Remodeling the Working-Kaldor Curve: The Roles of Scarcity, Time to Maturity and Time to Harvest*

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Abstract

We show that, when modeling the relation between the convenience yield and current scarcity, time to maturity and time to harvest should interact with current scarcity, *i.e.* the two should enter multiplicatively. In implementing this idea we also compare three models for current scarcity, based on inventory levels, the spot price or both. We use data for corn, wheat and soybeans, 1/1986 to 7/2007.

The multiplicative model performs noticeably better than the traditional version, which only focuses on the measurement for scarcity and not on the nature of the relation between convenience yield and scarcity, time to maturity, and harvest. More importantly, though the combination of spot price and inventory provides a better proxy for scarcity than either spot price or inventory separately, the pure spot-price version performs nearly as well as the spot/inventory combination, and much better than the pure inventory version. This is useful because inventory data are typically hard to obtain and increasingly noisy. Our model still exhibits a clear "Working curve", albeit with slopes changing with time to maturity and pre- versus post-harvest periods.

The multiplicative model still works better even during the government loan program period in the middle 1980's; in fact, the program seems to have little impact on modeling the convenience yield. Nor did the change in the delivery system for corn and soybeans, as of 2000, have a big influence.

JEL classification: G12, G14, G1, N32, N52, Q11
Key words: backwardation, convenience yield, theory of storage, scarcity

Introduction and research question

The theory of storage explains normal backwardation[1] in commodity prices through the notion of a convenience yield, that is, the stream of benefits that stem from having the commodity at hand and that, therefore, accrue to inventory holders but not to holders of futures contracts (Kaldor, 1939; and Working, 1948,1949). These benefits arise, among others, from increased production flexibility, from avoided re-ordering costs and from the option of market timing. The value of the premium should rise fast when markets are tightening, *i.e.* when inventory positions are low. We loosely refer to such a situation as one of scarcity. Familiarly, Working (1948,1949) proposes inventory levels as the variable underlying the convenience yield. At about the same time, however, Hayek (1945) wrote that producers and consumers just need the price to take rational decisions; in line with this, Brennan (1958) and others have proposed the spot price as a sufficient statistic for scarcity, in the sense that there should be a one-to-one relation between the spot price and the available inventories. Price data could also avoid measurement issues in inventories, as we argue below. Middle-of-the-roaders, lastly, would hold that both prices and inventory data contain useful information, and that data other than price and inventories may also be useful to capture and understand the value of convenience.

One common feature of these propositions is that there is no mention of time to maturity. In the empirical work, likewise, time to maturity is often ignored or, at best, brought in as an additional, additive effect. A second feature of standard modeling of scarcity is that the occurrence of a harvest, if any, during the life of the contract is deemed to be negligible; only Fama and French (1987) mention this, and as a substitute to the explicit modeling of scarcity rather than a consideration to be taken into account when interpreting inventory or price data. In our view, however, n million bushels m months prior to expiration may mean something very different depending on whether the next harvest comes during, or right after, or far after the contract's life. Thus, prior to empirically studying the roles of inventory and price data as measures of scarcity we want to give the structure of the equation more thought. We show analytically that under fairly general circumstances the convenience yield depends on the product of time to maturity and a function of current scarcity, $\phi(x, S, ...)$, involving *e.g.* inventories x and/or the spot price S. In addition, if there is a harvest during the contract's

[1]Normal backwardation is the premium in the spot price relative to the futures price, after taking out the effects of storage costs and time value. Another common name is "net convenience yield", but this is already more an explanation than a pure label. Yet another name is risk-adjusted spread, which refers to the futures price as the risk-adjusted expectation.

life, a new term comes up involving a product of a similar function of scarcity, $\psi(x, S, ...)$ and the time to maturity beyond harvest time:

$$Y(x, S, t; T, T_h) = \phi(x_t, S_t, ...) \cdot (T - t) + \psi(x_t, S_t, ...) \cdot \max(T - T_h, 0), \tag{1}$$

where Y is the convenience yield, t current time, x the inventory, S the spot price, T the delivery date, and T_h the time of the first harvest following t. As of now we refer to this as the multiplicative version of the model. The first issue in this paper is whether this model does better than the traditional versions, $Y = \phi(x_t, S_t)$ or $Y = \phi(x_t, S_t) + b \cdot (T - t)$. The second issue is what the best specification is of $\phi(x_t, S_t)$: is it a function of just x, or just S, or both, or possibly extra variables.

On the basis of R^2 or either of the standard Information Criterions, our empirical work supports the multiplicative regression specification against the classical versions: for any choice of the function $\phi(x, S)$ it outperforms the above traditional models, and usually substantially so. Regarding the issue of the specification of $\phi(x, S)$, we find that a function $\phi(x, S)$ involving both a spline in the inverse of inventory and a linear term in S does best. From a practitioner's perspective, however, the good news is that a simplified version with just the price, $a + S\,[b \cdot (T - t) + c \cdot \max(T - T_h, 0)]$, nearly always does almost as well as the complete version, at least more recently. This is interesting for hedgers and speculators because it is not obvious how current inventory data should be measured (e.g. Chicago *v* Chicago and Toledo) and because around harvest time the stock data are demonstrably lagging behind reality and, therefore, unreliable.

We test these two issues on data between 1/1/1986 and 31/07/2007, for corn, wheat and soybeans, from the electronic records of the CBOT. The major empirical findings have already been mentioned: the good performance of our multiplicative models compared to the traditional models and the success of the spot price as the sufficient measure of scarcity in our new model. Another notable result, however, arises when we compare pure price and quantity models, treating them as alternatives rather than complements. From 1989 to 1999, the spot price used to capture the backwardation premium quite well, and far better than did inventory, which means a marked reversal of the situation that prevailed in the days of Kaldor (pre-1935). As of 2000, in contrast, while the spot price still does better than inventory and almost as well as the mixed model, the success of all specifications, including the multiplicative one, in capturing the backwardation has dropped. The drop is moderate for wheat, but more pronounced for corn and soybeans. One possible explanation is the introduction of new delivery system, as of 2000: delivery is now via a shipping certificate instead of a warehouse receipt from either Chicago or Toledo, and the barge that holds the product can be anywhere in the Illinois or Mississippi rivers. With such a fuzzy product, the meaning of both inventory and price data is bound to become unclear. But we also have evidence that this is definitely not the sole explanation: also for wheat, where no change was implemented during the sample period, explanatory power goes down in the 2000s.

The remainder of this article is structured as follows. Section 1 reviews the standard models and proposes our new variant. Section 2 provides empirical results. In Section 2.3, we verify whether the splines obtained from our new model have the shape of "Working-Kaldor curves" and we present robustness checks. Section 3 concludes.

1 Modeling convenience yield and current scarcity

In this section we review the traditional test equations and derive an alternative model that introduces time to maturity in a multiplicative way. Strictly speaking, our characterization of futures prices applies to forward prices only, but the two are now recognized to be virtually undistinguishable.

We first motivate our study of backwardation. Normal backwardation is defined as:

$$Y_{t,T} := [S_t + \mathrm{PV}(C, \mathbf{r}, t, T)](1 + r_{t,T}) - f_{t,T}, \tag{2}$$

where t denotes current time (the moment of valuation) and T the moment the futures contract expires; $Y_{t,T}$ is the backwardation—the premium paid for cash positions; S_t the current spot price; $\mathrm{PV}(C, \mathbf{r}, t, T)$ the present value of paying the storage cost C over the contract's life; $r_{t,T}$ the simple percentage rate of return on a risk-free investment maturing at T; and $f_{t,T}$ the current futures price. Interpreting the futures price as a risk-adjusted expectation of the future spot price, the traditional finding of positive empirical values for Y suggests negative risk-adjusted returns from holding inventory. This seemingly negative return, Kaldor and Working argue, is an illusion because the convenience yield, a non-monetary dividend, is ignored.

In this paper we take for granted that backwardation does reflect a convenience aspect, and we accordingly refer to Y as the convenience yield. It is important to understand the determinants of the convenience yield because this helps to evaluate the performance and benefits of commodity futures markets. The performance and the economic benefits of the futures market to an industry depend on how well the market responds to the fundamentals in the industry. According to Ward and Dasse (1976), the rationality of the basis (the difference between futures price and spot price) is considered as an index of performance because "basis patterns are the key to commercial utilization of futures contracts" and there is reason for concern about the market's performance and its benefits if the determinants of the basis cannot be explained. Holders of inventory need to understand the price they are paying, implicitly, for having the inventory at hand rather than just holding a claim on future delivery. Likewise, the success of hedgers in futures markets depends on how well they understand and forecast the convenience yield. Understanding the mechanism and the determinants of the convenience yield helps market participants in making successful production and marketing decisions.

1.1 Traditional Models of the Convenience Yield

Since Kaldor (1939) and Working (1948, 1949), most of the literature tests the Net Convenience Yield theory by verifying whether $Y_{t,T}$ is related to time-t inventory (e.g. Working, 1948; Telser, 1958; Brennan, 1958; Thompson, 1986; Fama and French, 1987; Yoon and Brorsen, 2002; Colin and Carter, 2007; and Gorton, Hayashi and Rouwenhorst, 2007). The typical finding is that there is, in fact, a convex negative relationship (the "Working curve") between the inventory level and the convenience yield, as expected under the theory of storage. The most extreme proposition would be that the convenience yield depends on just current inventory. This 'strong' version of the theory would require that the distribution of future inventory data depend only on the current level, and likewise for the expected future period-by-period convenience yields that are presumably related to these future inventories. Testable hypotheses would include the following: (i) there is a detectable effect from inventories; and (ii) no other indicator of scarcity plays any significant role. However, there is also no consensus on the nature of the inventory/convenience-yield relation. While Garcia and Good (1983) and Karlson, Anderson and Dahl (1993) adopt concise functions like $a + bx$ or $a + b\ln x$ or $a + bx^{-1}$, Telser (1958), Brennan (1958), Yoon and Brorsen (2002), and Gorton, Hayashi and Rouwenhorst (2007) take multi-term nonlinear functions. Among these, the spline inventory function applied by Gorton *et al.* is the most flexible one, and it captures the time series and term structure of backwardation better than the others:

$$(\text{I model:})\ \ Y_{t,T} = \alpha + \theta_1 x_t + \theta_2 x_t^2 + \theta_3 x_t^3 + \theta_4\, \mathbf{1}_{x_t>k}\, (x_t - k)^3. \tag{3}$$

where x_t is inventory, and $\mathbf{1}_{x_t>k} = 1$ if $(x_t > k)$, otherwise $\mathbf{1}_{x_t>k} = 0$. This model, augmented with time to maturity, is one of the standard contenders against our own version.

