Constructing the US interest rate volatility index

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Abstract

Mention the word volatility to most traders, and VIX comes to mind. Surprisingly, much less attention has been paid to the introduction of equivalent leading indicators of expected future volatility in the fixed-income market. We suggest for the first time the construction of an implied volatility index of forward interest rates from the U.S. cap (floor) market based on the methodology developed in equity derivatives markets. From the results we notice that by September 2006 predictions regarding future interest rate volatility suddenly become more variable. That is, approximately one year before the origin of the current financial crisis.

Keywords: Implied volatility, implied volatility indexes, caps

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1. Introduction

Volatility is a basic feature of financial markets whose importance of modelling and predicting is a growing research topic in modern finance. As far as forecasting performance of future realized volatility (*ex-post* empirical measure of daily return variability) of asset prices is concerned, Poon and Granger (2003) show a review of 93 papers focused on this topic. Broadly speaking, there are two categories of methods widely used in making forecasts of future realized volatility: time series volatility forecasting models (based on historical price information) and option-based volatility forecasts (volatility implied from option prices on a particular underlying). Overall, results from the previous study suggest that forecasts based on implied volatility often beat forecasts based on historical returns.

Implied volatility measures the market's assessment at time *t* of the uncertainty regarding the future development of the asset underlying an option, as implied volatilities are determined via the prices of traded options with a concrete time to maturity, which is the forecast horizon of the implied volatility of the asset. Thus, this volatility is forward looking.

According to Giot (2005), the significance of implied volatility as a rational forecast of future realized volatility and the information content of implied volatility with respect to historical volatility are two important research topics in the academic literature. In practice, these research topics have been widely exploited in stock markets through the construction of implied volatility indexes from options on a particular stock market index (see Fleming *et al*. (1995), Moraux *et al*. (1999), Bluhm and Yu (2001), Giot (2002), and Corrado and Miller (2005)). However, we can hardly find research focused on these topics in the fixed-income market. Thus, this is one of the motivations of this paper.

The model-based methodology applied for the construction of volatility indexes in the equity market consists of a weighting scheme of the implied volatilities of a set of options computed within the overall context of Black-Scholes (1973) option pricing model or a similar model. In particular, implied volatilities are weighted in such a way that the index represents the annualized implied volatility of a particular stock market index underlying an *at the money* (ATM) option with constant time to maturity (i.e., constant forecast horizon of future expected volatility). Thus, at any time *t* the selection of options is done by taking as point of reference the nearness of the time to maturity and the strike of the traded options to the constant time to expiration established for the construction of the index and to the ATM strike, respectccely. In practice, the process of construction of the implied volatility indexes is carried out by considering that the value of the ATM strike is either the spot price (current value) or the forward price of the stock market index on which the option is written. [1]

In 1993 the Chicago Board Options Exchange (CBOE) introduced the first implied volatility index on a stock market index using data from options on the S&P100 Index: the S&P 100 Volatility Index (VIX). VIX very quickly became the benchmark risk measure for stock market volatility. It represents the implied volatility of an ATM "synthetic" S&P 100 option with constant time to maturity (30 calendar days) at any point in time (details regarding the construction of VIX are available in Fleiming *et al.* (1995)).[2]

Following the example of the CBOE, other options markets introduced their own volatility indexes in Europe. In 1994 the Deutsche Börse created a volatility index for the German stock market: VDAX, from options on the DAX Index (see Lyons (2005) for a detailed description of the construction process of VDAX). In 1997 the MONEP (Marché des Options négociables de Paris) introduced VX1 and VX6 indexes to

[1] Black (1976) extended the Black-Scholes model to price European options on an asset in terms of the future (or forward) price for a contract maturing at the same time as the option.
[2] In 2003 the CBOE introduced the new VIX, computed from options on the S&P 500 rather than the S&P 100.

measure the uncertainty concerning the French stock market from options on the CAC-40 Index (see Moraux *et al.* (1999) for further details about these indexes).[3]

Mention the word volatility to most traders, and VIX comes to mind. Surprisingly, much less attention has been paid to the introduction of equivalent leading indicators of expected future volatility in the fixed-income market. To the best of our knowledge, up to now, the Merrill Lynch Option Volatility Index (MOVE) and the Lehman Brothers Swaption Volatility Index (LBPX), constructed as a weighted average of implied volatilities of Treasury bond options and a basket of liquid swaptions for different terms to maturity of the underlying instrument: Treasury bonds and swaps, respectively, have been the only attempts to measure expectations of future volatility from traded interest rate derivatives.

In this study we suggest for the first time the construction of a “pure” measure of the expected future volatility referred to a particular forward interest rate (with a concrete period of reference) based on the model-based methodology applied in equity derivatives markets. The interest rate volatility index (IRVIX) is constructed from data of the U.S. cap (floor) market. Caps (floors) are portfolios of options on interest rates traded in the over-the-counter (OTC) interest rate derivatives market, one of the most liquid OTC derivatives markets in the world.

Information provided by the market consists of implied *flat* volatility quotes of caps (floors), where implied volatilities are computed by equalling the market prices of such derivatives and the Black (1976) model price applied to all the caplets (floorlets) that compose the cap (floor) by assuming that the volatility of forward interest rates underlying every option is constant. Thus, *flat* volatilities do not enable us to know what the period of reference of the underlying forward interest rate whose implied volatility has been estimated is.

[3] An attempt to create implied volatility indexes in the context of emerging markets can also be found in Skiadopoulos (2004) for the Greek derivatives market.

We focus on this item in order to construct implied volatility indexes of forward interest rates with a concrete period of reference by using *spot* volatilities recovered from *flat* volatility quotes (i.e., implied volatilities of caplets (floorlets) with a concrete period of reference) and then applying the methodology developed in equity markets.

The interest rate volatility index constructed like that aims to represent the annualized implied volatility of the forward interest rate underlying an ATM caplet with a fixed time to maturity, which is the forecast horizon of the expected future interest rate volatility. From our point of view, a wide range of applications can arise from the introduction of these indexes. Next we consider some broad lines of potential applications.

On the one hand, analyzing whether interest rate volatility as measured by the index contains additional useful information about the future state of economy to that broadly documented of the term structure of interest rates is probably one of the most attractive applications.

On the other hand, this measure of forward interest rate volatility might also be applied in the study of the impact of monetary policy on interest rate volatility, as well as used in the valuation of more complex interest rate derivatives such as swaptions. Finally, one of the potential applications of the implied volatility indexes is the possibility of introducing futures and options on such indexes, as occurred in the US after the launch of the VIX. On February 2006 options on VIX began trading on the CBOE, following the previous introduction of VIX futures on the CBOE Futures Exchange (CFE) in 2004. According to Areal (2008), in practice these derivatives can be used in turn to create hedge strategies against changes in volatility, or to speculate on changes in the market volatility.

In this paper we analyze the behaviour and statistical properties of four implied volatilities indexes covering four different forecast horizons over the period from July 30, 2004 to January 30, 2009. The fact of the sample period comprising the origin of the current financial crisis is especially relevant in order to visualize the information content of the indexes as leading indicators of business cycle.

The structure of the paper is as follows. The next section is focused on caps (floors) valuation within the LIBOR Market Model (LMM) framework. In section three we present the construction process of volatility indexes in equity derivatives markets and how to implement such methodology from cap (floor) market data. Section four is aimed at the description of the database and the methodology applied for the construction of the interest rate volatility index (IRVIX). In section five the behaviour and statistical properties of the volatility indexes are analyzed. Finally, section six provides a summary of the study.

