Treasury Bill Yields : Overlooked Information*

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Abstract

Suppose a risk premium factor denotes the time-varying market prices of risk in the Treasury bond market. The question is whether the risk premium factor affects bond prices. Equivalently, we may ask whether the factor is spanned by the cross-section of term structure. This paper finds that the factor is almost but not completely hidden from term structure. Particularly, Treasury bill yields are shown to have unique information about the risk premium factor, which is missing from Treasury bonds. Moreover, the factor is found to be visualized as a wedge shape on Treasury bill yields. The factor predicts a decrease in the level factor of term structure, but survey forecasts do not seem to be aware of its existence. The risk premium factor also predicts negative economic growth meanwhile the slope factor predicts positive growth. This implies that the risk premium and slope factors have qualitatively different information. The risk premium factor also improves the out-of-sample forecasts of future term structure by wielding its forecastability of the level factor.

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1 Introduction

One can earn excess returns from holding long-term Treasury bonds versus short-maturity Treasury interest rates, and the literature shows that the excess returns are predictable. The existence of predictability proves the time-varyingness of the market prices of risk in the bond market. If the market prices of risk were constant, the excess returns would not have been predicted by any measure. Let the time-varying market prices of risk be denoted by a state variable. The question is whether the state variable affects bond prices. To put it differently, we may ask if the market prices of risk are spanned by the cross-section of bond yields.

The early literature such as Fama and Bliss (1987) and Campbell and Shiller (1991) shows that the slope of term structure is able to predict excess returns. However, a growing number of recent papers raise doubt about the slope factor's forecastability. For example, Cochrane and Piazzesi (2005, 2008) find that a substantial amount of predictive content is located beyond the first three principal components (PCs) of yield curves. Considering that the first three PCs can explain more than 99% of total variation of yield curves, the predictive content can be deemed hidden. Duffee (2009b) and Joslin, Priebsch, and Singleton (2010) pursue the existence of a hidden risk premium factor even further, and explain that the factor becomes hidden since its physical process is canceled by risk premium compensation, leaving the factor an *iid* random variable under risk-neutral probability measure. The two papers are based on the same theory, but they suggest different estimation strategies. Duffee (2009b) estimates the hidden factor using the Kalman filtering method, and Joslin, Priebsch, and Singleton (2010) relate the factor to macroeconomic risks and estimate it using data of realized inflation and industrial production growth.

This paper contributes to the literature by showing that the hidden factor is actually not completely hidden. The risk premium factor is found to be visibly revealed through the very short end of yield curves, i.e., the yields of Treasury bills. Note that a forward rate is nothing but a risk-neutral expectation of future one-period riskfree interest rates, $f_t^{(n)} = E_t^Q [r_{t+n-1}]$,[1] and suppose the one-period riskfree interest rates are a function of state variables at the moment, $r_{t+n-1} = f(X_{t+n-1})$. The risk premium factor, RP_t, is an element of the state variables, X_t. According to Duffee (2009b) and Joslin, Priebsch, and Singleton (2010), the risk premium factor has zero persistence under risk-neutral probability measure, thus $E_t^Q [RP_{t+n-1}] = \overline{RP}$ for all $n > 1$. The zero persistence explains why the risk

[1] In fact, the forward rate has one more term to adjust Jensen's inequality. For the sake of simple notation, however, the adjustment term is temporarily omitted here. Section 3.1 explains the theory in detail.

premium factor is hidden from yield curves. In comparison, I claim that the risk premium factor has small but non-zero persistence, $E_t^Q\left[RP_{t+n-1}\right] \rightarrow \overline{RP}$ as n increases. It is why the risk premium factor is expected to be seen from the very short end of term structure.

My hypothesis is that the risk premium factor has weak but non-zero persistence. Its null hypothesis is that either the persistence is zero or the persistence is strong enough to survive for long periods. These hypotheses can be tested by comparing the informativeness of T-bill and T-bond yields. If my hypothesis were right, T-bill yields would have unique risk premium information which is not spanned by T-bond yields. If the persistence were zero, the risk premium factor would be spanned by neither T-bill nor T-bond yields. If the persistence were strong, the information of T-bills would be redundantly overlapped with the information of T-bonds. Thus, the risk premium factor could be spanned by T-bonds alone.

Therefore, the main task of this paper is to compare the informativeness of risk premiums between T-bonds and T-bills. I am not the first one who shows that T-bills are informative of risk premiums. For example, Campbell (1987) shows that the spread between 2- and 1-month bill rates is able to predict excess returns. However, there has been no previous research comparing their informativeness systematically. The closest work is Pearson and Sun (1994), who estimate a two-factor Cox, Ingersoll, and Ross (1985) term structure model and conclude that "estimates based on only bills imply unreasonably large price errors for longer maturities." However, they did not pursue the difference thereafter and left behind its implication with regard to risk premiums.

In a nutshell, T-bill yields are found to have unique risk premium information which is missing from T-bond yields. 98% of total variation of T-bill and T-bond yields can be explained by the so-called shape factors–level, slope and curvature–but these shape factors are not informative of risk premiums. What is informative is the remaining 2% of variation. I estimate the risk premium factor in four different ways: three estimates with T-bonds alone and one estimate with both T-bonds and T-bills. The last estimate of risk premium factor always outperform the former three with strong statistical significance. The risk premium factor is also found to be significantly correlated to Duffee (2009b)'s hidden factor.[2] I also add several tests to make sure that the outperformance is not caused by mechanical tautology due to a larger number of explanatory variables.

Particularly, the risk premium factor is visualized as a wedge-shaped factor of T-bill

[2]Duffee (2009b)'s hidden factors are downloaded from his webpage at `http://www.econ.jhu.edu/People/duffee/estimatedFactors.mat`

yields, which reminds of Cochrane and Piazzesi (2005)'s tent-shaped risk premium factor. One interesting finding is that the wedge-shape pattern is observed even when the risk premium factor is estimated without T-bill yields. It is thanks to the cross-sectional restriction imposed by the affine term structure model. Even when the risk premium factor is imperfectly estimated by T-bonds alone, the affine model induces a wedge pattern on the short end of term structure.

From the mechanical point of view, the risk premium factor could predict high excess returns since it predicts a decrease in the level factor. Considering the fact that the level factor alone can account more than 95% of total variation of term structure, strong level factor forecastability can be directly translated into strong excess return forecastability. However, the market participants do not seem to be aware of the risk premium factor. Survey forecasts of professionals for 10-year T-bond yields are significantly correlated to the slope factor but not to the risk premium factor. However, the risk premium factor outperforms both the slope factor and survey forecasts when they are regressed on realized yield changes in the next quarter. As a matter of fact, survey forecasts fail to show any significance in the prediction of realized yield changes.

