Credit Spreads and Incomplete Information

Snorre Lindset, Arne-Christian Lund, and Svein-Arne Persson*

E-mail: snorre.lindset@hist.no

arne-christian.lund@nhh.no

svein-arne.persson@nhh.no

May 14, 2008

Abstract

A company's credit spreads and default policy are analyzed in a structural model of credit risk. Agents have incomplete information about the company's EBIT (Earnings Before Interest and Taxes) process and observe it with time delays. When all agents observe the state variable with the same delay, the delay has a minor effect on credit spreads and default policy. Asymmetric information occurs when different agents observe the EBIT process with different time delays. Our

*Lindset is an Associate Professor at Trondheim Business School and an Adjunct Associate Professor at the Norwegian University of Science and Technology, Department of Industrial Economics, Lund is a post. doc fellow at the Norwegian School of Economics and Business Administration, and Persson is a Professor at the Norwegian School of Economics and Business Administration and an Adjunct Professor at Trondheim Business School. The authors would like to thank Darrel Duffie, Hans Marius Eikseth, Steinar Ekern, Jørgen Haug, Kristian Miltersen, Aksel Mjøs, Øyvind Norli, Per Östberg, and participants at faculty seminars at Trondheim Business School, the Norwegian School of Economics and Business Administration, Princeton University, and the University of Stavanger for useful comments and discussions. *First version: March 4, 2008.*

simple model with neither noisy accounting information nor discrete arrival of information, but with asymmetric information between bond- and equity holders, produces qualitatively similar results as Duffie and Lando (2001). Wider credit spreads are obtained in another model where we allow for trade of equity, and where the information asymmetry is between the management and the financial market.

Keywords and phrases: Credit risk, credit spreads, delayed information, asymmetric information.

JEL-classification: G12, G33

1 Introduction

We analyze a company's credit spreads and default policy in a model where agents have incomplete information about the company's EBIT (Earnings Before Interest and Taxes) process, the only state variable. The agents observe the state variable with time delays. If all agents observe the state variable with identical time delay, the delay has a minor effect on credit spreads and default policy compared to the case with complete information. Asymmetric information occurs if different agents observe the state variable with different time delays and leads to wider credit spreads. This effect is especially pronounced for bonds with short maturities, and short-term credit spreads do not converge to zero. Introducing a simplified Duffie and Lando (2001) model with asymmetric information between bond- and equity holders, we show that this asymmetry produces positive credit spreads for short-term maturities. Finally, a model relaxing non-tradability of equity and bonds, but with information asymmetry between management and the financial market produces wider credit spreads than the simplified model.

The risk that a debtor will not honor his contractual obligations with the creditor is called credit risk. This topic has received attention in both the academic literature and among practitioners. There are two dominating approaches to credit risk in the finance literature; *structural models* and *reduced form models*. The first was pioneered by Merton (1974). He models the value of a company's assets by a stochastic process and debt and equity are considered as contingent claims on the total asset value. Some of the papers in this tradition include Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Leland (1994), and Duffie and Lando (2001). The second approach assumes the existence of a default arrival intensity. This approach was pioneered by Jarrow and Turnbull (1992), for extensions see e.g., Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), and Schönbucher (1998).[1] Coculescu, Geman, and Jeanblanc (2008) and Guo, Jarrow, and Zeng (2008) analyze technical aspects of credit risk and incomplete information.

As a bond's time to maturity approaches zero, the credit spread approaches zero in traditional structural models. This property is not in line with observations in financial markets and is considered a problem with the structural models. The reduced form approach is typically able to produce strictly positive credit spreads also for short term maturities, but is not founded on economic models of the company. However, they seem to be more useful than structural models when it comes to practical use and calibration to market data.

We analyze four cases. The first case assumes complete information and builds on the model of Leland (1994). It serves as a natural benchmark for the other three cases.

In the second case we assume that all agents receive the same information with identical time delay. This case is motivated by Jarrow and Protter (2004) who state that real life values of companies are typically not observable, and that implications of this fact for default policy and credit spreads are not well understood. We show that the most important difference between case 2 and case 1 is that the value of a *bankruptcy lottery* must be accounted for when a bankruptcy decision is made. Furthermore, for realistic parameter values we show that the value of this lottery is too low to have a significant effect on default policy and credit spreads.

The third case is the first of two cases with asymmetric information. In

[1] Comprehensive treatments of these two approaches can be found in the enclopedic monograph by Bielecki and Rutkowski (2002) or in the more accessible monograph by Duffie and Singleton (2003).

this case equity holders are better informed than bond holders, i.e., equity holders receive information about the value of the state variable earlier than the bond holders. This assumption is in line with Duffie and Lando (2001). To keep these two groups of agents separated, equity is by assumption not traded, eliminating bond holders' access to the equity market. Equity holders are by assumption precluded from buying corporate debt. The latter assumption is justified by insider-trading regulation. This case produces wider credit spreads than case 1 and 2. In particular, short-term credit spreads do not converge to zero. Although our model is simpler than the model of Duffie and Lando (2001), i.e., it contains neither noisy accounting information nor discrete arrival of information, it produces qualitatively similar results. Asymmetric information and our simpler model with a continuous flow of information are sufficient to obtain short-term credit spreads that do not converge to zero.

In case 4 we introduce a new agent called management who is better informed than the financial market, i.e., the bond- and equity holders. We disregard all agency problems between the management and the equity holders. All participants in the financial market have access to the same information at any point in time. We relax the non-tradability assumption in case 3 and do not restrict equity holders from buying or selling bonds and equity and vice versa for bond holders. This is a reasonable and tractable property of our model and corresponds nicely with what is observed in real markets. These assumptions are superior to the assumptions in case 3 for the following reasons:

1. Tradability of equity extends the model's applicability to a much wider set of companies. There are relatively few companies with a secondary

market for corporate debt whose equity is not traded.[2]

2. When the company needs more capital to run its operations, non-tradability of equity can cause equity holders to file for bankruptcy due to liquidity problems, not because it is economically optimal. The reasons are:

    - The equity holders cannot finance infusion of capital by selling or diluting their stocks.
    - Equity holders might have problems borrowing money to finance any infusion of capital by using the equity as collateral since the true value of equity is only known by themselves and not by the lender. Moreover, this information cannot be revealed to the lender (otherwise, bond holders would have the same information as equity holders and the information asymmetry disappears).

3. Asymmetric information leads to wider credit spreads, and thereby a higher cost of debt financing. This result implies a lower debt ratio than the optimal ratio in the case of fully informed bond holders. This again leads to a lower total value of the firm, and, thus, asymmetric information also reduces the value of the equity. There is therefore no economic rationale for keeping information away from bond holders in these types of models.

Credit spreads are wider in case 4 than in case 3.

The model of Duffie and Lando (2001) is the first structural model which is equivalent to a reduced form model. In case 3 and 4 we present two more examples of structural models which are equivalent to reduced form models.

[2] Examples are Special Purpose Vehicles, mutually owned companies (banks and insurers), foundations, and municipals.

We do not address the problem of optimal capital structure, but instead focus our analysis on the effects of incomplete information on credit spreads and default policy.

The paper is organized as follows: In section 2 we present the economic model and some preliminaries. The classical case with complete information (case 1) is reviewed in section 3. The case with delayed information (case 2) is analyzed in section 4. The case with delayed and asymmetric information between bond- and equity holders (case 3) is analyzed in section 5. In section 6 we analyze the case with delayed and asymmetric information between management and the financial market (case 4). Section 7 contains some concluding remarks. Technical results and proofs are relegated to the appendices.

2 The Economic Model and Basic Results

2.1 The Set-up

We use the EBIT (earnings before interest and taxes) version of the economic model of Leland (1994), see e.g., Goldstein, Ju, and Leland (2001). The EBIT process of the company is given by the stochastic differential equation

$$d\delta_t = \mu\delta_t dt + \sigma\delta_t dB_t, \tag{1}$$

where the drift and volatility parameters μ and σ are constants. We assume that $\mu < r$, where r represents the constant risk free interest rate and that the initial value of the process δ_0 is a positive constant. Here B_t is a standard Brownian motion that is defined on a fixed, filtered probability space $(\Omega, \mathcal{F}, P)$. Furthermore, P represents the original probability measure, and all agents are risk neutral.