2. Caps and floors valuation. The LIBOR Market Model and the Black formula

A forward rate agreement (FRA) is the underlying of one of the simplest interest rate options: the caplets (floorlets). A FRA can be defined as an agreement between two parties at time t to exchange at time $T_i+\tau$ an amount of money proportional to the difference between a strike, K, agreed upon at time t, and the floating interest rate, $R(T_i, T_i + \tau)$, that resets at time T_i. The proportionality factor is given by the product of the notional principal, NP, and the *tenor* interval, τ.

The additive sum of interest rate options on FRAs gives rise to caps (floors), one of the most popular interest rate derivatives offered by financial institutions in the OTC market. Each option composing the cap (floor) has the same strike and the same period of life, tenor (time distance between floating interest rate resets), as the others, but a different expiration date (the expiration date of an option, $T_i+\tau$, is the same as the exercise date, T_i, of the following one). Typically, the expiration dates for the caplets (floorlets) are on the same cycle as the frequency of the underlying floating rate (Longstaff *et al.*, 2001).

Caps are designed to hedge the interest rate risk created by the variability of the floating rate in some financial contracts where market participants pay cash flows tied to some floating rate. Next we describe the way payoffs take place in a cap. On the first reset date of the cap, the floating rate of the contract is observed and compared to the strike. If the floating rate is greater than the strike, then on the second reset date the seller of the cap pays the holder the difference between the floating rate and the strike multiplied by the notional principal and the tenor (if the floating rate is less than the strike, there is no payoff from the cap). Thus, through the life of a cap, payments are done at the end of each tenor interval although its amount is fixed at the reset date (at the beginning of the tenor interval) when the interest rate is observed.[4]

Analogously to caps, a floor provides a payoff when the interest rate in some financial contract tied to a floating rate falls below a certain rate. That is, the floor provides insurance against the interest on the floating rate of a contract falling bellow a certain level.

Next we show a brief overview of the LIBOR Market Model (LMM) valuation framework, which leads to the Black (1976) pricing formula for caps (caplets) and floors (floorlets) used by market practitioners.

As described previously, the payoff derived from a caplet at maturity, $T_i+\tau$, is given by[5]:

$$Payoff_{\mathrm{Ti+\tau}} = NP \cdot \left[f\left(T_i, T_i, T_i + \tau\right) - K \right]^+ \cdot \tau \qquad [1]$$

$$Payoff_{\mathrm{Ti+\tau}} = \left[NP \cdot f(T_i, T_i, T_i + \tau) \cdot \tau - NP \cdot \tau \cdot K \right]^+ \qquad [2]$$

where $f(t, T_i, T_i+\tau)$ denotes the time t forward rate applying between T_i and $T_i+\tau$, with t prior to T_i. Notice that at reset, T_i, the forward rate is set by definition to be equal to the corresponding interest rate, $R(T_i, T_i + \tau)$:

[4] Caps are usually defined so that the initial floating rate, even if it is greater than the cap rate, does not lead to a payoff on the first reset date (Hull, 2009).

[5] Analogous expressions for floorlets can easily be derived.

$$f\left(T_i,T_i,T_i+\tau\right)\equiv R\left(T_i,T_i+\tau\right) \quad [3]$$

In order to obtain the price of a caplet at time t before T_i, we consider that the value of the forward rate $f(t, T_i, T_i+\tau)$ can be replicated from a portfolio of traded assets (see Díaz *et al.*, 2009). The present value of this portfolio $\pi(t)$ is given by assuming, for each unit of the notional principal NP, a long position on a zero coupon maturing at T_i and a short position on a zero coupon bond maturing at $T_i+\tau$. Thus, we have

$$\pi(t)=NP\cdot\left[P(t,T_i)-P(t,T_i+\tau)\right] \quad [4]$$

Now it follows that if we reinvest the principal payment of the shorter bond in the zero coupon bond maturing at $T_i+\tau$, this portfolio produces the same payoff as the floating leg of the caplet: the first term on the right hand side of Equation (2). That is,

$$\pi(T_i+\tau)=NP\cdot\left[(1+R(T_i,T_i+\tau)\cdot\tau)-1\right]=NP\cdot f(T_i,T_i,T_i+\tau)\cdot\tau \quad [5]$$

Then, by applying a no-arbitrage argument, we should verify the following equality:

$$P(t,Ti)-P(t,Ti+\tau)=f(t,Ti,Ti+\tau)\cdot\tau\cdot P(t,Ti+\tau) \quad [6]$$

$$f(t,T_i,T_i+\tau)\cdot P(t,T_i+\tau)=\frac{P(t,T_i)-P(t,T_i+\tau)}{\tau} \quad [7]$$

The LMM assumes that the forward interest rate $f(t, T_i, T_i+\tau)$ follows a lognormal stochastic process and, therefore, under the forward measure Q^i, the arbitrage portfolio $f(t,T_i,T_i+\tau)\cdot P(t,T_i+\tau)$ discounted by the numeraire $P(t,T_i+\tau)$, that is, the forward rate $f(t, T_i, T_i+\tau)$, must follow a martingale (a zero drift-stochastic process):

$$\frac{df(t,T_i,T_i+\tau)}{f(t,T_i,T_i+\tau)} = \sigma(t,T_i)dz \qquad [8]$$

where dz is a standard Wiener process. Concerning the volatility function, $\sigma(t,T_i)$, the LMM approach is characterized by imposing that the volatility functions of the forward rates should be restricted to being deterministic functions of time (Rebonato, 2002).

Next, taking into account that the forward interest rate equals the expected future interest rate in a world that is forward risk neutral with respect to a zero-coupon bond maturing at time $T_i+\tau$, $P(t,T_i+\tau)$, we show the conditional distribution of the natural logarithm of the forward rate after applying *Itô´s lemma*:

$$\ln[f(Ti,Ti,Ti+\tau)] \sim G(\ln[f(t,Ti,Ti+\tau)] - \frac{1}{2}\sigma^2_{i,Black}(Ti-t);\sigma^2_{i,Black}(Ti-t)) \qquad [9]$$

where $G(\cdot)$ denotes the Gaussian distribution, and the volatility of changes (from t to T_i) in the logarithm of the forward rate, $\sigma_{i,Black}$, can be understood as an average of the forward instantaneous volatility, $\sigma(t,T_i)$, over the period [t, T_i]:

$$\sigma^2_{i,Black}(T_i-t) = \int_t^{T_i} \sigma^2(u,T_i)du \qquad [10]$$

Finally, when the payoff of the caplet is integrated over the log-normal distribution, one recovers the market-standard Black formula for caplets valuation at time t prior to T_i:

$$Caplet(t, Ti, \tau, NP, \sigma_{i,Black}) = NP\left[f(t, Ti, Ti+\tau)N(h_1) - K \cdot N(h_2)\right] \cdot P(t, Ti+\tau) \cdot \tau \quad [11]$$

where

$$h_1 = \frac{\ln\left[f(t, Ti, Ti+\tau)/K\right] + \frac{1}{2} \cdot \sigma_{i,Black}^2 \cdot (Ti - t)}{\sigma_{i,Black} \cdot \sqrt{(Ti - t)}} \quad [12]$$

and

$$h_2 = \frac{\ln\left[f(t, Ti, Ti+\tau)/K\right] - \frac{1}{2} \cdot \sigma_{i,Black}^2 \cdot (Ti - t)}{\sigma_{i,Black} \cdot \sqrt{(Ti - t)}} \quad [13]$$

and $P(t, T_i + \tau)$ denotes the value at time t of a zero coupon bond paying 1 unit at time $T_i+\tau$.

