Speculative Bubbles Dynamics and the Role of Anchoring

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Abstract:

We investigate the role played by the anchoring-and-adjustment heuristic in the speculative bubbles dynamics. In order to link anchoring bias and price deviations from fundamental value, we develop a stock market equilibrium model with heterogeneous investors: fundamental investor anchoring to past stock market prices and noise traders. The equilibrium model we derive suggests that price is a function of fundamental value, past price, noise and anchoring level. Based on our model, we run a set of Monte Carlo experiments with various anchoring levels: no anchoring, low anchoring and high anchoring. We bring the evidence that large speculative bubbles can only occur when fundamental traders highly anchor to stock market prices. Noise cannot in itself cause such phenomenon. Our findings also suggest that a high anchoring level is consistent with slowly mean reverting bubbles lasting many years.

Keywords: Anchoring, Behavioral finance, Bubbles, Experiment, Stock market

JEL Classification: G12, G14

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Tulip mania, South Sea bubble, stock market booms and crashes[1], etc.: "speculative bubbles have a long history, their importance unquestioned" (Stanley, 1997: 612). A huge academic literature demonstrates that stock market prices often deviate from intrinsic value before reverting to mean (Shiller 1981a, 1981b; LeRoy and Porter, 1981). Indeed, stock prices are found to be more volatile than the dividend streams that drive fundamental value. Bubbles, that consist of "price deviation from intrinsic value" (Camerer, 1989: 3), can be positive when stock markets are over valuated, or negative when they are under valuated.

The understanding of speculative bubbles dynamics – how they bust and burst – appears to be an important issue in the asset pricing literature. However, the "etiology of speculative bubbles" (Shiller, 1999) remains unclear. The first explanation proposed, the rational bubbles theory (Hahn, 1966; Blanchard and Watson, 1982; Tirole, 1982, 1985), is only consistent with positive bubbles and requires strong conditions of existence (Diba and Grossman, 1987).

On the other hand, behavioral finance fails to formally link biases and bubbles or proposes explanations that are often *ad hoc*. For example, noise trading (Kyle, 1985; Black, 1986) and irrational fads resulting from sociological trends (Shiller, 1984; Summers, 1986) are supposed to be stationary (Froot and Obstfeld, 1991: 1190)[2]. Therefore, mean reversion in stock market prices is tautological since it is based on a statistical assumption. Furthermore, Camerer (1989: 29) argued that "fads are not well understood theoretically".

This articles aims to investigate the role played by the anchoring bias in the speculative bubble dynamics. The anchoring-and-adjustment heuristic fits into the decision making and judgment literature. This behavioral bias was first documented by Tversky and Kahneman (1974); in an experiment, subjects were asked to assess in percentage the number of African countries members of the United Nations Organization. The authors found that answers were highly influenced by an arbitrary chosen anchor consisting of a random number spun on a wheel-of-fortune between 0 and 100. But anchors, to which individuals refer when assessing a value, do not only consist of random numbers.

In an "information-rich"[3] context, Northcraft and Neal (1987) bring the experimental evidence that property listing prices influence the assessment of fair value of both experts and amateurs. Marsat and Williams (2009) obtain similar results when they ask subject to assess the fundamental value of a "real world" stock using a rich informative dataset: both market listing and manipulated prices ("anchors") influence their estimates. Their results are consistent with the anchoring-and-adjustment hypothesis.

In order to analyze the role of anchoring in the speculative bubble dynamics, we first model the fundamental value assessment process when fundamental investors anchor to past market prices. We then develop a stock market equilibrium model[4] with heterogeneous investors: fundamental traders and noise traders. Fundamental traders are "semi rational" investors falling into the anchoring trap when they assess the fundamental value of stocks while noise traders are irrational investors.

[1] e.g.: 1929, 1987, late 1990s and early 2000s.
[2] "It is often argued that stationary fads or noise trading lie behind departures from present-value prices", (Froot and Obstfeld, 1991: 1190).
[3] Northcraft and Neale (1987: 96)
[4] We build an equilibrium model à la Cutler et al. (1990). This is the reason why the title of our article refers to the one of Cutler et al. (1990): "Speculative Dynamics and the Role of Feedback Traders".

The equilibrium model we derive from the two demand functions is then used to run Monte Carlo experiments. In order to investigate the role played by the anchoring heuristic in the speculative bubbles dynamics, we run a set of Monte Carlo experiments with various values for the anchoring levels: no anchoring, low anchoring and high anchoring to past market prices. For each set, we compute simulated prices, simulated fair values and the corresponding valuation index that allows us to measure both magnitude and length of speculative bubbles. The valuation index we use is defined as price to value minus one (Lee et al., 1999).

The paper contributes to the understanding of speculative bubble dynamics. We find evidences that both large bubbles – those with an absolute valuation index superior to 50% – and persistent bubbles – those lasting many years – can only occur when fundamental traders highly anchor to stock prices. Noise trading itself[5] cannot lead to large and lasting speculative bubbles. Our model also suggests that a high anchoring level is consistent with slowly mean reverting bubbles.

The paper is organized as follows. In section 1 we present the anchoring-and-adjustment literature and the fundamental value assessment process. In section 2 we develop an equilibrium model of the stock market with heterogeneous investors: fundamental traders that anchor to past market prices and noise traders. In section 3, we discuss the Monte Carlo experiments implementation method, our results and the contribution of the anchoring-and adjustment heuristic to the understanding of speculative bubbles dynamics. Section 4 concludes.

1. The fundamental value assessment process with anchoring

Before modeling the fundamental value assessment process when investors anchor to past market prices, we need to present the academic literature dedicated to the judgment bias known as anchoring-and-adjustment heuristic.

