On Downside Risk Predictability through Predetermined Variables: A Quantile Regression Approach.[1]

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Abstract

In this paper, we analyze the role played by market liquidity and trading-related variables in forecasting downside risk. In particular, we analyze the empirical relevance of spreads and volume variables to forecast one-day ahead Value-at-Risk (VaR) of the value-weighted market portfolio in the US market using the Quantile Regression. This methodology allows us to appraise directly and in a natural way the effects of these predictive variables on the tail distribution of returns. The usual backtesting VaR analysis, based on unconditional and conditional coverage tests, reveals that liquidity and trading variables seem to considerably enhance the VaR performance.

Key words: Value at Risk, CAViaR, Quantile Regression, Bid-Ask Spreads, Volume.

1 Introduction

Implementing risk control and monitoring systems requires quantitative procedures to appraise the level of underlying uncertainty and construct accurate predictions. The Basel Committee on Banking Supervision has largely contributed to popularize certain international standards, known as Basel I and II Accords, in the financial services industry. This regulatory setting entitles financial institutions to use internal models based on the Value-at-Risk (VaR) measure for meeting market risk capital requirements. VaR is simply a statistical estimate on how much a portfolio can lose, within a certain horizon, and given an arbitrary confidence level. Despite the limitations of the VaR paradigm, it is widely agreed that, without the efforts made to comply with the Basel standards, the financial industry would likely be facing an even deeper crisis. Nevertheless, the economic turmoil has risen the convenience of developing alternative procedures for quantifying market risk, or revising the methods already existing, particularly, the VaR. The present paper is motivated by this concern.

The existing literature on VaR modelling and forecasting has suggested a number of procedures to forecast VaR. The methods differ mainly in the degree of sophistication: From the simple EWMA approach to the more advanced settings based on the Extreme Value Theory. Previous research has shown that most of these methods do not seem to perform successfully in practice, which underlines the practical complexity that lies behind the simple notion of VaR; see, for instance, the empirical evidence in Kuester *et al.* (2006). In spite of the large methodological differences involved, all these methods share a common characteristic: They rely almost exclusively on the information conveyed by historical returns to forecast the conditional loss distribution of a portfolio. For practical purposes, this may turn out to be unnecessarily restrictive. The implicit belief that returns subsume all the relevant information to determine market risk may be originated in a conservative interpretation of the Efficient Market Hypothesis (EMH). The EMH forbids the systematic predictability of returns on the basis of the available information, *i.e.*, posits an orthogonal condition on the first-order conditional moment of returns. However, it remains silent about higher-order moments, such as conditional volatility, or other distributional features of the returns time-series process, like conditional percentiles. Moreover, in practice financial markets largely departure from the complete-market and symmetric-information hypotheses that, as in the case of the EMH, underlie most of the theoretical models in asset pricing. Returns may exhibit non-trivial links with the endogenous variables that characterize the market environment and the trading process. Whereas most of the existing literature is devoted to the modelling and forecasting of downside risk based on returns and their volatility, observable variables which are related to liquidity, trading activity, and private information arrivals may exhibit forecasting power as well.

This paper analyzes empirically whether there exist predictive relations between certain endogenous variables and the conditional loss function of a portfolio. More specifically we study if different measures related to bid-ask spreads volume contain relevant information to forecast daily VaR of the market portfolio return at the usual levels of confidence. Like returns, these variables are available on the trading-basis and are highly sensitive to the information flow. Like volatility, trading activity and liquidity are believed to reflect and subsume market sentiments, collective expectations and market conditions, and so they have a major influence on the decisions of investors. In contrast to returns and volatility, however, these variables seem to have been relegated to a secondary plane, if not ignored, in the existing literature related to modelling and forecasting downside risk. The main aim of this paper is to address empirically the premise that market-risk forecasts may be improved by using available information which is not necessarily constrained to returns.

We compare the performance of several potential predictors using daily market data from the US Stock Exchange in the period 01/1988 through 12/2002. This data set includes value-weighted market portfolio returns as well as different measures of market bid-ask spreads (quoted and effective bid-ask spread in both absolute and relative terms) and market volume (trading volume in thousand of shares, number of trades, number of sell trades, shares sold in thousands and volume in dollars). Three main reasons prompted us to consider this specific sample in our study: *i*) market-portfolio data allow us to eliminate the idiosyncratic noise that may affect the main conclusions drawn from the analysis on individual stocks, *ii*) the period sample is particularly interesting for risk management purposes, as it includes a stress scenario of great volatility originated in the burst of the technological bubble in 2000, and *iii*) we can analyze in this sample the aggregate measures of liquidity and volume that have been used previously in several studies (see Chordia *et al.*, 2001; 2002; 2008) and which are freely available, thereby enabling further research on the same sample for comparative purposes.[1]

Paralleling the literature devoted to the analysis of predictability in returns, the most simple and direct way to appraise the forecasting ability of a set of variables is through predictive linear regressions in a least-squares analysis; see, for instance, Cochrane (2005) and references therein. The main difficulty in our context is that the actual level of VaR (*i.e.*, the dependent variable in such an analysis) is unobservable and has to be modelled as a latent process, which makes it infeasible the least-squares analysis. Alternatively, the Quantile Regression theory provides us with the appropriate methodology to analyze the dynamics of the conditional

[1]The data are graciously provided by Prof. A. Subrahmanyam on his website, http://www.anderson.ucla.edu/x1921.xml. Update are not avalaible. In any case, we are interested in this period to analyze the performance of the models in the dot-com crisis.

percentiles directly and without departing significantly from the intuitive spirit that characterizes predictive regressions; see Koenker and Bassett (1978). We can model directly the tail of the conditional distribution of returns by specifying a functional form that relates the time-varying dynamics of VaR to its own past and lagged values of the variables used as predictors, building on the so-called CAViaR setting in Engle and Manganelli (2004). A restricted version of this model, which considers solely returns-related information (so-called Symmetric Absolute Value CAViaR model, see Section 3.1), can be taken as a proper benchmark to assess the incremental value of liquidity- and volume-related variables. Furthermore, the asymptotic theory for quantile regression models is well established and allows us to address formally the statistical significance of the estimated coefficients. Even more importantly, the CAViaR setting is specifically designed to construct conditional VaR forecasts, so we can use standard backtesting techniques (e.g., Christoffersen 1998) to analyze the actual out-of-sample performance of the models extended with lagged endogenous variables. Our main empirical conclusions largely support the suitability of the liquidity- and volume-related variables in forecasting daily VaR.

The remaining of the paper is organized as follows. Section 2 surveys the main literature related to the topic and discusses the empirical findings and the theoretical settings that support the main hypothesis in this paper. Section 3 reviews the basic elements in VaR modelling and briefly describe the quantile regression approach. Section 4 describes the main features in the data set and develops the empirical analysis. Finally, Section 5 summarizes and concludes.

2 Literature review and previous considerations

The paper is related to the stream of literature that has used the quantile regression approach in the context of VaR modelling and forecasting; see, for instance, Taylor (1999), Kouretas and Zarangas (2005), Bao *et al.* (2006), Kuester *et al.* (2006), and Huang *et al.* (2010). The distinctive feature of our analysis is the special focus given to the predictable role of certain variables related to liquidity and trading activity. As a consequence, this paper can also be related to the literature in risk management focused on modelling liquidity risk. It is argued that the maximum expected loss in a portfolio or a financial instrument should not ignore the transaction costs and, more generally, the liquidity costs that stem from closing the positions of a portfolio. Liquidity costs depend on the future market conditions and, hence, has to be estimated, which introduces an additional source of non-diversifiable risk for investors. Remarkably, the liquidity costs can be particularly sizeable in stress conditions, so neglecting this source of uncertainty can result in substantial underestimates of the VaR measures (Lawrance and Robinson, 1996). This consideration has given rise to so-called liquidity-adjusted VaR models aimed to take adequate account of the expected liquidity costs; see, among others, Jorion (2007), Zheng and Shen (2008) and Jarrow and Protter (2005). As in this literature, our study acknowledges that the variables which are widely accepted to be related to liquidity risk are likely relevant to estimate VaR measures. In contrast with this literature, however, we do not attempt to appraise the expected cost due to liquidity risk as an incremental, different component in the total risk of the portfolio (which is an interesting topic left for future research), but rather use both liquidity- and volume-related information to forecast market risk itself. The implied belief, therefore, is that there exists an interaction between these risk factors (*e.g.*, Zheng 2006) such that the variables that characterize liquidity risk may be used to predict market risk.