At the other extreme, one could follow Hayek and argue that the ultimate summary statistic for scarcity should be the price. In this view, inventory only seems to work because it is proxying for price, the omitted variable in the Working curve. Brennan (1959), for instance, argues that time-t inventory is an exact function of the current spot price; therefore the spot price should be an equally sufficient statistic for scarcity. Garcia and Good (1983) similarly assert that the convenience yield of corn is affected by the flow of corn to the market or by "the rate at which producers deliver corn to the market" and "the rate at which the market is consuming corn". They then argue that current cash price is probably the best indicator of the corn flow to the market because it influences the storage and marketing decisions of the farmers and thus leads to relative shifts from demand to supply. Thus the traditional price/convenience yield model is:[2]

$$\text{(Spot model:)} \quad Y_{t,T} = \alpha + \beta S_{t-1} + \epsilon_t. \tag{4}$$

A third group of researchers adopt a more middle-of-the-road view. Some of these still adhere to the inventory logic but admit that the interpretation of storage data depends on circumstances. For instance, time to maturity would probably matter too (for instance, Jiang and Hayenga, 1997), or the timing within the year (Fama and French, 1987). Some authors, like Jiang and Hayenga (1997) simply add time to maturity to a traditional nonlinear inventory model as an independent variable: $Y = a + \phi(x) + b \cdot (T - t)$. Augmenting Equation (3) in this way, we get the third competitor to our own specification:[3]

$$\text{(I+T model:)} \; Y_{t,T} = \alpha + \theta_1 x_t + \theta_2 x_t^2 + \theta_3 x_t^3 + \theta_4 \, \mathbf{1}_{x_t > k} \, (x_t - k)^3 + \gamma \cdot (T - t). \tag{5}$$

Other middle-of-the-roaders such as Garcia and Good (1983) use the spot price, inventory and time to maturity as combined measurements for scarcity:

$$\text{(S+I+T model:)} \; Y_{t,T} = \alpha + \beta S_{t-1} + \theta_1 x_t + \theta_2 x_t^2 + \theta_3 x_t^3 + \theta_4 \, \mathbf{1}_{x_t > k} \, (x_t - k)^3 + \gamma \cdot (T - t). \tag{6}$$

Brennan (1995), when testing his model, suggests that maybe the convenience yield depends on the rate of depletion of the inventories rather than their level. In the same spirit, Garcia and Good (1983) use exports and imports, and similar variables are introduced by Jiang and Hayenga (1997).

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[2] Garcia and Good (1983) use the current spot price (S_t) in their (different) model, but we cannot use (S_t) as it is already involved in the calculation for the convenience yield, as can be seen from Equation (2); thus to avoid spurious correlation stemming from microstructural noise in the spot price we use S_{t-1} instead. Garcia and Good (1983) also point out that the lagged price affects the marketing decision at $t - 1$ and, therefore, influences the inventory carried over from $t - 1$ to t—hence the link with $Y_{t,T}$.

[3] In fact, Jiang and Hayenga use either the inverse of inventory or the log of inventory not a spline. We adopt the spline function because of its flexibility, for instance its capacity to discover non-convexities if these would be present.

It is clear that all of these hypotheses mainly focus on the measurement of scarcity, not on the nature of the relation between scarcity and convenience yield. There is no room for interactions between scarcity and time to maturity nor for any impact of a harvest occurring during the life of the contract. These considerations lead to the research questions already listed above: in what form should time to maturity and time to harvest enter the model, and how does one best measure scarcity.

1.2 A more structured model for convenience yields

Current time is denoted as t, and the futures contract expires at T. We define the cost of storage (C) in dollar terms per period, not annualized percentages of the spot value, and we define the effective risk-free rate of return for horizon $T - t$ in the financial markets, $r_{t,T}$, as the simple percentage growth in the value of a one-period risk-free investment. The term structure of these (un-annualized) rates is denoted by the vector $\mathbf{r}_t$. We denote the present value of paying the storage cost over many periods by $\mathrm{PV}(C, \mathbf{r}, t, T)$. Spot and futures prices, S and $f_{t,T}$, are observable, and so are the dollar cost of storage per period and the interest rates. Convenience, measured in dollars, is the part of (minus) the basis $f - S$ that cannot be explained by storage costs and interest expenses from buying and holding inventory:

$$Y_{t,T} := [S_t + \mathrm{PV}(C, \mathbf{r}, t, T)](1 + r_{t,T}) - f_{t,T}. \tag{7}$$

In terms of pricing theory, one of the variables on the right-hand side can be pinned down immediately: the futures price is the expectation 'under the measure q'—the risk-adjusted expectation—of the future spot price:

$$f_{t,T} = \mathrm{E}_t^q(\tilde{S}_T). \tag{8}$$

Let's now turn to the convenience premium. In the presence of a positive price for convenience, the holders of inventory must be the users of the commodity who need a buffer stock. In fact, other potential holders, namely users of the commodity who merely want to hedge more distant price risks or 'speculators' (the players with a purely financial interest), would not be willing to pay the premium for an advantage that has no real value for then. In the case of a representative holder of inventory, the marginal cost of holding inventory or one period, $Y_{s,s+1}$, must equal the marginal benefit from holding inventory during this period. This benefit is the marginal expected out-of-stock cost X avoided by buying an inventory Q immediately rather than buying $Q - \mathrm{d}Q$ now and postponing the purchase of $\mathrm{d}Q$ for one period, and other similar benefits. This marginal benefit must be non-negative:

$$\begin{aligned} Y_{s,s+1} &:= (S_s + C)(1 + r_{s,s+1}) - \mathrm{E}_s^q(\tilde{S}_{s+1}) && (9) \\ &\stackrel{e}{=} \frac{\partial \mathrm{E}(\tilde{X}_{s,s+1})}{\partial Q_s} \\ &\geq 0. && (10) \end{aligned}$$

The marginal expected cost of running out of stock during the next period is known at the beginning of the period, but obviously depends on circumstances.[4] We close by linking this period-by-period price of convenience to the multiperiod one defined in Equation (7). Consider the futures contract that has one period to go; or, if no such contract is traded, work with the futures price we would have seen in a complete market, namely, the risk-adjusted expectation about the next spot price. Starting from Equation (8) for time t, $t+1$, we can use Equation (7) to obtain:

$$\begin{aligned} \mathrm{E}_t^q(\tilde{S}_{t+1}) &= f_{t,t+1} \\ &= (S_t + C)(1 + r_{t,t+1}) - Y_{t,t+1}. && (11) \end{aligned}$$

This will hold also for the period starting at $t+1$, and so on. Repeatedly take expectations of the time $(t+1)$ until T version, conditional on time-t until $T-1$ information, and substitute Equation (9) into it. By comparing again to Equation (7), we obtain that the multiperiod price of convenience is the capitalized future value of a stream of period-by-period prices of convenience (see the Appendix for a proof):

$$Y_{t,T} = \mathrm{E}_t^q \left(Y_{t,t+1} \Pi_{l=2}^{T-t}(1 + r_{t+l-1,t+l}) + \tilde{Y}_{t+1,t+2} \Pi_{l=3}^{T-t}(1 + r_{t+l-1,t+l}) + ... + \tilde{Y}_{T-1,T} \right). \quad (12)$$

From Equation (12), in order to build up a more informative scarcity model that fits the convenience yield, we need to specify the functional forms for the function $Y(x_t, T - t)$, with x referring to an adequate measure of scarcity. In the strict form of the Kaldor-Working hypothesis all expected future period-by-period convenience yields would be functions of current scarcity x_t. In addition, we know that the term structure of expected future $Y_{s,s+1}$ must be smooth unless there is a harvest during the contract's life. Any unusual jump or bump in the *ex ante* Ys would be smoothed out when traders alter their buying plans to

[4] To further characterize the equilibrium we would also need to specify the supply: what makes holders of inventory sell how much? But at this stage all what is needed is that the equilibrium period-by-period price of convenience is non-negative and its future value is uncertain and, plausibly, higher the lower inventories are.

take advantage of cheap premiums or to avoid excessive ones; the only predictable jumps that cannot be arbitraged away are those caused by a harvest.

Assuming, initially, no harvest, a functional form suitable for estimation is obtained by approximating the term structure of expected future period-by-period yields via an exponential, starting from the spot convenience:

$$\mathrm{E}_t(\tilde{Y}_{t+l,t+l+1}) \approx Y_{t,t+1}(1+g_{t,T})^l, \tag{13}$$

with both $Y_{t,t+1}$ and $g_{t,T}$ being functions of current scarcity only. We can show that the capitalized value, at T, of the future period-by-period convenience yields depends, to a first approximation, on the expected final level times the time to maturity (see the Appendix for a proof):

$$\begin{aligned} Y_{t,T} &\approx Y_{t,t+1}(1+g_{t,T})^{T-t} \cdot (T-t) \\ &= \phi(x_t, S_t, ...) \cdot (T-t). \end{aligned} \tag{14}$$

where $\phi(x_t, S_t, ...) := Y_{t,t+1}(1+g_{t,T})^{T-t}$ or $\mathrm{E}_t(Y_{T-1,T})$.

The smooth approximation in Equation (13) is implausible for contracts where there is a harvest in the interval $[t, T]$. If a harvest does occur, say at T_h, we would expect the convenience yield to drop around that time to a lower level $\mathrm{E}_t^q(Y_{T_h,T_h+1})$, and then resume a new growth path from that new base. In that case,

$$\begin{aligned} Y_{t,T} &\approx Y_{t,t+1}(1+g_{t,T})^{T_h-t}(T_h-t) + \mathrm{E}_t^q(Y_{T_h,T_h+1})(1+g_{T_h,T})^{T-T_h}(T-T_h), \\ &= Y_{t,t+1}(1+g_{t,T})^{T_h-t}(T-t) \\ &\quad + \left[\mathrm{E}_t^q(Y_{T_h,T_h+1})(1+g_{T_h,T})^{T-T_h} - Y_{t,t+1}(1+g_{t,T})^{T_h-t}\right](T-T_h). \end{aligned} \tag{15}$$

In the Kaldor-Working tradition, both expected marginal premiums are functions of current scarcity only, but we can be more general and specify the model as:

$$Y_{t,T} = \phi(x_t, S_t, ...) \cdot (T-t) + \psi(x_t, S_t, ...) \cdot \max(T-T_h, 0). \tag{16}$$

In versions narrowed down to pure inventory functions, the usual Working-curve pattern (positive-valued, negative-sloped and convex) should apply to the entire expression and to $\phi(x)$. About $\psi(x)$, which is actually a difference between two Working curves, we have no priors.