1.1. The anchoring-and-adjustment literature

The anchoring-and-adjustment heuristic fits into the decision making and judgment literature. It depicts the facts that when assessing the value of a good, of a property or of a stock, individuals can be influenced by an exogenous data (the anchor) that have a suggestive power. According to Northcraft and Neale (1987: 85), "the psychological literature on the 'anchoring-and-adjustment' heuristic suggests that (a) an arbitrarily chose reference point (anchor) will significantly influence value estimates, and (b) value estimates will be insufficiently adjusted away from the reference point toward the true value of the object of estimation (Slovic and Lichtenstein, 1971)".

The anchor can be an external (e.g.: random number, public information) or an internal data (self-generated anchor). In Tversky and Kahneman's (1974) seminal research, anchor was given by a random number spun on a wheel-of-fortune between 0 and 100. It was proved that the answers of subjects that were asked to assess the number[6] of African countries members of the United Nations Organization were influenced by the random anchor. Similarly, Ariely et al. (2003) find experimental evidences that subject's maximum willingness-to-pay for

[5] i.e.: with fundamental investors not falling into the anchoring trap.
[6] This number was expressed in percentage.

ordinary products[7] is influenced by a number derived from their personal social security number.

But anchors are not necessarily random numbers[8]. They can consist of public information such as past market prices. Northcraft and Neale (1987) brought a major contribution to the anchoring literature since they demonstrated that in an informative-rich context[9], both experts and amateurs were influenced by listing prices when they assessed the Fair Market Value (FMV) of a "real world" property that was located in Tucson, Arizona. For example, the mean appraisal value given by expert subjects was $ 67,811 with a $ 65,900 listing price and $ 75,190 with an $ 83,900 listing price.

Surprisingly, subjects who were given "extreme anchors" (±12% from the actual listing price) appeared to suffer the same biasing influence that subjects who were given "moderate anchors" (±4% from the actual listing price). The experiment found evidences against the hypothesis that "the biasing influence of the listing price on estimates of FMV (…) decrease as the listing price becomes a less credible estimate of FMV" (Northcraft and Neale, 1987: 86). It does mean that individuals fall into the anchoring trap even if they are confronted to "unreal" manipulated listing prices.

In an experimental research, Marsat and Williams (2009) found similar results when asking subjects to assess the fundamental value of a "real world" stock. The median estimate was EUR 26.0 when subjects were given the actual market price (EUR 36.76) and rose to EUR 39.1 when subjects were given a highly manipulated price (EUR 60.11). Both credible and "less credible"[10] anchors prove to have a biasing influence on the fundamental value assessment process. It suggests that investors suffer the biasing influence of stock market prices even during speculative bubble periods when large misevaluations occur.

In most experiments, subjects are students from business schools or from social sciences departments (e.g.: Ariely et al., 2003; Marsat and Williams, 2009). It raises a question: are real world professionals less likely victims of the anchoring bias than university students? The researches that asked both amateurs and professionals give contradicting answers. Kaustia et al. (2008) find evidences that financial market professionals are less likely victims of the anchoring bias than student when they are asked to expect the long-term stock return. The anchoring effect "nevertheless remains statistically and economically significant, even when [authors] restrict the sample to more experienced professionals" (Kaustia et al., 2008: 391).

However, Northcraft and Neal (1987) find contradictory findings that should be "enlightening"[11] for the understanding of speculative bubbles dynamics since "experts are susceptible to decision bias… [and] …are less likely than amateurs to admit to (or perhaps understand) their use of heuristics in producing biased judgments" (Northcraft and Neale, 1987: 95).

[7] Cordless trackball, cordless keyboard, average wine, rare wine, design book and Belgian chocolates (Ariely et al., 2003: 76).
[8] See also Ariely et al. (2006).
[9] Northcraft and Neale (1987: 87) detail the design overview of their experiment: "Each subject visited a piece of property currently for sale in Tucson, Arizona and filled out a short questionnaire concerned with estimating the value of the property. Each subject was provided with a 10 page packed of information which included (…) 4. Information (including listing price, square footage, (…), etc.) about other property located in the same neighborhood as the property being evaluated (…)."
[10] See Northcraft and Neale (1987: 86).
[11] Northcraft and Neale (1987: 95)

We can now turn these findings into a model of fundamental value assessment we will use in section 2 to develop a stock market equilibrium model.

1.2. Modeling the fundamental value assessment process

Without anchoring-and-adjustment

When investors are purely rational *homo œcomicus*, fundamental value assessment rely on the mere Fisher (1930) rule: “capital, in the sense of capital value, is simply future income discounted or, in other words, capitalized. The value of any property, or rights to wealth, is its value as a source of income and is found by discounting that expected income”, (Fisher, 1930: 12). This principle, known as the Discounted Cash Flows (DCF) model, applies to a wide variety of assets: bills, bonds, mortgages, stocks, etc. When applied to stocks, Fisher rule becomes[12] the Williams’ (1938) Dividend Discounted Model (DDM):

$$V_t = \sum_{i=1}^{\infty} \frac{\mathrm{E}_t\, D_{t+i}}{(1+r)^i} \qquad (1)$$

Where: V_t, fundamental/intrinsic value at time t; D_t, dividend per share at t; E_t, rational expectation operator based on information available à t and r, cost of equity capital.

Samuelson (1973) proved “that properly discounted present values of assets vibrate randomly”. More accurately, assuming a constant payout ratio and a constant discount rate, he demonstrated[13] that the fundamental value V_t follows a geometric random walk with drift when earnings grow at constant rate μ:

$$\mathrm{Ln}\, V_t = \mathrm{Ln}\, V_{t-1} + \mu + u_t \qquad (2)$$

Where: $\mu > 0$, positive drift and u_t, Gaussian white noise with standard deviation σ_u: $u_t \sim N\,(0, \sigma_u)$. The μ drift is the secular trend growth of earnings.