The information filtration $\mathcal{F}_t$ is generated by the process $\{\delta_s, 0 \leq s \leq t\}$

(augmented with all sets of measure zero). To incorporate delayed information we assume that two distinct classes of agents have information filtrations $\mathcal{F}_t^m$ and $\mathcal{F}_t^l$, where m and l signify *more* and *less* information, respectively. We define

$$\mathcal{F}_t^m = \mathcal{F}_{t-m}, \text{ for all } t \geq m,$$

$$\mathcal{F}_t^l = \mathcal{F}_{t-l}, \text{ for all } t \geq l,$$

and $0 \leq m \leq l$. Clearly, from this specification the filtrations can be nested as

$$\mathcal{F}_t^l \subseteq \mathcal{F}_t^m \subseteq \mathcal{F}_t.$$

Let the assessment of the value of the company at time t by an agent with information delay $k \in \{0, m, l\}$ be denoted by V_t^k. The assessed value of the company is the expected discounted value of the EBIT stream. An important feature of our model is that at time t the owner of the EBIT stream receives the delayed payment rate δ_{t-k}. Thus

$$V_t^k = E\Big[\int_t^\infty e^{-r(s-t)}\delta_{s-k}ds\big|\mathcal{F}_t^k\Big] = \frac{\delta_{t-k}}{r-\mu}, \tag{2}$$

where $t \geq k$. With a time lag k in the filtration, any other payment rate δ_{t-j}, $j < k$ could be used by the owner to reduce the information lag to j. We remark that V_t^k is simply δ_{t-k} multiplied by a constant, and is therefore also a geometric Brownian motion.

For $k = 0$ the value is identical to the value when there is no delay in the flow of information. Both the expectation and the righthand side in expression (2) clearly reflect the delay through the presence of δ_{t-k} instead of δ_t that is present under complete information. Note that $V_t^k = V_{t-k}^0$, i.e., the value assessed at time t by an agent with information lag k is identical to the value assessed at time $t-k$ by an agent with complete information.

As in Leland (1994) we assume that the company has issued perpetual debt with face value D. The debt is serviced by a constant rate of coupon payments C. These payments are tax deductible (only interest is paid on perpetual debt). The tax benefit rate is θC, where θ is the tax rate.

In our model the bankruptcy decision is based on the information of the better informed agent. In correspondence with practice, we let the equity holders make the bankruptcy decision in the following way in the four cases we consider: In the first three cases the equity holders are better informed and the bankruptcy decision is based on their information set. In the fourth case the management of the company, which always acts in the best interest of the equity holders, is better informed than the equity holders and the bankruptcy is therefore based on its information set.

We define the stopping time τ with respect to the filtration $\mathcal{F}_t^m$ for fixed $t \geq m$ as

$$\tau = \inf\{u \geq t : V_u^m \leq V_B^m\}, \tag{3}$$

where from expression (2) $V_t^m = \frac{\delta_{t-m}}{r-\mu}$, and V_B^m is a constant. In this model the company is bankrupt and liquidated the first time $V_t^m = V_B^m$, i.e., τ represents the time of bankruptcy.

In addition to the information contained in $\mathcal{F}_t^l$, the least informed agent also observes whether the company is bankrupt or not. Formally we define

$$\mathcal{G}_t^l = \mathcal{F}_t^l \vee \sigma(1_{\{s>\tau\}}, s \leq t),$$

where $1_{\{\cdot\}}$ denotes the usual indicator function.

The better informed agents are restricted from trading in the financial market, and the market therefore only consists of the least informed agents. Prices of debt instruments are therefore set by the least informed agents.

We denote the complete-information value of the company upon bankruptcy by V_τ, i.e., from equation (2),

$$V_\tau \equiv V_\tau^0 = \frac{\delta_\tau}{r-\mu}.$$

Upon bankruptcy a cost of αV_τ occurs. Here α is assumed constant, and the bankruptcy cost is therefore proportional to the complete-information value. Also, in case of bankruptcy, the debt holders require the face value of the debt D to be repaid. In general V_τ is different from V_τ^m, i.e., the complete-information value of the assets at time τ is different from the value upon which the bankruptcy decision is made. In particular, there is a positive probability that $V_\tau - D - \alpha V_\tau > 0$. In this case the time τ value of the company is sufficient to cover debt and bankruptcy costs, and any proceeds are paid to the equity holders.

2.2 Corporate Bond Pricing

We analyze a zero coupon bond maturing at time s with recovery rate $R(\tau, s)$ in the case of default at time τ. Duffie and Lando (2001) explain the connection between the perpetual debt and the following unit discount bond. The time t price of a unit discount corporate bond maturing at time s is the conditional expected discounted payoff, i.e.,

$$\begin{aligned} \varphi(t,s) &= E\Big[e^{-r(s-t)}1_{\{\tau>s\}} + e^{-r(\tau-t)}R(\tau,s)1_{\{\tau\leq s\}}\big|\mathcal{G}_t^l\Big] \\ &= e^{-r(s-t)}P(\tau>s\big|\mathcal{G}_t^l) + \int_t^s R(u,s)e^{-r(u-t)}f_\tau(u)du, \end{aligned} \tag{4}$$

where $f_\tau(u)$ is the probability density of the stopping time τ conditional on $\mathcal{G}_t^l$. Define

$$P(s) = P(\tau > s\big|\mathcal{G}_t^l). \tag{5}$$

In the remaining of the paper we calculate $P(s)$ under different assumptions about the information structures. For notational conveinience we distinguish between the different cases by adding superscripts to $P(s)$.

In the special case considered in Duffie and Lando (2001), $R(u,s) = (1-\alpha)e^{-r(s-u)}$, $u \in (t,s]$, and the pricing expression (4) simplifies to

$$\varphi(t,s) = e^{-r(s-t)}P(s) + (1-\alpha)e^{-r(s-t)}(1-P(s)). \tag{6}$$

We use this recovery rate throughout the paper because it leads to tractable analytical expressions. Other, possibly more realistic recovery functions have much of the same qualitative properties as the one above, but they may require numerical solutions.

Based on the definition of the credit spread η and expression (6) we have that

$$e^{-(r+\eta)(s-t)} = e^{-r(s-t)}P(s) + (1-\alpha)e^{-r(s-t)}(1-P(s)),$$

so

$$\eta = \frac{-\ln\Big(\alpha P(s) + (1-\alpha)\Big)}{s-t}. \tag{7}$$

Notice that the credit spread vanishes as $\alpha \to 0$. If there is no economic loss in case of bankruptcy, there is of course no credit risk. Further more, the credit spread tightens as $P(s)$ increases.

2.3 Credit Default Swap (CDS)

CDSs are the most common form of credit derivatives. A CDS is a default insurance contract. For each unit of face value of debt it pays the amount

$$X = 1 - \frac{(1-\alpha)V_B^m}{D} \tag{8}$$

at the time of default, if default happens before the CDS matures at some time T. Notice that X is $\mathcal{F}_t^m$-measurable, but in general not $\mathcal{G}_t^l$-measurable. The cost of the insurance is covered by coupon payments to the issuer of the CDS, known as the CDS spread. The CDS spread is the annualized coupon rate $c(t,T)$ that implies a total market value of the swap of zero at the time of issue. Assuming semi-annual coupons and that $n = 2T$,

$$c(t,T) = \frac{2E[Xe^{-r(\tau-t)}1_{\{\tau\leq T\}}|\mathcal{G}_t^l]}{\sum_{i=1}^n e^{-0.5ri}P(t+0.5i)}, \tag{9}$$

where $P(\cdot)$ is defined in expression (5).

3 Case 1: Complete Information

Let us start by looking at the classical case where there is no delay in the flow of information. This case is essentially the Leland (1994) model. Under complete information $l = m = 0$ and $\mathcal{F}_t^l = \mathcal{G}_t^l = \mathcal{F}_t^m = \mathcal{F}_t$. Thus, in the case with complete information all agents are equally informed. For notational simplicity we let $V_B^m = V_B$, where $V_B = V_B^0$. In this section we denote the initial time by t, where $t \geq 0$.

3.1 Equity Holders' Optimization Problem

The equity holders are faced with the following optimal stopping problem (see e.g., Duffie (2001), chapter 11.C)

$$\phi(v) = \sup_{\tau\in\mathcal{T}} E\Big[\int_t^\tau e^{-r(s-t)}(\delta_s - (1-\theta)C)ds\big|\mathcal{F}_t\Big], \tag{10}$$

where $\mathcal{T}$ is the set of $\mathcal{F}_t$-adapted stopping times. The value function satisfies the ordinary differential equation (ODE)

$$\mu v\phi_v + \frac{1}{2}\sigma^2 v^2\phi_{vv} - r\phi + (r-\mu)v - (1-\theta)C = 0, \tag{11}$$

where subscripts denote partial derivatives. The general solution to this equation is

$$\phi(v) = A_1 v^{\gamma_1} + A_2 v^{\gamma_2} + v - (1-\theta)\frac{C}{r},$$

where A_i, $i = 1, 2$, are constants to be determined from boundary conditions and

$$\gamma_i = \frac{\frac{1}{2}\sigma^2 - \mu \pm \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2},$$

with $\gamma_1 < 0$ and $\gamma_2 \geq 1$. Differentiating ϕ with resepct to v yields

$$\phi'(v) = \gamma_1 A_1 v^{\gamma_1 - 1} + \gamma_2 A_2 v^{\gamma_2 - 1} + 1.$$

When the value of the company approaches infinity, only equity holders benefit from any further increase in asset value, thus

$$\lim_{v \to \infty} \phi'(v) = 1.$$

As $\gamma_2 \geq 1$, this condition implies that $A_2 = 0$, i.e.,

$$\phi(v) = A_1 v^{\gamma_1} + v - (1-\theta)\frac{C}{r}. \tag{12}$$

We impose the usual value matching and high contact conditions

$$\phi(V_B) = 0 \tag{13}$$

and

$$\phi_v(V_B) = 0. \tag{14}$$

Equations (13) and (14) can be solved for A_1 and V_B. The solution for V_B is[3]

$$V_B = \frac{\gamma_1}{\gamma_1 - 1}\frac{(1-\theta)C}{r}. \tag{15}$$

3.2 Corporate Bond Pricing

In the case of complete information the corporate bond price is given by expression (6), using that

$$P(s) = P^1(s) = P(\tau > s \big| \mathcal{F}_t) = \Psi(s - t, V_t, V_B),$$

[3]In the special case considered by Leland (1994) where $\mu = r$, $V_B = (1-\theta)C/(r+0.5\sigma^2)$.