The risk premium factor is also shown to be related to macroeconomic activities, but it is so in a puzzling way. The literature shows that the slope factor, which is considered a conventional risk premium factor, predicts high economic growth. In contrast, my risk premium factor predicts low economic growth. This result suggests that the slope and risk premium factors have qualitatively different information. All in all, the risk premium factor simultaneously predicts low economic growth, low level factor, and high excess returns. This set of predictabilities is consistent with the perspective of the business cycle as well as the negative market beta of government bonds.[3]

The risk premium factor is also found to improve the out-of-sample forecastability of future term structure. Note that the term structure dynamics consist of two modules: time-series and cross-section. The time-series module explains how state variables evolve over time, and the cross-section module explains how state variables are mapped to bond yields. The affine term structure model imposes restrictions on the cross-section, thus it has little to do with forecastability as shown by Duffee (2002), Duffee (2009a) and Joslin, Singleton, and Zhu (2011). What is important for better forecasts is the better understanding of the time-series dynamics. Here the risk premium factor contributes by wielding its forecastability

[3]Gregory Mankiw discusses the negative beta of Treasuries in his blog post, "Treasuries as Negative Beta Assets?" The article can be found at `http://gregmankiw.blogspot.com/2008/11/treasuries-as-negative-beta-assets.html`.

of the level factor. The average RMSE of one-month out-of-sample forecasts from the benchmark random walk model is 30.32 basis points. The risk premium factor lowers the RMSE by 2.33 bps to 27.99 bps.

I also check the model's validity with a couple of tests. The first test is based on Campbell and Shiller (1991) and Dai and Singleton (2002) who show that the change of bond yields should be predicted by their slopes with the coefficient of unity after adjusting expected excess returns. The second test is the Sharpe ratios of holding long-term bonds conditional on their maturities. Duffee (2010) documents the problem as "conditional maximum Sharpe ratios implied by fully flexible four-factor and five-factor Gaussian term structure models are astronomically high." My model passes both tests securely.

The rest of this paper is organized as follows. Section 2 reports preliminary results. Section 3 describes the affine term structure model and its estimation. Section 4 explains the model's implications. Section 5 shows how the risk premium factors are related to macroeconomic activities and survey forecasts. Section 6 concludes.

2 Notation and Preliminary Results

2.1 Notation

Let $p_t^{(n)}$ denote the log price of an n-period maturity discount bond at time t.

$$p_t^{(n)} = \log E_t^Q \left[e^{-\sum_{s=0}^{n-1} r_{t+s}} \right] \tag{2.1}$$

where r_t denotes one-period risk-free interest rate at time t. The unit length of a period is a month. The continuously compounded bond yield is

$$y_t^{(n)} = -\frac{1}{n} p_t^{(n)} \tag{2.2}$$

Note that $y_t^{(n)}$ denotes monthly yields. The log forward rate at time t for a loan between time $t+n-h$ and $t+n$ is

$$f_t^{(n,h)} = p_t^{(n-h)} - p_t^{(n)} \tag{2.3}$$

Lastly, the excess return of holding n-period maturity bonds from time t for h periods is given as

$$\begin{aligned} exr^{(n)}_{t,t+h} &= \left\{p^{(n-h)}_{t+h} - p^{(n)}_{t}\right\} - \left\{0 - p^{(h)}_{t}\right\} \\ &= -(n-h)\, y^{(n-h)}_{t+h} + n\, y^{(n)}_{t} - h\, y^{(h)}_{t} \end{aligned} \tag{2.4}$$

All bond yields and excess returns in this paper are nominal.

2.2 Data

U.S. Treasury bond yields are downloaded from the webpage of the Federal Reserve Board.[4] The Fed approximates observed bond yields using a Svensson curve, and posts the data on their webpage at a daily frequency.[5] Though the data are available from 1961 to the present, I take the data only from January 1980 because the Svensson curve has been approximated in a full format with six parameters since then. Until 1979, the Svensson curve had been approximated with only four parameters.

The Svensson curve approximation is a crucial component of this paper. It allows one to calculate a yield for any maturity. Thus, the curve makes it possible to estimate excess returns for one-month holding period. Caution needs to be made that the Svensson curve also has a problem. As pointed out by Cochrane and Piazzesi (2008), it misses some information from the observations of bond yields since the curve is approximated by only six parameters.

The data of Treasury bills are downloaded from the Federal Reserve Bank of St. Louis.[6] They provide secondary market rates of 1-month, 3-month, 6-month and 1-year Treasury bills. Since the data on 1-month Treasury bills are available only since July 2001, I fill the missing values of previous periods with the risk-free interest rates downloaded from Kenneth French's website.[7]

The sample horizon spans from January 1980 to March 2011. All values are taken on the last trading day of each month. The data of T-bond and T-bill yields are recorded on a discount basis. Thus, one needs to convert them into continuously compounded yields to make the data consistent with the model.

[4] http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html
[5] See Gurkaynak, Sack, and Wright (2006).
[6] http://research.stlouisfed.org/fred2/categories/116
[7] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

2.3 Preliminary Results

This section raises doubt about the slope factor's forecastability. The literature shows that the slope factor has forecastability when excess returns are estimated over overlapping one-year holding periods. However, this section finds that its forecastability disappears when excess returns are estimated over non-overlapping one-month periods. Rather than the slope factor, I show that Treasury bill yields are significantly informative of bond risk premiums.

Before explaining the results, it is important to comprehend the implication of bond risk premium forecastability. By way of contradiction, suppose that the market price of risk is constant. Since all risk premiums are proportional to the multiplication of a risk (volatility) and its price, constant volatility and constant price implies constant risk premiums. If risk premiums were constant, they would not be predicted by any measure. However, the literature almost unanimously agrees that risk premiums can be predicted by a measure that is not related to stochastic volatility. The slope factor is one example. Thus, we can conclude that the market price of risk in the bond market is time-varying. Moreover, anything with strong forecastability of excess returns is supposed to be related to the time-varying market price if it were not related to stochastic volatility.[8]

To begin, Table 1 estimates excess returns over one-year holding periods and regresses them on the slope factor. Panel A replicates Fama and Bliss (1987)'s work by using forward rate spreads $\left(f_t^{(n,1y)} - y_t^{(1y)}\right)$ as a proxy of the slope factor. The magnitudes of the coefficients in this panel are similar to theirs. I find 0.792 and 0.964 from the excess returns of holding 3-year and 5-year maturity bonds while Fama and Bliss (1987) find 1.13 and 0.93 respectively. However, the coefficients in this panel are shown to be barely significant. The difference of significance may be due to three reasons. First, I report Newey-West t-statistics to control for the autocorrelation of dependent and independent variables. Second, my sample periods span from 1980 to 2011 while theirs do from 1964 to 1985. Third, different methods are used to approximate yield curves.

In Panel B, I regress the excess returns on 5Y–1Y bond spreads. The spreads are chosen since they are often used by the literature as a tentative proxy of the slope factor.

[8] Campbell and Shiller (1991) and Dai and Singleton (2002) propose a similar test with the change of bond yields instead of excess returns. However, the results should be interpreted in different ways. In Dai and Singleton (2002)'s test, the slope factor is supposed to deliver the coefficient of unity after adjusting for expected excess returns. In this risk premium forecastability test, however, the coefficient of the slope factor can be zero if it were not related to a risk premium factor. In fact, the slope factor's coefficient can be any value depending on its relation to risk premiums. Section 4.2 of this paper covers Dai and Singleton (2002)'s test in detail.