This conjecture may be formalized heuristically as follows. Consider that the price of a financial asset at time t, S_t, obeys a general jump-diffusion process:

$$dS_t = \mu(S_t, t)\, dt + \sigma(S_t, t)\, dB_t + [S_t - dQ_t] \tag{1}$$

where B_t is a standard Brownian motion that captures the arrival of 'normal' information, and

$$Q_t = \sum_{i=1}^{\mathcal{N}_t} J_{i,t} \tag{2}$$

is a jump process, independent of B_t, that characterizes the 'abnormal' flow of information and which may cause large movements in prices; see, for instance, Merton (1976). The jump component is defined on a Poisson counter process, $\mathcal{N}_t$, that controls the arrival rate of jumps, each of one causing a random shock $J_{i,t}$ with mean $E(J_{i,t}) = \mu_J$ and variability $Var(J_{i,t}) = \theta_J$. The likelihood of arrivals is governed by a certain intensity parameter $\lambda_t \geq 0$, which may generally obey time-varying dynamics (Maheu and McCurdy, 2004). Since prices are prone to large movements when liquidity dries up, the expected size of an extraordinary shock and/or its additional variability in the price are expected to be dynamically related to state variables that reflect market conditions, such as those that characterize the liquidity risk. For instance, the parameters $(\lambda, \mu_J, \theta_J)'$ that control the probabilistic distribution of jumps likely obey time-varying dynamics as a function, among other potential drivers, of liquidity costs, because of the impact of the same piece of news cannot be expected to be the same independently of the market conditions: When liquidity costs are large, the total variability due to abnormal news should be expected to be larger as well, everything else constant. As a result, liquidity and volume variables, which are highly related to the flow of information and are widely considered as natural proxies for the liquidity risk, may exhibit predictive power on the standard measures of market risk.

Our paper can also be related to the empirical literature in market microstructure and asset pricing devoted to the analysis of the joint dynamics of volatility and the main variables related to the information flow. This literature gives us an alternative, but closer view on how and why certain variables, such as liquidity and volume, may be able to forecast downside market risk. In particular, a number of studies have underlined the predictive role of volume and volume-related variables on volatility. Since market risk measures are tightly linked to the standard desviation of the distribution, these variables may have predictive power on the latent VaR process via volatility. The information content in trading volume has been emphasized in Campbell *et al.* (1993), Blume *et al.* (1994), Wang (1994) and Suominen (2001) yet, as remarked by Jondeau *et al.* (2007), little research effort has been made to use forecasting models of volume or trading activity to help predicting the future variance of an asset. This paper contributes to this literature by providing empirical evidence on the forecasting suitability of these variables from the perspective of downside risk.

More specifically, the mixture of distributions hypothesis in Clark (1973) alleges that some of the stylized features in returns (such as non-normality and time-varying volatility) are generated through a persistent mixing process that measures the rate at which new information is transmitted into the markets. The information arrival rate may be seen as a latent process that may also affect volume and other observable variables related to trading activity (Tauchen and Pitts, 1983). Given that the trading volume exhibits a strong degree of serial correlation, the implication of this theory is that lagged volume is also correlated to volatility and could anticipate market movements; see, among others, Lamoureux and Lastrapes (1990), Andersen (1996) and Gerlach *et al.* (2006). Predictability may also be supported in terms of the sequential information arrival hypothesis by Copeland (1976) and Smirlock and Starks (1988). These authors argue that investors react to new information differently such that the price adjustment is not instantaneous, which eventually generates causality relationships between volume and volatility. Although the adjustment process is expected to occur over a short-period of time during the trading session, the stock market may take longer periods to adjust prices when massive shocks arrive, or in periods of particular instability. The recent events in the financial markets during, with prices changing wildly from one day to another, perfectly exemplify this statement.

The literature on market microstructure has also raised connections between bid-ask spreads and volatility. It is widely accepted that bid-ask spreads reflect inventory, order processing, and adverse selection costs, so they can proxy for information asymmetry (Glosten and Milgrom, 1985 and Stoll, 1989). The adverse selection and inventory risk components imply a positive correlation between the spread and the volatility of the traded asset according to the evidence shown in, among others, Roll (1984) and Black (1991) and Admati and Pfleiderer (1988). Previous empirical studies supporting this positive relationship include Hasbrouck (1999), Bollerslev and Melvin (1994) and Kalimipalli and Warga (2002), who show that this phenomenon exists in common stocks, foreign exchange rates and corporate bonds, respectively; see also Easley *et al.* (1997), Geoffrey and Gurun (2008).

3 Modelling and forecasting downside risk: Value at Risk

We start out our analysis by introducing some notation and several assumptions. Let $\{r_t\}$ be the daily return time-series of a financial asset. Also, let $\mathcal{F}_t$ be the natural filtration including all the available information at time t, such as any measurable transformation on the past observations of r_t as well as any other observable variable. The VaR of a financial asset is the maximum loss over a horizon of h periods (in days) which is expected at the $(1-\lambda)\%$ confidence level given $\mathcal{F}_t$, *i.e.*, the λ-quantile of the conditional loss distribution of a portfolio, with $\lambda \in (0,1)$. Formally, we denote:

$$\begin{aligned} VaR_{\lambda,t+h} &= -\{x \in \mathbb{R} : \Pr(r_{t+h} \leq x|\mathcal{F}_t) = \lambda\} \\ &= -\{Q_{\lambda,t}(r_{t+h})\} \end{aligned} \tag{3}$$

with $Q_{\lambda,t}(r_{t+h})$ defined implicitly and where, typically, h ranges from 1 to 10 days and $\lambda = \{0.01,\ 0.05\}$. For instance, the financial firms Bank Trust and JP Morgan report 1% and 5% daily VaRs, respectively. For simplicity but no loss of generality, we assume in the sequel that returns behave as a stationary martingale difference sequence such that $E(r_t|\mathcal{F}_{t-1}) = 0$, with bounded moments $E\left(|r_t|^k\right) < \infty$ for some $k \geq 2$.[2]

The financial econometrics literature has suggested very different procedures to forecasts VaR. Throughout the following subsection, we discuss the main characteristics of the quantile regression approach. Appendix A sketches the main features of several alternative procedures that shall be used later in Section 4 for comparative purposes.

[2] In practice, it is costumary to demean returns previously so that the resultant series behaves as a martingale difference sequence. In the empirical analysis carried out in Section 4, we will filter out the predicatable pattern in the market portfolio by fitting an AR(1) process.

3.1 Quantile regression: CAViaR models

The conditional λ-quantile of the return time series r_t, say $Q_{\lambda,t-1}(r_t)$, can formally be defined as $\Pr(r_t \leq Q_{\lambda,t}(r_t)|\mathcal{F}_{t-1}) = \lambda$. Given (3), and considering a one-day holding period, $h = 1$, it follows immediately that

$$VaR_{\lambda,t} = -Q_{\lambda,t-1}(r_t) \tag{4}$$

Following Koenker and Bassett (1978) and Basset and Koenker (1982), the conditional quantile could be written as a linear function of a set of k explanatory variables, $X_t = (x_{1t}, ..., x_{kt})'$, and a $(k \times 1)$ vector of unknown coefficients β_λ that depends on the λ-quantile, namely, $Q_{\lambda,t-1}(r_t) = X'_{t-1}\beta_\lambda$. This formulation is equivalent to assume the Quantile Regression model

$$r_t = X'_{t-1}\beta_\lambda + u_{t,\lambda}, \tag{5}$$

where $u_{t,\lambda}$ is an error term satisfying $E(u_{t,\lambda}|X_{t-1}) = 0$. Note that we do not impose any particular restriction on the distribution of the data.

Model (5) is highly reminiscent of the standard linear regression model. Whereas the least-squares setting attempts to characterize the conditional mean of the distribution, the quantile regression allows us to model directly the dynamics of the λ-quantile of the conditional distribution. A well-known particular case arises for $\lambda = 1/2$, *i.e.*, the median of the distribution. The so-called Least Absolute Deviation regression model is intended to provide estimates of the slope coefficients for the median of the process (rather than the mean in the OLS context) seeking to robustify estimates against outliers. In this case, the regression coefficients can be estimated consistently by minimizing the sum of the absolute values of the residuals. More generally, given an arbitrary value of λ, the unknown β_λ vector of parameters can be estimated consistently as:

$$\widehat{\beta}_\lambda : \arg\min_{\beta_\lambda \in \mathbb{R}^k} \left\{ \sum_{t=1}^{T} \lambda \left| u_{t,\lambda} \right| \mathbb{I}_{\left(r_t \geq X'_{t-1}\beta_\lambda\right)} + \sum_{t=1}^{T} (1-\lambda) \left| u_{t,\lambda} \right| \mathbb{I}_{\left(r_t < X'_{t-1}\beta_\lambda\right)} \right\}. \tag{6}$$

where $\mathbb{I}_{(\cdot)}$ is an indicator function.

