among those only CE CoCos which are sufficiently dilutive. In this way, one minimizes the chance that the loss absorption capacity has to be used in the first place.

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Appendix

A Mathematical foundations: the call option function and its derivatives

Denote a call option with strike price D and expected return R as $C[R,\ D]$. The full expression for $C[R,\ D]$ is

$$\begin{aligned} C[R,\ D] &= \exp(-r)\left[R\exp(r)\Phi(d_1) - D\Phi(d_2)\right] \\ &= R\Phi(d_1) - \exp(-r)D\Phi(d_2) \end{aligned}$$

where r is the risk-free rate, $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution, $d_1 = \frac{1}{\sigma}\left[\ln\frac{R}{D} + r + \frac{1}{2}\sigma^2\right]$ and $d_2 = \frac{1}{\sigma}\left[\ln\frac{R}{D} + r - \frac{1}{2}\sigma^2\right]$. We use the following first and second-order partial derivatives of $C[R,\ D]$ in the paper.

A.1 Vega

Vega is the sensitivity of the option value with respect to the volatility of its underlying assets. It is calculated by taking the derivative of the call option with respect to volatility σ:

$$V[R,\ D] = \frac{\partial C[R,\ D]}{\partial\sigma} = R\phi(d_1) > 0$$

where $\phi(\cdot)$ is the probability density function of the standard normal distribution.

A.2 C_D: The derivative of the call option with respect to the strike price D

$$C_D = \frac{\partial C[R,\ D]}{\partial D} = -\exp(-r)\Phi(d_2) < 0$$

A.3 V_σ: The second-order derivative of $C[R,\ D]$ with respect to σ

The second-order derivative of $C[R,\ D]$ with respect to σ is the first-order derivative of vega with respect to σ. We refer to this shorthand as V_σ in the text.

$$V_\sigma = \frac{\partial^2 C[R,\ D]}{\partial\sigma^2} = \frac{\partial V[R,\ D]}{\partial\sigma} = R\phi'(d_1)\frac{\partial d_1}{\partial\sigma} = -R\phi(d_1)d_1\left(1 - \frac{d_1}{\sigma}\right)$$

which is negative for values of $\sigma^2 > 2\left(\ln\frac{R}{D} + r\right)$.

A.4 V_D: The cross-order partial derivative of $C[R,\ D]$ with respect to σ and D

The cross-order partial derivative of $C[R,\ D]$ with respect to σ and D is also the first-order derivative of vega with respect to the strike price D. We refer to this shorthand as V_D in the main text.

$$V_D = \frac{\partial^2 C[R,\ D]}{\partial\sigma\partial D} = \frac{\partial V[R,\ D]}{\partial D} = R\phi'(d_1)\frac{\partial d_1}{\partial D} = -R\phi(d_1)\,d_1\left(-\frac{1}{\sigma D}\right) = \frac{R}{D}\,\phi(d_1)\,\frac{d_1}{\sigma} > 0$$

Note that V_D can be written in terms of V_σ as follows:

$$V_D = -\frac{V_\sigma}{D(\sigma - d_1)}$$

which is positive whenever $\sigma^2 > 2\left(\ln\frac{R}{D} + r\right)$, precisely the same condition that keep $V_\sigma < 0$. Moreover, $V_D \to 0$ as the gap between σ and d_1 widens: as σ increases, V_D shrinks to 0. For a given σ, V_D goes to zero as D rises. We refer to $\sigma^2 > 2\left(\ln\frac{R}{D} + r\right)$ and the widening of the gap between σ and d_1 as high fragility conditions: $\sigma^2 > 2\left(\ln\frac{R}{D} + r\right)$ is necessary but not sufficient.

B Proof for various results in the paper

B.1 Proof that $\frac{\partial^2 p^c}{\partial\tau\partial\sigma} < 0$

$$\begin{aligned}
\frac{\partial^2 p^c}{\partial\tau\partial\sigma} &= \frac{\partial}{\partial\sigma}\left(\frac{\partial p^c}{\partial\tau}\right) \\
&= \frac{\sigma(1-\tau)\,\phi(-d_c)\,d_c\left(\frac{\partial d_c}{\partial\sigma}\right) - \phi(-d_c)(1-\tau)}{\sigma^2(1-\tau)^2} \\
&= \frac{\phi(-d_c)(1-\tau)\left[\sigma d_c\frac{\partial d_c}{\partial\sigma} - 1\right]}{\sigma^2(1-\tau)^2} \\
&= \frac{\phi(-d_c)(1-\tau)\left[-\sigma d_c\left(1+\frac{d_c}{\sigma}\right) - 1\right]}{\sigma^2(1-\tau)^2} \\
&< 0
\end{aligned}$$