Table 1: Excess Returns from Holding Bonds for One Year

The dependent variable is the excess return from holding bonds for one year, $exr^{(n)}_{t,t+1y}$. n denotes bond maturities, which are shown on the first row. $f^{(n,1y)}_t$ denotes the log forward rate at time t for a loan between time $t+n-1$ year and $t+n$. Bond yields and forward rates in this table are annualized. Numbers in parentheses are Newey-West t statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

maturity	3Y	5Y	10Y	15Y
	Panel A. Regression on forward rate spread			
$f^{(n,1y)}_t - y^{(1y)}_t$	0.792*	0.964*	1.270**	1.488
	(1.894)	(1.926)	(1.975)	(1.588)
obs	364	364	364	364
R^2	0.057	0.053	0.039	0.027
	Panel B. Regression on 5Y–1Y spread			
$y^{(5y)}_t - y^{(1y)}_t$	0.926*	2.026**	4.590***	6.671***
	(1.864)	(2.294)	(3.008)	(3.168)
R^2	0.054	0.085	0.125	0.133

For example, Cochrane and Piazzesi (2005) use 5Y yields and 5Y–1Y spreads to denote the level and slope factors respectively. Ang, Piazzesi, and Wei (2006) also use the 5Y–1Y spreads to test whether the slope factor forecasts future GDP growth. This panel shows that the 5Y–1Y spreads have significant forecastability of annual excess returns. Moreover, the forecastability gets stronger for longer maturity bonds.

However, the slope factor's predictability vanishes in Table 2 in which excess returns are estimated for non-overlapping one-month periods. The forward rate spreads[9] in Panel A completely lose statistical significance. The 5Y–1Y spreads in Panel B are now barely significant, and their R^2's decrease from 5.4–13.3% in the previous table to 0.5–0.9% in this one.

This result implies that the slope factor's predicability is probably spurious due to its persistence. The slope factor is a persistent process with the half life of 11.34 months, and the annual excess returns for overlapping holding periods are mechanically autocorrelated.

[9]The forward rate spreads in Table 2 are measured on a monthly frequency to match with the holding periods of excess returns.

Table 2: Excess Returns from Holding Bonds for One Month

The dependent variable is the excess return from holding bonds for one month, $exr^{(n)}_{t,t+1m}$. n denotes bond maturities, which are shown on the first row. $f^{(n,1m)}_t$ denotes the log forward rate at time t for a loan between time $t+n-1$ month and $t+n$. The variable "wedge" in Panel C is defined as $\left\{\frac{1}{5}y^{(1m)}_t + \frac{4}{5}y^{(6m)}_t\right\} - y^{(3m)}_t$. Bond yields and forward rates in this table have monthly values without annualization. Numbers in parentheses are Newey-West t statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

maturity	3Y	5Y	10Y	15Y
Panel A. Regression on forward rate spread				
$f^{(n,1m)}_t - y^{(1m)}_t$	0.521	0.971	1.559	1.578
	(0.670)	(1.256)	(1.532)	(1.034)
obs	375	375	375	375
R^2	0.003	0.006	0.006	0.003
adj. R^2	0.000	0.003	0.004	0.000
Panel B. Regression on 5Y–1Y spread				
$y^{(5y)}_t - y^{(1y)}_t$	1.368	2.539	4.777*	5.909*
	(1.178)	(1.601)	(1.920)	(1.714)
R^2	0.005	0.008	0.009	0.007
adj. R^2	0.003	0.005	0.006	0.004
Panel C. Regression on slope as well as Treasury bill yields				
$y^{(5y)}_t - y^{(1y)}_t$	0.170	0.690	1.347	0.817
	(0.124)	(0.364)	(0.446)	(0.192)
$y^{(1m)}_t$	7.295**	9.194**	12.896**	17.301**
	(2.340)	(2.243)	(1.995)	(2.013)
$y^{(3m)}_t$	-29.746***	-41.041***	-68.704***	-101.604***
	(-3.288)	(-3.572)	(-3.737)	(-4.020)
$y^{(6m)}_t$	22.309***	31.620***	55.519***	84.117***
	(3.469)	(3.842)	(4.093)	(4.372)
R^2	0.108	0.090	0.079	0.085
adj. R^2	0.098	0.080	0.070	0.075
Panel D. Regression on slope and wedge factors				
$y^{(5y)}_t - y^{(1y)}_t$	0.126	0.830	1.901	1.618
	(0.097)	(0.479)	(0.711)	(0.437)
Wedge	29.225***	40.227***	67.707***	101.006***
	(3.276)	(3.590)	(3.867)	(4.224)
R^2	0.105	0.088	0.079	0.084
adj. R^2	0.100	0.084	0.074	0.079

When both dependent and explanatory variables are persistent, not only are regression coefficients likely to be biased toward significance but also R^2's tend to increase, both of which are found from the comparison of the two tables. It is why Boudoukh, Richardson, and Whitelaw (2007) warn for using excess returns of overlapping periods for a predicability regression. This concern is called long-horizon predictability bias.

According to Panel C, however, all T-bill yields are shown to be significant in predicting monthly excess returns. Moreover, their coefficients show very interesting patterns. First, the sum of coefficients of 1- and 6-month yields are very close to the coefficient of 3-month yields. Second, the ratio of the coefficients between 1- and 6-month yields seems to be stable, floating approximately around 1 : 4. Based on these patterns, I define a wedge factor as $\left\{\frac{1}{5}y_t^{(1m)} + \frac{4}{5}y_t^{(6m)}\right\} - y_t^{(3m)}$ and test its predicability.[10] The wedge factor can be interpreted as the difference between 3-month yield and the weighted average of 1- and 6-month yields. The notion of the wedge factor reminds of Cochrane and Piazzesi (2005)'s tent-shaped risk premium factor. Treasury bill yields would make the shape of a big smile when the wedge factor is high.

Panel D shows that the wedge factor has strong forecastability of monthly excess returns. R^2's decrease very little from Panel C to Panel D, and adjusted R^2's actually increase thanks to the smaller number of explanatory variables. These two panels imply that the informativeness of T-bill yields can be well summarized by a single factor.

The wedge factor is one of the main topics of this paper. By using the fact that a bond yield is an average of forward rates, $y_t^{(n)} = \frac{1}{n}\left\{y_t^{(1m)} + f_t^{(2m)} + \cdots + f_t^{(n)}\right\}$, the wedge factor can be rewritten as

$$\text{Wedge} \equiv \left\{\frac{1}{5}y_t^{(1m)} + \frac{4}{5}y_t^{(6m)}\right\} - y_t^{(3m)} \tag{2.5}$$

$$= \frac{2}{5}\left(\bar{f}_t^b - \bar{f}_t^a\right) \tag{2.6}$$

where $\bar{f}_t^a$ denotes the average of forward rates in this quarter, $\bar{f}_t^a \equiv \frac{1}{2}\left\{f_t^{(2m)} + f_t^{(3m)}\right\}$, and $\bar{f}_t^b$ denotes the average of forward rates in the next quarter, $\bar{f}_t^b \equiv \frac{1}{3}\left\{f_t^{(4m)} + f_t^{(5m)} + f_t^{(6m)}\right\}$. It is interesting that the 1-month T-bill yield is completely canceled out in equation (2.6). The new expression can be interpreted as the spread of average forward rates between the next two quarters. Since a forward rate is nothing but a risk-neutral expectation of future

[10] I run MLE with the SURs (seemingly unrelated regressions) of monthly excess returns for all maturities from 2 to 15 years, and find that the likelihood is maximized when the ratio is 1 : 4.3. However, this paper sticks to the ratio of 1 : 4 for easy notation.

one-month riskfree interest rate, $f_t^{(n)} = E_t^Q\left[r_{t+n-1}\right]$, the wedge can be also considered the expected change of one-month interest rates in two quarters under risk-neutral probability measure.