Engle and Manganelli (2004) proposed a family of models belonging to this general setting particularly intended to infer the dynamics of the VaR of a portfolio. The distinctive feature is that the conditional quantile is seen as a latent autoregressive process which may also depend on a number of covariates, the so-called Conditional Autoregressive Value at Risk (CAViaR). Thus, recalling (4) , and following Engle and Manganelli (2004), we consider that the conditional quantile obeys dynamics given by:

$$VaR_{\lambda,t} = \beta_{\lambda,0} + \beta_{\lambda,1} VaR_{\lambda,t-1} + \beta_{\lambda,2}|r_{t-1}| + \beta^*_\lambda \; f\left(X^*_{t-1}\right) \tag{7}$$

with $\beta_\lambda = \left(\beta_{\lambda,0}, \beta_{\lambda,1}, \beta_{\lambda,2}, \beta_\lambda^*\right)'$, where X_t^* is a certain variable used as a predictor, and $f(\cdot)$ is a measurable function, such as $f\left(X_{t-1}^*\right) = |\log\left(\left|X_{t-1}^*\right|\right)|$.[3] Under the parametric restriction $\beta_\lambda^* = 0$, the resultant model is the called Symmetric Absolute Value CAViaR (SAV-CAViaR henceforth) in Engle and Manganelli (2004). The restricted model is fed solely with the information conveyed by the returns time-series, so testing the null hypothesis $\mathrm{H}_0 : \beta_\lambda^* = 0$ in the unrestricted specification (7) provides formal evidence on the empirical suitability of the predictor X_t^* to forecast the λ-quantile of the conditional loss distribution.[4] The extent of predictability may vary on the particular value of λ. We are particularly interested in $\lambda \in \{0.01,\ 0.05\}$, as these quantiles are the most relevant cases for empirical purposes on risk modelling and management. Finally, it should be remarked that CAViaR models are specifically intended to generate VaR forecasts owing to its autoregressive nature, so we can further analyze the predictive ability of the regressor in the more interesting context of the out-of-sample performance. We shall discuss both issues in greater detail later on in Section 4.

Some further comments on this specification follow. First, the functional form of CAViaR-type models attempts to explode parsimoniously the statistical information conveyed by the past of the conditional quantile and a set of predetermined variables. The autoregressive structure ensures that the conditional quantile changes smoothly over time. Since VaR dynamics are expected to be highly persistent, the lag of the dependent process could be seen as an instrumental variable that proxies for the true latent process. Second, the variable $|r_{t-1}|$ proxies for the unobservable volatility of the returns, as we can expect a major influence of this latent process on the VaR dynamics owing to the persistence nature of the latent process. Furthermore, this process introduces short-term variation in the VaR dynamics which is related to the arrival of news or shocks in the return process. At this point, the similitudes of the SAV-CAViaR and the GARCH-type models used to model conditional variance are totally evident. Finally, we could consider a set of $N > 1$ potential predictors to extend the basic SAV-CAViaR equation, although we should note that the existing literature has not discussed yet which variables should be included in such an analysis. The model in which the simple SAV-CAViaR specification is extended with lagged values of a single

[3] In our analysis, all the series are strictly positive. The conventional logarithmic transformation on the absolute value of the series can be used to smooth the process and reduce the statistical problems related to outlying observations and heteroskedasticity. Furthermore, because such a transformation yields either a strictly positive or negative series in our empirical setting, (see Section 4.1 for details), we shall consider $f\left(X_{t-1}^*\right) = |\log(\left|X_{t-1}^*\right|)|$.

[4] The empirical analysis in Kuester et al., (2006) shows that the SAV model has a good performance in relation to more sophisticated CAViaR-type specifications that include asymmetries. Hence, we shall use the SAV model in the current paper as a baseline specification. Note that the model can be generalized as $VaR_{\lambda,t} = \beta_{\lambda,0} + \sum_{i=1,p} \beta_{\lambda,i} VaR_{t-i} + \sum_{j=1,q} \beta_{\lambda,j}^* X_{t-1,j}$.

predictive variable can be seen as a low-order individual autoregressive distributed lag model, which is particularly useful in the forecasting analysis; see, for instance, Stock and Watson (2003) and Rapach and Strauss (2009). This strategy allows us to examine how a variety of liquidity and volume-related effects perform relative to the restricted SAV model.

Under suitable regularity assumptions and as the sample size is allowed to grow unbounded, it can be shown (cf. Engle and Manganelli 2004, Thms. 1 and 2) that:

$$\sqrt{T}(\widehat{\beta}_\lambda - \beta_\lambda) \xrightarrow{d} \mathcal{N}(0, V_\lambda) \tag{8}$$

i.e., $\widehat{\beta}_\lambda$ is a $\sqrt{T}$-consistent estimate of the unknown vector β_λ, and the (suitably re-scaled) estimation bias is asymptotically distributed as a normal distribution with zero mean and finite covariance matrix V_λ. In order to estimate consistently the asymptotic covariance matrix, Engle and Manganelli (2004) propose a robust estimator which is defined as the matrix product of the sample analog of the outer product of the gradient of the objective function and an estimator that combines kernel density estimation (e.g., k-nearest neighbor estimators) with the heteroskedasticity-consistent covariance matrix estimator of White (1980); see Engle and Manganelli (2004, Thm. 3) for details. We shall use this approach to compute the covariance matrix and the standard errors in our analysis.

4 Empirical analysis

4.1 Data

Our data set comprises continuously compounded returns from the value-weighted portfolio in the US market over the period 01-04-1988 to 12-31-2002, totalling 3782 daily observations. These data are available from CRSP. In addition, we observe daily data for the aggregate liquidity and volume-related variables which are constructed from individual firm bid-ask spreads and volume data; see Chordia *et al.* (2001) for details. This data set includes:
i) Trading-related variables: Trading Volume (V) measured in thousand of shares; Number of Trades (NT) calculated as the sum of sell and buy trades; Number of Sell trades (NS); Number of Shares Sold in thousands (NSS) and Traded Volume in Dollars (TVD).
ii) Liquidity variables: Quoted Spread (QS) measured as the dollar difference between ask and bid prices; Effective Spread (ES) given by the signed difference between trade price and bid-ask midpoint; Relative Quoted Spread (RQS), defined

as QS/MP, and Relative Effective Spread (RES), defined as ES/MP.[5]

[Insert Table 1 around here]

Table 1 displays the descriptive statistics for the demeaned returns and all the explanatory variables (in logarithms) used in our analysis. Returns exhibit the characteristic stylized features in daily samples: Excess of kurtosis, mild degree of skewness and negligible autocorrelation, whereas the most salient feature of the predictors is the strong degree of persistence as measured by the first-order autocorrelation coefficient. Returns are contemporaneously correlated to all the variables analyzed (correlations are not shown for saving space but are available upon request). In particular, returns are positively correlated with the variables in the volume group (the average correlations are around 39%) and negatively correlated to liquidity-related variables (the average correlations are around -25%). The variables within each group are strongly correlated among themselves, and largely and negatively correlated with the variables in the other group. The cross-correlations range from -79%, for TVD and QS, to -88%, for TVD and QS.

[Insert Figure 1 around here]

It is interesting to note that the total sample spans different periods in terms of market activity and volatility, as depicted in Figure 1 (note that this figure displays the sample used later in the out-of-sample analysis; see Section 4.3 for further details). The beginning of this sample corresponds to the period that followed the market crash in October 1987. After the extraordinary crash, the volatility of the market decreased progressively and reverted to much lower values. In 1998, the Long-Term Capital Management (LTCM) failure in the hedge fund industry led to a massive bailout by other major banks and investment houses that generate an excess of volatility in the market and that preceded the burst of the dot-com firms in 2000. Finally, the data from 2000 on show the large excess of volatility that characterized the market after the burst of the technological bubble. It should be noted that the final part of the sample roughly matches the out-of-sample period analyzed in this paper.

[5] In addition to these variables, we considered alternative variables which did not led to qualitative changes over those reported in the next section. For instance, considering the logarithm transformation of the volume or the unexpected volume –measured as the residuals from an AR(1) model– does not seem to have a major change in the out-of-sample ability of the model. These results are not presented to save space but are available from the authors upon request. MP refers to the midpoint.