1.3 Modeling Issues *re* Current Scarcity

As there is less consensus on the measurement of current scarcity, we apply our multiplicative model with all three different scarcity proxies: inventory, spot price and both of them (the combined proxy).

Measuring scarcity by inventory:

We follow Gorton *et al.* (2007) and others to model $\phi(x)$ and $\psi(x)$ as cubic spline functions. Departing from the tradition, however, we take the inverse of inventory instead of the inventory. We prefer inverses on *a priori* grounds: in this specification, $Y_{t,t+1}$ automatically goes to zero when inventory goes to infinity, and to infinity when inventory approaches zero. Not surprisingly, then, also in terms of *ex post* goodness of fit this specification does better than a spline in x. Thus, $\phi(x_t)$ for instance is operationalized as:

$$\phi(x_t, ...) \quad = \quad a_1 x_t^{-1} + a_2 x_t^{-2} + a_3 x_t^{-3} + a_4 \, \mathbf{1}_{x_t^{-1} > k_1} \, (x_t^{-1} - k_1)^3 + \ldots . \tag{17}$$

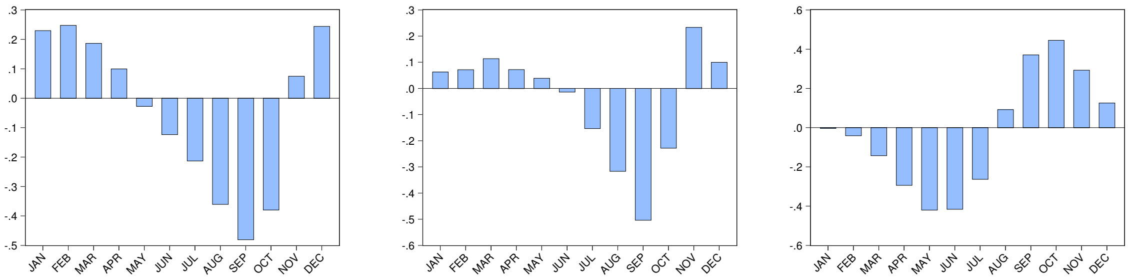
where x is inventory (or, in fact, normalized inventory, see below) and k_1 is the 'knot' where the third derivative is allowed to change. (Specifications with more than one knot do not improve performance.) We define k_1 in our empirical model later. $\mathbf{1}_{x_t^{-1} > k_1}$ is equal to unity if $x_t^{-1} > k_1$, otherwise it equals zero.

The above still ignores the problems with the definition of inventory. Strictly-Chicago inventory numbers ignore availabilities that may be quite nearby, so that they would be noisy proxies of a true variable. But numbers for a wider area are probably noisy too since they ignore the location issues: corn 200 miles away is not the same as corn available in the Chicago harbor. We propose to go on using Chicago data,[5] but we add time dummies that may provide positive evidence that these Chicago data systematically miss part of the story.

There would, of course, be no problem if Chicago's stocks were always a constant fraction of the wider availabilities corrected for location effects. We have strong suspicions that the Chicago data actually lag behind the true availabilities. Figure 1 shows the monthly average of the normalized inventory for corn, wheat and soybeans in Chicago from 1/1986 to 8/2008. Clearly, the graphs indicate that inventories peak in December (corn), in October (wheat) and

[5]It is argued that from the end of the 1970's, Toledo was added as the main market for grain delivery, thus inventories in Toledo should be added to Chicago number to study the model. We also test our model with the total inventories of Chicago and Toledo, but inventories in Chicago still gives us better result.

Figure 1: Seasonality of normalized total inventory from 1986 to 2008 in Chicago



(a) Corn (b) Soybeans (c) Wheat

Note: Given that inventory data are volatile and have a trend, it is not possible to measure seasonality on the raw inventory data. Instead, we compute the deviation from the long-term trend ((raw inventory - trend)/trend) and fit monthly dummies to this deviation to find the seasonality.

in November (soybeans) which is one or two months after the actual harvest times.[6]

We can expect the price of convenience to fall as soon as the harvest begins, which is before reported inventories rise. Misreporting could thus lead to seasonal mistakes in the fitted values. To test for this we add dummies into the spline inventory model $\phi(x)$ and $\psi(x)$:

$$\begin{aligned}(\text{I}{\times}\text{T model:})\ Y_t &= C + \left[\sum_{m=1}^{12} \delta_m \mathbf{1}_{M(t)=m} + \phi(x_t)\right] \cdot (T-t) \\ &\quad + \left[\sum_{m=1}^{12} \rho_m \mathbf{1}_{M(t)=m} + \psi(x_t)\right] \cdot \max(T - T_h, 0). \qquad (18)\end{aligned}$$

where $M(t) = \{1, 2, ..., 11, 12, 1, 2, ...\}$ identifies the month that is associated with observation t. If inventories are under-reported right after the harvest time, then actual convenience yields should be below the average pattern that the regression is trying to predict on the basis of the data it has. Thus, if Chicago inventories are late in reflecting the new harvests, we expect negative regression coefficients for the dummies for the months right after the harvest.

Measuring scarcity by the spot price:

In this case, $\phi(x, S, ...)$ and $\psi(x, S, ...)$ simplify to $a + bS_{t-1}$. It may look unfair to use this simple affine model while inventories enter via a much more flexible spline, but our finding is that powers of prices are not significant; and even with the simple specification, the spot price already beats a spline of x.

[6]Actual harvest time in most of the US and other big producing countries starts from middle of July for corn, middle of May for wheat and September for soybeans.

One potentially useful feature of prices, if they are set in efficient markets, is that they should see through any shortcomings in published inventory data. Thus, the dummies that we inserted into the inventory model to remedy lags in reporting the true availabilities should now be redundant. Therefore, we test for this by also adding dummies, beside the spot price, to get:

$$\begin{aligned}(\text{S}\times\text{T model:})\ Y_t \;&=\; C + \left[\sum_{m=1}^{12} \delta_m \,\mathbf{1}_{M(t)=m} + \beta S_{t-1}\right] \cdot (T-t) \\ &\quad + \left[\sum_{m=1}^{12} \rho_m \,\mathbf{1}_{M(t)=m} + \zeta S_{t-1}\right] \cdot \max(T-T_h, 0). \qquad (19)\end{aligned}$$

Measuring scarcity by both inventory and the spot price:

The multiplicative model for combined proxy is:

$$\begin{aligned}(\text{S+I}\times\text{T model:})\ Y_t \;&=\; C + \left[\sum_{m=1}^{12} \delta_m \,\mathbf{1}_{M(t)=m} + \beta S_{t-1} + \phi(x_t)\right] \cdot (T-t) \\ &\quad + \left[\sum_{m=1}^{12} \rho_m \,\mathbf{1}_{M(t)=m} + \zeta S_{t-1} + \psi(x_t)\right] \cdot \max(T-T_h, 0). \qquad (20)\end{aligned}$$

One problem with the cash prices is that they can differ dramatically even less than 30 miles away; that is, there is no such thing as 'the' spot price that corresponds well to Chicago futures. A common solution is to replace the spot price by the nearest future price, which does not differ from other futures prices in terms of location. It is easily verified that, in the proposed model, the difference between two futures prices has the same format as the spot-futures premium.[7]

We now proceed to the empirical work.

2 Data and main results

2.1 Data and estimation procedures

Data

We use weekly data for corn, wheat and soybeans from 1/1986 to 7/2007 for the March, May, July, September and December contracts (corn and wheat) and for the January, March, May,

[7] We did try cash prices in Illinois region 1, which Chicago is part of. However, the cash price models does worse than the nearest futures prices models. We can provide the results upon request.

July, August, September and November contracts (soybeans). Data for inventory and storage cost are from CBOT (provided by the University of Illinois). Because deliverable inventories are for delivery only, this number is a big understatement of actual inventories since they do not reflect stocks available for delivery that can easily be shipped to official warehouses. Thus, we use the total inventories in selected warehouses[8] in Chicago instead of only the deliverable inventory in our test.[9] In the late 1970's, Toledo was added as a main delivery market for grain. Therefore, one could argue that Toledo inventories should be added to Chicago inventories when considering the scarcity in Chicago. We also test our models with such data and in fact, the total Chicago and Toledo number does worse than the Chicago number for the period from 1986 to 2001 for corn and soybeans (we only have Toledo data in this period for corn and soybeans) and from 1986 to 2007 for wheat.

By a futures price we mean the daily settlement price reported at The Chicago Board of Trade (CBOT). For the interest rate, we use the 3-month LIBOR rates.

Model selection and data preprocessing

When comparing the multiplicative version with the traditional model for the three different scarcity proxies, we use the adjusted-R^2, the Akaike information criterion (AIC) and the Schwarz or Bayesian information criterion (BIC) to test whether the multiplicative model is better than the traditional model and to see which variable is the best proxy for scarcity.[10] In addition, we apply the P-test proposed by Davidson and Mackinnon (1981) (henceforth referred to as the DM test) for simultaneously testing the truth of a base model against several non-nested alternative models. DM's P-test is a standard likelihood-ratio test on whether the fitted-value forecasts from the alternative models improve upon the base model's prediction. When the base model is tested against the other models, the P statistic is asymptotically Chi-squared distributed with degrees of freedom equal to the number of alternative non-nested models if the base model is true. Thus, the lower the P statistic, the more plausible it becomes that the base model might be the true one.

[8]These warehouses are selected by the CBOT.

[9]In fact, as a robustness test, we also applied deliverable data, but the result confirms our argument that the total inventories turns out in a better result than the deliverable one. We can provide the results of this test upon request.

[10]Based on our test results, the I+T model as defined in Equation (5) always outperforms the simpler I model. Thus, we always use the I+T model as our traditional inventory model to compare with the others, instead of the I model itself.

The inventory data and the prices used in the regressions need some pre-processing: quantities are normalized, and prices are deflated. Normalized inventory equals inventory divided by its Hodrick-Prescott trend. On the basis of eyeball tests we settled for a smoothness parameter of 8 E+8. To interpret this, one can refer to the standard values for the smoothness parameter λ_q in quarterly series: 1,600 for quarterly series with peak-to-peak cycles of short duration (roughly 10 years), and 160,000 for cycles of about 30 years or longer. Gorton *et al.* (2007), for instance, use 160,000. Correcting for the effect of frequency (we have weekly instead of quarterly data), our λ is in-between these standards.[11]

We now turn to the second main regressor, prices. Familiarly, these are potentially unit-root variables, which of course would create statistical problems. For commodities, the priors in favor of unit-root characteristics are not as strong as for financial assets. In fact, for the period from 1/1986 to 7/2007, the Phillips-Perron test statistic rejects the null of unit-root properties for corn and soybeans (with *p*s of 2 and at 4%, respectively) but not for wheat (*p* = 59.3%). The Augmented Dickey-Fuller (ADF) test produces nearly the same results as the Phillips-Perron test. In older data, however—not discussed in this paper—unit roots are a problem not just for wheat but for all series. Fortunately, it turns out that simple deflation by the CPI takes care of the unit root for wheat in this period, and also in the older data. The Phillips-Perron test comfortably rejects a unit root for deflated prices: the probabilities are 1.18% for corn, 6.77% for soybeans and 8.63% for wheat. Deflation makes sense on economic grounds too: scaling the prices by the CPI implicitly also deflates the convenience yields; so a dollar paid for convenience is corrected for the near-doubling of the CPI over the 1986-2007 period.