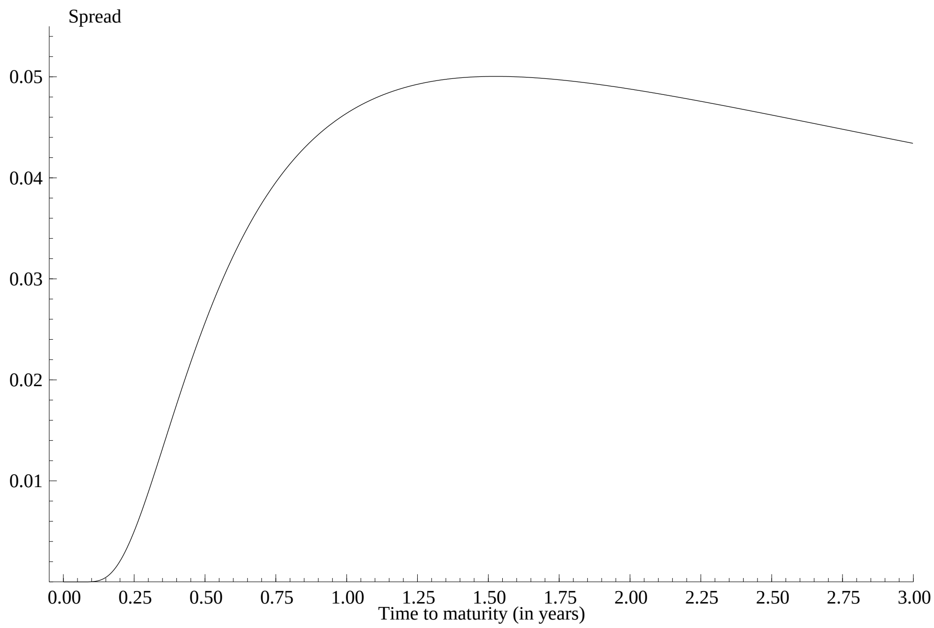

Figure 1: **Credit spreads classical case** The figure shows the credit spreads for zero-coupon bonds with up to three years to maturity.

where an analytical expression for $\Psi(\cdot, \cdot, \cdot)$ is given in expression (25) in appendix A.

Example 1. *Assume that* $\delta_t = 3.5$*,* $r = 0.08$*, and* $\mu = 0.045$*. From expression* (2) *these parameters give* $V_t = 100$*. Furthermore, assume that* $\sigma = 0.3$*,* $\theta = 0.3$*,* $\alpha = 0.3$*, and* $C = 13$*. From expression* (15) $V_B = 65$*. With the recovery policy in expression* (6) *the credit spreads for zero-coupon bonds with maturities of up to three years are plotted in Figure 1. From Figure 1 we clearly see that the credit spread approaches zero as the time to maturity approaches zero, a typical property of structural models of credit risk.*

3.3 Credit Default Swap

Under complete information $P^1(t+0.5i) = \Psi(0.5i, V_t, V_B)$, where $\Psi(\cdot,\cdot,\cdot)$ is given in expression (25) in appendix A. In this section X defined in expression (8) is a constant (i.e., also $\mathcal{G}_t^l$-measurable), and the CDS spread is given by

$$c(t,T) = \frac{2X\Upsilon(T-t, V_t, V_B)}{\sum_{i=1}^{n} e^{-0.5ri}\Psi(0.5i, V_t, V_B)},$$

where $\Upsilon(T-t, V_t, V_B) = E[e^{-r(\tau-t)}1\{\tau \leq s-t\}\big|\mathcal{F}_t]$ is given in expression (26) in appendix B.

We present a numerical example illustrating the CDS spreads in subsection 5.3 where we also include delayed and asymmetric information.

4 Case 2: Delayed Information

4.1 The Equity Holders' Optimization Problem

In the case with delayed information all agents have access to the same information, but they receive the information with a time lag $l = m > 0$. This assumption implies that $\mathcal{F}_t^l = \mathcal{G}_t^l = \mathcal{F}_t^m \subset \mathcal{F}_t$. In this section the initial point in time is denoted by $t \geq l$. Thus, at time t agents observe the state variable (i.e., the EBIT process) at time $t-m$. This assumption changes the optimization problem for the equity holders. From standard properties of geometric Brownian motions follow that the complete-information value of the assets at the bankruptcy time τ is given by the log-normally distributed random variable

$$V_\tau = V_\tau^m e^{(\mu - \frac{1}{2}\sigma^2)m + \sigma(B_\tau - B_{\tau-m})}. \tag{16}$$

From the definition of the barrier V_B^m in expression (3), the time τ value of the assets in expression (16) can also be written as

$$V_\tau = V_B^m e^{(\mu - \frac{1}{2}\sigma^2)m + \sigma(B_\tau - B_{\tau-m})}.$$

In the case where the value of the assets is sufficiently high to cover both repayment of the debt and bankruptcy costs, i.e., $V_\tau - D - \alpha V_\tau > 0$, the equity holders get the payoff $V_\tau - D - \alpha V_\tau$. By deciding to file for bankruptcy at time τ, the equity holders enter a bankruptcy lottery[4] with payoff $((1-\alpha)V_\tau - D)^+$. The time τ value of this lottery is

$$\begin{aligned}\pi(V_B^m) &= E[((1-\alpha)V_\tau - D)^+ \big| \mathcal{F}_\tau^m] \\ &= (1-\alpha)e^{\mu m} V_B^m N(z) - DN(z - \sigma\sqrt{m}),\end{aligned} \tag{17}$$

where

$$z = \frac{\ln\left(\frac{(1-\alpha)V_B^m}{D}\right) + (\mu + \frac{1}{2}\sigma^2)m}{\sigma\sqrt{m}}$$

and $N(\cdot)$ is the cumulative standard normal probability distribution function.

Proof. The result follows from the standard Black-Scholes-Merton formula for a European call option, but without discounting since the payoff is received instantaneously when $V_\tau^m = V_B^m$. □

The equity holders' optimization problem is now given by

$$\phi(v) = \sup_{\tau \in \mathcal{T}_m} E\Big[\int_t^\tau e^{-r(s-t)}(\delta_{s-m} - (1-\theta)C)ds + e^{-r(\tau-t)}\pi(V_B^m)\big|\mathcal{F}_t^m\Big], \tag{18}$$

where $\mathcal{T}_m$ denotes the set of $\mathcal{F}_t^m$-adapted stopping times. There are three differences between expressions (18) and (10). The first is the inclusion of

[4]This lottery has some resemblance to the *wild card play* that is present when trading in the CBOT Treasury bond futures, see e.g., Hull (2006).

the bankruptcy lottery in the optimization problem. Second, the lagged state variable δ_{t-m} enters, and third, the information set at time t is lagged.

In appendix C we show that the equity holders' optimization problem is equivalent to a standard stopping problem. Thus, also in the case with delayed information the value function $\phi(v)$ is the solution to the ODE (11) and has general solution given by expression (12). However, the value matching and the high contact conditions now change to

$$\phi(V_B^m) = \pi(V_B^m) \tag{19}$$

and

$$\phi_v(V_B^m) = \pi_v(V_B^m) = (1-\alpha)e^{\mu m}N(z), \tag{20}$$

respectively.

Using expressions (19) and (20), we are not able to find analytical expressions for A_1 and V_B^m. However, equations (19) and (20) can easily be solved numerically.

In Table 1 we illustrate the effect of different delays on the default barrier and the value of the bankruptcy lottery. The effect is relatively small for delays less than one year. For instance, for $m = 0.5$, the value of V_B^m is only changed at the second decimal place and the value of the bankruptcy lottery is zero with two digits accuracy. For longer delays, the value of the bankruptcy lottery is larger and, thus, more important for the equity holders' bankruptcy decision. The value function ϕ is increasing in the assessed value of the assets (V_t^m). Combining this observation with expression (19), it is clear that the assessed value V_t^m that makes (19) hold (i.e., V_B^m) must be higher for higher values on the righthand side. Thus, longer delays increase the default barrier.