It is now a simple step to compute the present value of a cap as the sum of the present values of its caplets. That is,

$$Cap_n = \sum_{i=1}^{n} caplet_i(\sigma) \quad [14]$$

Note that the price of the cap is computed by assuming that the volatility of all the caplets that compose the cap is constant. Indeed, the market convention for caps (floors) is to quote cap prices in terms of the implied value of σ which sets the Black model price equal to the market price (Longstaff *et al.*, 2001). These volatilities are then referred to as *flat* volatilities.

3. Implementing the equity market methodology from cap market data

In this section first we describe the process applied in equity derivatives markets to construct implied volatility indexes of stock market indexes returns. In particular, the calculation methodology of the US implied volatility index VIX is presented. Then, we show how to implement the equity market methodology for the construction of interest rate volatility indexes from cap (floor) market data.

The key idea under this methodology is the selection of a set of options according to their time to maturity and strike in order to obtain at any point in time the market's assessment of the expected future volatility of the asset underlying an ATM option with constant time to maturity (i.e., constant forecast horizon of future volatility).

VIX is based on a weighting scheme of the Black-Scholes implied volatilities on eight nearest-to-the-money S&P 100 call and put options at the two nearest maturities to the constant time to expiration established for the construction of the index: 30 calendar days (22 trading days). That is, two pairs of call and put options are selected for the nearby and second nearby options. The value of the underlying denoting the ATM strike is the current index level.

According to Poon and Granger (2003), since different implied volatilities are recovered from options with the same time to maturity but different strikes, a decision has to be made about which of these implied volatilities should be used in order to achieve the best forecast of future realized volatility. In this sense, the most common strategy consists of selecting the implied volatilities derived from ATM options since these are the most liquid options and hence measurement errors are less probable to occur. In case that ATM options were not available (which is a very common situation when we are interested in computing daily implied volatilities along a concrete period of time), then nearest-to-the-money options are used instead through a weighting scheme with the aim of obtaining a value for the implied volatility that is approximately ATM. Thus, this is the decision adopted in the construction of implied volatility indexes from options on stock market indexes.

The weighting process is carried out through three steps. First, the implied volatilities of the four pairs of call and put options within the four categories of options are averaged. Second, at each maturity, the two average volatilities at the two strikes that straddle the spot level and are nearest to it are linearly interpolated to obtain ATM implied volatilities. Finally, the nearby and second nearby ATM volatilities are linearly interpolated to create a constant 30-calendar day (22-trading day) implied volatility index, which constitutes the VIX.

The main differences across countries in the implementation of the methodology for the construction of volatility indexes are those regarding the number and type of options (call and/ or put options, European or American options…) selected, and the value of the underlying asset denoting the ATM strike of the option: the spot price (current value of the asset) or the forward price of the asset.

Implementing this methodology to construct the interest rate volatility index from cap (floor) market data implies first recovering *spot* volatilities from *flat* volatility quotes (i.e., implied volatilities of forward interest rates underlying a particular caplet (floorlet)). This process is commonly known as *stripping* process. The implied volatilities of caps (floors) involved in the *stripping* process must be chosen according to the selection criteria established by the methodology applied in equity derivatives markets in order to obtain a measure of the expected future volatility of the forward interest rate underlying an ATM caplet with constant time to maturity.

Unlike stock markets, where implied volatilities from options can be computed at any time before the expiration date of the option, implied volatilities of forward interest rates recovered from caplets (floorlets) have a fixed time to maturity: from t to T_i (exercise date of the option).

Thus, since implied forward rate volatilities have a constant term to maturity, the only criteria we must consider in order to implement the equity market methodology within the *stripping* process is the strike. That is, the only criteria to select the *flat* volatility quotes of caps (floors) involved in such process is the nearness of their strikes to the ATM strike of a caplet (floorlet).

According to the Black pricing formula, a caplet (floorlet) is said to be ATM if the forward rate, $f(t, T_i, T_i+\tau)$, involved in such option is equal to the strike of its corresponding cap (floor). The value of the forward interest rate is computed according to the following formula:

$$f(t,T_i,T_i+\tau) = \left(\frac{P(t,T_i)}{P(t,T_i+\tau)} - 1 \right) \cdot \frac{1}{\tau} \qquad [15]$$

with $P(t,T_i)$ and $P(t,T_i+\tau)$ denoting the values at time t of two zero coupon bonds paying 1 unit at maturity: T_i and $T_i+\tau$, respectively.

The construction process of the interest rate volatility index (IRVIX) is described in detail in the next section.

4. Data and methodology

In conducting this study, we use two types of data from the U.S. fixed-income market provided by *Reuters*: market-implied *flat* volatilities of caps (floors) for different strikes and terms to maturity[6], and the zero coupon curves (discount factors bootstrapped from the most liquid rate instruments that are available: a combination of deposits, liquid futures and interest rate swaps). Daily data have been collected for the period from July 30, 2004 to January 30, 2009.

Flat implied volatilities are recovered from caps (floors) with maturities from 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15 to 20 years, and for the following range of strikes: 0,01; 0,015; 0,02; 0,025; 0,03; 0,035; 0,04; 0,05, and 0,06. These strikes represent values above and below the ATM strikes for caps (floors). According to Hull (2009), a cap

[6] Information provided by *Reuters* consists of *flat* volatility quotes of caps/ floors: at a particular strike and for a concrete term to maturity, traders may contract the same instrument as a cap or a floor depending on the circumstances.

(floor) is said to be ATM if the strike of such an instrument equals the swap rate for a swap that has the same payment dates as the cap.

Next we describe the process of construction of the interest rate volatility index. As stated in the previous section, the process according to which *spot* volatilities are recovered from *flat* volatility quotes is commonly known as *stripping* process. The *stripping* process consists of obtaining at any time t the price of the caplet $caplet(t, T_i, T_i + \tau)$ by subtracting the prices of two consecutive caps computed from *flat* volatility quotes of the corresponding terms to maturity.[7] That is,

$$Cap(t, T_i + \tau) - Cap(t, T_i) = Caplet(t, T_i, T_i + \tau) \quad [16]$$

with $Cap(t, T_i + \tau)$ and $Cap(t, T_i)$ denoting the prices of the caps maturing at $T_i+\tau$ (expiration date of the caplet) and T_i (exercise date of the caplet), respectively. Then, the implied volatility of the caplet, $\sigma(t, T_i, T_i + \tau)$, is extracted from the Black pricing formula[8].

Implementing the equity market methodology for the construction of the interest rate volatility index, IRVIX, first implies computing at any time t the value of the forward interest rate, $f(t, T_i, T_i+\tau)$, that represents, according to the Black formula, the ATM strike of a caplet. As presented in Equation (15):

$$f(t, T_i, T_i + \tau) = \left(\frac{P(t, T_i)}{P(t, T_i + \tau)} - 1 \right) \cdot \frac{1}{\tau} \quad [17]$$

[7] Only caps notation is used in the description of the methodology.
[8] US caps (floors) have a 3-month tenor. Notice that we need all the caps to have the same tenor in order to apply the required interpolation techniques to obtain intermediate points along the term structure of interest rate volatility from available maturities of caps.

Next, the two *flat* volatility quotes involved in the *stripping* process, $\sigma(t,T_i)$ and $\sigma(t,T_i+\tau)$, are selected at the two strikes closest to the value of the forward rate: the strike just above that level, strike *out the money* (OTM), and the strike just below that level, strike *in the money* (ITM), respectively. [9]

Notice that for maturities different from those quoted by the market for *flat* volatilities of caps, interpolation and extrapolation techniques must be applied. Following the research work developed by Hernández, L.G. (2005), we interpolate *flat* volatilities by using cubic splines in order to obtain smoother curves than by using linear interpolation (the market practice). This allows fitting better the typical "humped" pattern for *flat* and *spot* volatilities as a function of maturity. If at a given date the number of available *flat* volatility quotes for different maturities and for a particular strike is less than six then, if possible, we use linear interpolation. [10]

Then, from the two *flat* volatilities quotes selected at the two strikes closest to the value of the forward rate we can obtain the prices of the two caps implied in the *stripping* process and, thus, the prices of the two nearest-to-the money caplets.