4.2 Estimation results

We estimate model (7) considering $\lambda \in \{0.01,\ 0.05\}$ for any of the predictive variables described previously using the entire sample. Our main aim is to address the statistical significance of the estimated coefficients.[6] The objective function (6) is optimized using the Simulated Annealing optimization algorithm (Goffe, Ferier and Rogers, 1994). This is a local random-search search algorithm which exploits the analogy between the way a metal cool and freezes to obtain a minimum energy crystalline structure (annealing process) and the search for a minimum in real-valued problems. The search process can accept values that increase the objective function (rather than lower it) with a probability that decreases as the number of iterations increases. The main purpose of this routine is to prevent the search process from becoming trapped in local optima, which in addition provides low sensibility to the choice of the initial values. To minimize the possibility of getting convergence to local optima, the optimization process is repeated $1,000$ times over the whole sample.[7]

Tables 2 shows the estimated coefficients together with the one-sided robust p-values for the variables in the trading activity and liquidity groups. As described in Engle and Manganelli (2004), the asymptotic covariance matrix of the estimate parameter is inferred robustly using a sandwich-type estimator based on a k-nearest neighbor kernel with $k = 40$ and $k = 60$ for the 1% and 5% quantiles, respectively.

[Insert Table 2 around here]

Several features are worth of commenting. First, the empirical results show the strong degree of persistence in the VaR estimates as measured by the estimate of the autoregressive coefficient, $\widehat{\beta}_{\lambda,1}$. Not surprisingly, persistence is stronger for $\lambda = 0.05$, as extreme percentiles are more likely driven by outliers, *i.e.*, the jumping-component of the data generating process in returns. This is expected to exhibit a more random, irregular behavior. In addition, the average influence of the volatility process on the VaR estimates, as measured by the coefficient $\widehat{\beta}_{\lambda,2}$, becomes more important as the size of the quantile reduces. These results completely agree with the qualitative evidence discussed, among others, in Engle and Manganelli (2004). Turning our attention to the empirical relevance of the predictive variables, we note that all the variables analyzed have a positive effect on the conditional distribution of returns. Increments in the volume- or illiquidity-

[6]Estimation is carried out for $\lambda \in \{0.01, 0.025, 0.05, 0.075, 0.1\}$. Table 4 reports the main estimates related to the main coefficients in a further analysis. We do not present all these results for the sake of brevity, but they are available upon request.

[7]Optimization is carried out using Matlab R2008a in a PC with processor Intel Core 2, 2.40GHz, and 4.00GB RAM.

related variables tend to generate larger levels of VaR. As discussed previously, there are several reasons that may explain this feature, including but not limited to, the correlation of the conditional volatility process with the different proxies for liquidity and trading variables. The analysis on the significance of the estimated coefficients offers mixed results. Whereas the null hypothesis $\mathrm{H}_0 : \beta^*_\lambda = 0$ of no predictability is strongly rejected for any of the variables analyzed for the 5% quantile, it cannot be rejected at any of the usual confidence levels for the 1% percentile, even though the size of the estimated coefficient increases slightly.

The differences in the degree of statistical significance could be attributed to heterogeneity in the predictive ability of the model as a function of the particular quantile involved, suggesting that the actual possibility of forecasting the conditional quantile may vanish as the size of the quantile reduces. Given that extreme values are more likely driven by purely idiosyncratic and noisy effects, the empirical evidence is theoretically plausible. Alternatively, the failure to reject statistically the null hypothesis of no predictability may also be rooted in statistical problems related to the estimation of the covariance matrix in a finite sample, *i.e.*, power distortions stemming from finite-sample inefficiencies. The potential sources of inneficiency for a small value of λ in this context are twofold. First, the number of valid observations used to characterize the model decreases as so does λ. Furthermore, the theoretical arguments that support the consistency of the robust nonparametric estimate of the covariance matrix hold asymptotically. As in the case of the HAC-type estimators (from the Heteroskedasticity and Autocorrelation Consistent covariance matrix estimation) used in the standard regression analysis, the finite-sample nonparametric estimation of the covariance matrix is likely to be highly sensitive to the characteristics of the data generating process and the choice of the bandwidth parameter. For instance, stationary but strongly-correlated residuals largely bias nonparametric HAC-type estimates. Since CAViaR models are a relatively novel contribution to the literature, little is known about the small-sample performance.

In order to shed some light on this issue, we carry out a deeper analysis to appraise the sensitivity of the results to the choice of k. In particular, we consider estimates of the parameters and their covariance matrix for a larger set of quantiles $\lambda \in \{0.1,\ 0.075,\ 0.05,\ 0.025,\ 0.01\}$, and $k \in \{10,\ 30,\ 50,\ 70,\ 90\}$. Table 3 summarizes the main results from this analysis, showing the estimated coefficients and the p-values related to the individual test for significance of the volatility proxy variable and the predictor X^*_t involved in each quantile regression (the complete analysis is available upon request). Note that changing k only affects the estimation of covariance matrix and, hence, the likelihood of accepting or rejecting the null hypothesis, but not the estimated parameters. We can observe that, for most of the variables analyzed, the particular choice of k does not seem to generate large differences when it comes to accepting or rejecting the null hypothesis at the conventional significance levels, so the qualitative evidence is not particularly sensitive to this value. For relatively large values of λ, the empirical evidence strongly supports predictability independently of the value of k in the range analyzed. As we attempt to predict more extreme quantiles, the statistical evidence weakens and eventually vanishes out. Whereas the hypothesis could still be marginally rejected for $\lambda = 0.01$ for the predictive variables in the volume-related group, the overall evidence suggests that liquidity-related variables do not seem to play a significant role to forecast the 1% conditional quantile. Overall, the evidence suggests that, in this analysis, the main results are not particularly sensitive to the choice of k in the range considered.

The failure to reject the null hypothesis is likely due to an artifact due to loss of efficiency when modelling extreme quantiles. For instance, whereas the estimated coefficient related to the proxy of volatility, $\hat{\beta}_{\lambda,2}$, largely increases on λ as expected, paradoxically the level of statistical significance tends to decrease. As a result, the proxy of volatility is not significant at the conventional levels for $\lambda \in \{0.025,\ 0.01\}$. This counterintutive feature strongly suggests the presence of power distortions in the analysis owing to greater levels of noise inference for small values of the quantile. Given that the ultimate purpose of computing VaR estimates is to generate market risk forecasts, the out-of-sample analysis provides a more appropriate framework to disentangle whether including market-related information is valuable to predict VaR or not. Even if the in-sample contribution of a certain variable were marginally significant, it may still play a critical role in the out-of-sample performance, as a good in-sample performance is neither a necessary nor sufficient condition to ensure a good out-of-sample performance. This is studied in the following subsection through standard backtesting analysis.

[Insert Table 3 around here]

4.3 Backtesting analysis

We split the total sample into an in-sample and an out-of-sample period to perform a rolling-window backtesting analysis. Following Alexander and Sheedy (2008), we consider a relatively large number of observations (2700) to be included in the estimation rolling window, which allows us to construct over 1000 one-day ahead VaR forecasts (see Figure 1 above). Our main goal is to analyze the forecasting ability of the covariate-extended model (7) in relation to the restricted SAV-CAViaR model that imposes $\beta_{\lambda}^{*} = 0$. In addition we compare the relative performance of these models to the VaR forecasts obtained from the EWMA model (RiskMetrics), the Gaussian GARCH(1,1) model, and the Extreme Value Theory (Appendix A sketches the main features of these risk models). This analysis does not attempt to compare which model performs better in a horse race, but rather provide a broad framework to appraise the possible gains in the actual forecasting ability of the models that make use of additional information. Tables 4 reports the average values of the parameters estimates of equation (7) over the out-of-sample period for $\lambda \in \{0.01,\ 0.025,\ 0.05,\ 0.075\}$ using volume and spread variables. As with the whole sample, the average estimates reveal a strongly persistent process and the positive effect of volatility.