As described below, we also do tests over subperiods. In these subperiods, unit roots are occasionally a problem even with deflated data. When testing whether price peaks are responsible for some of our results, we discovered that omitting the peak episodes also solved the nonstationarity problem. As results hardly differ depending on whether the peaks are included, we show results for the pared-down samples. Details about the starting and ending dates of the subperiods, the omitted data, and the Phillips-Perron p-values in each period for each commodity are given in Table 1.

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[11] To translate λ_q into an equivalent for weekly data, Ravn and Uhlig (2002) recommend multiplication by the fourth power of the relative frequency. Taking 4 weeks in a month, we would get $\lambda_w = \lambda_q \times 12^4 = \lambda_q \times 20,736$, implying an equivalent quarterly smoothness of $\lambda_q = 38,500$—in-between the 1600 and 160,000 standards.

Eviews' default option is to multiply by the square of the relative frequency instead of the fourth power. This provides a much lower smoothness, and led to a trend that totally overfits the series.

Table 1: Details about periods using for the tests

Periods	'86-07'	'89-07'	'89-99'	'00-07'
			Panel A: Corn	
Starting date	1/01/86	1/01/89	1/01/89	1/12/99
Ending date	29/07/07	29/07/07	28/02/99	6/10/06
Deleting period			19/4/96 – 1/06/96	
No. Observations	1126	969	523	358
Phillips-Perron p-value	0.012	0.021	0.049	0.074
			Panel B: Soybeans	
Starting date	1/01/86	1/01/89	1/01/89	1/12/99
Ending date	29/07/07	29/07/07	28/02/99	29/07/07
Deleting period				3/12/03 – 2/7/04
No. Observations	1126	969	530	369
Phillips-Perron p-value	0.067	0.011	0.022	0.038
			Panel C: Wheat	
Starting date	1/01/86	1/01/89	1/01/89	7/01/00
Ending date	29/07/07	29/07/07	31/12/99	30/06/06
Deleting period			19, 26/4/96	
No. Observations	1126	969	572	339
Phillips-Perron p-value	0.086	0.055	0	0.076

The next issue to be settled is how to use the multiple close prices, one per traded contract, that are available on any given date. A common approach is to splice together the data into one long time series, but this means that at any given date just one price is used, even though there may be five (for corn and wheat) or seven (for soybeans) prices available. We can do better. Note that in our multiplicative model all commodity futures prices, regardless of their delivery date, are driven by the same term structure of expected one-period convenience yields. If our assumption holds that this term structure is either sloping upward or downward with, possibly, a jump at harvest time, then the same function $\phi() \cdot (T-t) + \psi() \cdot \max(T-T_h, 0)$ should fit all contracts. This implies a clue as to how optimally use the data. Instead of using a single spliced-together series, we splice together all contracts with the same delivery date into parallel time series. This gives us five time series for corn and wheat (March, May, June, September, December) and seven time series for soybeans (January, March, May, June, August, September, November). Each of these, at any given date, has its own time to maturity, $T-t$, and time beyond harvest, $T-T_h$, but all should still be driven by the same short-term convenience $Y_{t+1,t}$. Thus, we apply pooled estimation for the new models with the restriction that the current-scarcity coefficients in $\phi()$ and $\psi()$ are the same for all contracts. For the traditional models, in contrast, we let the impact of spot prices and time to maturity (TTM) vary across contracts (cross section specific coefficients).

Note also that we do not include the maturity month of the contract in the analysis because the delivery date is not fixed precisely,[12] and because the liquidity of a futures contract is very low in its delivery month. All models are estimated using Pooled Least Squares with fixed effects and correcting for period heteroscedasticity and serial correlation in the residuals of a given cross-section (Period SUR).

2.2 Main results

Table 3 summarizes the R^2, AIC, BIC and P statistic for traditional models (Spot, I+T and S+I+T) and our multiplicative models (S×T, I×T and S+I×T) for corn, soybeans and wheat. Next to results for the base period 1986-2007 ('86-07'), the table also presents these parameters for three variant sample periods: the subperiods 1989-1999 ('89-99') and 2000-2007 ('00-07'), *i.e.* pre and post the new delivery rules, and a sample without government inventory policy, 1989-2007 ('89-07'). The conclusions for each of the criteria are quite similar, so in the discussion we often just cite R^2 figures.

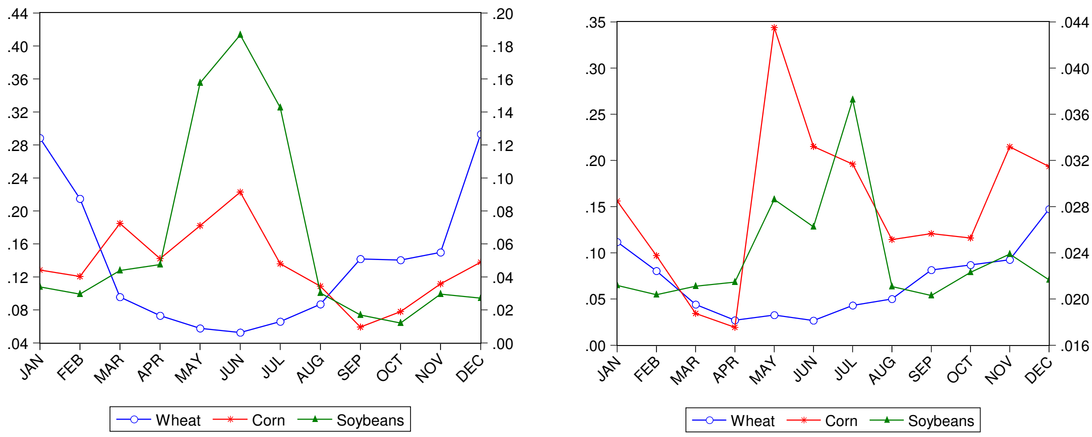
Comparing the results of the periods before and after the year 2000, we find that they are totally different for corn and soybeans but not for wheat. What exactly changed is not obvious. At this stage it just seems safe to separately report our results for the two periods, *i.e.* prior to and after 2000, but we do return to possible explanations toward the end of this section.

First we compare each multiplicative model to its additive counterpart. The finding is that in terms of increased R^2 and improved AIC, BIC and P statistics our multiplicative model outperforms the traditional model for each of the three scarcity proxies and for all commodities in both periods. Thus, given both the theoretical foundation and the empirical confirmation, we conclude the multiplicative model makes more sense than the traditional additive one. Accordingly, most of our diagnoses and comments below focus on the multiplicative versions.

The next issue is the relative performance of the three competing versions of the current-scarcity measures, *i.e.* the various ways of specifying $\phi()$ and $\psi()$. Unsurprisingly, the combined model, with both quantity and price data, does best even after correction for degrees of freedom;

[12] As long as the model is linear in TTM, errors in $T - t$ equally affect all observations, whether TTM is large or small. But in the non-linear models that follow, errors in $T - t$ disproportionably affect observations near the end of the contract's life. For the sake of comparability, we omit the final month everywhere.

Figure 2: Estimated monthly dummies from I×T model



(a) 1989-1999 (b) 2000-2007

Note: In these graph, we plot the dummy coefficients that interact with TTM in I×T model (Equation 18). The scales on the left vertical axis refer to wheat and soybeans, and the numbers on the right refer to corn.

but price always comes in second, and very often as a very close second. Inventory is third, and usually a distant third. Interestingly (to us), the conclusion regarding the performance of inventory would have been rather different if we had relied on the traditional, additive models: then of the six second prizes—two periods, three goods—two would have gone to inventory rather than none; and the distance between the runner-up and the leader, the combined model, would have seemed much more pronounced. Thus, the choice for a multiplicative model matters not just in terms of performance but also for qualitative conclusions: prices do contain almost all relevant information, while quantities add very little. Theoretically this might be a puzzle, as price and quantity are closely related endogenous variables (Brennan, 1958), but perhaps the answer is that, in reality, price is measured with less error than is inventory.

With respect to the quality of stock data, we indeed find clear evidence of problems in the pattern of the dummies: the Chicago inventories are actually lagging behind reality right after the harvest. After accounting for information from reported inventories, a systematic residual unexplained seasonality in the convenience yield is clearly observed from the coefficients of the monthly dummies of our multiplicative inventory model (I×T) in both periods. From Figure 2, it is clear that the dummies are systematically correcting the convenience yield downward right after the harvest time, and upward later in the year. This is consistent with the idea that after the harvest a lot of the output remains in storage in local silos, with very littlle of it being shipped already to the official warehouses. Unexpectedly, however, we observe the same pattern for coefficients of the monthly dummies in the multiplicative price model (S×T). This result, together with the fact that the seasonal dummies tend to be significant, remains a bit of a puzzle, as spot prices ought to have seen through any shortcomings in the inventory data. Still, remember that prices achieve a lot more than quantities. Thus, from a practical point of view, the excellent performance of price as a measure of scarcity is convenient: it allows one to model or predict the convenience yield without needing any inventory data, where various candidates are available, all with different degrees of shortcomings.

Other factors than sluggish adjustments in the Chicago stock levels might have played a role too. We explored two more avenues: Chicago data may be too narrow; and the way we filter or normalize inventory, *i.e.* our choice of the Hodrick-Presscott lambda, may have been too arbitrary. These explanations are not supported. Specifically, the same results hold when we use the combined inventory of Chicago and Toledo or even the entire Great Lakes data, as already mentioned above; and using raw (un-filtered) data gives us the same outcomes, implying that more modest changes in the Hodrick-Presscott lambda would matter even less.

Subperiod results and the role of delivery options

It is well known that one confounding factor in commodity futures, relative to, say, currency contracts, is the existence of delivery options. If asked for delivery by the buyer, the seller can choose his preferred moment during an entire month. There is also a location option: traditionally, the goods could be provided in either Chicago (on Lake Michigan) or Toledo (on Lake Erie). Recently, the location option has been substantially widened.