From Table 2 it is clear that increasing the volatility has virtually no effect on the value of the bankruptcy lottery, except for high values of the volatility (say, above 100%). Note that we have taken into account that a more risky firm typically has less debt than a less risky firm.

4.2 Corporate Bond Pricing

In the case of delayed information the corporate bond price is given by expression (6), using that

$$P(s) = P^2(s) = P(\tau > s \big| \mathcal{F}_t^l) = \Psi(s - t + l - m, V_t^l, V_B^m) = \Psi(s - t, V_t^l, V_B^m),$$

where an analytical expression for $\Psi(\cdot, \cdot, \cdot)$ is given in expression (25) in appendix A. Observe that $P^2(s)$ neither depends on m nor l since they are equal and therefore cancel.

Assuming that the assessed asset values are the same under complete and delayed information, the only way delayed information can change credit spreads is if the default barrier $V_B^m \neq V_B$. As we saw in Table 1, this is the case for large delays in the flow of information and/or high asset volatilities, cf. Table 2. In Table 3 we report credit spreads for different delays in information and different levels of the volatility. The corresponding default barriers are taken from Tables 1 and 2. As Table 3 shows, for reasonable delays (typically less than one year) and levels of the volatility, any changes in the credit spreads are negligible.

The above observations may shed light on two important aspects:

1. The effect of not being able to observe the process for the value of the assets in a structural model has a minor effect on the optimal default policy, cf., the discussion in Jarrow and Protter (2004). This effect becomes noticeable for larger delays and high levels of volatility.

2. Delayed information has a negligible effect on credit spreads.

Table 1: **Effect of delayed information** The table shows how the default barrier V_B^m and the price of the bankruptcy lottery $\pi(V_B^m)$ varies for different lengths of the information lag m. The parameter values are $\alpha = 0.3$, $r = 0.08$, $\mu = 0.045$, $\sigma = 0.3$, $\theta = 0.3$, $C = 13$, and $D = 90$.

m	V_B^m	V_B	$\pi(V_B^m)$
0.0	65.0000	65.0000	0.0000
0.2	65.0000	65.0000	0.0000
0.5	65.0289	65.0000	0.0036
1.0	65.6098	65.0000	0.1291
1.5	67.0580	65.0000	0.5834
2.0	69.2680	65.0000	1.4737
2.5	72.1945	65.0000	2.8798
3.0	75.8860	65.0000	4.9026
3.5	80.4840	65.0000	7.6939
4.0	86.2511	65.0000	11.4961

Table 2: **Effect of volatility under delayed information** The table shows how the default barrier V_B^m and the price of the bankruptcy lottery $\pi(V_B^m)$ varies for different levels of the volatility σ. The parameter values are $\alpha = 0.3$, $r = 0.08$, $\mu = 0.045$, $m = 0.2$, $\theta = 0.3$, and $C = 13$. D is approximately equal to the market value of corporate debt when $\delta_t = 3.5$.

σ	D	V_B^m	V_B	$\pi(V_B^m)$
0.15	92	93.2899	93.2899	0.0000
0.25	99	73.6273	73.6273	0.0000
0.50	81	39.9644	39.9643	0.0000
0.75	62	23.3913	23.3901	0.0001
1.00	48	14.9151	14.9079	0.0009
2.50	16	3.0370	2.8094	0.0836
5.00	6	1.2562	0.7214	0.3902

Table 3: **Effect on credit spreads** The table shows credit spreads when information is delayed (η^2) and not delayed (η^1) for different delays m and levels of the volatility σ. The parameter values are $\delta_{t-m} = 3.5$, $\alpha = 0.3$, $r = 0.08$, $\mu = 0.045$, $\theta = 0.3$, and $C = 13$. D is approximately equal to the market value of corporate debt when $\delta_t = 3.5$.

					Time to maturity					
					0.1		1		3	
m	σ	D	V_B^m	$\pi(V_B^m)$	η^2	η^1	η^2	η^1	η^2	η^1
0	0.30	90	65.0000	65.0000	0.0000	0.0000	0.0464	0.0464	0.0434	0.0434
0.5	0.30	90	65.0289	65.0000	0.0000	0.0000	0.0465	0.0464	0.0435	0.0434
2	0.30	90	69.2680	65.0000	0.0003	0.0000	0.0686	0.0464	0.0518	0.0434
4	0.30	90	86.2511	65.0000	0.3650	0.0000	0.2067	0.0464	0.0884	0.0434
0.2	0.75	62	23.3913	23.3901	0.0000	0.0000	0.0285	0.0285	0.0479	0.0479
0.2	2.50	16	3.0370	2.8094	0.0002	0.0001	0.1868	0.1817	0.1131	0.1128

4.3 Credit Default Swap

Under delayed information $P^2(t+0.5i) = \Psi(0.5i, V_t^l, V_B^m)$, where $\Psi(\cdot,\cdot,\cdot)$ is given in expression (25) in appendix A. Also in this section X is a constant. The CDS spread is then given by

$$c(t,T) = \frac{2X\Upsilon(T-t, V_t^l, V_B^m)}{\sum_{i=1}^{n} e^{-0.5ri}\Psi(0.5i, V_t^l, V_B^m)},$$

where $\Upsilon(T-t, V_t^l, V_B^m) = E[e^{-r(\tau-t)}1\{\tau \leq s-t\}\big|\mathcal{F}_t^l]$ is given in expression (26) in appendix B.

5 Case 3: Asymmetric Information between Bond- and Equity holders

We now analyze the first case with delayed and asymmetric information. Equity holders have access to information earlier than the bond holders. In order to keep these two groups of agents separated, equity holders are neither allowed to buy nor sell bonds in the secondary market nor trade equity. This assumption is also used by Duffie and Lando (2001). The information structure is now as follows; $0 \leq m < l$, i.e., $\mathcal{F}_t^l \subset \mathcal{G}_t^l \subset \mathcal{F}_t^m \subseteq \mathcal{F}_t$. As before, the initial point in time $t \geq l$.

Collin-Dufresne, Goldstein, and Helwege (2003) find that since 1937 only four companies have defaulted on bonds with an investment grade rating from Moody's. This suggests that the time lag $m-l$ cannot be too long. With a sufficiently large time lag, even a company issuing bonds rated investment grade may have enough time to move into default.[5]

[5]This observation also gives some justification for using a diffusion model instead of a jump-diffusion model. Few companies "jump" into default when their bonds are rated investment grade.

5.1 The Equity Holders' Optimization Problem

The equity holders' optimization problem is now given by

$$\phi(v) = \sup_{\tau \in \mathcal{T}_m} E\Big[\int_t^\tau e^{-r(s-t)}(\delta_{s-m} - (1-\theta)C)ds + e^{-r(\tau-t)}\pi(V_B^m)\Big|\mathcal{F}_t^m\Big]. \quad (21)$$

This problem is identical to the problem in expression (18) in case 2, and thus have the same solution. This means that asymmetric information, as defined in case 3, has a minor effect on default policy compared to case 1 with complete information. It is important to emphasize that the default barrier V_B^m is not a function of the state variable. In particular this means that also the bond holders who have less information than the equity holders can calculate V_B^m. More formally, V_B^m is $\mathcal{G}_t^l$-measurable.

5.2 Corporate Bond Pricing

In case 1 (case 2) the bond holders observe that the value of the assets (lagged asset value) approaches and eventually hits the default barrier V_B (V_B^m). In contrast to this situation, under asymmetric information they may observe that the assessed asset value V_t^l approaches V_B^m, but they never observe that it hits V_B^m because the bankruptcy decision is based on the value V_t^m with $l > m$. Formally, the stopping time τ is *totally inaccessible*, as also is the case in the model of Duffie and Lando (2001). Thus, also our model is an example of a structural model that is equivalent to an intensity based model.

For the bond holders to calculate the bond prices, they need to estimate the survival probability conditional on their information set.

1. the lagged asset value observed by the management is higher than the default barrier V_B^m (otherwise the company would already have been declared bankrupt), and

2. the lagged information set $\mathcal{F}_t^l$ which contains the assessed asset value V_t^l.