[Insert Tables 4 around here]

We follow Christoffersen (1998) to gauge the accuracy of out-of-sample VaR forecasts. For any of the VaR forecasts in the out-of-sample period and any of the risk models considered, we define the exception variable $H_{\lambda,t}$ as a dummy variable taking value one if the actual return at time t exceeds the predicted VaR value, and zero otherwise. Thus, the variable signals whether the market falls below the expected VaR threshold. The main purpose is to test for the hypothesis of perfect conditional coverage, *i.e.*, $\mathrm{H}_0 : E\left(H_{\lambda,t}|\mathcal{F}_{t-1}\right) = \lambda$, which implies that $H_{\lambda.t}$ is uncorrelated with any measurable function of the information set $\mathcal{F}_{t-1}$, and that VaR exceptions will approximately occur with the nominal conditional and unconditional probability. More specifically, we account for tests that address whether, *i*) the unconditional likelihood of an exception matches the expected frequency, $\mathrm{H}_0 : E\left(H_{\lambda,t}\right) = \lambda$, using a test statistic labelled LR_{UC}; *ii*) exceptions are serially uncorrelated, using a test statistic labelled LR_{IND}; and *iii*) the conditional likelihood of an exception equals λ, using the test statistic labelled LR_{CC}. These tests statistics and their asymptotic distribution are described in detail in Appendix B. Table 5 reports the results for the benchmark models (VaR-GARCH, VaR-EWMA, EVT-BM and SAV-CAViaR).[8]

[Insert Table 5 around here]

The overall performance of the VaR-GARCH and the SAV-CAViaR models is very similar in the period analyzed. In general, both models are biased towards yielding over-conservative VaR forecast (*i.e.*, the estimated exception ratio tends to be positive biased in relation to the nominal level). This result agrees with the usual finding in the literature, which tends to show a relatively large proportion of exceptions in parametric models due to parameter biases originated in the occasional outliers that typically contaminate the estimation window. As a result, the backtesting analysis strongly rejects the hypothesis of perfect unconditional coverage hypothesis for both models when analyzing $\lambda = \{0.05, 0.075\}$. Also, both

[8] We do not report the outcomes from the estimation process of these models for saving space. They are available upon request.

the simplest VaR-EWMA and the more sophisticated ETV models seem to produce better results, although the EWMA model overestimates the true VaR for the quantiles $\lambda = \{0.01, 0.025\}$, and the ETV model tends to underestimate the VaR process for $\lambda = 0.075$. Overall, none of the risk models considered in this analysis seems able to pass simulteneously the backtesting analysis convincingly and uniformly across the quantiles analyzed.

[Insert Tables 6 around here]

Next, we turn our attention to the results from the backtesting analysis from the covariate-extended CAViaR models. These are shown in Table 6 (volume and liquidity extended CAViaR models). The most remarkable finding is that, whereas the standard SAV-CAViaR model can be largely biased towards underestimate the true Value at Risk, the inclusion of volume-related and liquidity variables provides a suitable correction such that most of the departures from the nominal level are corrected. Remarkably, those biases are removed and the empirical values tend to stabilize around the nominal level. For all the variables analyzed, this improvement in the unconditional properties of the VaR forecast time-series is achieved without increasing the degree of serial dependence in the exceptions variable. As a result, all the extended CAViaR models are able to pass the backtesting analysis at any of the usual confidence levels. The p-values of the decisive LR_{CC} tests statistics are well above the conventional statistical significance levels for the vast majority of tests, particularly for the set of liquidity-extended CAViaR risk models. As an example, the mean value of exception for the baseline SAV-CAViaR model for $\lambda = 0.075$ is slightly greater than 10%, with the GARCH(1,1) closely matching this value. In sharp contrast, the largest mean value for the set of variables analyzed is 8.7%, with some variables yielding even a larger bias reduction. For instance, including the Effective Spread (ES) and Relative Effective Spread (RES) leads to an unconditional coverage around 8%. Overall, among all the predictors considered, the ES and RES variables in the liquidity group seem to provide the best results in the backtesting analysis.

Figure 2 plots the one-day forecasts from the standard SAV-CAViaR model and the RQS-extended CAViaR model for the 5% quantile. We can observe that the forecasts from the liquidity-extended model tend to be much higher, so the inclusion of liquidity variables tends to generate large changes in the VaR forecasts.

[Insert Figure 2 around here]

More importantly, the differences in the VaR forecast have the correct sign and, hence, adding the information conveyed by these variables is able to reduce significantly the excessive number of exceptions in the baseline SAV-CAViaR model.

Similar results arise when analyzing volume-extended models.[9] In view of the empirical results from the backtesting analysis, we can conclude that the correction provided by the inclusion of microstructure variables is useful for VaR forecasting.

5 Concluding remarks

We have analyzed the predictability of downside market risk using different variables related to the trading-activity and liquidity categories. Our approach has mainly built on the CAViaR quantile regression model proposed by Engle and Manganelli (2004). The predictive analysis over the whole sample and the standard backtesting VaR analysis support the suitability of these variables to forecast one-day ahead VaR, improving purely returns-based models. The overall evidence is robust against the inclusion of different measures of liquidity and trading-activity, as well as the consideration of different quantiles.

Our methodological approach can be related to the so-called LVaR (see Jarrow and Subramanian, 1997 and Jorion, 2007), although we provide a different perspective to the problem of forecasting VaR. In particular, while LVAR determines the total VaR as the sum of the "traditional" VaR (as termed in Jorion 2007, pp. 354) plus an additional component related to transaction costs, we use the information conveyed by bid-ask spreads, trading volume, and other variables to forecast the market VaR itself. The fundamental premise, therefore, is that these variables convey useful information to forecast the tail distribution of conditional returns. Our empirical findings strongly support this hypothesis therefore the volume and liquidity variables contain relevant information in forecasting downside risk.

[9] We do not report these results for saving space but they are available from the authors upon request.

Appendix A: VaR models

In Section 4.3 we compare the performance of CAViaR model with other standards procedures to compute VaR. These include the EWMA, GARCH and EVT (Extreme Value Theory) methods. The common setting in these models assumes that returns obey dynamics given by

$$r_t = \sigma_t \eta_t, \quad \eta_t | \mathcal{F}_{t-1} \sim iid(0,1) \tag{A.1}$$

where σ_t denotes the conditional volatility of the process. We briefly discuss the main settings of these approaches in the sequel.

A. VaR EWMA

RiskMetrics (1996) popularized the Exponential Weighting Moving Average (EWMA) scheme as an easy way to model the volatility process. The latent volatility dynamics are assumed to obey the recursive dynamics:

$$\widehat{\sigma}_t^2 = \delta\ \widehat{\sigma}_{t-1}^2 + (1-\delta) r_{t-1}^2, \quad t = 1, ..., T \tag{A.2}$$

with the initial condition $\widehat{\sigma}_0^2 = r_0^2 = E\left(r_t^2\right)$. The smoothing parameter δ can be estimated, although RiskMetrics advises to set $\lambda = 0.95$ for data recorded on a daily basis. Then, the one-day ahead forecast given $\mathcal{F}_T$ is simply given by $\widehat{\sigma}_{T+1|T}^2 = \delta\ \widehat{\sigma}_T^2 + (1-\delta) r_T^2$.

The RiskMetrics approach assumes the particularly strong assumption that the innovations η_t are conditionally Normal distributed. Then, the EWMA-VaR forecast would be determined as $-\mathcal{Z}_\lambda \widehat{\sigma}_{T+1|T}$, with $\mathcal{Z}_\lambda$ denoting the λ-quantile of the standard normal distribution. In order to ensure robustness against likely departures from normality, we proceed in a slightly different way. Let $\widehat{\eta}_t = r_t / \widehat{\sigma}_t$ be the estimated innovations given the estimates of the EWMA volatility process, and let $Q_\lambda(\widehat{\eta}_t)$ be the unconditional λ-quantile of the empirical distribution. Then, an alternative VaR forecast that does not make any distributional assumption is:

$$VaR_{\lambda,t+1}\left(EWMA\right) = -Q_\lambda(\widehat{\eta}_t) \widehat{\sigma}_{T+1|T} \tag{A.3}$$

B. VaR GARCH

The simplest GARCH (1,1) model is by far one the most popular approach to model and forecast market risk due to its impressive performance (Hansen and Lunde 2005) and simplicity. The standard GARCH(1,1) model assumes that daily returns obey dynamics given by:

$$\begin{aligned} r_t &= \sigma_t \eta_t, \quad \eta_t | \mathcal{F}_{t-1} \sim iid\mathcal{N}(0,1) \\ \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{A.4}$$

with the restrictions $\omega > 0$, α, $\beta \geq 0$ ensuring that the conditional variance process is well-defined. Although financial returns are known to be non-normally distributed, the Gaussian assumption is particularly convenient because it ensures parameter consistency (Bollerslev and Woolridge, 1992). We use this assumption to generate consistent estimates of the conditional volatility process:

$$\widehat{\sigma}^2_{T+1|T} = \widehat{\omega} + \widehat{\alpha} r^2_T + \widehat{\beta}\ \widehat{\sigma}^2_T, \tag{A.5}$$

and, as in the previous case, we avoid the distributional assumption in constructing the VaR forecast. Given the GARCH estimates of the volatility process $\widehat{\sigma}_t$ and the resultant standardized innovations, $\widehat{\eta}_t = r_t/\widehat{\sigma}_t$, the "robust" one-day VaR-GARCH forecast is determined as:

$$VaR_{\lambda,T+1}\left(GARCH\right) = -Q_\lambda(\widehat{\eta}_t)\widehat{\sigma}_{T+1|T}. \tag{A.6}$$

C. VaR EVT

We also adopt Extreme Value Theory (EVT) to estimate the one-day ahead VaR using the block maxima (BM) method. This method requires of the choice of a block length, say n, and previous estimates of the conditional volatility process. Since it is widely agreed that GARCH estimates tend to overperform other procedures when it comes to forecast volatility, we determine $\widehat{\eta}_t = r_t/\widehat{\sigma}_t$ use the GARCH(1,1) estimates as previously discussed.