Whit anchoring-and-adjustment

When investors anchor to stock market prices, as suggested by the anchoring literature and Shiller (1999), fundamental value assessment rely on both “true” rational fundamental value and past market price. Since adjustment away from the anchor is always insufficient (Slovic and Lichtenstein, 1971; Epley and Gilovitch, 2006), the *biased* fundamental value assessed by investors can be written in first approximation as:

$$\mathrm{Ln}\, V_t^* = (1 - \alpha)\, \mathrm{Ln}\, V_t + \alpha\, \mathrm{Ln}\, P_{t-1} \qquad (3)$$

[12] If the transversality condition holds: $\mathrm{Lim}_{\, i \to +\infty}\, \mathrm{E}_t\, P_{t+i}\, (1 + r)^{-i} = 0$.

[13] See equation (16) in Samuelson (1973: 31). This property derives from Theorem 3 when earnings follow themselves a geometric random walk with drift.

Where: V_t^*, biased fundamental value at t; V_t, true rational fundamental value given at t by Williams' (1938) DDM; P_{t-1}, market prices at $t-1$ and $0 < \alpha < 1$, anchoring parameter. For the sake of convenience, equation (3) is log-linear[14].

Campbell and Sharpe (2007: 8) propose a "model of forecast anchoring" similar to our value assessment model. They define the biased forecast as λ time the unbiased rational prediction plus $(1-\lambda)$ time the forecasted series moving average (the anchor). The λ parameter they use equals $1-\alpha$ in our model.

The anchoring parameter satisfies: $0 < \alpha < 1$; $\alpha > 0$ means that investors always fall into the anchoring trap while $\alpha < 1$ means that investors always take account of the true rational value. The higher α the more investors anchor to past market price P_{t-1}. In other words, the higher α the less investors adjust[15] their value estimates away from the anchor P_{t-1} to the true rational fundamental value V_t. In the remainder of the article "high anchoring" will be taken as a synonym of "low adjustment" and "low anchoring" as a synonym of "high adjustment".

However, the hypothesis of a constant anchoring parameter appears to be unrealistic according to experimental evidences. For example, Marsat and Williams (2009: 10) bring the evidence that subjects are more likely to anchor when the anchor is "less credible" (manipulated price)[16]. The psychological literature suggests that the level of anchoring should grow as the gap between rational value and market price increases in order to reduce the cognitive dissonance. Festinger (1957: 4) states that cognitive dissonance "arises" when "persons sometimes find themselves doing things that do not fit with what they know, or having opinions that do not fit with other opinions they hold". Akerlof and Dickens (1982) were the first to investigate "the economic consequences of cognitive dissonance".

The value assessment process should effectively be a source of cognitive dissonance when investors "[have] opinions" (i.e.: fundamental value V_t) "that do not fit with other opinions they hold" (i.e.: past price P_{t-1}). Indeed the market price may be viewed as the aggregate opinion of all market participants. If some of these participants use "false" information (noise), the gap between fundamental value and market price may grow and cognitive dissonance may arise.

Festinger (1957: 3) states that "1. The existence of dissonance, being psychologically uncomfortable, will motivate the person to try to reduce the dissonance and achieve consonance. 2. When dissonance is present, in addition to trying to reduce it, the person will actively avoid situations and information which would likely increase the dissonance". Following Festinger (1957) we state that, in order to reduce dissonance, investors will be more likely to anchor to market price when the gap between true fundamental value and market price is large (i.e.: speculative bubble). It means that anchoring is not a constant parameter but a time time-varying α_t function. This stating should be translated into equation[17] as following:

[14] In section 2, the demand function of fundamental investors refers to the logarithm of biased fundamental value. One should note that we would obtain similar results with: $V_t^* = (1-\alpha)\,V_t + \alpha\,P_{t-1}$, since the following ratio is by definition close to one: $\exp\,[(1-\alpha)\,\mathrm{Ln}\,V_t + \alpha\,\mathrm{Ln}\,P_{t-1}] \,/\, [(1-\alpha)\,V_t + \alpha\,P_{t-1}]$.

[15] $\lambda = 1-\alpha$ can be viewed as the adjustment parameter.

[16] Similarly, the authors find that subjects assert using less fundamental information when confronted to a manipulated price.

[17] For the sake of convenience, we use a "semi-linear" function.

$$\alpha_t = \text{Min}\left(\,\alpha_{\text{Min}} + |\text{Ln}\, P_{t-1} - \text{Ln}\, V_t\,|, \alpha_{\text{Max}}\,\right) \qquad (4)$$

Where: α_t, anchoring level at t; $\alpha_{\text{Min}} > 0$, floor value for the anchoring level (i.e.: minimum anchoring level) and $\alpha_{\text{Max}} < 1$, cap value for the anchoring level (i.e.: maximum anchoring level). When the gap between past prices and actual fundamental value is null (consonance), the anchoring level is minimum and α_t equals α_{Min}. Figure 1 depicts the anchoring function:

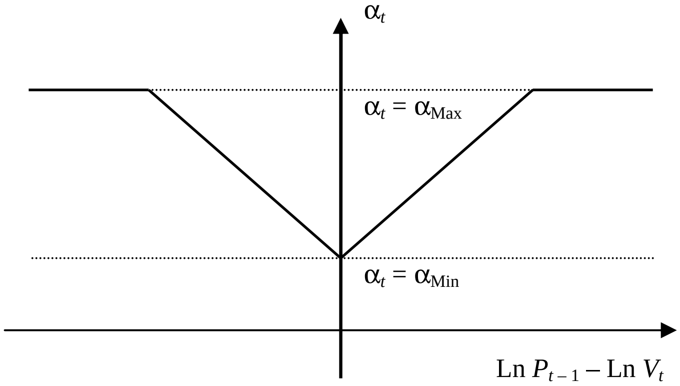

FIGURE 1. ANCHORING FUNCTION α_t

Therefore the fundamental value assessment process first described in equation (3) becomes more sophisticated since it takes account of the degree of anchoring that is itself a function of the relative gap ν_t between rational fair value and past price:

$$\begin{aligned} \text{Ln}\, V_t^* &= (1-\alpha_t)\,\text{Ln}\, V_t + \alpha_t\,\text{Ln}\, P_{t-1} \\ \alpha_t &= \text{Min}\left(\,\alpha_{\text{Min}} + |\nu_t|, \alpha_{\text{Max}}\,\right) \\ \nu_t &= \text{Ln}\, P_{t-1} - \text{Ln}\, V_t \end{aligned} \qquad (5)$$

Our research brings a contribution to the anchoring-and-adjustment literature by modeling the value assessment process, equation (5). We can now use this model to develop a stock market equilibrium model with heterogeneous investors: fundamental traders falling into the anchoring trap and noise traders. It will allow us to understand the role played by anchoring in the speculative bubble dynamics.