To summarize, this table suggests that excess returns in the bond market are predictable not by the slope factor but by the wedge of T-bill yields. Caution needs to be made that the table does not suggest the complete lack of informativeness from T-bond yields. It only discredits the slope factor, which has been considered a proxy of risk premium both in the bond and stock markets.

However, the informativeness of T-bill yields is relatively new to the literature since they have been usually overlooked or lightly handled by the search of risk premium predictors in the bond market. Several papers such as Ang and Bekaert (2007) and Henkel, Martin, and Nardari (2011) document that short-term interest rates are informative of risk premiums in the stock market. However, as far as I know, this paper is the first one to compare the risk premium informativeness of short- and long-term bond yields.

This section shows only the preliminary results. Thus, it is still premature to conclude that the information of T-bill yields is so unique that it is not spanned by T-bonds. To provide more convincing evidence that T-bills span an independent dimension of T-bonds, I compare various specifications of risk premium factors under the framework of an affine term structure model in the following sections. The comparison leads to the conclusion that T-bill yields indeed have unique information with regard to risk premiums, and this information is hidden from the cross-section of T-bond yields.

The results of the following sections can be summarized as follows. First, T-bonds and T-bills span independent dimensions of risk premium information. They are equally important in terms of forecastability. Second, the risk premium information can be summarized by a single factor, like a vertex in a two-dimensional space, and this factor is visually realized as a wedge (smile) of T-bill yields. Interestingly, the wedge is observed even when the risk premium factor is estimated without T-bill yields.

3 Affine Term Structure Model and Its Estimation

3.1 Affine Term Structure Model

Since Duffie and Kan (1996)'s seminal work, the affine model becomes a norm of term structure research. Thus, I will only briefly outline the model. According to Duffee (2002)'s specification, my model belongs to the essentially affine class since I assume Gaussian factors. Gaussian factors are chosen based on the findings of Dai and Singleton (2000), Collin-Dufresne, Goldstein, and Jones (2009) and Andersen and Benzoni (2010) who show that stochastic volatilities of interest rates are not spanned by the cross-section of bond yields.

Let X_t denote a column vector of state variables. It will be soon specified which state variables are used for this research. The variables are assumed to follow VAR(1) process.

$$X_{t+1} = \mu + \Phi X_t + \epsilon_{t+1}, \qquad \Omega \equiv E\left[\epsilon_{t+1}\epsilon_{t+1}^\top\right] \tag{3.1}$$

One-period risk-free interest rate, r_t, is given as a linear function of state variables.

$$r_t = \delta_0 + \delta^\top X_t \tag{3.2}$$

The market price of risk, λ_t, is a column vector which is also linearly proportional to state variables.

$$\lambda_t = \lambda_0 + \Lambda X_t \tag{3.3}$$

where $\lambda_0 \in \mathcal{R}^n$ and $\Lambda \in \mathcal{R}^{n\times n}$. Thus, the log nominal pricing kernel can be derived as

$$m_{t+1} = -r_t - \frac{1}{2}\lambda_t^\top \Omega\, \lambda_t - \lambda_t^\top \epsilon_{t+1} \tag{3.4}$$

Note that the price of a discount bond is equal to the expectation of its future value discounted by the pricing kernel. This recursive form can be written as

$$\begin{aligned} p_t^{(n)} &= \log E_t\left[\exp\left(m_{t+1} + p_{t+1}^{(n-1)}\right)\right] \\ &= E_t\left[m_{t+1} + p_{t+1}^{(n-1)}\right] + \frac{1}{2}\operatorname{var}_t\left(m_{t+1} + p_{t+1}^{(n-1)}\right) \end{aligned} \tag{3.5}$$

By combining all these equations, the solution of log bond price can be derived as

$$p_t^{(n)} = A_n + B_n^\top X_t \tag{3.6}$$

$$A_{n+1} = -\delta_0 + A_n + B_n^\top \mu^Q + \frac{1}{2} B_n^\top \Omega\, B_n \tag{3.7}$$

$$B_{n+1}^\top = -\delta^\top + B_n^\top \Phi^Q \tag{3.8}$$

where $\mu^Q \equiv \mu - \Omega\,\lambda_0$ and $\Phi^Q \equiv \Phi - \Omega\,\Lambda$. μ^Q and Φ^Q denote the dynamics of state variables under risk-neutral probability measure. Note that the solution of log bond price is also a linear function of state variables.

The expectation of excess returns can be also derived as a linear function of state variables.

$$E_t\left[exr^{(n)}_{t,t+1}\right] = B^\top_{n-1}\Omega\left(\lambda_0 + \Lambda X_t\right) - \frac{1}{2}B^\top_{n-1}\Omega B_{n-1} \tag{3.9}$$

ΛX_t in the above equation denotes the time-varying market price of risk. If some state variables are not related to the time-varying price, their corresponding columns in Λ will be zero. However, it does not imply that their uncertainties are not priced by the market. It is important to understand the difference between a risk and the market price of a risk. Section 3.5 explains the difference in detail.

Lastly, it can be shown that a forward rate is nothing but a risk-neutral expectation of future one-month riskfree interest rate as

$$\begin{aligned} f^{(n)}_t &\equiv p^{(n-1)}_t - p^{(n)}_t \\ &= \delta_0 + \delta^\top\left\{\left(I + \Phi^Q + \cdots + \Phi^{Q(n-2)}\right)\mu^Q + \Phi^{Q(n-1)}X_t\right\} - \frac{1}{2}B^\top_{n-1}\Omega B_{n-1} \\ &= E^Q_t\left[r_{t+n-1}\right] - \frac{1}{2}B^\top_{n-1}\Omega B_{n-1} \end{aligned} \tag{3.10}$$

where the last term of the above equation, $\frac{1}{2}B^\top_{n-1}\Omega B_{n-1}$, adjusts Jensen's inequality.

3.2 State Variables

The popularity of affine term structure model is largely due to its flexibility. The model can lead to hundreds of different implications depending on the selection of state variables. For example, Ang, Bekaert, and Wei (2008) select inflation and two latent factors, and conclude that the upward sloping term structure is due to the fact that inflation risk premium increases with maturity. Ang and Piazzesi (2003) is another example, which selects inflation and macroeconomic growth factor and thereby incorporates the Taylor rule into the term structure model.

My model uses four state variables: three shape factors (PC1, PC2 and PC3) and one risk premium factor (RP).

$$X_t = \begin{bmatrix} PC1_t & PC2_t & PC3_t & RP_t \end{bmatrix}^\top \tag{3.11}$$

Three shape factors–level, slope and curvature–are chosen since, as shown by Litterman and Scheinkman (1991), they are sufficient to explain most of the total variations of yield curves. The shape factors are estimated by applying principal component analysis to bond yields with the maturities of 1 month, 3 months, 6 months, 1 year, 2 years, $\cdots$ and 15 years. Thus, 18 bond yields are used in total at each time: 3 yields from Treasury bills and 15 yields from Treasury bonds. One can refer to Cochrane and Piazzesi (2008) for the details about the principal component analysis approach.

The key element of this paper lies in how the risk premium factor is measured. I estimate the risk premium factor using four different ways and compare their implications. Each of the estimation approaches is explained by corresponding subsections below.