Let $g = [T/n]$, where $[\cdot]$ denotes the integer part of the argument. We divide the total sample into $g = 740$ blocks of equal length, $n = 5$, and record the maximum value for each block. Then, a Generalized Error Distribution (GEV) is fitted to the block maxima by optimizing a maximum likelihood procedure that yields three parameters that characterize ρ_1 (shape), ρ_2 (scale) and ρ_3 (location). The generalized representation is particularly useful when maximum likelihood estimates have to be computed, as in general, we do not know in advance the type of limiting distribution of the sample maxima. Given the resultant estimates, we determine

$$VaR_{block} = \hat{\rho}_3 - \frac{\hat{\rho}_2}{\hat{\rho}_1}\left[1 - \left\{-\ln\left(1 - \frac{1}{n\lambda}\right)\right\}^{-\hat{\rho}_1}\right] \tag{A.7}$$

from which we can obtain the one-day ahead VaR EVT forecast

$$VaR_{\lambda,T+1}\left(EVT\right) = -\widehat{\sigma}_{T+1|T} VaR_{block} \tag{A.8}$$

with $\widehat{\sigma}_{T+1|T}$ determined as in $(A.5)$. The estimations are unbiased, asymptotically normal and of minimum variance under proper assumptions; see Embrechts *et al.* (1997) and Coles (2001).

Appendix B: Backtesting analysis

I) Unconditional test:

The most basic assumption is that the market risk model provides a correct unconditional coverage, *i.e.*, $H_0 : E\left[H_{\lambda,t}\right] = \lambda$. The null hypothesis is be rejected for large values of the Likelihood Ratio (LR) test (see Kupiec, 1995) defined as

$$LR_{UC} = 2(N - N_\lambda)\left[\log(1 - \frac{N_\lambda}{N}) - \log(1-\lambda)\right] + 2N_\lambda\left[\log\frac{N_\lambda}{N} - \log\lambda\right] \sim \chi^2_{(1)} \tag{B.1}$$

where $\chi^2_{(1)}$ stands for a Chi-squared distribution with one degree of freedom, N_λ is the number of exceptions, and N is the total number of out-of-sample observations. Note that N_λ/N is simply the sample mean of $H_{\lambda,t}$, *i.e*, the sample equivalent of $E\left[H_{\lambda,t}\right]$.

II) Independence test:

If exceptions are serially correlated, the conditional coverage will be defective even if the unconditional coverage is correct, because the risk of bankruptcy is higher if so. If the risk model tends to yield clustered exceptions, then the risk manager should increase the VaR in order to lower the conditional probability of an exception to the expected λ. Christoffersen (1998) proposes the analysis of the first-order serial correlation in $H_{\lambda,t}$ through a binary first-order Markov chain with transition probability matrix

$$\Pi = \begin{pmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{01} \end{pmatrix}, \text{ with } \quad \pi_{ij} = \Pr(H_{\lambda,t} = j \mid H_{\lambda,t-1} = i), i,j \in \{0,1\} \tag{B.2}$$

The approximate joint likelihood conditional on the first observation is

$$\mathcal{L}(\Pi; H_{\lambda,t} \mid H_{\lambda,1}) = (1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}, \tag{B.3}$$

where n_{ij} represents the number of transitions from state i to state j. The maximum-likelihood estimators under the alternative hypothesis are $\widehat{\pi}_{01} = n_{01}/\left(n_{00}+n_{01}\right)$, and $\widehat{\pi}_{11} = n_{11}/\left(n_{10}+n_{11}\right)$. Under the null hypothesis of independence, we have $\pi_{01} = \pi_{11} = \pi_0$, with $\pi_0 = \lambda$, from which the conditional binomial joint likelihood is

$$\mathcal{L}(\pi_0; H_{\lambda,t} \mid H_{\lambda,1}) = (1-\pi_{01})^{n_{00}+n_{10}}\pi_{01}^{n_{01}+n_{11}}. \tag{B.4}$$

Note that π_0 can be estimated as $\widehat{\pi}_0 = N_\lambda/N$. The likelihood ratio test for the hypothesis of independence is given by

$$LR_{IND} = 2\left[\log \mathcal{L}(\hat{\Pi}; H_{\lambda,t} \mid H_1) - \log \mathcal{L}(\widehat{\pi}_0; H_{\lambda,t} \mid H_{\lambda,1})\right] \sim \chi^2_{(1)} \tag{B.5}$$

III) Conditional test:

Finally, we can study simultaneously whether the VaR violations are independent and occur with the correct probability. Because $\widehat{\pi}_0$ is unconstrained, the test in equation (B.5) does not impose the correct coverage. We can readily devise a joint test for independence and correct coverage (i.e., correct conditional coverage) by combining the previous tests as

$$LR_{CC} = 2\left[\log \mathcal{L}(\hat{\Pi}; H_{\lambda,T} \mid H_1) - \log \mathcal{L}(\lambda; H_T \mid H_1)\right] \sim \chi^2_{(2)} \tag{B.6}$$

This is equivalent to testing if the sequence of $H_{\lambda,t}$ is independent and the probabilities to observe an exception given the set of information is equal to the nominal level λ, namely, $\pi_{01} = \pi_{11} = \lambda$. Therefore, we can write

$$LR_{CC} = LR_{UC} + LR_{IND}, \tag{B.7}$$

which provides the suitable test statistic to check whether $\{H_{\lambda,t}\}$ fulfills the correct conditional coverage properties. Since the test involves two restrictions, the asymptotic convergence to a $\chi^2_{(2)}$ distribution.

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Tables

Table 1: Descriptive Statistics.

X_t^*	MEAN	MEDIAN	MAX.	MIN.	VAR	SKEW.	KURT.	$\rho_{(1)}$
$r_{\mathbf{t}}$ ($\times 100$)	0	0.01	5.54	-6.7	0.98	-0.2	7.78	-0.06
V	7.39	7.21	9.69	5.52	0.73	0.36	1.86	0.96
NT	6.85	6.69	8.76	5.07	0.65	0.3	1.64	0.98
NS	6.09	5.94	8.02	4.24	0.65	0.29	1.67	0.98
NSS	6.51	6.33	8.8	4.5	0.73	0.35	1.86	0.96
TVD	11.1	11.04	13	9.13	0.76	0.17	1.66	0.96
QS	-1.89	-1.74	-1.2	-3.4	0.21	-1.64	4.79	0.99
ES	-2.29	-2.11	-1.5	-3.8	0.2	-1.53	4.48	0.99
RQS	-5.39	-5.39	-4.8	-6.9	0.2	-1.02	3.23	0.99
RES	-5.96	-5.76	-5.2	-7.2	0.2	-0.96	3.08	0.99

This table shows the descriptive statistics (mean, median, maximum, minimum, variance, skewness and kurtosis) of the demeaned returns and all the explanatory variables involved in the analysis (in logarithms). The last row indicates the first-order autocorrelation of the variables. The variables included are V (trading volume); NT (number of trades); NS (number of sell trades); NSS (number of shares sold in thousands); TVD (traded volume in dollars); QS (quoted spread); ES (effective spread); RQS (relative quoted spread) and RES (relative effective spread).