Finally, the two implied volatilities of the caplet, $\sigma(t,T_i,T_i+\tau)$, recovered from the strike OTM (K^A) and the strike ITM (K^B), are linearly interpolated to create the interest rate volatility index, IRVIX, according to the following expression:

$$IRVIX(t,T_i,T_i+\tau) = \sigma_{K^{\mathbf{B}}}^{Caplet}\left(t,T_{i,}T_i+\tau\right)\cdot\left(\frac{K^A - f(t,T_i,T_i+\tau)}{K^A - K^B}\right) + \sigma_{K^A}^{Caplet}(t,T_{i,}T_i+\tau)\cdot\left(\frac{f(t,T_i,T_i+\tau) - K^B}{K^A - K^B}\right)$$

[18]

[9] Note that volatility quotes of ATM caps (the most liquid caps traded) can not be used as a rational approximation to the value of the forward rate *f(t,* T_i*,* $T_i+\tau$*)* as *flat* cap volatilities quotes involved in the stripping process for the terms to maturity T_i and $T_i+\tau$ must have the same strike. And ATM cap volatilities have a different strike for every term to maturity.

[10] If at a particular date, neither cubic spline interpolation nor linear interpolation can be applied, the implied volatility of the caplet is computed as the one of the previous day.

where $IRVIX(t, T_i, T_i + \tau)$ represents the annualized (according to the actual/360 day count convention) implied volatility of the forward interest rate $f(t, T_i, T_i+\tau)$ underlying an ATM caplet with constant time to maturity (from t to T_i, exercise date of the option).

According to Fleming *et al.* (1995), the linear interpolation of implied volatilities from OTM and ITM options to create an ATM implied volatility implicitly assumes that the "volatility smile" is well approximated by a line. Thus, this approximation is considered reasonable when the interpolation is made for a small range of strikes. In this case, the two strikes closest (above and below) to the ATM strike of a caplet for a concrete term to maturity.

In this study we daily construct four implied volatility indexes of forward interest rates for the following tenor intervals: 1 year to 1 year and 3 months (1Y, 1Y+3M), 1 year and 3 months to 1 year and 6 months (1Y+3M, 1Y+6M), 1 year and 6 months to 1 year and 9 months (1Y+6M, 1Y+9M), and 1 year and 9 months to 2 years (1Y+9M, 2Y).[11] That way, the implied volatility index $IRVIX(t, 1Y, 1Y + 3M)$ measures the market's assessment at any time t of the uncertainty regarding the 3-month interest rate from t to t plus 1 year.

5. Empirical analysis

From July 30, 2004 to January 30, 2009 we analyze the evolution and statistical properties of the four implied volatility indexes that have been constructed: $IRVIX(t, 1Y, 1Y + 3M)$, $IRVIX(t, 1Y + 3M, 1Y + 6M)$, $IRVIX(t, 1Y + 6M, 1Y + 9M)$, and $IRVIX(t, 1Y + 9M, 2Y)$.

Figures 1 to 4 plot the daily levels of the indexes. Graphs of the series in levels are provided in order to show more clearly the behavior and evolution of the indexes

[11] These periods represent the four closest forecast horizons of future interest rate volatility. According to Duarte *et al.* (2007), the most-liquid cap maturities are one, two, three, four, five, seven and ten years, thus, it is the intention of the authors constructing indexes covering these particular forecast horizons.

over the period, as it may help to better understand the statistical properties of the series after being transformed. Besides, the information content of the daily evolution of the indexes increases due to the sample period comprising the origin of the current financial crisis. Thus, next we analyze the behaviour of the indexes in levels during the sample period.

[INSERT FIGURES 1 - 4]

As suggested by the graphs, during the sample period the implied volatility indexes are far from being stationary. Moreover, the first order autocorrelation of 99% supports the idea that the series appear to be near-random walk.

We can observe that the trend in the evolution of the four volatility indexes is quite similar across the whole sample. At the beginning of the sample period the evolution of the indexes is characterized by slight decreasing trend from August 2004 up to August 2005. From that date to approximately September 2006, evolution drawn by the volatility indexes shows a period of maximum stability. Then, from September 2006 onwards the series start to show frequent small-sized spikes. That is, approximately one year before the origin of the current financial crisis, when the levels of the volatility indexes remarkably increase and larger (up and down) spikes are observed, predictions regarding future interest rate volatility become more variable.

Thus, the empirical evidence seems to suggest the existence of a change in the behavior of the volatility indexes around September 2006. Within the context of the current financial crisis, it seems to be a remarkable sign of the potential application of implied volatility indexes of forward rates as leading indicators of business cycle.

In this sense, in order to analyze the statistical properties of the indexes we divide the sample period into two subsamples: the first subsample (from July 30, 2004 to August 31, 2006) and the second subsample (from September 01, 2006 to January 30, 2009).

Next, we implement an analysis of the statistical properties of the indexes based on first differences (daily volatility changes). According to Fleming *et al.* (1995), the variable of interest for academics and practitioners is changes or innovations to expected volatility as they want to know how changes in expected volatility influence changes in security valuation.

Table 1 shows the summary statistics of the first differences in the implied volatility indexes over the whole sample (Panel A) and for the first and second subsamples (Panel B and C, respectively).

[INSERT TABLE 1]

The average value of daily changes in the volatility indexes reported for the second subsample is higher than for the first subsample. Furthermore, within the second subsample, the average future expected volatility decreases over the forecast horizon. As well as the mean, the standard deviation (volatility of volatility) is higher in the second subsample. Moreover, since the series of daily levels of the implied volatility index $IRVIX(t,1Y+9M,2Y)$ evidences more frequent small-sized spikes over the time, the volatility of daily volatility indexes changes is higher for the index with furthest away forecast horizon in both subsamples.

The series show slight negative skewness (except for the volatility index maturing in 1 year and 9 months) and significant excess kurtosis (leptokurtosis). The first order autocorrelation coefficients and the Augmented Dickey Fuller (ADF) test values are also provided. The autocorrelation structure of the daily volatility indexes changes varies over the forecast horizon. Finally, the values of the Augmented Dickey

Fuller (ADF) test evidence that the implied volatility indexes are stationary in the first differences.

From the evidence reported regarding non-normality in the first differences of the indexes we introduce a first ln-difference transformation in the series in levels. Daily evolution of first differences of ln implied volatility indexes (i.e., the day to day percentage change in the volatility indexes) is shown through Figures 5 to 8. The graphs of the series suggest again the existence of a change in the behavior of the series around September 2006.

[INSERT FIGURES 5 TO 8]

Table 2 reports the statistical properties of the series in first ln-differences over the whole sample (Panel A) and for the first and second subsamples (Panel B and C, respectively).

[INSERT TABLE 2]

As expected, the excess kurtosis reported for the four indexes in the first differences has decreased, but it still remains. According to Dotsis *et al.* (2007), the evidence of non-normality may be attributed to the presence of jumps in implied volatility. Figure 9 shows the empirical distribution of the series in first ln-differences. The first order autocorrelation coefficients reveal a statistically significant negative autocorrelation (expect for the volatility index maturing in 1 year and 3 months). This degree of correlation is stronger for the volatility index maturing in 1 years and 9 months. The evidence of negative autocorrelation suggests the presence of mean reversion in the daily ln implied volatility indexes changes. The same results (excess kurtosis and negative first-order autocorrelation) are usually reported for most of the implied volatility indexes introduced in stock markets (see Dotsis *et al.* (2007) and

Konstantinidi *et al.* (2008)). Finally, the ADF test allows rejecting the null hypothesis of a unit root in the series.