2. Stock market equilibrium with anchoring

The model we develop is derived from the one proposed by Cutler et al. (1990). They postulate the existence of three heterogeneous populations: smart, fundamental and feedback traders. Both smart and fundamental traders are rational investors[18] while feedback traders are irrational noise traders, "buying after price increases"[19] (positive feedback traders) or "buying after price declines" [20] (negative feedback traders). We choose to develop a model à la Cutler et al. (1990) since it proposes a very parsimonious framework.

[18] Except in the case fundamental traders use lagged fundamental values.
[19] Cutler et al. (1990: 65).
[20] Cutler et al. (1990: 65).

Our model postulates the existence of two heterogeneous populations on the stock market: "semi rational" fundamental traders and irrational noise traders. Fundamental traders are not fully rational investors since they anchor to past market prices when they assess equity assets intrinsic value. Fundamental traders exhibit "less-than-perfectly-rational human behavior" (Shiller, 1999). Noise traders are irrational investors; they trade based on "pseudo-signals that [they] believe convey information about future returns but that would not convey such information in a fully rational model (Black, 1986)", (Shleifer and Summers, 1990: 23). These pseudo-signals that are not correlated with fundamental information can consist of technical analysis, popular trading strategies, brokers' recommendations, financial gurus' advices or even Internet chats (Hirshleifer, 2001: 1552).

Fundamental traders base demand $S_{1,t}$ on actual price P_t relative to the estimated fundamental value V_t* assessed using equation (5):

$$S_{1,t} = \beta\,(\text{Ln}\, P_t - \text{Ln}\, V_t^*); \quad \beta < 0 \tag{6}$$

Where: $S_{1,t}$, fundamental traders' stocks demand; $\beta < 0$, demand parameter; P_t, actual stock price and V_t*, biased fundamental value.

Fundamental traders follow an investment strategy that is extremely popular among asset management practitioners: they buy stocks when actual price is low relative to their estimated fundamental value and sell stocks when actual price is high relative to their estimated fundamental value. This "buy cheap, sell dear" investment strategy is also known as value style investing.

Noise traders base demand $S_{2,t}$ on their irrational expectation for the next period stock return:

$$S_{2,t} = \delta\,(\text{E}_t^* R_{t+1} - \rho); \quad \delta > 0 \tag{7}$$

Where: $S_{2,t}$, noise traders' stocks demand; $\delta > 0$, demand parameter; E_t*, irrational expectation operator based on noise available at t; R_{t+1}, *ex-post* stock return from t to $t + 1$ and ρ, required rate of return on stocks.

We postulate that noise traders' irrational expectations are unpredictable and uncorrelated to fundamental value changes u_t. Therefore, their demand for stocks $S_{2,t}$ exhibits a random behavior (stochastic trades). This "pure (independent) noise trading" hypothesis (Hirshleifer, 2001: 1566-sqq.) is also used in DeLong et al. (1990). Hence the irrational expectation for the next period stock return can be written as:

$$\begin{aligned} \text{E}_t^* R_{t+1} &= \varepsilon_t \\ \text{E}\,(\varepsilon_t) &= \mu \\ \sigma\,(\varepsilon_t) &= \lambda\sigma_u;\ \lambda > 1 \\ \text{Cov}\,(\varepsilon_t, \varepsilon_{t-i}) &= 0 \text{ for } i \neq 0 \\ \text{Cov}\,(\varepsilon_t, u_t) &= 0 \end{aligned} \tag{8}$$

Where: ε_t, independent stationary random variable (noise) with μ mean and $\lambda\sigma_u$ standard deviation.

Equation (8) depicts the way noise traders expect next period stock returns. Their irrational expectations are supposed to have the same mean than the fundamental value innovation process[21] u_t; this is the only "true" information about V_t noise traders possess[22]. We do postulate that noise trader's expectations are more volatile than changes in the true rational fundamental value, hence: $\sigma(\varepsilon_t) > \lambda\sigma_u$; since $\lambda > 1$. Finally, according to the "pure (independent) noise trading" hypothesis, irrational expectations are unpredictable and uncorrelated to fundamental value changes u_t; hence: $\mathrm{Cov}(\varepsilon_t, \varepsilon_{t-i}) = 0$ for $i \neq 0$ and $\mathrm{Cov}(\varepsilon_t, u_t) = 0$.

By substituting equation (8)'s relation into noise traders demand equation (7) we obtain:

$$S_{2,t} = \delta\,(\varepsilon_t - \rho);\ \ \delta > 0 \tag{9}$$

Assuming "that the [equity] asset is in zero net supply" and assuming "a constant required rate (ρ) of zero" (Cutler et al., 1990: 65), stock market equilibrium requires:

$$S_{1,t} + S_{2,t} = 0 \tag{10}$$

Solving equation (10), we can derive the stock price that leads to market equilibrium at t:

$$\begin{aligned} &\mathrm{Ln}\,P_t = (1-\alpha_t)\,\mathrm{Ln}\,V_t + \alpha_t\,\mathrm{Ln}\,P_{t-1} + \gamma\,\varepsilon_t; \quad \gamma = -\,\delta/\beta > 0 \\ &\quad \alpha_t = \mathrm{Min}\left(\,\alpha_{\mathrm{Min}} + |\nu_t|,\ \alpha_{\mathrm{Max}}\,\right) \\ &\quad \nu_t = \mathrm{Ln}\,P_{t-1} - \mathrm{Ln}\,V_t \end{aligned} \tag{11}$$

The price is a function of true fundamental value, past price, noise and anchoring level α_t. During periods of low anchoring (i.e.: α_t close to α_{Min}), the influence of fundamental value relative to past price is high. Similarly, during periods of high anchoring (i.e.: α_t close to α_{Max}), the influence of fundamental value relative to past price is high.