3.2.1 Risk Premium Factor from the Slope Factor

The first candidate of a risk premium factor is the de facto slope factor. Previous literature documents that the slope factor predicts excess returns at a one-year horizon. Thus, simply using the slope as a risk premium factor would provide a good benchmark model. Throughout this paper, SL_t will be used to denote the slope factor, $y_t^{(5y)} - y_t^{(1y)}$.

$$\text{Model 1}: \quad RP_t = SL_t$$

3.2.2 Risk Premium Factor from One-Year Holding Excess Returns Projected on T-bonds

The second candidate is based on Cochrane and Piazzesi (2005)'s seminal work. They estimate excess returns from holding bonds for one year and regress these returns on the forward interest rates of the first five yearly maturities.

$$exr_{t,t+1y}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} y_t^{(1y)} + \beta_2^{(n)} f_t^{(2y)} + \beta_3^{(n)} f_t^{(3y)} + \beta_4^{(n)} f_t^{(4y)} + \beta_5^{(n)} f_t^{(5y)} + \epsilon_{t+1y}^{(n)} \quad (3.12)$$

for $n = 2$, 3, 4 and 5 years.

What is interesting is that the predicted excess returns of four different maturity bonds share substantial commonality. There are four separate time series of excess returns ($n = 2, 3, 4, 5$), but their predicted components could be explained by a single factor. In fact, this is also an empirical foundation why this paper assumes only one risk premium factor.

Following Cochrane and Piazzesi (2008), I expand the right-hand variables from 5 to 15 forward rates, and estimate excess returns for 14 different maturities.

$$exr^{(n)}_{t,t+1y} = \beta^{(n)}_0 + \beta^{(n)}_1 y^{(1y)}_t + \beta^{(n)}_2 f^{(2y)}_t + \cdots + \beta^{(n)}_{15} f^{(15y)}_t + \epsilon^{(n)}_{t+1y} \tag{3.13}$$

for $n = 2, \cdots, 15$ years. Principal component analysis is applied to the predicted excess returns, and it is shown that 94.05% of total variations of the predicted excess returns can be explain by the first principal component alone. I use this principal component as a risk premium factor, and let it be denoted by CP_t.[11]

$$\text{Model 2}: \quad RP_t = CP_t$$

3.2.3 Risk Premium Factor from One-Month Holding Excess Returns Projected on T-bonds

The third risk premium factor is estimated by the same method as the CP factor except that the dependent variable is replaced with the excess returns from one-month holding periods. As explained in the preliminary result section, the one-year holding excess returns are subject to the long-horizon predictability bias problem. This problem can be mitigated by replacing one-year holding returns with one-month returns.

$$exr^{(n)}_{t,t+1m} = \beta^{(n)}_0 + \beta^{(n)}_1 y^{(1y)}_t + \beta^{(n)}_2 f^{(2y)}_t + \cdots + \beta^{(n)}_{15} f^{(15y)}_t + \epsilon^{(n)}_{t+1y} \tag{3.14}$$

Let MX_t denote the risk premium factor estimated by this approach. This approach is intended to show how model implications are changed depending on the holding horizon of excess returns. This also works as an interim bridge between the CP factor and the last full-fledged risk premium factor.

$$\text{Model 3}: \quad RP_t = MX_t$$

[11] The estimation of CP and two other following risk premium factors–MX and MX2–are subject to look ahead bias. But the bias does not pose a significant threat to the validity of the model since the main question is whether future excess returns are spanned by current bond yields. Indeed, this approach is used by many papers such as Cochrane and Piazzesi (2005), Cochrane and Piazzesi (2008), Ludvigson and Ng (2009) and Huang and Shi (2009).

3.2.4 Risk Premium Factor from One-Month Holding Excess Returns Projected on T-bonds and T-bills

The last risk premium factor is estimated by projecting monthly excess returns on the yields of T-bonds as well as T-bills. Let $MX2_t$ denote the risk premium factor estimated by this approach.

$$exr_{t,t+1m}^{(n)} = \beta_0^{(n)} + \gamma_1^{(n)} y_t^{(1m)} + \gamma_2^{(n)} y_t^{(3m)} + \gamma_3^{(n)} y_t^{(6m)} + \beta_1^{(n)} y_t^{(1y)} + \cdots + \beta_{15}^{(n)} f_t^{(15y)} + \epsilon_{t+1y}^{(n)}$$
$$\text{Model 4}: \quad RP_t = MX2_t$$

This estimate is intended to test whether the addition of Treasury bills can indeed improve predictability. If Treasury bills did not have any unique information which is not spanned by Treasury bond yields, the performance of MX2 would have been indistinguishable from the performance of MX. Thus, the informativeness of Treasury bills can be tested by comparing MX and MX2.

In sum, each risk premium factor expands on its predecessor. Model 2 estimates risk premiums from predictability regressions instead of choosing an arbitrary combination of bond yields. Model 3 replaces annual excess returns with monthly ones. Model 4 utilizes data on Treasury bills in addition to Treasury bonds. By comparing these models, we can easily infer how each component contributes to different model implications.

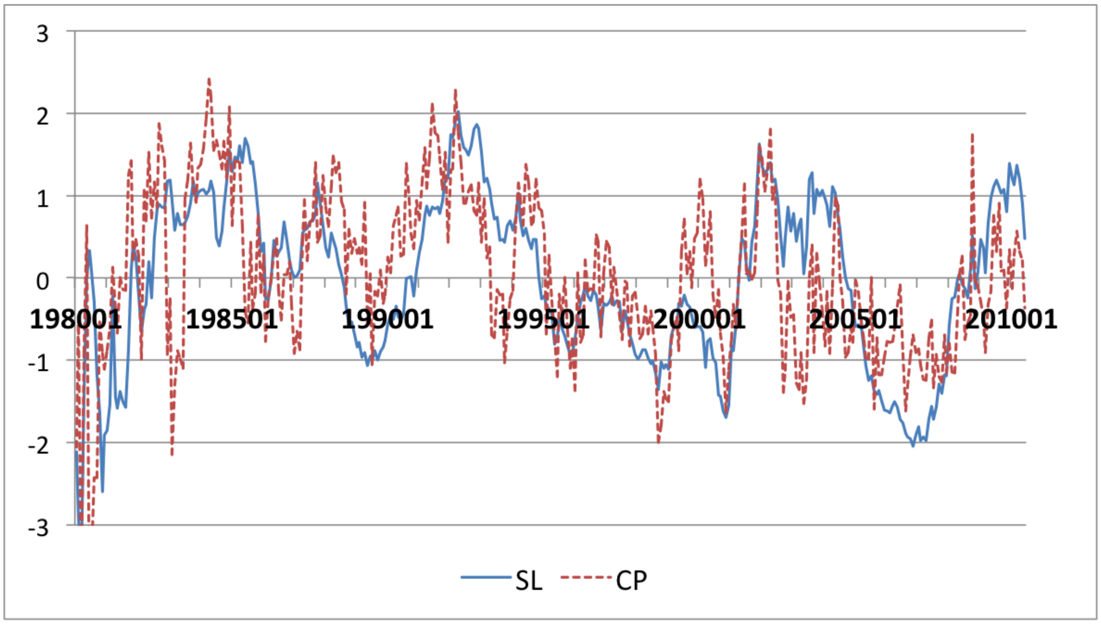
3.3 Estimation of State Variables

As explained in the previous section, my model uses four state variables: three shape factors and one risk premium factor. The shape factors are estimated by applying principal component analysis to bond yields. Since the shape factors have been documented numerous times by previous literature, I will not show their estimates in this paper. Instead, this section presents estimates of risk premium factors since it is where our interest lies.