[INSERT FIGURE 9]

6. Summary and conclusions

The model-based methodology applied for the construction of volatility indexes in the equity market consists of a weighting scheme of the implied volatilities of a set of options computed within the overall context of Black-Scholes (1973) option pricing model or a similar model. In particular, implied volatilities are weighted in such a way that the index represents the annualized implied volatility of a particular stock market index underlying an *at the money* (ATM) option with constant time to maturity (i.e., constant forecast horizon of future expected volatility). VIX in the US, VDAX in Germany, and VX1 in France, are some benchmark risk measures for stock market volatility.

In this study we suggest for the first time the construction of a “pure” measure of the expected future volatility referred to a particular forward interest rate (with a concrete period of reference) based on the methodology applied in equity derivatives markets. The interest rate volatility index (IRVIX) is constructed from data of the U.S. cap (floor) market.

Information provided by the market consists of implied *flat* volatility quotes of caps (floors), where implied volatilities are computed by equalling the market prices of such derivatives and the Black (1976) model price applied to all the caplets (floorlets) that compose the cap (floor) by assuming that the volatility of forward interest rates underlying every option is constant. Thus, *flat* volatilities do not enable us to know what the period of reference of the underlying forward interest rate whose implied volatility has been estimated is.

We focus on this item in order to construct implied volatility indexes of forward interest rates with a concrete period of reference by using *spot* volatilities recovered from *flat* volatility quotes (i.e., implied volatilities of caplets (floorlets) with a concrete period of reference) and then applying the methodology developed in equity markets. The interest rate volatility index (IRVIX) constructed like that aims to represent the annualized implied volatility of the forward interest rate underlying an ATM caplet with a fixed time to maturity, which is the forecast horizon of the expected future interest rate volatility.

Some of the potential applications of the index are included next. The analysis of the information content of the volatility indexes as leading indicators of business cycle is perhaps one of the most attractive applications. The indexes might also be applied for the study of the impact of monetary policy on interest rate volatility and for the valuation of more complex interest rate derivatives such as swaptions. Finally, the volatility indexes might give rise to the introduction of futures and options on such indexes, as occurred in the US after the launch of the stock volatility index VIX.

Over the period from July 30, 2004 to January 30, 2009 we daily construct four implied volatility indexes of forward interest rates for the following tenor intervals: 1 year to 1 year and 3 months (1Y, 1Y+3M), 1 year and 3 months to 1 year and 6 months (1Y+3M, 1Y+6M), 1 year and 6 months to 1 year and 9 months (1Y+6M, 1Y+9M), and 1 year and 9 months to 2 years (1Y+9M, 2Y).

From the behaviour of the four indexes over the sample period, we notice that from September 2006 onwards, after approximately one year of maximum stability, the series start to show frequent small-sized spikes. That is, predictions regarding future interest rate volatility become more variable approximately one year before the origin of the current financial crisis. It seems to be a remarkable sign of the potential application of implied volatility indexes of forward rates as leading indicators of business cycle.

Finally, the statistical properties of the series after the introduction of a first ln-difference transformation show excess kurtosis (leptokurtosis) and significant negative first-order autocorrelation. The same evidence holds for most of the implied volatility indexes in stock markets, where the non-normality is sometimes attributed to the presence of jumps and the negative first-order autocorrelation supports the modelling of implied volatility indexes as mean reverting processes.

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FIGURE 1. Daily levels of the lmplied Volatility Index $IRVIX(t, 1Y, 1Y+3M)$ during the period from July 30, 2004 to January 30, 2009.

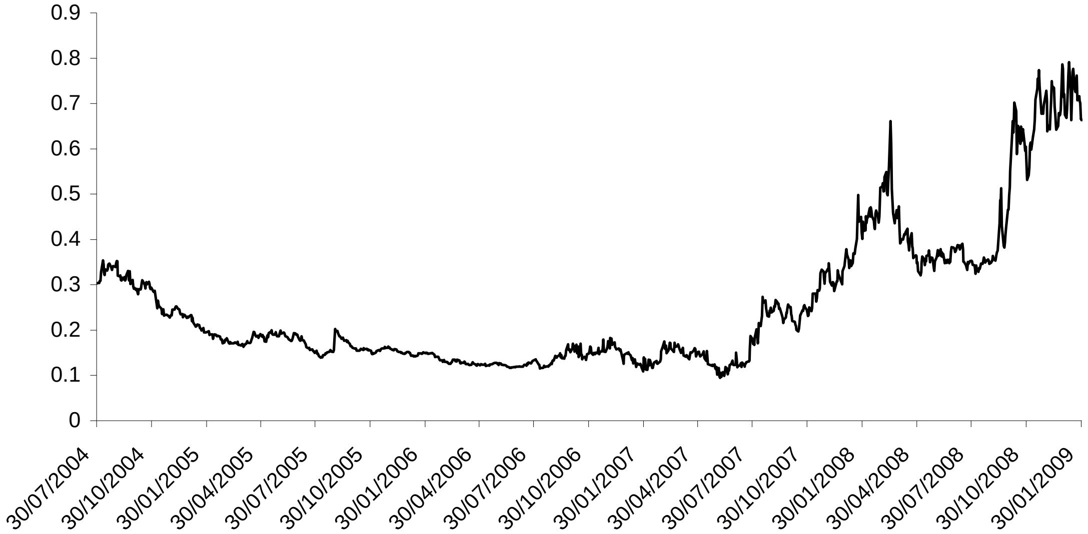

FIGURE 2. Daily levels of the Implied Volatility Index $IRVIX(t, 1Y+3M, 1Y+6M)$ during the period from July 30, 2004 to January 30, 2009.

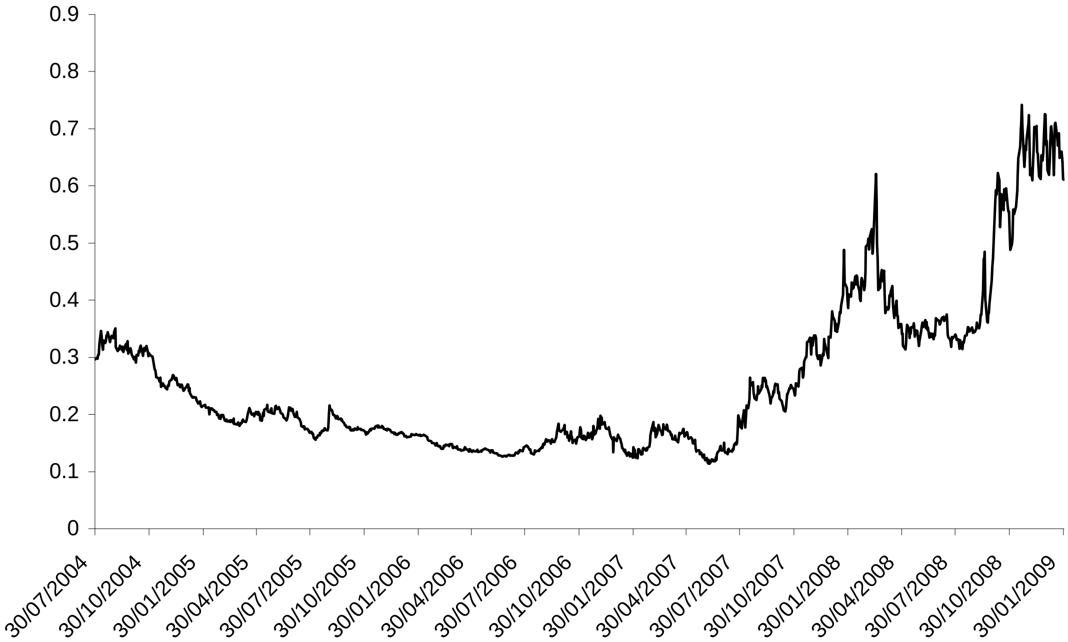

FIGURE 3. Daily levels of the Implied Volatility Index $IRVIX(t,1Y+6M,1Y+9M)$ during the period from July 30, 2004 to January 30, 2009.