Until the 1999 December contract, corn and soybeans were delivered via a warehouse receipt or warehouse certificate, whereas as of the 2000 March contract, delivery is via a shipping certificate. For wheat, a similar change took place as of the September 2008 contract. With a shipping certificate, deliverable grain is no longer stored at the shipping points; so this certificate is like a call on cargo that is somewhere being barged down the Illinois and Mississippi rivers.

The change in the delivery systems could affect the informativeness of both inventory and price data. First, Chicago warehouse stocks are possibly less relevant since the introduction of shipping certificates. Second, with the product becoming increasingly fuzzy, even the interpretation of futures prices becomes more difficult. Thus, we expect all models to become worse at explaining the observed yield.

To test for this, we split the '89-07' period into two sub periods, pre- and post-2000. As of

1/3/1999 the new delivery system has been in vogue, starting with the March 2000 contract; thus, the '1989-1999' period starts from 1/1989 and ends on 28/2/1999.[13] The '2000-2007' period runs from 12/1999 until 7/2007 because from 12/1999 all quoted futures contracts were 2000 contracts for which the new delivery system was effective. Recall the change in delivery only applies to corn and soy; for wheat it occurred outside the sample period. We nevertheless apply the same subperiod test to wheat too, by way of control.

The picture that emerges from comparing the two middle panels in Table 3 is unclear, We do see that the explanatory power of the multiplicative inventory model (using warehouse stocks) does go down for corn, from 50 to 40 percent; but we see the same for wheat (from 46 to 39) even though there the delivery system did not change, and we see no drop in R^2 for soybeans, where the change was similar to that for corn. For all three goods we also note a drop in the performance of price-based models and combined versions. For corn and soy this is consistent with confounding effects from the new delivery system, but the fact that we also see this for wheat tells us that there must have been other factors at work too.

We can sum up our findings, thus far, as follows: (i) the multiplicative model does much better than the additive contender; (ii) prices do much better than inventory data—almost as good, in fact, as the combined models; and (iii) the post-2000 period seems to be harder to model. To close the discussion of the results from this sample, we discuss some additional checks.

2.3 Validity and robustness checks

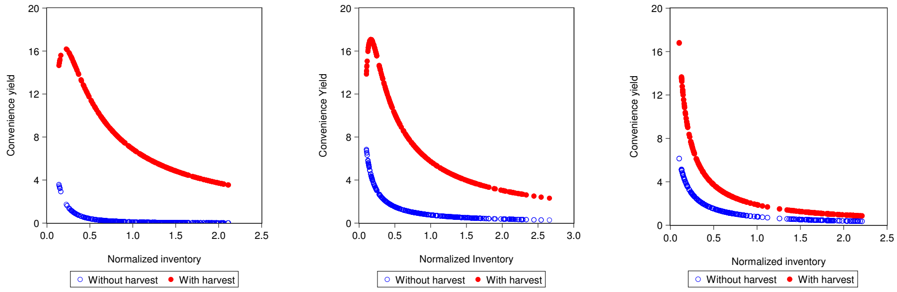
The first diagnostic question is whether inventory models still exhibit the Working-curve shape, the convex negative relation that Working predicted when plotting convenience against inventory. We then end with a discussion of some additional validity and robustness checks.

Is there still a Working curve?

The fact that the price seems to be more informative than the storage data does not mean that pure inventory-based data have no merit. In fact, their performance is still quite respectable in terms of R^2 etc. Yet, at this stage we have just considered measures of fit and significance, not the shape of the yield/inventory relation. A good fit does not automatically mean that the

[13] We test for the delivery system effect from 1989 instead from 1986 because we also want to be free from the government loan program's effect in this analysis (see below, robustness checks).

Figure 3: Convenience yield-inventory curve for wheat, corn and soybeans 2000 - 2007



(a) Corn (b) Soybeans (c) Wheat

Note: The graphs show the fitted value of the convenience yields based on Equation (18) with the pooled estimation results. The graphs are split by commodity (corn, wheat or soybeans) for the 2000-2007 period. In one graph, there are two curves: (i) for contracts mature before the harvest time (without harvest); (ii) for after the harvest time (with harvest).

resulting curve has the Working shape, nor does a relatively poor fit mean that the Working pattern is absent.

To calculate fitted values for the curve, we keep just the terms in the spline that are significant. Results from the pared-down estimations are shown in column ‘I×T, pruned’ of Tables 4. We calculate two different fitted values for the convenience yield of each commodity. The first fitted function is $\phi(x_t) \cdot (T-t)$, and is applicable when the contract matures before the harvest time, while the second fitted function, $\phi(x_t) \cdot (T-t) + \psi(x_t) \cdot \max(T-T_h, 0)$, applies when the contract matures after the harvest. The significance of the $\phi(x_t) \cdot (T-t)$ and $\psi(x_t) \cdot (T-T_h)$ terms indicates that the Working curve depends on time to maturity and also, when relevant, on time to harvest. We show plots of fitted values for the December contract for corn, November contract for soybeans and September contract for wheat with TTM $= 260$ days, using the 2000-2007 estimates.[14] Because in this period the corn December contract has $T-T_h$ equal 165, the set time to maturity gives 95 days for the period beyond the harvest for corn. These numbers are 170 for soybeans and 125 for wheat.

Figures 3[15] show plots of the fitted values against the normalized inventories. The top curve refers to a situation with a harvest intervening during the contract's life, the bottom one has no harvest in-between. Clearly, a Working curve can be observed for each commodity in

[14] Working curves for other contracts and for the period ‘89-99’ are available upon request.

[15] In these figures, we delete some outliers where the reported normalized inventories are less than 0.05. Prices in such low inventory conditions are likely to be driven more by underreporting or stock-out effects than by inventory levels.

each of the graph. In addition, the shape of the curve changes for different time to maturities and when the contract goes through the harvest.

We close with two remarks. First, for very low inventory levels the slope of the curve anomalously changes slope. Our guess is that these data points reflect under-reported inventories. That is, Chicago stocks may occasionally be quite low relative to availabilities nearby, so that the yield looks low relative to the reported inventory. Stated differently, if we had more representative inventory figures, these data points would have been more to the right and would have fitted in with the regular Working curve. Our second remark is that the plots are obtained from the pure inventory model. Yet, comparing the coefficients for the inventory terms in the regressions with and without price data in Tables 4, we recognize that the curves change drastically between the two models, especially for the period 1989-1999 when the spot price is more informative. Exactly what the residual role of inventories is, once the spot price has captured its part of the yields, is not clear.

Additional validity and robustness checks

We close with a discussion of issues related to (i) a potentially disturbing outside factor, the government loan program; (ii) the price spikes in the 1990s; and (iii) the regression specification.

During the mid-1980's, a government loan provided strong incentives to farmers to store, so it led to huge stocks in this period. While these stocks were not directly available to the market, they were still known to exist. As a result, this factor may create a contamination of the convenience yield. After the 1988 drought, the government sold off all stocks and changed it stockholding program. Thus, in order to see whether the government loan program has any effect on our analysis, we also do our test for the period after this program (from 1/1989 to 7/2007). The results are shown alongside the main results, in the lower panel of Table 3, and do not reveal any remarkable impact from omitting the potentially confounding years.

Having ditched the idea that the delivery system upset the old relations (see subperiod results, above), we explored the idea that the power of the spot-price model may very well depend on its variability in the sample. Notably, there were extreme peaks of the spot price in the '89-99' period. Logically, during such episodes the futures prices would rise less, as they reflect future expected prices, so extreme spot prices would tend to go hand in hand with high backwardation. But we get essentially the same results, with the same good R^2s for the price-based model, whether we include the peak periods into the sample or not.

In terms of regression specification, lastly, one question we raise is whether the improved performance of the multiplicative models is really due to the interaction of time to maturity and scarcity, or whether it is, instead, mostly ascribable to the seasonal dummies, which are absent from the standard models. We test for this by estimating the following two incomplete versions of the multiplicative models, one with just the interactive term and no dummies, and one without the interactive term but with dummies. For the spot-price model, this would read as:

$$\begin{aligned} &\text{(Model 1:)} \quad Y_t = C + \beta S_{t-1} \cdot (T-t) + \zeta S_{t-1} \cdot \max(T - T_h, 0) \\ &\text{(Model 2:)} \quad Y_t = C + \textstyle\sum_{m=1}^{12} \delta_m \mathbf{1}_{M(t)=m} + \beta S_{t-1} \end{aligned} \tag{21}$$

We find that for all commodities and periods, Model 1 always beats Model 2 in terms of R^2, AIC, BIC and P statistics.[16] This result indicates that the interaction terms are more important than the seasonal effects.

The second issue relates to the internal validity of our model. The question is whether the interaction between scarcity and time to maturity succeeds in capturing all time-related effects in the convenience yield. Specifically, by construction the average residual is zero for each regression and sample, but zero averages should also be observed in subsets of observations, regardless of how the subsets are composed. Thus, we sort all observations into buckets defined by commodity and time to maturity. Specifically, for corn and wheat we have ten buckets per sample, corresponding to 9, 13, 17, 22, 26, 30, 31, 35, 39, 43 weeks to maturity, and for soybean we have 13 buckets (4, 8, 9, 13, 17, 22, 26, 30, 35, 39, 43, 44, 48 weeks to maturity). We retrieve the residuals from the regressions, and compute average residuals for each bucket. The null is that all these means are equal to each other and to zero. In Table 2, we report the p-values from the Anova F-test and Welch F-test for the equality of the mean of these error term series for each commodity from the S+I×T model. The Anova F-test assumes that the series also have equal variance while the Welch test relaxes this assumption. Table 2 shows that even the best-performing multiplicative model still misses part of the time-to-maturity effects: of the six[17] standard Anova tests, two reject the hypothesis, and the Welch test identifies an additional suspect case. Thus, even though the results are not disastrous, there is still room for future research. The prime area of potential improvement is probably the theoretical assumption that the term structure of expected future week-by-week convenience yields is approximately

[16] We can provide this estimation result on request.

[17] three goods, two subperiods

Table 2: P-values (%) from the F-test for the equality of the mean error across time-to-maturity classes

	Corn		**Wheat**		**Soybeans**	
	'89-99'	'00-07'	'89-99'	'00-07'	'89-99'	'00-07'
Anova F-test	12.45	18.90	0.38	96.38	2.07	73.62
Welch F-test	33.68	1.25	1.39	34.15	0.00	8.47

Key Residuals are retrieved and put into buckets defined by time to maturity. For corn and wheat we have ten buckets per sample, corresponding to 9, 13, 17, 22, 26, 30, 31, 35, 39, 43 weeks to maturity, and for soybean we have 13 buckets (4, 8, 9, 13, 17, 22, 26, 30, 35, 39, 43, 44, 48 weeks to maturity). The Anova F-test assumes that the series also have equal variance while the Welch test relaxes this assumption.

exponential and that its present value is not very sensitive to the ever-changing discount rate and slope of that term structure.