Equation (11) suggests that the anchoring-and-adjustment heuristic has a biasing influence on the informational content of stock prices since high anchoring is by definition the consequence of past noise and anchoring itself. Furthermore, equation (11) suggests that high anchoring lead to large misevaluations and speculative bubbles, while low anchoring lead to period of higher informational efficiency with small price deviations from fundamental value.

Finally equation (11) suggests that, in the particular case where they are no noise traders on the stock market (i.e.: $\gamma = 0$, since $\delta = 0$), stock market prices are an exponential smoothing of past fundamental values. In this "unrealistic" case, the anchoring bias leads to a delayed information phenomenon and no speculative bubbles can appear.

Indeed, assuming $\gamma = 0$ and $\alpha_t = \alpha$ for the sake of convenience, the equilibrium model becomes:

$$\mathrm{Ln}\,P_t = (1-\alpha)\,\mathrm{Ln}\,V_t + \alpha\,\mathrm{Ln}\,P_{t-1}; \quad 0 < \alpha < 1 \tag{12}$$

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[21] See: geometric random walk for fundamental value defined in equation (2).

[22] We may also postulate optimistic (resp.: pessimistic) believes by supposing $\mathrm{E}(\varepsilon_t) > \mu$ (resp.: $\mathrm{E}(\varepsilon_t) > \mu$). We do not make these hypotheses since our research aims to study the role played by anchoring in speculative bubbles dynamics, not the one of market sentiment (i.e.: optimism/pessimism).

Solving equation (10) backward, we obtain[23]:

$$\text{Ln } P_t = (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \text{ Ln } V_{t-i} \tag{13}$$

Since we postulate a positive drift μ in fundamental value[24], the forward solution defined by equation (13) suggests that anchoring leads to undervaluation when $\gamma = 0$. This is a key finding since we show that the anchoring-and-adjustment heuristic cannot lead to large speculative bubbles if noise traders are not present on the stock market[25]. However since this case appears to be "unrealistic", we will not go further.

The stock market equilibrium defined by equation (11) allows us to investigate the role played by the anchoring heuristic in speculative bubble since we can analyze the dynamics of stock market prices relative to their fundamental values using a simple valuation index ι_t defined as follows (Lee et al., 1999):

$$\iota_t = \frac{P_t}{V_t} - 1 \tag{14}$$

By substituting equations (11) and (2)'s relations into equation (14) we derive the speculative bubble dynamics. However, the equilibrium model we developed has a complex mathematical formulation than cannot be easily studied within an analytical framework. It is for this reason that we turn to numerical experiments. In the following section, we run a set of Monte Carlo experiments with various anchoring levels.

3. Explaining speculative bubbles dynamics

This section presents the methodology we used to run Monte Carlo experiments, the results of the experiments and the contribution of the anchoring-and-adjustment heuristic to the understanding of speculative bubble dynamics.

3.1. Methodology

In order to investigate the role played by anchoring in speculative bubbles dynamics, we run a set of Monte Carlo experiments. We consider three hypotheses: no anchoring, low anchoring and high anchoring. The no anchoring hypothesis ($\alpha_t = 0$) will allow us to control for others hypotheses. Since the anchoring function defined in equation (4) has two parameters $\alpha_{\text{Min}} > 0$ and $\alpha_{\text{Max}} < 1$, we will only allow the floor parameter α_{Min} to vary while the cap parameter α_{Max} will remain constant at 0.99. The experiments use $\alpha_{\text{Min}} = 0.50$ for the low anchoring hypothesis and $\alpha_{\text{Min}} = 0.75$ for the high anchoring one. In other words, the low anchoring hypothesis refers to a low minimum level of anchoring while the high anchoring one refers to a high minimum level of anchoring.

[23] If the following condition holds: $\text{Lim}_{i \to +\infty} \; \alpha^{i+1} \text{ Ln } P_{t-i-1} = 0$

[24] See equation (2).

[25] This stating is confirmed by the run of Monte Carlo experiments. The results are not presented in the article.

In each hypothesis we run 10,000 experiments simulating fundamental value and price time series, each series counting 500 observations. The time tick of the model is month[26]. We chose a very large sample size (5×10^6 observations) in order to obtain robust results. Finally, the same dataset of fundamental value and noise series was used in each hypothesis[27].

The 10,000 fundamental value series were simulated using equation (2) with the following parameters: $\mu = 0.005$ and $\sigma_u = 0.03$. It corresponds to an annual earning growth rate of 6.2 percent with an annualized volatility of 10.4 percent. The 10,000 noise series were simulated using equation (8) with Gaussian distribution and $\lambda = 2$: $\varepsilon_t \sim N\,(0.005, 0.06)$. It means that the volatility of noise traders' irrational expectations is twice as much as the volatility of fundamental value changes.

In order to give both fundamental and noise traders the same weighting on the stock market we set $\gamma = 1$. Finally, we fixed the following initial condition ($t = 1$) for each experiment: $P_1 = V_1 = 100$. It does mean that no mispricing and hence no bubbles exist in the beginning. Table 1 reports the parameters and settings we used:

TABLE 1–MONTE CARLO EXPERIMENTS: PARAMETERS AND SETTINGS

Experiments: 10,000; Observations: 500; Time tick: month

Anchoring		No	Low	High
Anchoring parameters	α_{Min}	0	0.50	0.75
	α_{Max}	0	0.99	0.99
Fundamental value	μ		0.005	
	σ_u		0.03	
Noise trading	λ		2	
Population weighting	γ		1	
Initial condition	ι_1		0	

Table 2 reports the stock return distributions in the three hypotheses. We computed the moments' mean value on the overall 10,000 experiments in each hypothesis[28]. Both emplacement and dispersion estimates appear to be plausible compared to "real world" stock markets. However, stock returns from experiments do not exhibit leptokurtic distribution since kurtosis is always close to 3 (i.e.: theoretical value for a Gaussian distribution).