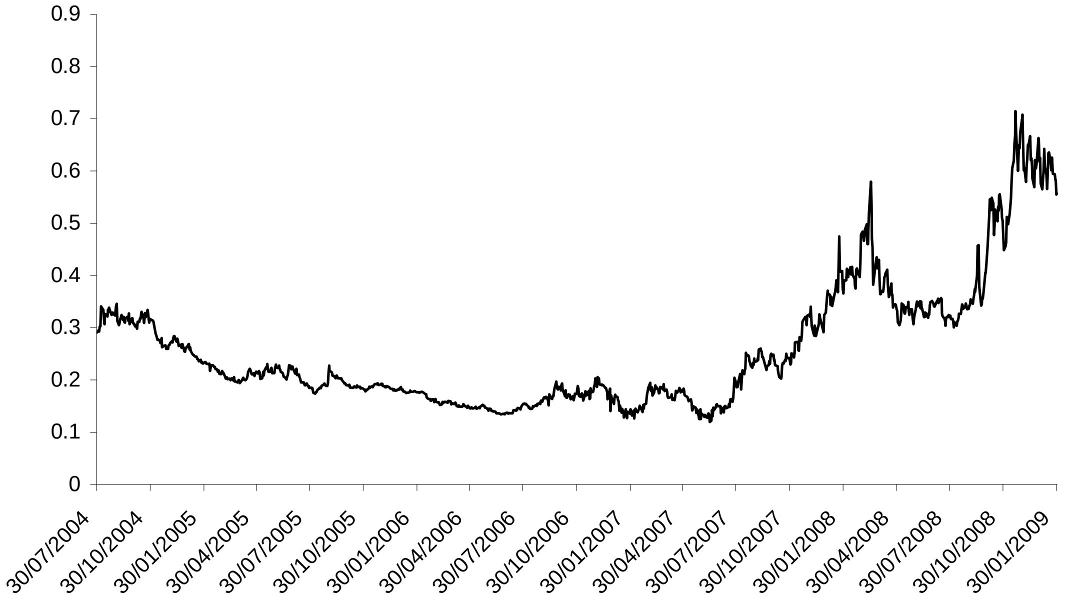

FIGURE 4. Daily levels of the Implied Volatility Index $IRVIX(t,1Y+9M,2Y)$ during the period from July 30, 2004 to January 30, 2009.

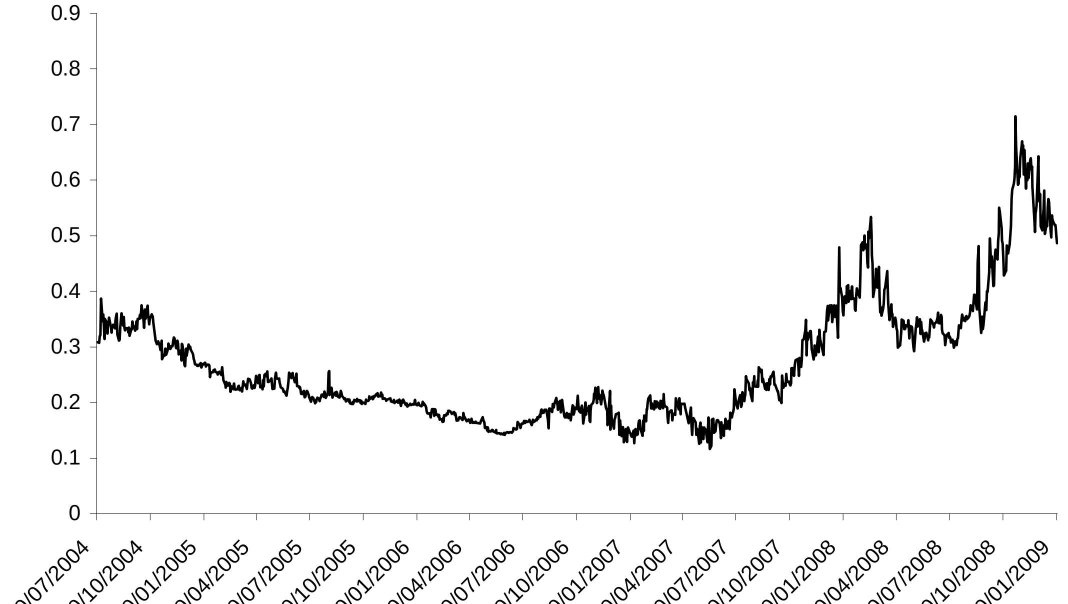

FIGURE 5. First ln-differences in the Implied Volatility Index $IRVIX(t, 1Y, 1Y+3M)$ during the period from July 30, 2004 to January 30, 2009.

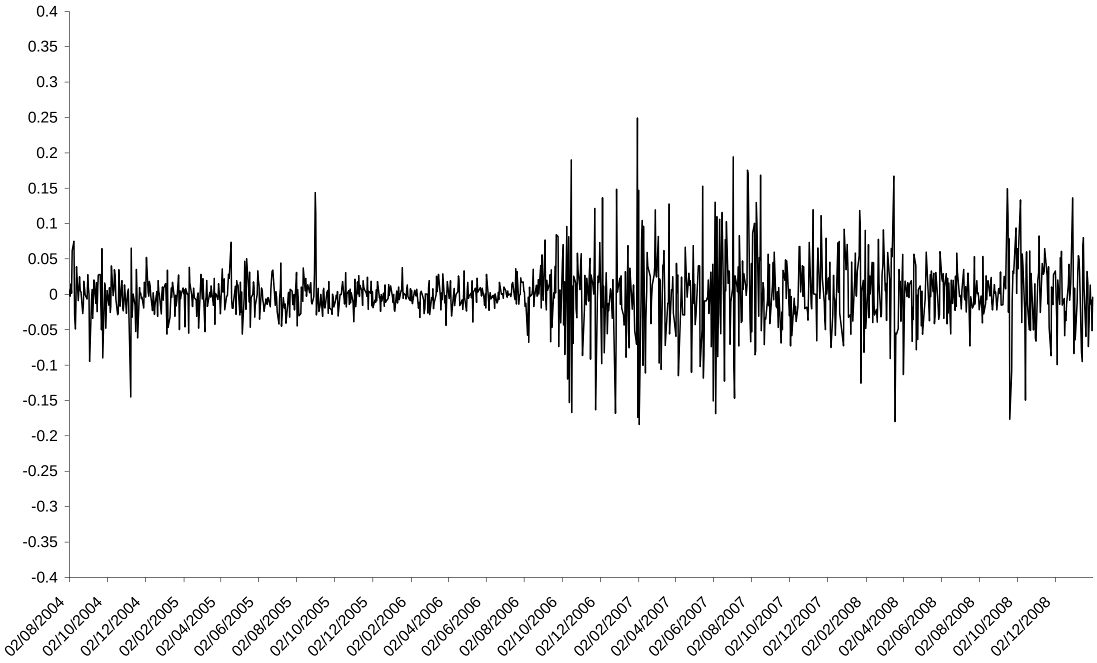

FIGURE 6. First ln-differences in the Implied Volatility Index $IRVIX(t, 1Y+3M, 1Y+6M)$ during the period from July 30, 2004 to January 30, 2009.