The survival prbability is calculated as

$$P(s) = P^3(s) = P(\tau > s \big| \mathcal{G}_t^l) = P(\tau > s \big| V_t^m > V_B^m; \mathcal{F}_t^l),$$

where the last equality follows from the Markov property of V_t^m. Using Baye's rule, we have that

$$\begin{aligned} P^3(s) &= \frac{P(\tau > s \big| \mathcal{F}_t^l) \cdot P(V_t^m > V_B^m \big| \tau > s; \mathcal{F}_t^l)}{P(V_t^m > V_B^m \big| \mathcal{F}_t^l)} \\ &= \frac{P(\tau > s \big| \mathcal{F}_t^l)}{P(V_t^m > V_B^m \big| \mathcal{F}_t^l)}. \end{aligned}$$

When $m \neq l$, $P(\tau > s \big| \mathcal{F}_t^l) = \Psi(s - t + l - m, V_t^l, V_B^m)$ and is given in expression (25) in appendix A. Combining this result with standard properties of log-normal random variables, we write

$$P^3(s) = \frac{\Psi(s - t + l - m, V_t^l, V_B^m)}{N\left(\frac{-\ln y + \nu(l-m)}{\sigma\sqrt{l-m}}\right)}, \tag{22}$$

where $y = V_B^m / V_t^l$ and $\nu = \mu - \frac{1}{2}\sigma^2$.

In appendix D we show that the spread under asymmetric information is wider than the spread under symmetrically distributed information. Taking the limit as $s \to t$, we have that

$$\lim_{s \to t} P^3(s) = 1 - y^{2\nu\sigma^{-2}} \frac{N\left(\frac{\ln y + \nu(l-m)}{\sigma\sqrt{l-m}}\right)}{N\left(\frac{-\ln y + \nu(t-m)}{\sigma\sqrt{t-m}}\right)}. \tag{23}$$

The limit in (23) is strictly positive and less than 1, thus it must be the case that

$$\lim_{s \to t} \eta^3 = \infty.$$

The intuition for this result is that if the defaultable zero-coupon bond has a price less than the default-free zero-coupon bond when the time to maturity vanishes, this can only be achieved for a "very high" credit spread (i.e., an infinite credit spread).

Example 2. *Assume that* $\delta_{t-l} = 3.5$*,* $\sigma = 0.3$*,* $\mu = 0.045$*,* $r = 0.08$*,* $C = 13$*,* $\theta = 0.3$*,* $\alpha = 0.3$*, and* $l = 0.25$*. These parameter values give* $V_B^m = 65$*. With the recovery policy in expression (6) the credit spreads for zero-coupon bonds with maturities of up to three years are plotted in Figure 2. Starting from the top, the graphs are for* $m = 0.06$*,* $m = 0.1$*,* $m = 0.14$*, and finally for complete information (the credit spreads from Example 1) The figure clearly demonstrates that asymmetric information leads to wider credit spreads. Increasing the asymmetry (i.e., reducing* m*) increases the credit spreads.*

In contrast to the credit spreads under complete information, the spreads under asymmetric information in the short end of Figure 2 do not converge to zero. The credit spreads under asymmetric information to the far left are for bonds maturing in half a day. In practice soon to mature corporate bonds are not analyzed in terms of their credit spread because "the discount" mostly reflects the probability for immediately bankruptcy, adjusted for the recovery rate (see e.g., the discussion on page 14-15 in Lando (2004)). Although not commented by the authors, in similar figures in Duffie and Lando (2001) it seems like the x-axes are truncated, possibly to exclude such "high" short-term credit spreads.

Example 3. *Consider a company which funds parts of its activities by short-term debt, for instance a financial institution borrowing in the interbank market. Assuming the same parameter values as in Example 2, Figure 3 and Figure 4 show that short-term credit spreads on overnight borrowing can be highly sensitive to asset volatility and assessed asset values. This can be relevant for explaining the effect the sub-prime crisis dating back to*

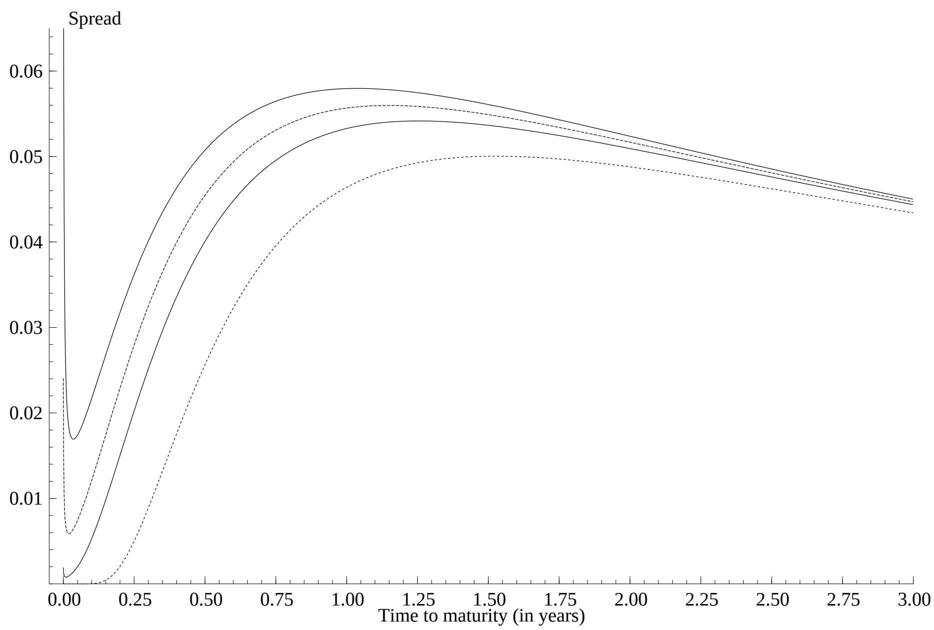

Figure 2: **Effect of asymmetric information between bond- and equity holders on credit spreads** The figure shows credit spreads for zero-coupon bonds with up to three years to maturity. The upper graph represents the case where the equity holders' delay $m = 0.06$. The second, third, and fourth graphs from the top represent the cases of $m = 0.1$, $m = 0.14$, and complete information, respectively. The bond holders' delay $l = 0.25$.

the summer of 2007 had on the interbank market. A key component was the information asymmetry between different banks about their exposure to the sub-prime market. Relatively small changes in the assessed value of a bank's assets or in the volatility of a bank's assets, could very well lead to high short-term credit spreads.

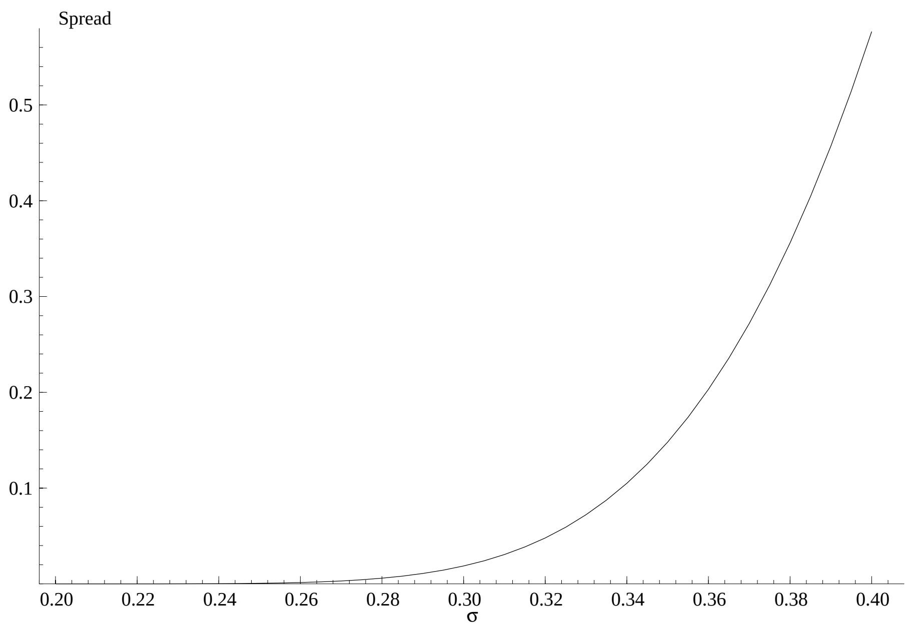

Figure 3: **Effect of volatility on overnight credit spreads** The figure shows the overnight credit spreads for different values of the underlying volatility σ. Other parameter values are: $\delta_{t-l} = 3.5$, $\mu = 0.045$, $r = 0.08$, $\theta = 0.3$, $\alpha = 0.3$, $m = 0.1$, and $l = 0.25$.

5.3 Credit Default Swap

In this case of delayed and asymmetric information X is also a constant and the CDS spread is given by

$$c(t, T) = \frac{2X\Gamma(T - t, V_t^m, V_B^m)}{\sum_{i=1}^{n} e^{-0.5ri} P^3(t + 0.5i)},$$

where $\Gamma(T - t, V_t^m, V_B^m) = E[e^{-r(\tau - t)}1\{\tau \leq T\} \big| V_t^m > V_B; \mathcal{F}_t^l]$ is given in expression (30) in appendix E and $P^3(\cdot)$ is defined in expression (22).