B.2 Proof that $WF_{pwd} \to 0$

The risk-shifting incentive for a PWD CoCo (17) has two terms: the conversion probability factor CF_{pwd} and the wealth transfer factor WF_{pwd}. WF_{pwd} can be rewritten as the difference between the vegas of two call options that differ only in the strike price. Using the definition of vega from A.1 and V_D from A.4, we can use the mean value theorem to rewrite this difference as follows:

$$
\begin{aligned}
WF_{pwd} &= p^c \left(V\left[R,\ D_d + \varphi D_s\right] - V\left[R,\ D_d + D_s\right] \right) \\
&= -p^c \left(\left(1 - \varphi\right) D_s V_D \left[R,\ D'\right] \right)
\end{aligned}
$$

for some $D' \in \left[D_d + \varphi D_s,\ D_d + D_s\right]$. In A.4, we have noted that when the high fragility condition holds, we have that V_D goes to zero, such that WF_{pwd} goes to zero as well.

B.3 Impact of φ on the risk-shifting incentives of PWD CoCos.

Since $C\left[R,\ D_d + D_s\right]$ and $V\left[R,\ D_d + D_s\right]$ are not functions of φ, we may express (28) as

$$
\begin{aligned}
\frac{\partial RSI_{pwd}}{\partial \varphi} &= \underbrace{\frac{\partial p^c}{\partial \sigma} \frac{\partial C\left[R,\ D_d + \varphi D_s\right]}{\partial \varphi}}_{\partial CF_{pwd}/\partial \varphi} + \underbrace{p^c \frac{\partial V\left[R,\ D_d + \varphi D_s\right]}{\partial \varphi}}_{\partial WF_{pwd}/\partial \varphi} \\
&= \underbrace{-\frac{\partial p^c}{\partial \sigma} \exp\left(-r\right) \Phi\left(d_2^*\right) D_s}_{\partial CF_{pwd}/\partial \varphi} + \underbrace{p^c \frac{R\phi\left(d_1^*\right) D_s}{D_d + \varphi D_s} \frac{d_1}{\sigma}}_{\partial WF_{pwd}/\partial \varphi} \\
&= \underbrace{-\frac{\partial p^c}{\partial \sigma} \exp\left(-r\right) \Phi\left(d_2^*\right) D_s}_{\partial CF_{pwd}/\partial \varphi} + \underbrace{p^c V_D^* \frac{D_s}{D_d + \varphi D_s}}_{\partial WF_{pwd}/\partial \varphi}
\end{aligned}
$$

Line 2 follows from the fact that φ is a variant of D, enabling us to use the chain rule to link D and φ. A.2 and A.4 describe how to differentiate $C\left[\cdot\right]$ and $V\left[\cdot\right]$ with respect to D. The notations d_1^* and d_2^* indicate that the functions d_1 and d_2 were evaluated at strike price $D_d + \varphi D_s$ instead of a generic strike price D.

B.4 Proof that $\tilde{\psi} < \bar{\psi}$

The equation for RSI_{ce} is

$$RSI_{ce} = \underbrace{\frac{\partial p^c}{\partial \sigma}\left(\frac{C[R,\ D_d]}{1+\psi D_s} - C[R,\ D_d + D_s]\right)}_{CF_{ce}} + \underbrace{p^c\left(\frac{V[R,\ D_d]}{1+\psi D_s} - V[R,\ D_d + D_s]\right)}_{WF_{ce}}.$$

In Section 4.3 we have found that $\psi = \bar{\psi}$ sets the wealth transfer to 0, implying that $CF_{ce} = 0$, while WF_{ce} remains negative. As CF_{ce} and WF_{ce} are generally of opposite signs, we need only choose a ψ that makes CF_{ce} positive and exactly offsets the negative value of WF_{ce}. In other words, choose ψ such that $p^c\left(\frac{V[R,\ D_d]}{1+\psi D_s} - V[R,\ D_d + D_s]\right) = \frac{\partial p^c}{\partial \sigma}\left(\frac{C[R,\ D_d]}{1+\psi D_s} - C[R,\ D_d + D_s]\right)$. Let us call this value $\tilde{\psi}$. We claim that $\tilde{\psi} < \bar{\psi}$. The expression for $\bar{\psi}$ is

$$\bar{\psi} = \frac{1}{D_s}\left(\frac{C[R,\ D_d]}{C[R,\ D_d + D_s]} - 1\right).$$