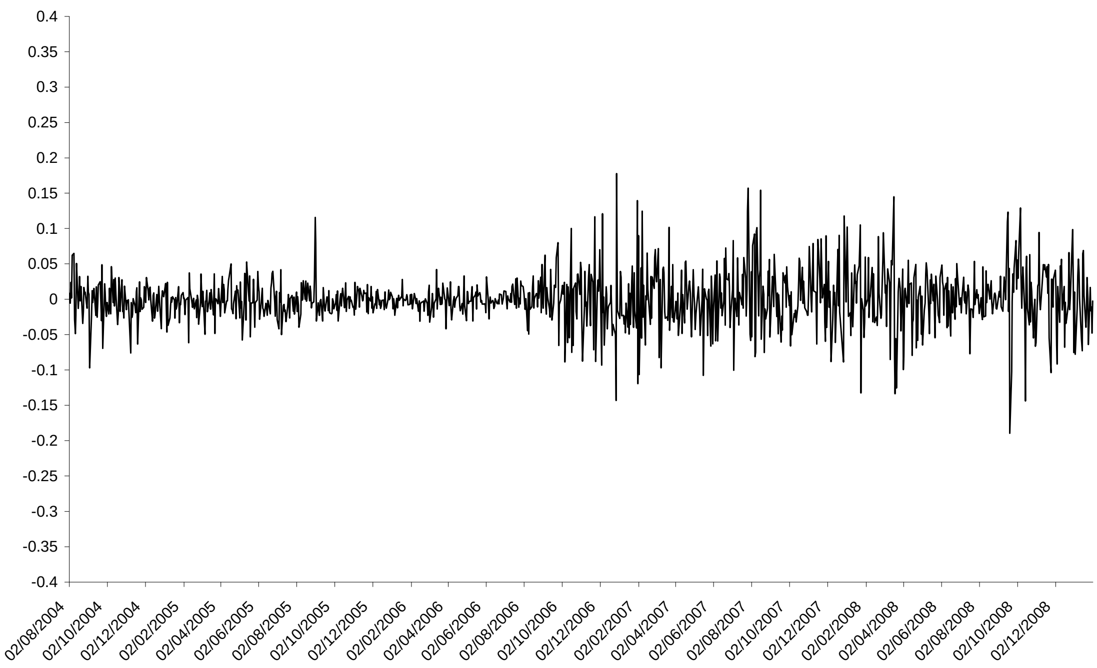

FIGURE 7. First ln-differences in the Implied Volatility Index $IRVIX(t, 1Y+6M, 1Y+9M)$ during the period from July 30, 2004 to January 30, 2009.

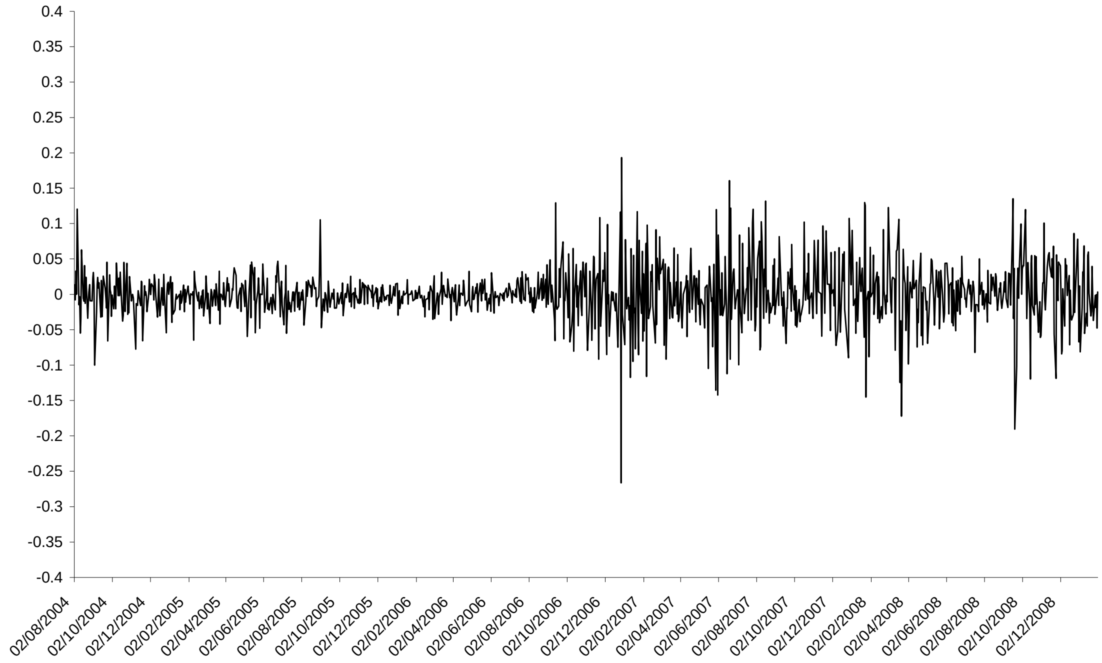

FIGURE 8. First ln-differences in the Implied Volatility Index $IRVIX(t, 1Y+9M, 2Y)$ during the period from July 30, 2004 to January 30, 2009.

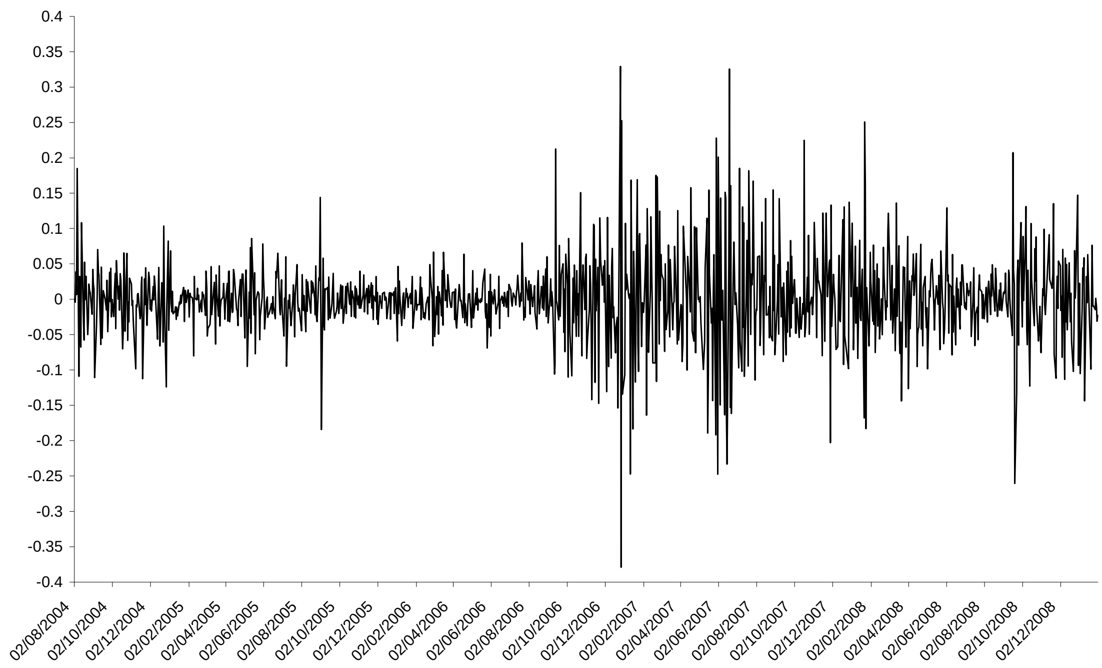

FIGURE 9. Histograms of First ln-differences in the Implied Volatility Indexes from July 30, 2004 to January 30, 2009. The continuous curves overlaying the histograms correspond to normal distributions with the same mean and standard deviation as the series.