TABLE 2–RETURN DISTRIBUTION

	No anchoring	Low anchoring	High anchoring
Mean	0.0091	0.0073	0.0069
Median	0.0050	0.0050	0.0050
Maximum	0.3217	0.2328	0.2094
Minimum	–0.2353	–0.1800	–0.1641
Std Dev.§	0.3152	0.2372	0.2121
Skewness	0.2649	0.1967	0.1813
Kurtosis	3.1086	3.0077	3.0475

(§) Annualized standard deviation

[26] The range represents a $41^{8/12}$ years period that will allow us to observe slowly mean reverting bubbles.
[27] For both fundamental values $\{V_t\}$ and noise $\{\varepsilon_t\}$, we simulated 10,000 series of 500 observations each. These series were then used to compute prices $\{V_t\}$ in each hypothesis.
[28] For example, "Maximum" refers to the maximum mean value on the overall sample (the 10,000 experiments).

Although the aim of this research is not to link anchoring bias and stock return distribution, one should note that both standard deviation and extrema estimates are more "realistic" when we consider the anchoring hypothesis. Indeed when fundamental traders do not anchor to stock market prices, we obtain less credible values. For example, the long term market volatility equals 31.52 percent.

3.2. Results and discussion

In order to investigate the role played by the anchoring bias in the speculative bubbles dynamics it would be useful to first consider the characteristic of the valuation index in terms of distribution (see: paragraph 3.2.1.). We will then measure both large bubbles frequency and length (see: paragraph 3.2.2.).

3.2.1. Valuation index distribution

Speculative bubbles induce large transitory price deviations from fundamental value. These deviations are positive when the stock market is over valued and negative when it is under valued. Table 33 reports the mean estimates for the valuation index distribution. A positive valuation index is synonym of overvaluation and a negative one of undervaluation.

TABLE 3–VALUATION INDEX DISTRIBUTION

	No anchoring	Low anchoring	High anchoring
Mean	0.0068	0.0087	0.0903
Median	0.0048	0.0051	0.0255
Maximum	0.2066	0.3084	1.2817
Minimum	–0.1621	–0.2252	–0.5084
Std Dev.	0.0604	0.0841	0.3639
Skewness	0.1801	0.2858	0.7174
Kurtosis	3.0531	3.5092	3.5665

When fundamental traders do not anchor to past market prices, no large speculative bubbles appear in the experiments we run: the maximum stock market overvaluation is 20.66 percent while the maximum stock market undervaluation is 16.21 percent. Low anchoring respectively moves these values to 30.84 percent and 22.52 percent.

On the other hand, a high anchoring level causes large speculative bubbles: the maximum stock market overvaluation rises to 128.17 percent and the maximum undervaluation to 50.84 percent. In other words, during positive bubbles price can be more than twice as much as fundamental value, while during negative bubbles it can be half as much as fundamental value. According to the experiments we run, this does mean that only high anchoring can lead to large bubbles. Noise itself cannot cause such phenomenon.

A high anchoring level implies a right sided valuation index distribution which means that positive bubbles are more frequent than negative ones (see also Table 4 below)[29]. This is consistent with the history of capital markets and recent stock market mispricing which are shown to be more often positive than not (Lee et al., 1999). One should finally note that the high anchoring hypothesis lead to a significant average overvaluation since the mean

[29] Similarly, extreme positive bubbles have a larger magnitude than extreme negative ones.

valuation index is 9.03 percent[30]. This result is consistent with Lee et al. (1999: 1712) which find evidence of significant overvaluation for the Industrial Dow Jones index on the January 1979 to June 1996 time period.

The first results of the Monte Carlo experiments lead one to believe that the anchoring-and-adjustment heuristics plays a major role in the speculative bubble dynamics. More accurately, large bubble should be caused by a high anchoring level in the fundamental value assessment process. Figure 2 is enough to show the explanatory power of the high anchoring hypothesis:

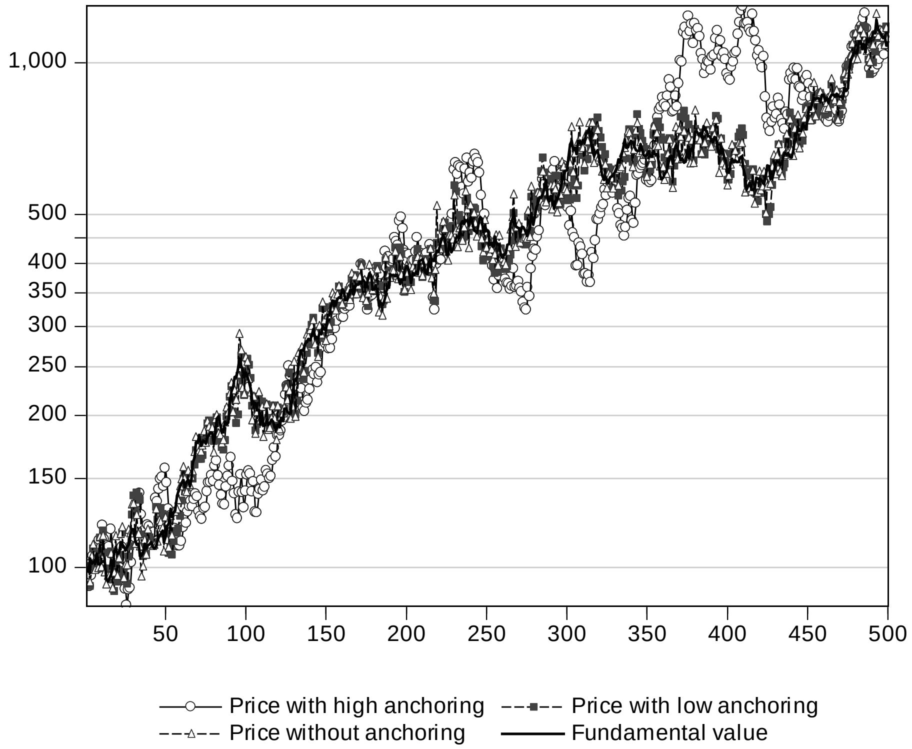

FIGURE 2. SIMULATED PRICE PATHS

Notes: Figure 2 depicts simulated fundamental value path and simulated price paths of one particular experiment. The simulated price paths base on both same simulated fundamental and noise paths. The characteristics of the experiment we chose are close to the one given for the “High anchoring” in Table 3. For example, the valuation index takes the following values: Mean = 0.0406, Max = 1.2270, Min = –0.4946 and Std Dev. = 0.3022. The Y axis scaling method is logarithmic.

During time periods of growing misevaluation, cognitive dissonance rises and the anchoring function α_t approaches to its cap value α_{Max}. At this stage, the informational content of stock market prices decrease since the fundamental value weighting in equation (11) drops. In other words, the model we developed suggests that fundamental traders falling into the anchoring trap make the noise created by noise traders last and hence contribute to the birth of large lasting price deviations from fundamental value.

Figure 3 depicts the valuation index and the dynamic anchoring function α_t when anchoring is high (α_{Min} = 0.75); both base on series used in figure 2.

[30] Contrary to the “No anchoring” and “Low anchoring” hypotheses.

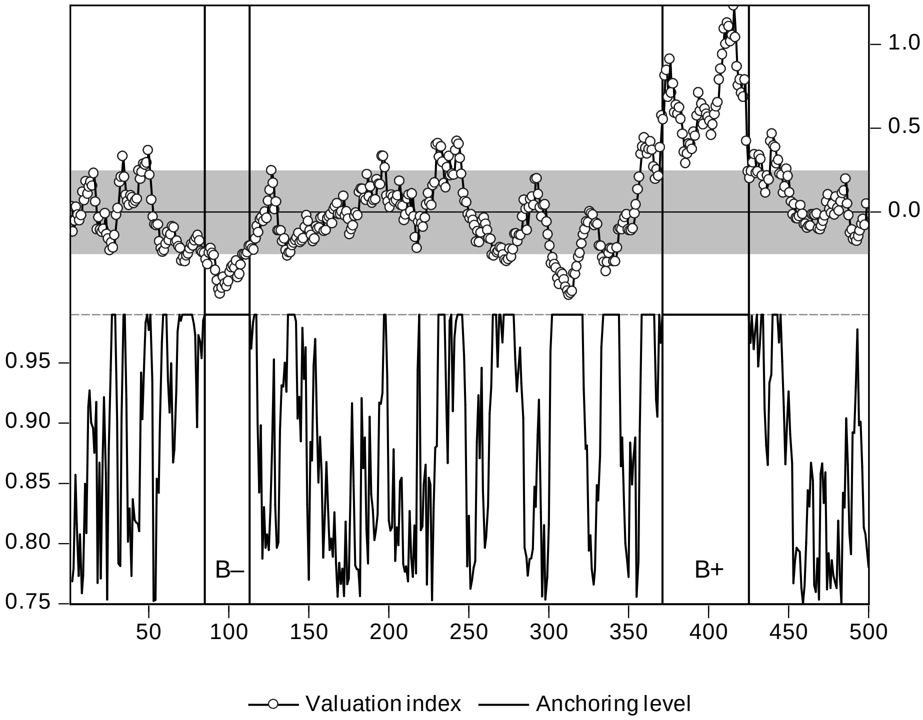

FIGURE 3. SIMULATED VALUATION INDEX AND ANCHORING LEVEL PATHS

Notes: Figure 3 depicts simulated valuation index and anchoring level paths of the experiment represented in figure 2. The shaded area on the right axis corresponds to a valuation index inferior to –25 percent and superior to 25 percent (i.e.: speculative bubble). The time period B– corresponds to a negative bubble with a valuation index inferior to –25 percent (length: 28 months). The time period B+ corresponds to a positive bubble with a valuation index superior to 25 percent (length: 58 months).

During speculative bubbles, fundamental traders do underweight fundamental data in the value assessment process and perform highly insufficient adjustments away from past market prices. For example (see figure 3) during negative bubble B– and during positive bubble B+, the dynamic anchoring level equals its cap value ($\alpha_{Max} = 0.99$) and adjustment is drastically low since $(1 - \alpha_t)$ equals 0.01. It does mean that during "high anchoring" bubbles true fundamental value V_t only represents 1 percent of the fundamental value V_t* assessed by fundamental while past market price P_{t-1} represents 99 percent of biased fundamental value. In other words, during speculative bubbles fundamental traders do not take account of economics news and other fundamental data since they highly fall into the anchoring trap.

3.2.2. Speculative bubbles: frequency and length

The statistical analysis of valuation indexes but lacks to document the characteristics of speculative bubbles in terms of both frequency and length. This paragraph investigates the role played by the anchoring bias in the busting and bursting of large and lasting bubbles. We assume for the sake of convenience that speculative bubbles correspond to an absolute valuation index superior to 25 percent. When the absolute ratio is superior to 50 percent, we speak of "large" speculative bubble. These are purely numerical conventions.

Table 4 reports the frequency of speculative bubbles. By construction cases where estimated extrema do no fit speculative bubbles are dropped[31].