Table 2: **Inference results from predictive quantile regressions.**

	$\lambda = 5\%$				$\lambda = 1\%$			
X_t^*	$\hat{\beta}_{\lambda,0}$	$\hat{\beta}_{\lambda,1}$	$\hat{\beta}_{\lambda,2}$	$\hat{\beta}_{\lambda}^*$	$\hat{\beta}_{\lambda,0}$	$\hat{\beta}_{\lambda,1}$	$\hat{\beta}_{\lambda,2}$	$\hat{\beta}_{\lambda}^*$
V	-0.04	0.96	0.05	0.01	0.02	0.92	0.11	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.37)	(0.00)	(0.06)	(0.16)
NT	-0.03	0.96	0.05	0.01	0.03	0.92	0.11	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.25)	(0.00)	(0.04)	(0.13)
NS	-0.02	0.96	0.05	0.01	0.03	0.92	0.11	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.24)	(0.00)	(0.04)	(0.16)
NSS	-0.03	0.96	0.05	0.01	0.03	0.92	0.11	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.27)	(0.00)	(0.06)	(0.19)
TVD	-0.06	0.96	0.05	0.01	-0.01	0.92	0.12	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.41)	(0.00)	(0.05)	(0.12)
QS	-0.01	0.97	0.05	0.01	0.07	0.92	0.14	0.01
	(0.01)	(0.00)	(0.00)	(0.00)	(0.12)	(0.00)	(0.08)	(0.32)
ES	-0.00	0.97	0.05	0.00	0.08	0.91	0.15	0.01
	(0.40)	(0.00)	(0.00)	(0.01)	(0.14)	(0.00)	(0.11)	(0.31)
RQS	-0.04	0.97	0.05	0.01	0.04	0.92	0.14	0.01
	(0.01)	(0.00)	(0.00)	(0.00)	(0.32)	(0.00)	(0.08)	(0.36)
RES	-0.08	0.96	0.05	0.02	-0.03	0.89	0.15	0.03
	(0.05)	(0.00)	(0.00)	(0.05)	(0.34)	(0.00)	(0.13)	(0.18)

This table shows the estimated parameters and robust p-values (in brackets) for the entire sample and the quantile regression model (7),

$$VaR_{\lambda,t} = \beta_{\lambda,0} + \beta_{\lambda,1} VaR_{t-1} + \beta_{\lambda,2}|r_{t-1}| + \beta_{\lambda}^* \mid \log(X_{t-1}^*)|),$$

given $\lambda = 0.05$ and $\lambda = 0.01$. The first column shows the volume-related and liquidity variables X_t^* analyzed.

Table 3: Sensitivity analysis of p-values to different k-bandwidth.

λ		V	NT	NS	NSS	TVD	QS	ES	RQS	RES	$\lvert r_{t-1}\rvert$
10%	$\hat{\beta}_\lambda$	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.02	0.06
	10	0.04	0.00	0.00	0.00	0.01	0.00	0.05	0.00	0.02	0.00
	30	0.00	0.00	0.00	0.00	0.05	0.03	0.00	0.00	0.05	0.00
k	50	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.07	0.00	0.00
	70	0.00	0.01	0.00	0.00	0.00	0.01	0.01	0.08	0.00	0.00
	90	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.01	0.00
7.5%	$\hat{\beta}_\lambda$	0.01	0.01	0.01	0.01	0.01	0.02	0.00	0.01	0.02	0.06
	10	0.00	0.00	0.00	0.00	0.24	0.00	0.02	0.00	0.13	0.00
	30	0.00	0.01	0.00	0.00	0.00	0.00	0.04	0.07	0.02	0.00
k	50	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.06	0.00
	70	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.04	0.00
	90	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.02	0.00
5.0%	$\hat{\beta}_\lambda$	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.02	0.05
	10	0.02	0.00	0.01	0.03	0.00	0.00	0.00	0.00	0.16	0.00
	30	0.00	0.01	0.00	0.00	0.00	0.04	0.00	0.00	0.06	0.00
k	50	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.00
	70	0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.07	0.00
	90	0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.01	0.05	0.00
2.5%	$\hat{\beta}_\lambda$	0.02	0.02	0.02	0.03	0.02	0.01	0.00	0.00	0.03	0.07
	10	0.04	0.14	0.43	0.00	0.00	0.04	0.34	0.39	0.46	0.01
	30	0.03	0.17	0.10	0.04	0.07	0.10	0.29	0.36	0.08	0.11
k	50	0.02	0.08	0.06	0.02	0.04	0.12	0.26	0.35	0.11	0.09
	70	0.03	0.03	0.04	0.02	0.02	0.11	0.27	0.35	0.06	0.07
	90	0.03	0.03	0.03	0.02	0.02	0.07	0.28	0.35	0.04	0.07
1.0%	$\hat{\beta}_\lambda$	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.14
	10	0.04	0.02	0.05	0.11	0.03	0.24	0.03	0.30	0.00	0.03
	30	0.15	0.16	0.12	0.15	0.14	0.34	0.33	0.38	0.24	0.05
k	50	0.12	0.12	0.11	0.14	0.11	0.33	0.28	0.38	0.15	0.03
	70	0.14	0.15	0.14	0.16	0.10	0.28	0.24	0.30	0.16	0.03
	90	0.11	0.13	0.11	0.12	0.10	0.28	0.24	0.32	0.15	0.03

This table shows the estimated coefficients $\hat{\beta}_{\lambda,2}$ and $\hat{\beta}^*_\lambda$ and robust p-values of the test for individual significance from model (7) and the entire sample when the robust asymptotic covariance matrix is estimated with a kernel with values of $k \in \{10,\ 30,\ 50,\ 70,\ 90\}$ in the covariance matrix estimation process for a larger set of quantiles. The $\hat{\beta}_{\lambda,2}$ estimates (last column) are from model (7) with $X^*_t = V$.

Table 4: Averaged estimates out-of-sample analysis.

VOLUME EXTENDED CAViaR						LIQUIDITY EXTENDED CAViaR				
X_t^*	λ	$\hat{\beta}_{\lambda,0}$	$\hat{\beta}_{\lambda,1}$	$\hat{\beta}_{\lambda,2}$	$\hat{\beta}_{\lambda}^*$	X_t^*	$\hat{\beta}_{\lambda,0}$	$\hat{\beta}_{\lambda,1}$	$\hat{\beta}_{\lambda,2}$	$\hat{\beta}_{\lambda}^*$
V	7.5%	-0.047	0.955	0.056	0.009	QS	-0.018	0.956	0.061	0.016
	5%	-0.055	0.960	0.046	0.011		-0.018	0.970	0.046	0.017
	2.5%	-0.099	0.934	0.076	0.023		-0.036	0.952	0.068	0.042
	1%	-0.003	0.822	0.331	0.028		0.044	0.836	0.349	0.065
NT	7.5%	-0.047	0.955	0.055	0.010	ES	-0.032	0.954	0.062	0.020
	5%	-0.034	0.965	0.045	0.008		-0.032	0.968	0.046	0.021
	2.5%	-0.052	0.952	0.058	0.015		-0.044	0.955	0.064	0.036
	1%	0.051	0.784	0.399	0.029		0.028	0.844	0.308	0.060
NS	7.5%	-0.041	0.954	0.056	0.010	RQS	-0.054	0.959	0.059	0.012
	5%	-0.037	0.962	0.046	0.010		-0.053	0.971	0.044	0.012
	2.5%	-0.079	0.937	0.065	0.025		-0.115	0.960	0.054	0.027
	1%	-0.060	0.778	0.382	0.056		-0.086	0.851	0.300	0.043
NSS	7.5%	-0.042	0.954	0.056	0.010	RES	-0.088	0.954	0.062	0.017
	5%	-0.041	0.963	0.045	0.010		-0.062	0.971	0.043	0.012
	2.5%	-0.118	0.920	0.084	0.033		-0.079	0.963	0.052	0.019
	1%	-0.167	0.775	0.369	0.073		-0.006	0.838	0.348	0.026
TVD	7.5%	-0.074	0.956	0.054	0.008					
	5%	-0.039	0.969	0.044	0.005					
	2.5%	-0.072	0.956	0.058	0.010					
	1%	0.059	0.789	0.435	0.015					

This table shows the average value of the out-of-sample parameters from model (7) estimates with volume-extended and liquidity variables. The column labelled as X_t^*denotes the volume-related and liquidity variables analyzed.

Table 5: Backtesting analysis. Benchmark models.