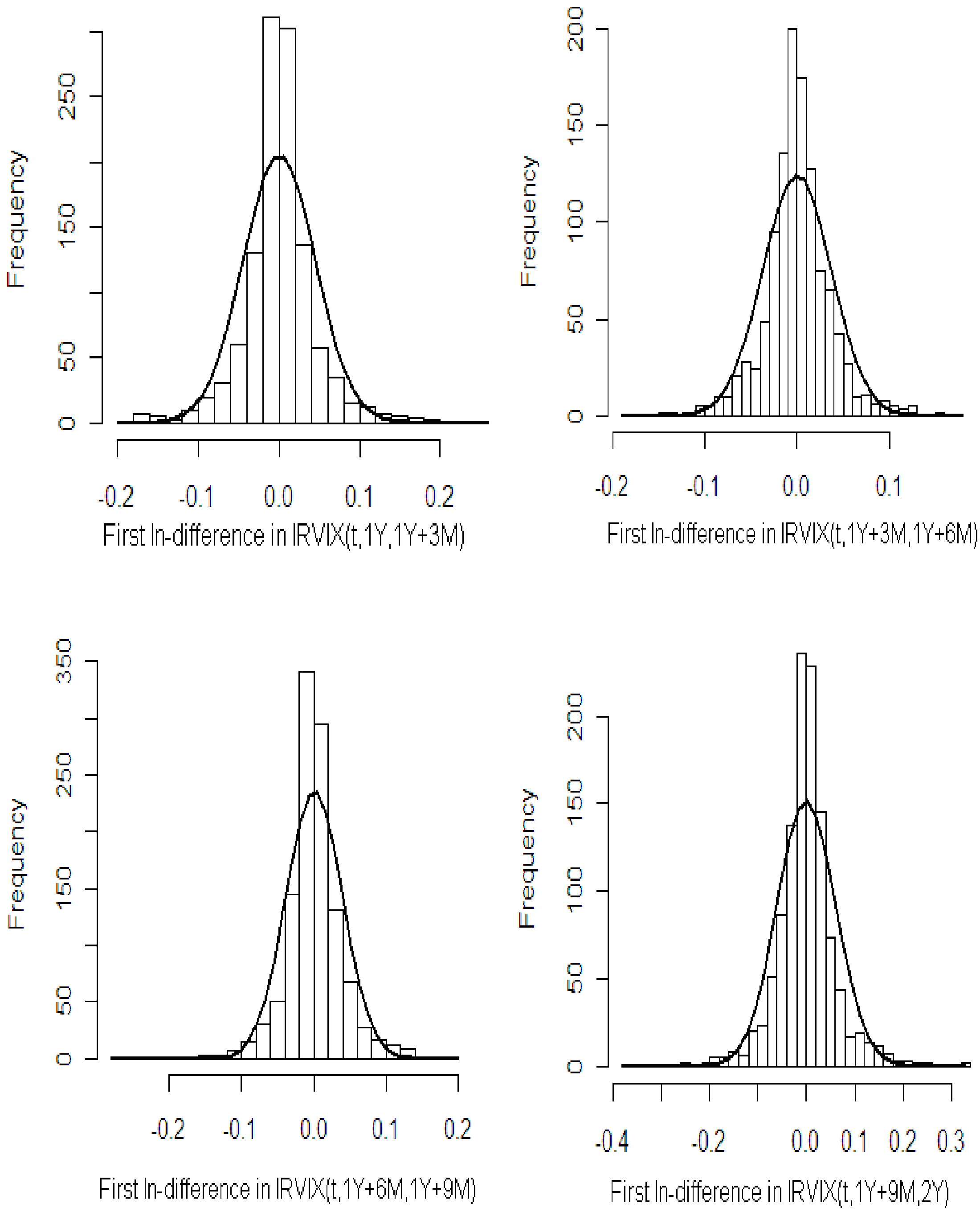

TABLE 1.- Summary statistics of First differences in the Implied Volatility Indexes.

	[1Y,1Y+3M]	[1Y+3M,1Y+6M]	[1Y+6M,1Y+9M]	[1Y+9M,2Y]
Panel A: Summary statistics from the whole sample: July 30, 2004 to January 30, 2009				
Mean	0.0003	0.0002	0.0002	0.0001
Standard Dev.	0.014	0.012	0.012	0.017
Skewness	-0.14	-0.39	-0.40	0.01
Kurtosis	14.92	13.29	11.61	9.41
ρ_1	0.07^{*}	0.08^{*}	0.02	-0.19^{*}
ADF	-26.56^{**}	-19.99^{**}	-20.28^{**}	-22.52^{**}
Panel B: Summary statistics from the first subsample: July 30, 2004 to August 31, 2006				
Mean	-0.0003	-0.0002	-0.0002	-0.0002
Standard Dev.	0.005	0.004	0.005	0.008
Skewness	-0.77	-0.37	0.21	0.20
Kurtosis	15.72	10.81	14.33	11.88
Panel C: Summary statistics from the second subsample: September 01, 2006 to January 30, 2009				
Mean	0.0008	0.0007	0.0006	0.0004
Standard Dev.	0.019	0.017	0.016	0.021
Skewness	-0.18	-0.38	-0.40	-0.03
Kurtosis	8.90	8.04	7.17	6.26

Entries report the summary statistics of the four implied volatility indexes in the first daily differences for the whole sample (Panel A) and for the first and second subsamples (Panel B and C, respectively). The first order autocorrelation ρ_1 and the Augmented Dickey Fuller (ADF) test values are also reported for the entire sample. One asterisk denotes statistical significance at a 5% confidence level. Two asterisks denote statistical significance at a 1% confidence level.

TABLE 2.- Summary statistics of First ln-differences in the Implied Volatility Indexes.

	[1Y,1Y+3M]	[1Y+3M,1Y+6M]	[1Y+6M,1Y+9M]	[1Y+9M,2Y]
Panel A: Summary statistics from the whole sample: July 30, 2004 to January 30, 2009				
Mean	0.0006	0.0006	0.0005	0.0003
Standard Dev.	0.044	0.037	0.039	0.060
Skewness	0.14	0.13	-0.13	0.13
Kurtosis	7.00	6.11	7.54	7.83
ρ_1	-0.12*	-0.05	-0.13*	-0.30*
ADF	-38.41**	-35.79**	-27.76**	-23.00**
Panel B: Summary statistics from the first subsample: July 30, 2004 to August 31, 2006				
Mean	-0.0015	-0.0012	-0.0010	-0.0009
Standard Dev.	0.023	0.020	0.020	0.033
Skewness	0.09	0.16	0.23	-0.04
Kurtosis	10.09	6.66	7.82	7.57
Panel C: Summary statistics from the second subsample: September 01, 2006 to January 30, 2009				
Mean	0.0026	0.0023	0.0020	0.0015
Standard Dev.	0.057	0.047	0.050	0.077
Skewness	0.04	0.02	-0.21	0.09
Kurtosis	4.62	4.21	5.10	5.38

Entries report the summary statistics of the four implied volatility indexes in the first daily ln-differences for the whole sample (Panel A) and for the first and second subsamples (Panel B and C, respectively). The first order autocorrelation ρ_1 and the Augmented Dickey Fuller (ADF) test values are also reported for the entire sample. One asterisk denotes statistical significance at a 5% confidence level. Two asterisks denote statistical significance at a 1% confidence level.