We continue the discussion of the 1986-2007 results with a digression on the assumedly additive effect of the seasonal and some additional checks.

Under the additive specification adopted above, the assumed effect of the seasonal on the yield is always the same, regardless of the level of inventories or the level of fitted values. We considered two alternatives. In the first, the unmodelled seasonal factors pertain not directly to the yield but to the short-term convenience over the current week. That is, the true marginal convenience consists of a term $\phi()$ related to current inventory and/or price plus unidentified seasonal factors. Depending on whether we also allow for information from the spot price, the extended models become:

$$\begin{aligned}(\text{I}\times\text{T-ext:})\ Y_t &= C + \left[\sum_{m=1}^{12} \delta_{1,m}\, \mathbf{1}_{M(t)=m} + \sum_{m=1}^{12} \delta_{2,m}\, \mathbf{1}_{M(t)=m} \cdot \phi(x_t)\right] \cdot (T-t) \\ &\quad + \left[\sum_{m=1}^{12} \rho_1 m\, \mathbf{1}_{M(t)=m} + \sum_{m=1}^{12} \rho_2 m\, \mathbf{1}_{M(t)=m} \cdot \psi(x_t)\right] \cdot \max(T-T_h, 0).\end{aligned} \tag{22}$$

and

$$\begin{aligned}(\text{S}+\text{I}\times\text{T-ext:})\ Y_t &= C + \left[\sum_{m=1}^{12} \delta_{1,m}\, \mathbf{1}_{M(t)=m} + \beta S_{t-1} + \sum_{m=1}^{12} \delta_{2,m}\, \mathbf{1}_{M(t)=m} \cdot \phi(x_t)\right] \cdot (T-t) \\ &\quad + \left[\sum_{m=1}^{12} \rho_1 m\, \mathbf{1}_{M(t)=m} + \zeta S_{t-1} + \sum_{m=1}^{12} \rho_2 m\, \mathbf{1}_{M(t)=m} \cdot \psi(x_t)\right] \cdot \max(T-T_h, 0).\end{aligned} \tag{23}$$

In this model, the slope and level of the Working curve varies across months, but the monthly curves are still restricted to be affine transforms of each other. To test whether the coefficients associated with the months are statistically different we apply a Wald test for H_0: $\delta_{1,m} = \delta_1$, $\forall m$ and $\delta_{2,m} = \delta_2$, $\forall m$. The Wald test for the coefficients from these two models confirm that the curves are significantly different across months. However, the explanatory power of the extended-seasonal model for the combined proxy is hardly better than that of our proposed model: the adjusted R^2 of the extended-seasonal model is about 2% to 4% higher for most of cases. Also, this specification seems to be more prone to overfitting, picking up nonsense effects: plots of Working curves are not as well-behaved as the curves we obtain in the base case.

A second variant we tried again related to the seasonal dummies. Inspired by the evidence that the seasonals seem to pick up the effects of systematic sluggishness in Chicago storage data, we tie the seasonal effect directly to the level of the inventory rather than to the current short-term convenience yield $\phi()$ or to the total convenience yield. That is, we conjecture that the current short-term yield ϕ depends on true storage levels x, which are equal to reported storage $\hat{x}$ data corrected for systematic seasonal errors that are multiplicative:

$$x_t = \hat{x}_t \prod_{m=1}^{12} (1 + \delta_m \, \mathbf{1}_{M(t)=m}), \tag{24}$$

and substitute this function into the spline. While intellectually appealing, this model seems to be too non-linear and complicated to estimate. We get nonsense results and, occasionally, failure to converge.

We close with two more robustness checks, or tests with slightly different specifications. First, in Equation (16), all convenience assumedly disappears when the contract expires; that is, spot-futures convergence is perfect. In reality, however, even at day T the cash position earns a premium over a futures contract because the seller of the futures contract can postpone delivery for up to a month, and choose a location and a quality that suits him best. Thus, an encompassing model, including the expected convenience at expiry denoted by $\chi(x_t)$, would be:

$$Y_{t,T} \quad = \quad C + \phi(x_t) \cdot (T - t) + \psi(x_t) \cdot \max(T - T_h, 0) + \chi(x_t), \tag{25}$$

possibly enriched with seasonal dummies. The results are not encouraging. The extra four coefficients are not jointly significant. Relatedly, the nice Working curves $\phi(x_t)$ we get from the basic (I×T) model often degenerate into nonsense patterns, presumably because three splines in the same variable x create too much collinearity. We dropped the idea in later applications.

Lastly we added flow variables, notably changes of inventories. They were not significant.

3 Conclusion

Empirical work on the relation between convenience yield and scarcity has mostly adopted regressions of the form $Y = f(x, S, ...)$ or $Y = f(x, S, ...) + b\,(T - t)$ and has focused on how to specify $f(x, S, ...)$, not on the impact of time to maturity and time to harvest on scarcity. Our analytical contribution is that the overall scarcity effect $f(x, S....)$ should be modeled as an interaction with time to maturity and that, whenever relevant, a similar term should be added for the part of time to maturity beyond harvest time: $Y = \phi(x, S, ...) \cdot (T - t) + \psi(x, S, ...) \cdot \max(T - T_h, 0)$.

In our empirical work we compare the multiplicative version with the traditional model for three types of scarcity measures: inventory, spot price and both of them. We choose these three approaches based on the three competing views from the literature on the determinants of backwardation. All three agree that backwardation reflects a convenience yield that rises when scarcity is higher, as proposed by Kaldor (1939) and Working (1948); but the competing propositions disagree as to how scarcity is best measured. The first view holds that the convenience yield only depends on time-t inventory. At the other extreme, others propose that the spot price is sufficient in representing all scarcity indicators, which would then leave no role for inventory. The more middle-of-the-road proposition predicts that the convenience yield is driven by a number of proxies for scarcity including the inventory level and time to maturity, and sometimes the spot price. We also examine whether there is one proposition that always outperforms the others, or whether each proposition has its merits for a specific condition or for a special period of the commodity markets. For this purpose, we use nearly 22 years of weekly data from 1/1986 to 7/2007, for corn, wheat and soybeans.

Our analysis shows that, for all commodities and all periods, the multiplicative specification does a better job than the traditional model, whether scarcity is measured by the spot price, inventory, or both. We also find that, in the quantity-based variants in our model the traditional "Working curve" is still very present, even though these curves do change depending on time to maturity and time beyond the harvest, if any. Our results also show that the government loan program in the middle 1980's has little impact on the model for convenience yield. Nor has the changing in the delivery system for corn and soybeans since 2000 had any clear impact on the explanatory power of Chicago storage data. A new finding that emerges from the better modeling of the yields is that prices in fact do a better job capturing the yield than do the storage data, even though the latter get a four-parameter spline to play with while the price term is just a simple, linear item. In fact, prices do almost as well as the combined model. This result is useful for hedgers, farmers and practitioners, as it is not obvious what kind of inventory data one should look for and available stock data seems to have some flaws.

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Appendices

Derivation of the multi-period convenience yield (Equation (12)): Starting from Equation (8) for time t, $t+1$, we can use Equation (7) to obtain:

$$\begin{aligned} \mathrm{E}_t^q(\tilde{S}_{t+1}) &= f_{t,t+1} \\ &= (S_t + C)(1 + r_{t,t+1}) - Y_{t,t+1}. \end{aligned} \tag{26}$$

This will hold also for the period starting at $t+1$, and so on. Take expectations of the time $(t+1)$ until T version, conditional on time-t information, and substitute Equation (9) into it:

$$\begin{aligned} \mathrm{E}_t^q(\tilde{S}_{t+2}) &= \mathrm{E}_t^q(\tilde{S}_{t+1}+C)(1+r_{t+1,t+2}) - \mathrm{E}_t^q(\tilde{Y}_{t+1,t+2}), \\ &= S_t\,(1+r_{t,t+1})(1+r_{t+1,t+2}) + C\,[(1+r_{t,t+1})(1+r_{t+1,t+2}) + (1+r_{t+1,t+2})] \\ &\quad -[Y_{t,t+1}(1+r_{t+1,t+2}) + \mathrm{E}_t^q(\tilde{Y}_{t+1,t+2})]. \end{aligned} \tag{27}$$

Repeated application yields

$$\begin{aligned} \mathrm{E}_t^q(\tilde{S}_T) &= S_t\,\Pi_{l=1}^{T-t}(1+r_{t+l-1,t+l}) + C\left(\Pi_{l=1}^{T-t}(1+r_{t+l-1,t+l}) + \Pi_{l=2}^{T-t}(1+r_{t+l-1,t+l}) + ... + (1+r_{T-1,T})\right) \\ &\quad -\mathrm{E}_t^q\left(Y_{t,t+1}\Pi_{l=2}^{T-t}(1+r_{t+l-1,t+l}) + \tilde{Y}_{t+1,t+2}\Pi_{l=3}^{T-t}(1+r_{t+l-1,t+l}) + ... + \tilde{Y}_{T-1,T}\right). \end{aligned}$$

Next we apply Equation (8) on the left-hand side while on the right-hand side we use obvious results for risk-free term structures, to obtain

$$\begin{aligned} f_{t,T} &= [S_t + \mathrm{PV}(C,\mathbf{r},t,T)]\,(1+r_{t,T}) \\ &\quad -\mathrm{E}_t^q\left(Y_{t,t+1}\Pi_{l=2}^{T-t}(1+r_{t+l-1,t+l}) + \tilde{Y}_{t+1,t+2}\Pi_{l=3}^{T-t}(1+r_{t+l-1,t+l}) + ... + \tilde{Y}_{T-1,T}\right). \end{aligned}$$

Comparing to Equation (7) we see that the multiperiod price of convenience is the capitalized future value of a stream of period-by-period prices of convenience:

$$Y_{t,T} = \mathrm{E}_t^Q\left(Y_{t,t+1}\Pi_{l=2}^{T-t}(1+r_{t+l-1,t+l}) + \tilde{Y}_{t+1,t+2}\Pi_{l=3}^{T-t}(1+r_{t+l-1,t+l}) + ... + \tilde{Y}_{T-1,T}\right).$$

Derivation of Equation (14):