Example 4. *Assume that* $\delta_{t-l} = 3.5$*,* $\sigma = 0.3$*,* $\mu = 0.045$*,* $r = 0.08$*,* $C = 13$*,* $\theta = 0.3$*,* $\alpha = 0.3$*,* $m = 0.1$*,* $l = 0.25$*, and the coupon payments on the CDS are paid semi-annually. These parameter values give* $V_B^m = 65$*. For these*

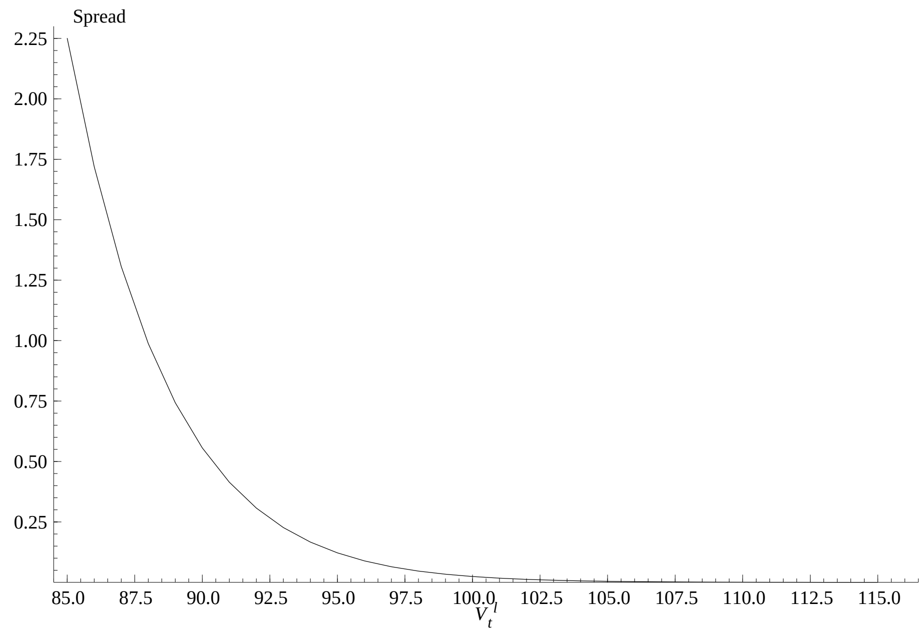

Figure 4: **Effect of assessed asset values on overnight credit spreads** The figure shows the overnight credit spreads for different values of the assessed asset value V_t^l. Other parameter values are: $\mu = 0.045$, $r = 0.08$, $\sigma = 0.3$, $\theta = 0.3$, $\alpha = 0.3$, $m = 0.1$, and $l = 0.25$.

parameters, the CDS rates for case 1 and case 3 are plotted in Figure 5. The highest rates are for case 3 with delayed and asymmetric information.

6 Case 4: Asymmetric Information between Management and Financial Market

In this case the management of the company is better informed than the bond- and equity holders who are the least informed agents. In particular, bond- and equity holders have the same information lag. Because management acts in the best interest of the owners, it solves the equity holders'

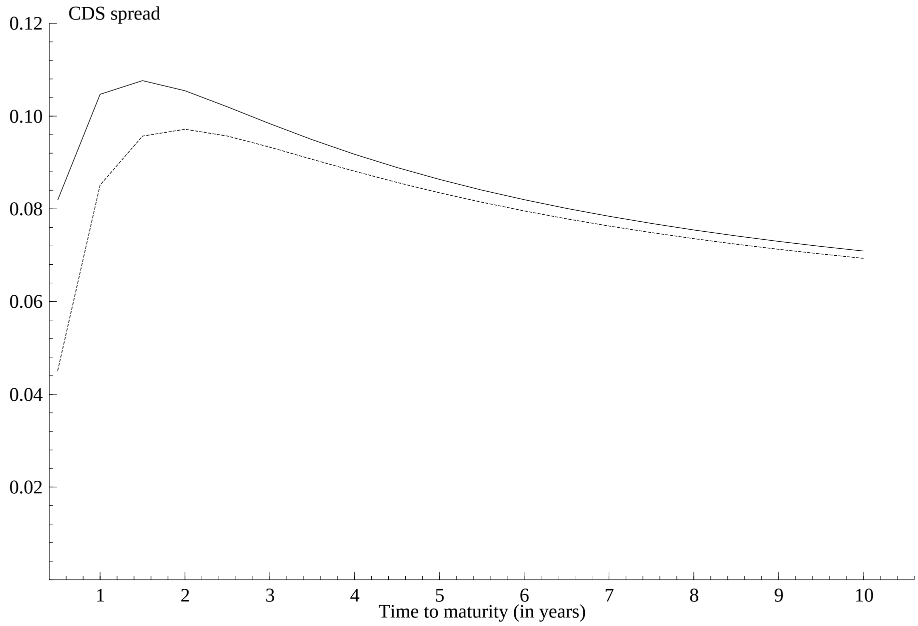

Figure 5: **CDS spreads** The figure shows the CDS spreads for different times to maturity. The top graph represents case 3 and the lower graph case 1.

optimization problem *as if* the owners had the same information set as the management. In many countries it is illegal for the management to run the company on the debt holders' expense if it knows that the company should have been declared bankrupt, partially justifying this assumption.

Financial distress and bankruptcy are characteristics of a highly extraordinary situation for a company. It is therefore reasonable to assume that such an event leads to an increased speed in the flow of information between the management and the equity holders. In our model when the management observes that the assessed asset value V_t^m hits the default barrier, the equity holders are informed and immediately file for bankruptcy.[6]

As argued in the introduction, as we show in appendix D, and further illustrate in Example 2, there are no reasons for keeping information away from the financial market in these kinds of models. The difference in time lag, $l - m$, therefore reflects the time it takes to inform the financial market. Reporting information right away may be costly. In practice, there can be strategic reasons (outside of our model) for the management to keep information away from the financial market for some time. In a financial market where both stocks and corporate bonds are traded without restrictions, both bond- and equity holders typically have the same information. If not, bond holders could buy one share of stock and equity holders could buy one share of bond to access the other party's information. Both these strategies eliminate any information asymmetry.

6.1 The Equity holders' Optimization Problem

When the management and the financial market have different time lags, it has to take into account that the equity holders receive a dividend payment consistent with their information set. I.e., at time t they receive dividends at a rate $\delta_{t-l} - (1-\theta)C$. Although δ_{t-l} is contained in the management's information set $\mathcal{F}_t^m$, we rewrite this in terms of the management's time t observable EBIT value δ_{t-m} as $\delta_{t-m} Z - (1-\theta)C$. Here

$$Z = e^{-(\mu - \frac{1}{2}\sigma^2)(l-m) - \sigma(B_{t-m} - B_{t-l})},$$

where Z is $\mathcal{F}_t^m$-measurable and unknown for the financial market. Conditional on no bankruptcy, i.e., $V_t^m > V_B^m$, the management solves the optimal

[6]The assumption that equity holders are immediately informed can be relaxed without adding much economic insight, but it will increase the notational burden. It is sufficient that equity holders are informed about the state variable and file for bankruptcy before time $\tau + (l-m)$. The stopping time is still totally inaccessible for the financial market.

stopping problem

$$\hat{\phi}(v) = \sup_{\tau \in \mathcal{T}_m} E\Big[\int_t^\tau e^{-r(s-t)}(\delta_{s-m}Z - (1-\theta)C)ds + e^{-r(\tau-t)}\pi(V_B^m)\big|\mathcal{F}_t^m\Big].$$

The only difference between this problem and the corresponding problem in expression (21) is the presence of Z. Since Z is $\mathcal{F}_t^m$-measurable, we can divede through by Z. The management can solve the equivalent problem, where $\phi(v) = \hat{\phi}(v)/Z$,

$$\phi(v) = \sup_{\tau \in \mathcal{T}_m} E\Big[\int_t^\tau e^{-r(s-t)}(\delta_{s-m} - (1-\theta)\frac{C}{Z})ds + \frac{1}{Z}e^{-r(\tau-t)}\pi(V_B^m)\big|\mathcal{F}_t^m\Big].$$

The value of the equity is then the solution to the ODE

$$\mu v\phi_v + \frac{1}{2}\sigma^2 v^2\phi_{vv} - r\phi + (r-\mu)v - \frac{(1-\theta)C}{Z} = 0.$$

The value matching and high contact conditions now become

$$\phi(V_B^m) = \frac{\pi(V_B^m)}{Z}$$

and

$$\phi_v(V_B^m) = \frac{(1-\alpha)e^{\mu m}N(z)}{Z},$$

respectively.