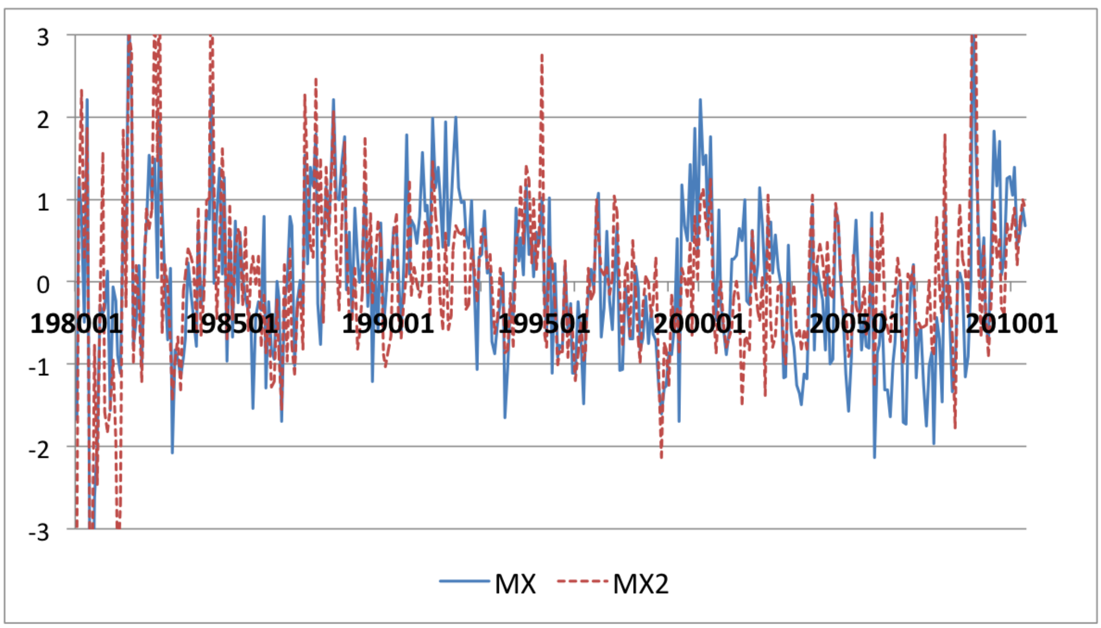
To begin, Figure 1 shows the time trends of estimated risk premium factors. They are all normalized by sample averages and standard deviations for the ease of comparison. The figure shows that MX and MX2 are most volatile, followed by CP factor. The estimates of CP are smoother than those of MX/MX2 since CP is estimated from annual excess returns of overlapping periods. Annual excess returns are mechanically autocorrelated month-to-month, thus it is natural that CP appears more stable than MX and MX2.

Figure 1: Time Trends of Risk Premium Factors

This figure shows the time trends of risk premium factors. All factors are normalized by sample averages and standard deviations.



(a) Risk Premium Factors : SL and CP



(b) Risk Premium Factors : MX and MX2

Table 3: Correlation of Risk Premium Factors

This table shows pairwise correlations among risk premium factors. Although not reported in the table, all correlations appear to be significant with zero p-values. The variable "wedge" is defined as $\left\{\frac{1}{5}y_t^{(1m)} + \frac{4}{5}y_t^{(6m)}\right\} - y_t^{(3m)}$. The last variable of this table is Duffee (2009b)'s hidden factor.

	SL	CP	MX	MX2	Wedge	Duffee (2009b)
SL	1					
CP	0.5851	1				
MX	0.2626	0.7541	1			
MX2	0.1947	0.5377	0.6990	1		
Wedge	0.2279	0.3549	0.2891	0.7030	1	
Duffee (2009b)	-0.3477	-0.5466	-0.3891	-0.4335	-0.4923	1

Note that the risk premium factors are extremely volatile during the early 1980's. The yield curves of this period were difficult to fit by the Svensson curve approximation. The parameters of Svensson curve often hit boundary values in the 1980's. In other words, the 1980's exemplify the limits of Svensson curve approximation. However, I do not discard the samples of the early 1980's. When the early 1980's were excluded, forecastability and all other model implications improved probably because of the large measurement errors due to the limits of Svensson curve approximation. If they were excluded, however, the sample would cover only the post-Volcker period and my claim would lose generality.

The figure also shows that the risk premium factors share substantial commonality. They appear to follow the same long-term trends. To further test the commonality, Table 3 presents pairwise correlations among risk premium factors. The wedge factor and Duffee (2009b)'s hidden factor are also added for comparison. Their correlations are all positive and significant with zero p-values.

One interesting fact is that the correlations are strongest between adjacent factors, i.e., SL and CP, CP and MX, and MX and MX2. It is because each risk premium factor is gradually imbued with one adjustment over its predecessor. In other words, the correlations can be interpreted as how far each risk premium factor changes from its predecessor.

Another interesting fact is the significant correlation of the wedge variable to all risk premium factors. Recall from Table 2 of the preliminary result section that the wedge of bill rates has strong forecastability. Thus, it is not a surprise that MX2 is significantly

Table 4: Excess Returns on Risk Premium Factors

The dependent variable is the excess return from holding bonds for one month, $exr^{(n)}_{t,t+1m}$. Bond maturities are shown on the first row. Numbers in parentheses are Newey-West t statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

maturity	2Y	3Y	5Y	10Y	15Y
			Panel A. Regression on SL		
SL_t	0.051	0.088	0.163	0.308*	0.381*
	(0.930)	(1.178)	(1.601)	(1.920)	(1.714)
R^2	0.004	0.005	0.008	0.009	0.007
			Panel B. Regression on CP		
CP_t	0.166***	0.253***	0.428***	0.765***	0.843***
	(2.820)	(3.208)	(3.791)	(3.908)	(3.068)
R^2	0.039	0.045	0.055	0.054	0.033
			Panel C. Regression on MX		
MX_t	0.257***	0.363***	0.563***	1.032***	1.215***
	(3.417)	(3.599)	(3.993)	(4.659)	(4.587)
R^2	0.094	0.093	0.096	0.099	0.068
			Panel D. Regression on MX2		
$MX2_t$	0.375***	0.534***	0.814***	1.395***	1.748***
	(4.085)	(4.565)	(5.540)	(6.619)	(6.728)
R^2	0.202	0.201	0.200	0.180	0.140

correlated to the wedge since MX2 is supposed to capture predictive contents of both T-bonds and T-bills, thereby including the wedge's contents. However, it is astonishing that other risk premium factors–SL, CP and MX–are also significantly correlated to the wedge even though they are estimated in the absence of T-bill yields. Later, Figure 3 will show that all risk premium factors indeed create a wedge shape among T-bill yields.

Moreover, Table 3 also shows that Duffee (2009b)'s hidden factor is significantly correlated to all of my risk premium factors. This result suggests that our variables capture similar information.

Lastly, to compare the forecastability of each risk premium factor, Table 4 regresses one-month holding excess returns on the risk premium factors. Each panel corresponds to each risk premium factor.

First of all, Panel A shows the results of the SL factor. Being consistent with Table 2 in the preliminary result section, the slope factor fails to show any significance except two, which are barely so at the 10% confidence level. Thus, it is hard to believe that the SL factor properly captures the information of risk premiums.