On the other hand, the expression for $\tilde{\psi}$ is

$$\tilde{\psi} = \frac{1}{D_s}\left(\frac{\frac{\partial p^c}{\partial \sigma}C[R,\ D_d] + p^c V[R,\ D_d]}{\frac{\partial p^c}{\partial \sigma}C[R,\ D_d + D_s] + p^c V[R,\ D_d + D_s]} - 1\right) = \frac{1}{D_S}\left(\frac{C[R,\ D_d]\left(\frac{\partial p^c}{\partial \sigma} + p_c\frac{V[R,\ D_d]}{C[R,\ D_d]}\right)}{C[R,\ D_d + D_s]\left(\frac{\partial p^c}{\partial \sigma} + p_c\frac{V[R,\ D_d + D_s]}{C[R,\ D_d + D_s]}\right)} - 1\right).$$

$\bar{\psi} = \tilde{\psi}$ if and only if $\frac{V[R,\ D_d]}{C[R,\ D_d]} = \frac{V[R,\ D_d + D_s]}{C[R,\ D_d + D_s]}$. However, we can write $\frac{V[R,\ D_d + D_s]}{C[R,\ D_d + D_s]}$ as follows:

$$\frac{V[R,\ D_d + D_s]}{C[R,\ D_d + D_s]} = \frac{V[R,\ D_d] + V_D D_s}{C[R,\ D_d] + C_D D_s} > \frac{V[R,\ D_d]}{C[R,\ D_d]} \tag{59}$$

where V_D and C_D are the derivatives of vega and the call option value with respect to the strike price, respectively. The inequality follows from $C_D < 0 < V_D$: the value of a call option falls when the strike price rises, while the vega of a call option rises when the strike price rises. Therefore we have shown that $\tilde{\psi} < \bar{\psi}$, as claimed.

B.5 Derivation of σ_s^* in terms of σ_e^*

We use the mean value theorem to write $V[R,\ D_d + D_s]\,|\sigma_s^*$ in terms of $V[R,\ D_d]\,|\sigma_e^*$, using V_σ and V_D:

$$V[R,\ D_d + D_s]\,|\sigma_s^* = V[R,\ D_d]\,|\sigma_e^* + (V_\sigma|\sigma_e^*)\,(\sigma_s^* - \sigma_e^*) + (V_D|D_d)\ D_s,$$

enabling us to write the first order conditions as

$$V\left[R,\ D_d\right]|\sigma_e^* + \left(V_\sigma|\sigma_e^*\right)\left(\sigma_s^* - \sigma_e^*\right) + \left(V_D|D_d\right)\ D_s = (\sigma_s^* - \sigma_e^*)bX + \sigma_e^* bX. \tag{60}$$

Subtracting (38) from (60) leads to

$$\sigma_s^* = \sigma_e^* + \frac{V_D|D_s}{bX - (V_\sigma|\sigma_e^*)} > \sigma_e.$$

We assume that the default coefficient bX is large enough such that $bX - V_\sigma > 0$. Actually, from A.3, $V_\sigma < 0$ whenever $\sigma^2 > 2\left(\ln\frac{R}{D} + r\right)$ holds, so the assumption that $bX > V_\sigma$ is always justified, as our analysis assumes it.

B.6 Derivation of σ_{pwd}^* and σ_{ce}^* in terms of σ_s^*

We may use the mean value theorem to rewrite (46), the first order condition of a PWD CoCo:

$$V\left[R,\ D_d + D_s\right]|\sigma_{pwd}^* + RSI_{pwd} = V\left[R,\ D_d + D_s\right]|\sigma_s^* + \left(V_\sigma|\sigma_s^*\right)\left(\sigma_{pwd} - \sigma_s\right) + RSI_{pwd}.$$

If we subtract (42) from it, we obtain

$$\sigma_{pwd}^* = \sigma_s^* + \frac{RSI_{pwd}}{bX - (V_\sigma|\sigma_s^*)} > \sigma_s^*.$$

Similarly, we may use the mean value theorem to write (48), the first order condition of a CE CoCo:

$$V\left[R,\ D_d + D_s\right]|\sigma_{ce}^* + RSI_{ce} = V\left[R,\ D_d + D_s\right]|\sigma_s^* + \left(V_\sigma|\sigma_s^*\right)\left(\sigma_{ce}^* - \sigma_s^*\right) + RSI_{pwd}$$

If we subtract (42) from it, we obtain

$$\sigma_{ce}^* = \sigma_s^* + \frac{RSI_{ce}}{bX - (V_\sigma|\sigma_s^*)} > \sigma_s^*.$$