6.2 Corporate Bond Pricing

In this case Z is unobservable for the financial market. The bond holders do therefore not know the optimal default barrier V_B^m when they calculate bond prices, i.e., V_B^m is not $\mathcal{G}_t^l$-measurable. In general, for a given value of Z, bond holders are able to determine V_B^m. The bond holders observe that the company is not bankrupt at time t, thus they infer that $V_t^l Z = V_t^m > V_B^m$. Using the definition of the bond price in expression (4), the law of iterated expectations, and our choice of recovery function, we can write the bond price as

$$\begin{aligned}\varphi(t,s) &= E\Big[E\Big[e^{-r(s-t)}1_{\{\tau>s\}} + e^{-r(\tau-t)}R(\tau,s)1_{\{\tau\le s\}}\big|\mathcal{F}_t^m\Big]\Big|\mathcal{G}_t^l\Big] \\ &= E\Big[e^{-r(s-t)}P(s) + R(t,s)e^{-r(s-t)}(1-P(s))\big|\mathcal{G}_t^l\Big] \\ &= e^{-r(s-t)}P^4(s) + R(t,s)e^{-r(s-t)}(1-P^4(s)),\end{aligned}$$

where

$$P(s) = P^4(s) = E\Big[P(\tau > s\big|\mathcal{F}_t^m)\Big|\mathcal{G}_t^l\Big] = E\Big[\Psi(s-t, V_t^m, V_B^m)\Big|\mathcal{G}_t^l\Big], \qquad (24)$$

where we remember that $V_t^m = V_t^l Z$.

A closed form expression for the bond price is not readily available, but the price can be estimated by numerical methods such as Monte Carlo simulations.

Compared to case 3, the increased uncertainy for the bond holders regarding the default barrier reduces the survival probability. This increased uncertainty leads to wider credit spreads in case 4 compared to case 3. The function $\Psi(\cdot,\cdot,\cdot)$ in expression (25) is concave in its second argument. In this case both the second and the third argument in $\Psi(\cdot,\cdot,\cdot)$, see expression (24), depend on Z. Numerical investigations indicate that $\Psi(\cdot,\cdot,\cdot)$, as a function of Z, i.e., $\Psi(Z)$ is concave. For realistic parameter values, $E[Z\big|\mathcal{G}_t^l] \approx 1$. By Jensen's inequality

$$P^3(s) \approx \Psi(E[Z\big|\mathcal{G}_t^l]) > E[\Psi(Z)\big|\mathcal{G}_t^l] = P^4(s).$$

Thus, credit spreads are wider in case 4 than in case 3, cf. expression (7).

Example 5. *Assume that* $\delta_{t-l} = 3.5$*,* $\sigma = 0.3$*,* $\mu = 0.045$*,* $r = 0.08$*,* $C = 13$*,* $\theta = 0.3$*,* $\alpha = 0.3$*, and* $l = 0.25$*. In Figure 6 we show the spread for four* *different delays m. Starting with the top graph,* $m = 0.1$*,* $m = 0.14$*,* $m = 0.18$*, and* $m = 0.22$*.*

In Figure 7 we plot the spreads for $m = 0.1$*,* $m = 0.14$*, and* $m = 0.18$ *for both case 3 and 4. The three upper graphs are for case 4, while the three lower graphs are for case 3 (the three year period is divided into 240 time steps). The increased uncertainty for the bond holders because they do no longer know the bankruptcy barrier* V_B^m *results in wider credit spreads, in particular for short-term bonds. The figure also illustrates the importanse of asymmetric information when determining short-term credit spreads.*

6.3 Credit Default Swaps

For the case in this section, the expression for X in expression (8) is $\mathcal{F}_t^m$-measurable (not $\mathcal{G}_t^l$-measurable). Thus, the CDS-spread is given by

$$c(t,T) = \frac{2E[Xe^{-r(\tau-t)}1_{\{\tau\leq T\}}\big|\mathcal{G}_t^l]}{\sum_{i=1}^{n} e^{-0.5ri}P^4(t+0.5i)}.$$

The fact that X is not $\mathcal{G}_t^l$-measurable precludes the use of expression (26) when estimating the expectation in the nominator. One way to estimate the expectation is by Monte Carlo simulation, but this is rather computational intensive because we cannot use closed form expression of the type in appendix B or E.

7 Conclusions

This paper analyzes the effect of delayed and asymmetric information on credit spreads on corporate bonds traded in a secondary market and on CDS spreads. In the case where the bankruptcy decision is based on delayed information we identified a potential gain for equity holders as a lottery with non-negative payoff. This payoff may be strictly positive in the case the

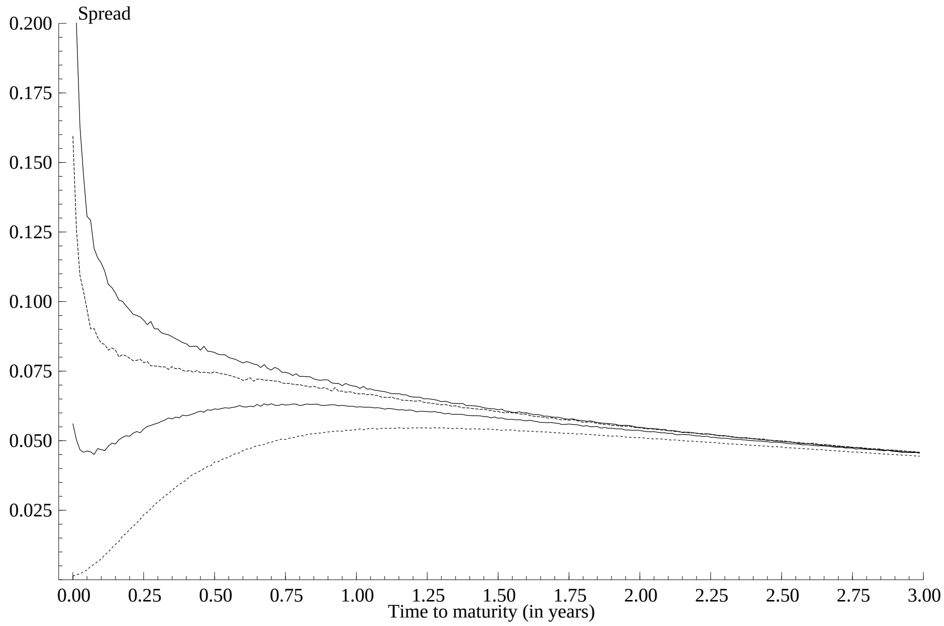

Figure 6: **Effect of asymmetric information between management and financial market on credit spreads** The figure shows credit spreads for zero-coupon bonds with up to three years to maturity. The upper graph represents the case $m = 0.1$. The second, third, and fourth graphs represent $m = 0.14$, $m = 0.18$, and $m = 0.22$, respectively. The bond holders' delay $l = 0.25$. The three year period is divided into 240 time steps and 100,000 simulations are used to calculate the credit spread at each time step.

actual market value of the company is significantly higher than the value of the company on which the bankruptcy decision was based. For realistic parameter values, this lottery has a rather small value and the effects on bankruptcy policy and credit spreads are also small.

Asymmetric information, on the other hand, has a substantial effect on credit spreads. Asymmetric information explains why credit spreads on bonds with short time to maturity do not approach zero. Our results are

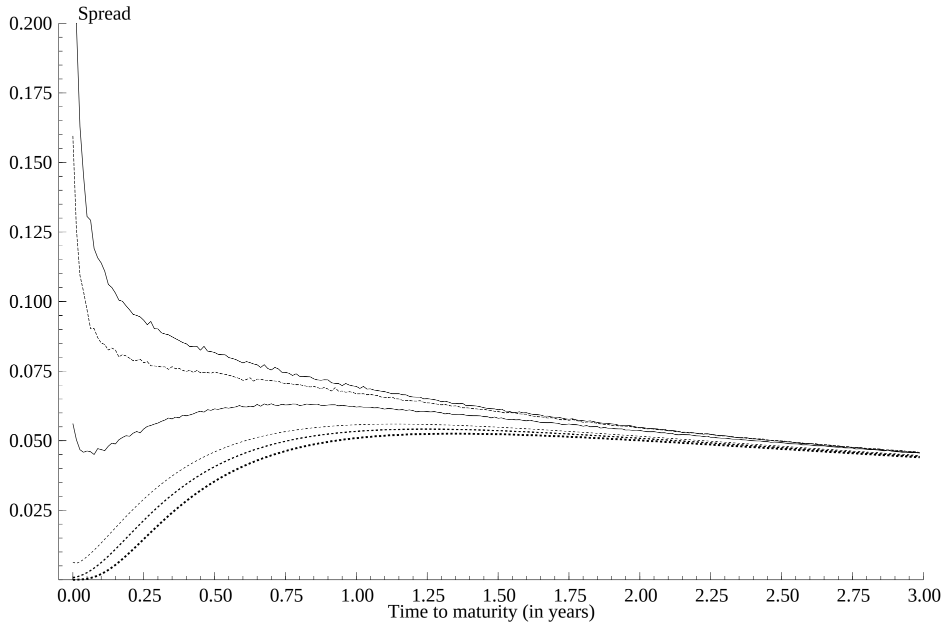

Figure 7: **Comparison of credit spreads for case 3 and case 4** The figure shows credit spreads for zero-coupon bonds with up to three years to maturity for case 3 and 4 for $m = 0.1$, $m = 0.14$, and $m = 0.18$. The upper three graphs are for case 4, while the lower three graphs are for case 3. The bond holders' delay $l = 0.25$. The three year period is divided into 240 time steps and 100,000 simulations are used to calculate the credit spread at each time step for case 4.

qualitatively similar to the results of Duffie and Lando (2001), which they derive in a model with incomplete accounting information. Our analysis shows that it is not the incomplete accounting information per se that causes the equivalence between the structural and reduced form approaches, but rather the information asymmetry between bond- and equity holders. The same conclusion applies in our case 4 where we relax some of their assumptions, e.g., non-tradability of stocks and bonds.