To show this, consider the future value computed from the yield forecasts and the interest rates. Once that value is known we can always compute the IRR of the operation. Let us denote the IRR (on a per-period basis) by R. Below, we first write the defining property of R, we rearrange by factoring out $(1+g_{t,T})^{T-t}$, and use some familiar properties for fixed-rate operations; lastly, we simplify:

$$
\begin{aligned}
Y_{t,T} &= Y_{t,t+1}\left[(1+R_{t,T})^{T-t} + (1+g_{t,T})(1+R_{t,T})^{T-t-1} ... + (1+g_{t,T})^{T-t-1}(1+R_{t,T}) + (1+g_{t,T})^{T-t}\right] \\
&= Y_{t,t+1}(1+g_{t,T})^{T-t}\left[\left(\frac{1+R_{t,T}}{1+g_{t,T}}\right)^{T-t} + \left(\frac{1+R_{t,T}}{1+g_{t,T}}\right)^{T-t-1} ... + \left(\frac{1+R_{t,T}}{1+g_{t,T}}\right) + 1\right] \\
&= Y_{t,t+1}(1+g_{t,T})^{T-t}\frac{(1+R'_{t,T})^{T-t}-1}{R'_{t,T}} \text{ where } 1+R'_{t,T} := (1+R_{t,T})/(1+g_{t,T}), \\
&\approx Y_{t,t+1}(1+g_{t,T})^{T-t}\frac{1+(T-t)R'_{t,T}-1}{R'_{t,T}}, \\
&= Y_{t,t+1}(1+g_{t,T})^{T-t}\cdot(T-t) \\
&= \phi(x_t)\cdot(T-t).
\end{aligned}
$$

where under the strong form of the hypothesis $\phi(x_t) := Y_{t,t+1}(1+g_{t,T})^{T-t} = \mathrm{E}_t(Y_{T-1,T})$ is assumed to be a function of just current scarcity.

Table 3: Comparisons of proposed models to traditional models for corn, soybeans and wheat for the recent sample 1986-2007

	Panel A: Corn						Panel B: Soybeans						Panel C: Wheat					
	Traditional models			Proposed models			Traditional models			Proposed models			Traditional models			Proposed models		
	Spot	I+T	S+I+T	S×T	I×T	S+I×T	Spot	I+T	S+I+T	S×T	I×T	S+I×T	Spot	I+T	S+I+T	S×T	I×T	S+I×T
	Full sample, 1986-2007																	
R^2(%)	52.38	32.59	73.65	78.21	32.91	78.31	30.65	30.95	58.21	58.28	28.12	60.40	48.74	29.44	70.55	75.79	30.58	76.02
AIC	7.488	7.838	6.898	6.710	7.837	6.707	8.902	8.896	8.395	8.397	8.934	8.343	8.138	8.461	7.586	7.391	8.449	7.384
BIC	7.502	7.858	6.925	6.749	7.884	6.757	8.916	8.914	8.420	8.429	8.977	8.384	8.152	8.481	7.613	7.430	8.496	7.434
P-stat	6644	10496	1706	362	10299	331	6830	6944	1862	1463	7400	1065	6624	10284	1925	485	9599	436
	Delivery system 1 (warehouse receipts): 1989-1999																	
R^2(%)	52.12	49.81	85.41	87.23	50.07	88.50	45.69	35.05	71.51	71.47	38.75	75.26	48.72	41.66	77.05	83.66	45.81	84.20
AIC	7.496	7.545	6.312	6.183	7.549	6.082	8.587	8.761	7.940	7.948	8.709	7.803	8.155	8.286	7.356	7.019	8.221	6.989
BIC	7.524	7.584	6.364	6.257	7.640	6.178	8.613	8.796	7.988	8.009	8.782	7.880	8.180	8.322	7.403	7.086	8.304	7.077
P-stat	9797	9404	1634	871	9240	572	5024	6475	1209	1155	6114	604	7534	8898	2204	589	7040	486
	Delivery system 2 (shipping certificate): 2000-2007																	
R^2(%)	29.96	38.21	65.27	71.61	40.59	72.59	19.30	36.56	55.95	52.24	38.27	64.20	48.27	38.45	70.38	75.18	38.50	76.80
AIC	5.995	5.873	5.300	5.104	5.846	5.074	8.166	7.927	7.566	7.650	7.909	7.365	6.820	6.997	6.269	6.098	7.009	6.036
BIC	6.032	5.924	5.370	5.203	5.968	5.203	8.202	7.974	7.630	7.732	8.007	7.468	6.858	7.051	6.342	6.202	7.132	6.166
P-stat	2387	1871	465	79	1677	21	3348	2433	1041	1253	2323	433	2605	2863	839	321	2727	205
	Sample without loan-program years: 1989-2007																	
R^2(%)	59.22	32.01	78.00	82.07	33.59	82.31	37.23	30.59	61.31	58.57	26.13	60.76	54.03	31.49	75.14	81.51	32.59	82.27
AIC	7.302	7.814	6.687	6.484	7.795	6.473	8.802	8.900	8.317	8.389	8.966	8.333	7.883	8.283	7.271	6.977	8.272	6.937
BIC	7.318	7.837	6.698	6.528	7.849	6.530	8.818	8.920	8.345	8.426	9.009	8.379	7.899	8.306	7.302	7.020	8.325	6.993
P-stat	6779	12542	1894	484	12046	418	5429	6780	1437	1708	7597	1332	9120	15346	3315	912	13252	705

Note: 1. The table summarizes results on convenience yields from six models: the traditional models for spot price (Spot), for inventory/time-to-maturity additive (I+T), for spot price and inventory (S+I+T) and the multiplicative models for spot price (S×T), for inventory/time-to-maturity additive (I×T), for spot price and inventory (S+I×T).

(Spot) : $Y_{t,T} = \alpha + \beta S_{t-1}$,

(I+T) : $Y_{t,T} = \alpha + \theta_1 x_t + \theta_2 x_t^2 + \theta_3 x_t^3 + \theta_4 \mathbf{1}_{x_t>k}\,(x_t - k)^3 + \gamma\,(T-t)$, with x = norminv;

(S+I+T) : $Y_{t,T} = \alpha + \beta S_{t-1} + \theta_1 x_t + \theta_2 x_t^2 + \theta_3 x_t^3 + \theta_4 \mathbf{1}_{x_t>k}\,(x_t - k)^3 + \gamma\,(T-t)$, with x = norminv;

(S×T) : $Y_{t,T} = \alpha + (\sum_m \delta_m \mathbf{1}_{M(t)=m} + \beta S_{t-1}) \cdot (T-t) + (\sum_m \rho_m \mathbf{1}_{M(t)=m} + \zeta S_{t-1}) \cdot \max(T - T_h, 0)$,

(I×T) : $Y_{t,T} = \alpha + (\sum_m \delta_m \mathbf{1}_{M(t)=m} + f(x_t)) \cdot (T-t) + (\sum_m \rho_m \mathbf{1}_{M(t)=m} + g(x_t)) \cdot \max(T - T_h, 0)$, with $x = 1/\text{norminv}$,

(S+I×T) : $Y_{t,T} = \alpha + (\sum_m \delta_m \mathbf{1}_{M(t)=m} + \beta S_{t-1} + f(x_t)) \cdot (T-t) + (\sum_m \rho_m \mathbf{1}_{M(t)=m} + \zeta S_{t-1} + g(x_t)) \cdot \max(T - T_h, 0)$, with $x = 1/\text{norminv}$.

norminv is normalized inventory with long-term trend of more than 20 years (smoothness parameter of 0.8b). $\mathbf{1}_{x>k_1} = 1$ if $norminv > k_1$, otherwise $\mathbf{1}_{x>k_1} = 0$ for I+T and S+I+T. $\mathbf{1}_{x>k_2} = 1$ if $1/\text{norminv} > k_2$, otherwise $\mathbf{1}_{x>k_2} = 0$ for S×T, I×T, and S+I×T.

2. In this table, $k_1 = 2$, 1.9 and $k_2 = 10$, 12 for corn and soybeans consecutively for both periods. For wheat, $k_1 = 2.2$, $k_2 = 60$ for the '89-99' period; and 1.9, 45 for the '00-07' period.

3. For corn, the harvest time in this period starts from 15 July, thus $T - T_h = 255$ for the March, 315 for the May, 375 for the July, 75 for the September and 165 for the December contract. For soybeans, the harvest time in this period starts from 1 September, thus $T - T_h = 150$ for the January, 210 for the March, 270 for the May, 330 for the July, 360 for the August, 30 for the September and 90 for the November contract. For wheat, the harvest time in this period starts from 15 May, thus $T - T_h = 315$ for the March, 375 for the May, 75 for the July, 135 for the September and 225 for the December contract.

4. P-stat is the P test statistic based on the DM test to simultaneously compare one model with other non-nested models. ***, ** and *indicate the significant coefficients at 1%, 5% and 10% consecutively.

Table 4: Estimation results for corn, wheat and soybeans for the periods '89-99' and '00-07'