TABLE 4–SPECULATIVE BUBBLE FREQUENCY (%)

	No anchoring	Low anchoring	High anchoring
Bubbles	–	0.63	42.40
¤ Large bubbles	–	–	18.50
Negative bubbles	–	–	18.91
¤ Large negative bubbles	–	–	4.58
Positive bubbles	–	0.63	23.49
¤ Large positive bubbles	–	–	13.92

Notes: "Bubbles" refer to an absolute valuation index superior to 25 percent: $|\iota_t| > 0.25$; "Large bubbles" refer to an absolute valuation index superior to 50 percent: $|\iota_t| > 0.50$; "Negative bubbles" refer to a valuation index inferior to –25 percent: $\iota_t < -0.25$; "Large negative bubbles" refer to a valuation index inferior to –50 percent: $\iota_t < -0.50$; "Positive bubbles" refer to a valuation index superior to 25 percent: $\iota_t > 0.25$; "Large positive bubbles" refer to a valuation index superior to 50 percent: $\iota_t > 0.50$.

The model we developed suggests that large speculative bubbles can only occur when fundamental traders highly anchor to past stock market prices. Noise trading itself cannot cause large price deviation from intrinsic value. Similarly a low anchoring level lead to rare non large bubbles with a frequency equal to 0.63 percent. In the high anchoring hypothesis, speculative bubbles are "endemic" since they do often appear on the stock market since their frequency equals 42.40 percent. Meanwhile, the frequency of large bubbles is 18.50 percent. It does mean that large speculative bubbles are not exceptional phenomenon when fundamental investors deeply fall into the anchoring trap. Finally, one should note that large bubbles essentially consist of positive ones; their frequency is three times as much as the frequency of large negative bubbles. This finding is consistent which the history of stock markets that do exhibit more large overvaluation than large undervaluation periods.

Table 5 reports the mean estimates for the length of both positive and negative speculative bubbles[32]. Cases where estimated extrema do no fit speculative bubbles are dropped.

TABLE 5–SPECULATIVE BUBBLE LENGTH (MONTHS)

	No anchoring	Low anchoring	High anchoring
Negative bubbles			
Mean	–	–	15.3
Maximum	–	–	56
Minimum	–	–	3
Positive bubbles			
Mean	–	1.6	16.8
Maximum	–	2	68
Minimum	–	1	3

[31] See: Table 2. The following cases were dropped: (a) "No anchoring, all bubbles" since Max = 0.2066 < 0.25 and Min = 0.1621 > –0.25; (b) "Low anchoring, negative bubbles" since Min = –0.2252 > –0.25 and (c) "Low anchoring, large positive bubbles" since 0.25 < Max = 0.3084 < 0.50. The frequencies we obtain in these hypotheses are no significant.

[32] It does mean the number of consecutive periods (months) with a valuation index superior to 25 percent (positive bubbles) or inferior to –25 percent (negative bubbles).

The high anchoring hypothesis is consistent with the existence of speculative bubbles lasting many years. The Monte Carlo experiments we run show that the maximum length for a positive bubble is 68 months while it is 56 months for a negative one. On average, both positive and negative bubbles last more than one year when fundamental investors highly anchor to past market prices. This is a key finding since our model is able to generate realistic mean reverting bubbles with large and lasting misevaluations (Lee et al., 1999).

The bubbles we simulate are not *ad hoc* ones since they do not rely on a statistical assumption of slowly mean reverting noise. Indeed, we assume a "pure (independent) noise trading"[33] and a geometric random walk for fundamental value. Therefore in our model, the slow mean reversion observed in the speculative bubble dynamics is the consequence of a high anchoring level.

4. Conclusion

This paper contributes to the understanding of speculative bubbles in stock markets. The equilibrium model we develop suggests that the anchoring-and-adjustment heuristic plays a major role in speculative bubble dynamics. Our research is also the first to formally link anchoring bias and stock market dynamics.

The anchoring-and-adjustment heuristic was first documented by Tversky and Kahneman (1974). In their seminal paper, the authors brought the experimental evidence that subjects were highly influenced by an arbitrary chosen anchor (a random number spun on a wheel-of-fortune) when they were asked to assess the number of African countries member of the United Nations Organization. Similarly, Northcraft and Neal (1987) bring the experimental evidence that property listed prices have a biasing influence on the fair value assessment; while Marsat and Williams (2009) find evidences that subjects that are asked to assess the intrinsic value of a stock are influenced by stock market prices.

In order to investigate the role played by the anchoring bias in the speculative bubbles dynamics we translate the psychological literature into an original model of fundamental value assessment. This model is then used to develop a stock market equilibrium model à la Cutler et al. (1990) with heterogeneous investors: fundamental traders that fall into the anchoring trap and noise traders. The equilibrium model we derive demonstrates that price is a function of true fundamental value, past price, noise and anchoring level. Our model also suggests that anchoring itself cannot lead to large speculative bubbles if noise traders are not present on the stock market.

The first order equilibrium model is then used to run a set of Monte Carlo experiments with various anchoring levels: no anchoring, low anchoring and high anchoring. Based on both simulated fundamental values and stock prices, we bring the numerical evidence that large speculative bubbles can only occur when fundamental traders highly anchor to stock market prices. The presence of noise traders is not a sufficient condition in itself for the birth of large speculative bubbles. Our findings also suggest that a high anchoring level is consistent with the following stylized facts: average overvaluation, predominance of positive bubbles and slow mean reversion (Lee et al., 1999).

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[33] (Hirshleifer, 2001: 1566-sqq.)

This paper suggests that fundamental traders falling into the anchoring trap have surprisingly a higher biasing influence than pure noise traders on the informational content of stock market prices. Paradoxically, these fundamental traders make stock prices revert to their mean value since they are the only ones to take account of the true rational fundamental value. This finding pleads for the existence of "semi rational" speculative bubbles and agrees with Camerer (1989: 30): "my personal views, shared by some, are that several new directions will prove useful: [among whom] theories of near rational fads or bubbles". One should finally note that the "behavioral capital asset pricing"[34] approach we use in this research could be extended in order to investigate other biases documented by the behavioral finance literature.

[34] We refer to an article written by Shefrin and Statman (1994): "Behavioral Capital Asset Pricing Theory".

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