In contrast, all other risk premium factors display strong and significant forecastability. Their p-values are all lower than 1%. Moreover, both Newey-West t-statistics and R^2's increase as the risk premium factor moves from CP to MX2. Note that R^2's increase from $3.3\% \sim 5.5\%$ for CP in Panel B to $6.8\% \sim 9.9\%$ for MX in Panel C, and to $14.0\% \sim 20.2\%$ for MX2 in Panel E. In fact, this increase is natural since MX and MX2 are directly estimated from the predicted excess returns of one-month holding periods.

However, it is important to note the difference of forecastability between MX and MX2. The R^2's of MX2 are almost twice as high as those of MX. The improvement of forecastability can be interpreted in two opposite ways. First, it can be a mechanical tautology due to the increasing number of explanatory variables. Since MX2 are estimated with three more variables than MX, it is obvious that MX2 would exhibit higher R^2's. Second, it is hopefully because T-bill yields contain unique information which is not spanned by T-bond yields. According to this second explanation, the difference between MX and MX2 is definitely more than mechanical.

Table 5 is intended to distinguish the two alternative explanations. It tests the null hypothesis that T-bill yields are no more informative than T-bond yields in terms of predicting risk premiums. Its dependent variable is the excess returns over one-month holding periods. Panel A and D of the table are identical with Panel C and D of Table 4.

First, Panel B of Table 5 regresses the excess returns on MX and T-bill yields. The panel shows that T-bill yields are still as significant as in Table 2 of the preliminary result section. This result supports the unique informativeness of T-bill yields since the panel controls for MX which is designed to summarize all risk premium information available from T-bond yields.

Furthermore, the coefficients of T-bill yields show similar patterns as in Table 2. The sum of coefficients of all T-bill yields is close to zero, and the ratio of coefficients of $y_t^{(1m)}$ and $y_t^{(6m)}$ is approximately one quarter. Thus, I again define the wedge factor as

Table 5: Informativeness of Treasury Bill Yields

The dependent variable is the excess return from holding bonds for one month, $exr^{(n)}_{t,t+1m}$. Bond maturities are shown on the first row. The variable "wedge" in Panel C is defined as $\left\{\frac{1}{5}y_t^{(1m)} + \frac{4}{5}y_t^{(6m)}\right\} - y_t^{(3m)}$. Numbers in parentheses are Newey-West t statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

maturity	2Y	3Y	5Y	10Y	15Y
Panel A. Regression on MX Alone					
MX_t	0.257***	0.363***	0.563***	1.032***	1.215***
	(3.417)	(3.599)	(3.993)	(4.659)	(4.587)
obs	375	375	375	375	375
R^2	0.094	0.093	0.096	0.099	0.068
Panel B. Regression on MX and Treasury Bill Yields					
MX_t	0.196***	0.279***	0.449***	0.839***	0.901***
	(3.226)	(3.349)	(3.600)	(4.016)	(3.432)
$y_t^{(1m)}$	0.379**	0.530**	0.642**	0.842*	1.191*
	(2.171)	(2.313)	(2.110)	(1.693)	(1.726)
$y_t^{(3m)}$	-1.439***	-1.998***	-2.687***	-4.359***	-6.942***
	(-3.127)	(-3.367)	(-3.479)	(-3.296)	(-3.631)
$y_t^{(6m)}$	1.055***	1.454***	2.021***	3.482***	5.727***
	(3.200)	(3.421)	(3.594)	(3.506)	(3.917)
R^2	0.163	0.158	0.145	0.139	0.119
Panel C. Regression on MX and Wedge					
MX_t	0.193***	0.276***	0.446***	0.840***	0.906***
	(3.104)	(3.204)	(3.496)	(3.972)	(3.411)
Wedge	1.414***	1.950***	2.620***	4.291***	6.923***
	(3.037)	(3.246)	(3.410)	(3.358)	(3.757)
R^2	0.159	0.154	0.143	0.138	0.118
Panel D. Regression on MX2 Alone					
$MX2_t$	0.375***	0.534***	0.814***	1.395***	1.748***
	(4.085)	(4.565)	(5.540)	(6.619)	(6.728)
R^2	0.202	0.201	0.200	0.180	0.140
Panel E. Bootstrapped R^2's of the MX2 Regression Under the Null Hypothesis that T-bill Yields are Not Informative					
.01 ptile	0.092	0.093	0.098	0.103	0.071
.05 ptile	0.094	0.095	0.099	0.104	0.072
.95 ptile	0.111	0.112	0.116	0.121	0.089
.99 ptile	0.121	0.120	0.124	0.129	0.096

$\left\{\frac{1}{5}y_t^{(1m)} + \frac{4}{5}y_t^{(6m)}\right\} - y_t^{(3m)}$ and test its forecastability in Panel C. Surprisingly, the wedge factor is shown to be even more significant than MX in two of the five columns. Considering the fact that MX is directly estimated from the predicted excess returns, the outperformance of the wedge factor is indeed impressive.

Lastly, the R^2's of MX2 in Panel D are tested using a bootstrap method. The null hypothesis is that T-bill yields are not more informative than T-bond yields. Under the hypothesis, T-bill yields should have been spanned by T-bond yields, Thus, I bootstrap T-bill yields as

$$y_t^{(s)} = \beta_0^{(n)} + \beta_1^{(n)} y_t^{(1y)} + \cdots + \beta_{15}^{(n)} y_t^{(15y)} + \epsilon_t^{(s)} \quad \text{for } s = 1, 3 \text{ and } 6 \text{ months}$$

The R^2's of the above equation are higher than 98%. In other words, the uniqueness of T-bill yields takes account of less than 2% of their total variation. MX2 is re-estimated using the bootstrapped T-bill yields along with original T-bond yields. The regressions in Panel D are repeated, and their R^2's are collected. The whole process is repeated 1,000 times.

Panel E of Table 5 shows the distribution of the bootstrapped R^2's. In almost all simulations, the bootstrapped R^2's are higher than those of MX in Panel A. However, they are far short of MX2's R^2's in Panel D. Not a single bootstrap simulation can achieve R^2's as high as of MX2. The bootstrapped R^2's are even less than those of Panel B and C. Therefore, we can confidently reject the null hypothesis.

Thus, Table 5 confirms that MX2's improved forecastability is more than mechanical tautology. The table shows that the unique information of T-bill yields is not spanned by the cross-section of T-bond yields. Moreover, the uniqueness takes into account of less than 2% of T-bill yields' total variation. The other 98% of variation is driven by the same factors of T-bond yields. However, the tiny proportion of 2% contains forecastability.

3.4 Estimation of Time-Series Parameters (μ, Φ, δ_0, δ and Ω)

It is important to note that there are two different dynamics of state variables. One is physical, and the other is risk-neutral. The physical dynamics determine time-series properties of state variables, explaining how yield curves are changed over time. The risk-neutral dynamics impose cross-sectional restrictions on term structure, turning a small number of state variables into bond yields of various maturities.

Accordingly, the model parameters can be divided into two groups. The first group is related to the time series of state variables, and the second group is related to the pricing kernel. I estimate the former group of parameters using VAR regressions, and the latter group by minimizing the sum of squared errors from fitting the cross-section of bond yields.

There are of course alternative strategies to estimate the parameters simultaneously. Kalman filtering is one example, and this method is actually being widely used by the literature. However, the simultaneous estimation strategies also pose several problems. First, the simultaneous method overestimates the persistence of state variables. It is because the method tends to put more weights on the cross-section of bond yields rather than their time-series properties. Though a weighting matrix can be used to make a balance between time-series and cross-section, it is hard to argue how much weight is optimal for estimation. Second, it takes exponentially longer time. The objective function of the simultaneous estimation needs to be optimized numerically, and the cost of this optimization increases exponentially with the number of parameters. For these reasons, I choose to estimate them separately.