MODEL	λ	Exc.	LR_{UC}	LR_{IND}	LR_{CC}	FV
EWMA	7.5%	8.9%	2.68(0.10)	0.67(0.41)	3.38(0.18)	1.90
	5%	5.5%	0.51(0.47)	0.00(0.99)	0.52(0.77)	2.24
	2.5%	1.5%	4.78(0.02)	0.46(0.49)	5.21(0.07)	2.98
	1%	0.5%	3.09(0.08)	0.05(0.82)	3.13(0.21)	3.94
GARCH(1,1)	7.5%	11.3%	18.22(0.00)	0.29(0.59)	18.59(0.00)	1.76
	5%	7.4%	10.63(0.00)	0.46(0.49)	11.15(0.00)	2.03
	2.5%	2.8%	0.35(0.55)	0.08(0.78)	0.44(0.80)	2.68
	1%	0.9%	0.10(0.75)	0.16(0.69)	0.27(0.87)	3.57
EVT-BM	7.5%	10.4%	10.85(0.00)	0.58(0.45)	11.49(0.00)	1.76
	5%	6.1%	2.36(0.12)	0.51(0.48)	2.89(0.23)	2.16
	2.5%	2.4%	0.04(0.83)	0.31(0.58)	0.35(0.84)	2.77
	1%	0.5%	3.10(0.08)	0.04(0.84)	3.13(0.21)	3.48
SAV-CAViaR	7.5%	10.1%	8.86(0.00)	0.46(0.49)	8.74(0.01)	1.82
	5%	7.4%	10.63(0.00)	0.50(0.48)	11.19(0.00)	2.04
	2.5%	3.2%	1.85(0.17)	0.00(0.99)	1.86(0.39)	2.55
	1%	1.3%	0.83(0.36)	0.31(0.57)	1.15(0.56)	3.23

This table shows the Backtesting analysis for the one-day forecasts of the VaR given the EMWA, Gaussian GARCH, EVT and SAV-CAViaR models. The second column shows the estimated ratio of empirical exceptions. LR_{UC}, LR_{IND}, and LR_{CC} denote the values of the test statistics for unconditional coverage, independence, and conditional coverage, respectively, (see Appendix B for details), whereas the p-values of the respective test statistics are exhibit in brackets. Finally, FV denotes the mean of the forecast VaR over the out-of-sample period. Bold letters are used to denote statistical rejection at any of the standard asymptotic nominal levels.

Table 6: Backtesting VaR analysis for volume and liquidity extended CAViaR models. See details in table 5.

X_t^*	λ	Exc.	LR_{UC}	LR_{IND}	LR_{CC}	FV
V	7.5%	8.1%	0.51(0.48)	0.06(0.80)	0.43(0.81)	1.95
	5%	5.5%	0.51(0.47)	0.36(0.55)	0.88(0.64)	2.21
	2.5%	2.2%	0.38(0.53)	0.94(0.33)	1.32(0.51)	2.72
	1%	0.7%	1.02(0.31)	0.08(0.77)	1.09(0.58)	3.25
NT	7.5%	8.1%	0.51(0.48)	0.06(0.80)	0.43(0.81)	1.95
	5%	6.0%	1.98(0.16)	0.61(0.43)	2.61(0.27)	2.16
	2.5%	2.3%	0.17(0.68)	1.03(0.30)	1.20(0.55)	2.68
	1%	0.9%	0.10(0.75)	0.14(0.70)	0.25(0.88)	3.27
NS	7.5%	8.2%	0.69(0.41)	0.03(0.85)	0.55(0.76)	1.94
	5%	5.5%	0.51(0.47)	0.36(0.55)	0.88(0.64)	2.18
	2.5%	2.0%	1.10(0.29)	0.78(0.38)	1.87(0.39)	2.69
	1%	0.6%	1.89(0.17)	0.06(0.81)	1.94(0.38)	3.35
NSS	7.5%	8.0%	0.35(0.55)	0.01(0.91)	0.25(0.88)	1.95
	5%	5.6%	0.73(0.39)	0.28(0.59)	1.03(0.60)	2.21
	2.5%	1.9%	1.61(0.20)	0.70(0.40)	2.29(0.32)	2.74
	1%	0.6%	1.89(0.17)	0.06(0.81)	1.94(0.38)	3.46
TVD	7.5%	8.3%	0.89(0.34)	0.01(0.91)	0.71(0.69)	1.92
	5%	6.1%	2.39(0.12)	1.44(0.23)	3.86(0.14)	2.13
	2.5%	2.8%	0.36(0.55)	0.08(0.78)	0.44(0.80)	2.62
	1%	1.2%	0.38(0.54)	2.45(0.12)	2.83(0.24)	3.31
QS	7.5%	8.6%	0.51(0.48)	0.06(0.80)	0.43(0.81)	1.92
	5%	5.3%	0.18(0.67)	0.55(0.46)	0.75(0.69)	2.27
	2.5%	2.1%	0.69(0.40)	0.86(0.35)	1.54(0.46)	2.84
	1%	1.0%	0.00(1.00)	0.18(0.67)	0.18(0.91)	3.53
ES	7.5%	8.0%	0.35(0.55)	0.10(0.75)	0.34(0.84)	2.00
	5%	5.0%	0.00(1.00)	0.92(0.34)	0.92(0.63)	2.32
	2.5%	2.2%	0.38(0.53)	0.94(0.33)	1.32(0.51)	2.85
	1%	0.9%	0.10(0.75)	0.14(0.70)	0.25(0.88)	3.44
RQS	7.5%	8.6%	1.67(0.19)	0.09(0.75)	1.50(0.47)	1.92
	5%	5.5%	0.51(0.47)	0.36(0.55)	0.88(0.64)	2.21
	2.5%	2.4%	0.04(0.84)	1.13(0.28)	1.17(0.55)	2.72
	1%	0.7%	1.02(0.31)	0.08(0.28)	1.09(0.58)	3.31
RES	7.5%	7.9%	0.23(0.63)	0.00(0.97)	0.13(0.93)	1.97
	5%	5.5%	0.51(0.47)	0.36(0.55)	0.88(0.64)	2.22
	2.5%	2.4%	0.04(0.84)	1.13(0.29)	1.17(0.56)	2.67
	1%	1.0%	0.00(1.00)	0.18(0.67)	0.18(0.91)	3.30

Figures

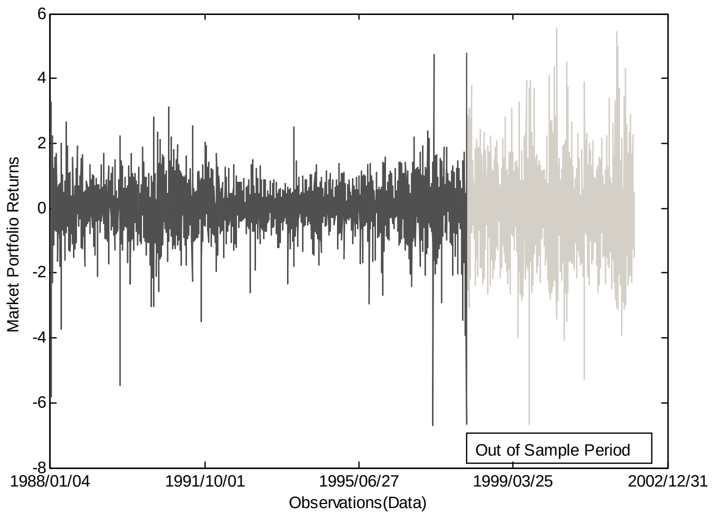

Figure 1: Returns of the market portfolio.

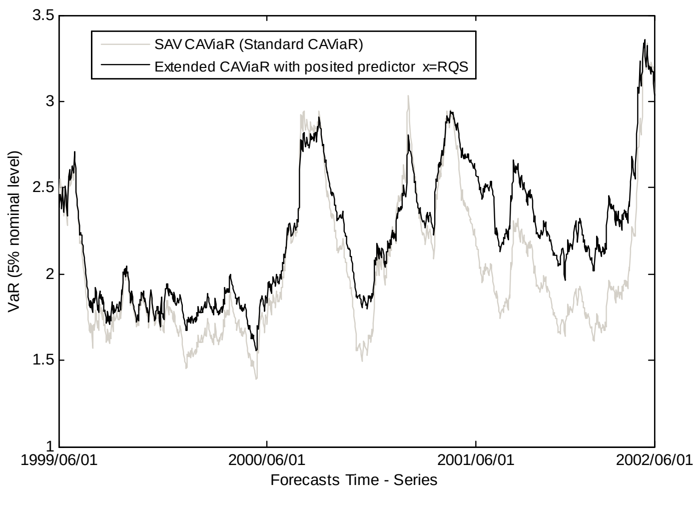

Figure 2: One-day forecasts from the SAV-CAViaR vs RQS-extended CAViaR for the 5% quantile.