	Panel A: Corn				Panel B: Soybeans				Panel C: Wheat			
	'89-99'		'00-07'		'89-99'		'00-07'		'89-99'		'00-07'	
	S+I×T	I×T, pruned	S+I×T	I×T, pruned	S+I×T	I×T, pruned	S+I×T	I×T, pruned	S+I×T	I×T, pruned	S+I×T	I×T, pruned
$Alpha - jan$					2.27	1.90	-0.08	0.10				
$Alpha - mar$	-1.14	-1.19	-0.34	-0.33	-1.69	-2.14	-1.85	-1.63	-5.72	-5.74	-4.26	-4.28
$Alpha - may$	-2.22	-2.10	-0.03	-0.03	-3.49	-3.75	-1.50	-1.38	-1.20	-1.28	-1.10	-1.24
$Alpha - jul$	-1.48	-1.54	0.63	0.62	-4.73	-4.54	-1.94	-2.05	7.42	7.44	5.43	5.47
$Alpha - aug$					-3.20	-2.71	-0.06	-0.31				
$Alpha - sep$	2.00	2.01	-0.04	-0.04	3.52	3.94	2.32	2.13	2.91	2.96	2.19	2.26
$Alpha - nov$					7.29	7.17	2.91	2.98				
$Alpha - dec$	2.84	2.82	-0.28	-0.28					-4.41	-4.37	-2.92	-2.86
$(T-t) * spot$	0.002^{***}		0.001^{***}		0.001^{***}		0.001^{***}		0.001^{***}		0.001^{***}	
$(T-T_h) * spot$	0.001^{***}		$3.1e-4^{**}$		0.001^{***}		$-6.3e-4^{**}$		0.001^{**}		0.001^{**}	
$(T-t) * x1$	0.015^{***}	0.029^{***}	-0.001		0.014^{***}	0.008^{***}	0.009^{***}	0.003^{***}	$6.1e-4^{***}$	0.013^{***}	0.001^{***}	0.005^{***}
$(T-t) * x2$	-0.002^{***}	-0.003^{***}	$6.9e-4$	$4.5e-4^{***}$	-0.001^{***}	$-4.0e-4^{***}$	-0.001^{***}		$-1.7e-5$	$-4.5e-4^{***}$	$-2.0e-5^{***}$	$-8.7e-5^{***}$
$(T-t) * x3$	$6.0e-5^{***}$	$1.2e-4^{***}$	$-4.1e-5$	$-2.6e-5^{***}$	$4.2e-5^{***}$	$4.2e-6^{***}$	$2.2e-5^{***}$	$-2.6e-6^{***}$	$1.1e-7$	$3.8e-6^{***}$	$1.1e-7^{***}$	$4.8e-7^{***}$
$(T-t) * x4$	$-6.0e-5^{***}$	$-1.2e-4^{***}$	$1.0e-4$		$-4.3e-5^{***}$		$-2.2e-5^{***}$	$3.4e-6^{***}$	$-5.4e-8$	$-9.6e-6^{***}$	$-1.1e-7^{***}$	$-4.8e-7^{***}$
$(T-T_h) * x1$	-0.009^{***}		-0.038^{***}	0.049^{***}	0.029^{***}	0.054^{***}	0.060^{***}	0.063^{***}	0.014^{***}	0.023^{***}	0.008^{***}	0.013^{***}
$(T-T_h) * x2$	0.001^{*}	$-1.5e-5^{***}$	0.012^{***}	-0.008^{***}	-0.003^{**}	-0.006^{***}	-0.007^{***}	-0.008^{***}	-0.001^{***}	-0.002^{***}	$-1.9e-4^{***}$	$-3.3e-4^{***}$
$(T-T_h) * x3$	$-3.4e-5^{*}$		$-7.4e-4^{***}$	$3.9e-4^{***}$	$7.1e-5^{**}$	$1.9e-4^{***}$	$2.2e-4^{***}$	$2.6e-4^{***}$	$9.4e-6^{***}$	$2.6e-5^{***}$	$1.1e-6^{***}$	$1.9e-6^{***}$
$(T-T_h) * x4$	$3.5e-5^{*}$		0.003^{***}	$-9.9e-4^{*}$	$-7.1e-5^{**}$	$-2.0e-4^{***}$	$-2.3e-4^{***}$	$-2.8e-4^{***}$	$-2.5e-5^{***}$	$-6.3e-5^{***}$	$-1.1e-6^{***}$	$-1.9e-6^{***}$
$(T-t) * Jan$	-0.199^{***}	0.044^{***}	-0.094^{***}	0.028^{***}	-0.236^{***}	0.112^{***}	-0.338^{***}	0.064^{***}	0.033	0.288^{***}	-0.008	0.231^{***}
$(T-t) * Feb$	-0.203^{***}	0.040^{***}	-0.098^{***}	0.024^{***}	-0.241^{***}	0.102^{***}	-0.348^{***}	0.054^{***}	-0.031	0.215^{***}	-0.056	0.176^{***}
$(T-t) * Mar$	-0.188^{***}	0.072^{***}	-0.107^{***}	0.019^{***}	-0.223^{***}	0.132^{***}	-0.355^{***}	0.063^{***}	-0.131^{***}	0.096^{***}	-0.133^{***}	0.088^{***}
$(T-t) * Apr$	-0.201^{***}	0.051^{***}	-0.109^{***}	0.017^{***}	-0.216^{***}	0.140^{***}	-0.348^{***}	0.068^{***}	-0.135^{***}	0.073^{***}	-0.138^{***}	0.068^{***}
$(T-t) * May$	-0.170^{***}	0.070^{***}	-0.082^{***}	0.043^{***}	0.030	0.361^{***}	-0.282^{***}	0.157^{***}	-0.161^{***}	0.058^{***}	-0.151^{***}	0.060^{***}
$(T-t) * Jun$	-0.163^{***}	0.091^{***}	-0.089^{***}	0.033^{***}	0.092	0.419^{***}	-0.321^{***}	0.127^{***}	-0.154^{***}	0.052^{***}	-0.152^{***}	0.055^{***}
$(T-t) * Jul$	-0.189^{***}	0.047^{***}	-0.087^{***}	0.031^{***}	0.027	0.330^{***}	-0.212^{**}	0.265^{***}	-0.138^{***}	0.066^{***}	-0.137^{***}	0.074^{***}
$(T-t) * Aug$	-0.190^{***}	0.034^{***}	-0.093^{**}	0.025^{***}	-0.241^{***}	0.105^{***}	-0.345^{***}	0.062^{***}	-0.144^{***}	0.087^{***}	-0.137^{***}	0.080^{***}
$(T-t) * Sep$	-0.209^{***}	0.010	-0.092^{***}	0.025^{***}	-0.265^{***}	0.079^{***}	-0.349^{***}	0.053^{***}	-0.103^{**}	0.142^{***}	-0.101^{***}	0.126^{***}
$(T-t) * Oct$	-0.201^{***}	0.020^{**}	-0.091^{***}	0.025^{***}	-0.263^{***}	0.070^{***}	-0.330^{***}	0.078^{***}	-0.103^{**}	0.140^{***}	-0.098^{***}	0.132^{***}
$(T-t) * Nov$	-0.192^{***}	0.035^{***}	-0.084^{***}	0.033^{***}	-0.235^{***}	0.103^{***}	-0.318^{***}	0.098^{***}	-0.092^{*}	0.150^{***}	-0.093^{**}	0.137^{***}
$(T-t) * Dec$	-0.190^{***}	0.048^{***}	-0.087^{**}	0.031^{***}	-0.243^{***}	0.099^{***}	-0.327^{***}	0.070^{***}	0.047	0.293^{***}	0.020	0.253^{***}
$(T-T_h) * Jan$	-0.167^{**}	0.042^{***}	0.015	-0.011^{**}	-0.599^{***}	-0.038	0.119^{**}	-0.057^{***}	-0.470^{***}	-0.188^{***}	-0.378^{***}	-0.159^{***}
$(T-T_h) * Feb$	-0.166^{**}	0.044^{***}	0.016	-0.007	-0.592^{***}	-0.032	0.106^{**}	-0.067^{***}	-0.416^{***}	-0.138^{***}	-0.335^{***}	-0.123^{***}
$(T-T_h) * Mar$	-0.193^{**}	0.030^{*}	0.015	-0.007	-0.633^{***}	-0.045	0.124^{***}	-0.061^{**}				
$(T-T_h) * Apr$	-0.178^{**}	0.038^{**}	0.015	-0.007	-0.638^{***}	-0.061^{**}	0.139^{***}	-0.047^{**}				
$(T-T_h) * May$					-0.897^{***}	-0.282^{***}	0.086	-0.113^{***}				
$(T-T_h) * Jun$					-0.953^{***}	-0.347^{***}	0.107^{**}	-0.102^{**}				
$(T-T_h) * Jul$					-0.838^{***}	-0.231^{**}	-0.016	-0.233^{***}	-0.291^{***}	0.015	-0.250^{***}	-0.044^{*}
$(T-T_h) * Aug$									-0.269^{***}	-0.026	-0.221^{**}	-0.017
$(T-T_h) * Sep$	-0.132^{*}	0.063^{***}	0.036^{*}	-0.008	-0.371^{**}	0.163^{*}	0.056	-0.106	-0.299^{***}	-0.044	-0.255^{***}	-0.043^{**}
$(T-T_h) * Oct$	-0.144^{**}	0.048^{***}	0.033^{*}	-0.012^{*}	-0.479^{***}	0.024	0.179^{**}	-0.001	-0.316^{***}	-0.060^{*}	-0.260^{***}	-0.044^{**}
$(T-T_h) * Nov$	-0.154^{**}	0.045^{***}	0.019	-0.011^{*}	-0.555^{***}	-0.029^{*}	0.208^{***}	0.018	-0.318^{***}	-0.063^{**}	-0.260^{***}	-0.045^{***}
$(T-T_h) * Dec$	-0.162^{**}	0.045^{***}	0.013	-0.017^{***}	-0.580^{***}	-0.045^{**}	0.104^{**}	-0.076^{***}	-0.452^{***}	-0.192^{***}	-0.382^{***}	-0.168^{***}

Note:
1. The table provides the detail estimation results from the proposed model for spot price and inventory (S+I×T) and the 'I×T, pruned' which is based on the I×T model but only the significant terms for inventory are kept. Two periods are reported in this table: from 1989 to 1999 and from 2000 to 2007.

$$(\text{I×T}) \quad : \quad Y_{t,T} = \alpha + (\sum_m \delta_m \mathbf{1}_{M(t)=m} + f(x_t)) \cdot (T-t) + (\sum_m \rho_m \mathbf{1}_{M(t)=m} + g(x_t)) \cdot \max(T - T_h, 0), \text{ with } x = 1/\text{norminv},$$

$$(\text{S+I×T}) \quad : \quad Y_{t,T} = \alpha + (\sum_m \delta_m \mathbf{1}_{M(t)=m} + \beta S_{t-1} + f(x_t)) \cdot (T-t) + (\sum_m \rho_m \mathbf{1}_{M(t)=m} + \zeta S_{t-1} + g(x_t)) \cdot \max(T - T_h, 0), \text{ with } x = 1/\text{norminv}.$$

norminv is normalized inventory with long-term trend of more than 20 years (smoothness parameter of 0.8b). $\mathbf{1}_{x>k_2} = 1$ if $1/\text{norminv} > k_2$, otherwise $\mathbf{1}_{x>k_2} = 0$.
2. In this table, $k_2 = 10, 12$ for corn and soybeans consecutively for both periods. For wheat, $k_2 = 60$ for the '89-99' period; and 45 for the '00-07' period.
3. For corn, the harvest time in this period starts from 15 July, thus $T - T_h = 255$ for the March, 315 for the May, 375 for the July, 75 for the September and 165 for the December contract. For soybeans, the harvest time in this period starts from 1 September, thus $T - T_h = 150$ for the January, 210 for the March, 270 for the May, 330 for the July, 360 for the August, 30 for the September and 90 for the November contract. For wheat, the harvest time in this period starts from 15 May, thus $T - T_h = 315$ for the March, 375 for the May, 75 for the July, 135 for the September and 225 for the December contract.
4. P-stat is the P test statistic based on the DM test to simultaneously compare one model with other non-nested models. ***, **and *indicate the significant coefficients at 1%, 5% and 10% consecutively.