A Survival Probability

Consider a geometric Brownian motion with dynamics as in expression (1) and initial value v. The probability of not crossing the barrier v_b in a time period of length s when $v > v_b$, is

$$\Psi(s, v, v_b) = N\Big(\frac{-\ln \frac{v_b}{v} + \nu s}{\sigma\sqrt{s}}\Big) - (\frac{v_b}{v})^{2\nu\sigma^{-2}} N\Big(\frac{\ln \frac{v_b}{v} + \nu s}{\sigma\sqrt{s}}\Big), \tag{25}$$

where $\nu = \mu - \frac{1}{2}\sigma^2$, see e.g., Musiela and Rutkowski (1997) Corollary B.3.4.

B Definition of Υ

Define

$$\Upsilon(s - t, v, v_b) = e^{b(z-w)} N(\frac{b - w(s-t)}{\sqrt{s-t}}) + e^{b(z+w)} N(\frac{b + w(s-t)}{\sqrt{s-t}}), \tag{26}$$

where

$$\begin{aligned} b &= \ln(v_b/v)/\sigma, \\ z &= (\mu - \frac{1}{2}\sigma^2)/\sigma, \end{aligned}$$

and

$$w = \sqrt{z^2 + 2r},$$

see e.g., Lando (2004), appendix B.

C The Equity Holders' Optimization Problem in the Case with Delayed Information

Observe that

$$
\begin{aligned}
& \sup_{\tau \in \mathcal{T}_m} E\Big[\int_t^{\tau} e^{-r(s-t)}(\delta_{s-m} - (1-\theta)C)ds + e^{-r(\tau-t)}\pi(V_B^m)\big|\mathcal{F}_t^m\Big] \\
= \quad & \sup_{\tau^* \in \mathcal{T}^*} E\Big[\int_{t-m}^{\tau^*} e^{-r(s-(t-m))}(\delta_s - (1-\theta)C)ds \\
& + e^{-r(\tau^*-(t-m))}\pi(V_B^m)\big|\mathcal{F}_{t-m}\Big],
\end{aligned}
$$

where the stopping time $\tau^* = \tau - m$ and $\mathcal{T}^*$ is the set of all $\mathcal{F}_{t-m}$-adapted stopping times. The expression in the last line we recognize as a standard optimal stopping problem and its connection to ODEs is known. For details, see Øksendal (2004).

D Proof that Asymmetric Information Increases Credit Spreads

In this appendix we prove that, *ceteris paribus*, asymmetric information leads to wider credit spreads. Let η^2 and η^3 be the credit spreads in case 2 and in case 3, respectively. We want to prove that $\eta^3 > \eta^2$. First observe that

$$
\begin{aligned}
\eta^3 = \frac{-\ln\Big(\alpha P^3(s) + (1-\alpha)\Big)}{s-t} \quad & > \quad \frac{-\ln\Big(\alpha P^2(s) + (1-\alpha)\Big)}{s-t} = \eta^2 \\
& \Leftrightarrow \\
P^3(s) \quad & < \quad P^2(s). \qquad (27)
\end{aligned}
$$

Remember that in case 2 $l = m$. To distinguish case 2 from case 3 we use m as the index for the bond holders' delay in case 2 and we reserve l to be the bond holders' information delay in case 3. We use the same time lag m in case 2 and in case 3. Let $\mathcal{A}$ be the event that no bankruptcy takes place on the time interval $[t, s]$ and $\mathcal{B}$ the event that no bankruptcy takes place on the time interval $[t - l + m, t]$. That is,

$$\begin{aligned} \mathcal{A} &= \{\omega : \inf_{s \in [t,s]} V_s^m > V_B^m\} \\ \mathcal{B} &= \{\omega : \inf_{s \in [t-l+m,t]} V_s^m > V_B^m\}. \end{aligned}$$

Also define

$$\hat{\mathcal{B}} = \{\omega : V_t^m > V_B^m\}.$$

We prove the result *ceteris paribus*. Therefore let $V_t^m = V_t^l = v$, for a positive constant v.

We then have that (conditional on $V_t^m = v$)

$$P^2(s) = P(\tau > s \big| \mathcal{F}_t^m) = P(\mathcal{A}) = P(\mathcal{A}|\mathcal{B}). \tag{28}$$

Because of the Markov property of V_t^m, $P^2(S)$ does not change when conditioning on $\mathcal{B}$. We further have that (conditional on $V_t^l = v$)

$$P^3(s) = P(\tau > s \big| \mathcal{G}_t^l) = P(\tau > s \big| \hat{\mathcal{B}}; V_t^l = v) = \frac{P(\mathcal{A} \cap \mathcal{B})}{P(\hat{\mathcal{B}})}. \tag{29}$$

Combining equations (28) and (29) with the inequality in (27), it is sufficient to show that

$$\frac{P(\mathcal{A} \cap \mathcal{B})}{P(\hat{\mathcal{B}})} < P(\mathcal{A}|\mathcal{B}).$$

Using Baye's rule, we get that

$$\begin{aligned} P(\mathcal{A}|\mathcal{B})P(\mathcal{B}) &< P(\mathcal{A}|\mathcal{B})P(\hat{\mathcal{B}}) \\ &\Leftrightarrow \\ P(\mathcal{B}) &< P(\hat{\mathcal{B}}). \end{aligned}$$

The last inequality is trivially satisfied, proving that credit spreads are wider under delayed and asymmetric information than under delayed information. By letting $m = 0$ (i.e., case 1), the proof still holds.

E Credit Default Swap

In this appendix we derive the expression for $\Gamma(T-t, V_t^m, V_B^m)$, a conditional version of the result in appendix B. By definition

$$\begin{aligned}
\Gamma(T-t, V_t^m, V_B^m) &= E\Big[e^{-r(\tau-t)}1\{\tau \le T\}\big|\mathcal{G}_t^l\Big] \\
&= E\Big[e^{-r(\tau-t)}1\{\tau \le T\}\big|V_t^m > V_B^m; \mathcal{F}_t^l\Big].
\end{aligned}$$

Trivially,

$$\begin{aligned}
& E\Big[e^{-r(\tau-t)}1\{\tau \le T\}\big|\mathcal{F}_t^l\Big] \\
= \; & E\Big[e^{-r(\tau-t)}1\{\tau \le T\}1\{V_t^m > V_B^m\}\big|\mathcal{F}_t^l\Big] \\
& + E\Big[e^{-r(\tau-t)}1\{\tau \le T\}1\{V_t^m \le V_B^m\}\big|\mathcal{F}_t^l\Big].
\end{aligned}$$

Observe that the last term in the previous expression can be written as

$$\begin{aligned}
& E\Big[e^{-r(\tau-t)}1\{\tau \le T\}1\{V_t^m \le V_B^m\}\big|\mathcal{F}_t^l\Big] \\
= \; & E\Big[e^{-r(\tau-t)}1\{V_t^m \le V_B^m\}\big|\mathcal{F}_t^l\Big] \\
= \; & E\Big[1\{V_t^m \le V_B^m\}\big|\mathcal{F}_t^l\Big] \\
= \; & P(V_t^m \le V_B^m\big|\mathcal{F}_t^l).
\end{aligned}$$

The second equality follows from the definition of the stopping time τ in expression (3).

From Baye's formula and the above two expressions follow that

$$\begin{aligned}
\Gamma(T-t, V_t^m, V_B^m) &= \frac{E\Big[e^{-r(\tau-t)}1\{\tau \le T\}\big|\mathcal{F}_t^l\Big] - P(V_t^m \le V_B^m\big|\mathcal{F}_t^l)}{P(V_t^m > V_B^m\big|\mathcal{F}_t^l)} \\
&= \frac{\Upsilon(s-t+l-m, V_t^m, V_B^m) - P(V_t^m \le V_B^m\big|\mathcal{F}_t^l)}{P(V_t^m > V_B^m\big|\mathcal{F}_t^l)} \qquad (30)
\end{aligned}$$

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