Table 6 shows the estimates of time-series parameters for Model 2, 3 and 4. The parameters of Model 1 are omitted due to the lack of space. The table presents the estimates of μ and Φ in the first four columns, and those of δ_0 and δ in the last column. Ω is estimated using the covariance matrix of residuals from the time-series estimation of state variables, but the estimates of Ω are also omitted to save space.

The diagonal elements of Φ represents the persistence of each state variable. As commonly known, the first shape factor (PC1) is almost as persistent as a random walk process. Its mean-reversion coefficient is even bigger than one in Panel A. However, the state variable is still stationary because of its interaction with other variables. The stationarity is tested by the eigenvalues of matrix Φ.

One interesting finding is that the risk premium factors have very strong mean reversion. The half-life of CP is barely one month, and those of MX and MX2 are even shorter than one month. Their weak persistence implies that the risk premium factors would be observed only at the very short end of yield curves. It is because, according to equation (3.8), the influence of a state variable decreases over maturities depending on its persistence under risk-neutral measure. For example, the level factor determines the level of yield curves since it is the most persistent factor, and the slope factor determines slope since it is less persistent than the level but more persistent than the others. This implication will be confirmed by Section 4.1.

Table 6: Time-Series Parameters (μ, Φ, δ_0 and δ)

The dependent variables are specified on the first row. The first column shows explanatory variables of each model. The middle four columns correspond to the estimates of μ and Φ, and the last column corresponds to δ_0 and δ. Time-series parameters of Model 1 are omitted due to the lack of space. Numbers in parentheses are Newey-West t statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

dep.var.	$PC1_{t+1}$	$PC2_{t+1}$	$PC3_{t+1}$	RP_{t+1}	$y_t^{(1m)} \times 100$
Panel A. Model 2 : Risk Premium Factor = CP					
$PC1_t$	1.002***	0.003	0.015	0.129***	3.076***
	(101.539)	(0.124)	(0.282)	(3.060)	(127.022)
$PC2_t$	-0.015*	0.928***	-0.060	-0.155***	1.098***
	(-1.944)	(60.980)	(-1.624)	(-3.777)	(60.538)
$PC3_t$	-0.010	-0.110***	0.712***	-0.042	0.430***
	(-1.116)	(-6.895)	(19.867)	(-1.501)	(23.845)
CP_t	-0.033***	-0.028	-0.062*	0.597***	0.121***
	(-4.060)	(-1.181)	(-1.931)	(11.809)	(6.532)
const	-0.008	-0.006	-0.002	0.004	5.075***
	(-1.170)	(-0.364)	(-0.048)	(0.113)	(303.856)
R^2	0.986	0.900	0.534	0.557	0.997
Panel B. Model 3 : Risk Premium Factor = MX					
$PC1_t$	0.997***	-0.003	-0.004	0.079	3.107***
	(119.655)	(-0.130)	(-0.075)	(1.488)	(117.429)
$PC2_t$	-0.009	0.938***	-0.026	-0.116**	1.047***
	(-1.188)	(63.461)	(-0.719)	(-1.998)	(67.803)
$PC3_t$	-0.005	-0.104***	0.728***	0.003	0.403***
	(-0.627)	(-6.676)	(20.495)	(0.062)	(18.900)
MX_t	-0.039***	-0.015	0.008	0.417***	0.038**
	(-3.629)	(-0.508)	(0.279)	(8.952)	(2.177)
const	-0.008	-0.006	-0.002	0.004	5.075***
	(-1.095)	(-0.358)	(-0.050)	(0.062)	(260.645)
R^2	0.986	0.900	0.531	0.230	0.996
Panel C. Model 4 : Risk Premium Factor = MX2					
$PC1_t$	0.997***	-0.001	0.007	0.071	3.107***
	(144.043)	(-0.034)	(0.131)	(1.171)	(119.929)
$PC2_t$	-0.013*	0.929***	-0.060	-0.135***	1.053***
	(-1.900)	(62.913)	(-1.540)	(-2.620)	(62.715)
$PC3_t$	-0.012*	-0.111***	0.708***	-0.114**	0.411***
	(-1.778)	(-6.760)	(20.326)	(-2.283)	(18.075)
$MX2_t$	-0.060***	-0.054	-0.125***	0.301***	0.068***
	(-5.243)	(-1.500)	(-2.845)	(6.311)	(3.578)
const	-0.008	-0.006	-0.002	0.008	5.075***
	(-1.131)	(-0.366)	(-0.048)	(0.162)	(248.219)
R^2	0.988	0.902	0.545	0.166	0.997

What is most important in the table is the fact that the risk premium factors are always shown to predict the future level factor (PC1_{t+1}) with strong significance in all panels. High risk premium factors predict the level factor to decrease in a following period. This is how the factors could predict future excess returns. Market prices of bonds over all maturities would increase as the level factor decreases. Thus, predicting lower level factor is consistent with predicting higher excess returns.

Moreover, considering that the level factor alone can explain 95% of total variations of yield curves, a better forecast of the level factor implies a better forecast of all bond yields. In fact, as shown by Duffee (2002) and Joslin, Singleton, and Zhu (2011), the lack of forecastability has been the most vexing problem of the affine term structure model literature. If a risk premium factor can indeed predict the level factor, it probably holds the key to improving the forecastability of the affine model and outperforming a random walk process as a benchmark. This conjecture will be vindicated in Section 4.4.

This table also suggests why the risk premium factor induces a wedge shape on T-bill yields. For example, suppose MX2 is one standard deviation high at time t. It raises the one-month riskfree interest rates (r_t) by 6.8 basis points. But it also increases the curvature by 0.125 standard deviations in the next period,[12] inducing a steep slope between 3-month and 6-month yields. As a result, a wedge shape becomes visualized at the short end of term structure.

3.5 Estimation of Pricing Kernel Parameters (λ_0 and Λ)

λ_0 and Λ determine the dynamics of state variables under risk-neutral probability measure as $\mu^Q \equiv \mu - \Omega\,\lambda_0$ and $\Phi^Q \equiv \Phi - \Omega\,\Lambda$, and these risk-neutral dynamics determine bond prices as shown by equation (3.6), (3.7) and (3.8). Thus, λ_0 and Λ are the key components to transform time-series properties (μ, Φ) into cross-sectional ones (μ^Q, Φ^Q).

Recall from equation (3.3) and (3.9) that ΛX_t denotes the time-varying market price of risk. If the i-th state variable were related to the time-varying price, the i-th column vector of Λ would have been non-zero. Vice versa, a zero column vector of Λ implies the i-th state variable is not related to the price. Thus, each column vector of Λ denotes which state variable is related to the time-varying price.

In comparison, each row of λ_0 and Λ denotes which uncertainty is priced by the market.

[12] High PC3 implies low curvature. Later, Figure 2 will show how each shape factor affects overall yield curves.

Risk premiums are essentially the covariance between a pricing kernel and uncertainties, $\text{cov}_t(\epsilon_{t+1}, m_{t+1}) = \Omega(\lambda_0 + \Lambda X_t)$. An uncertainty would be priced if and only if the i-th row of λ_0 and Λ is not completely zero. In other words, we can see which uncertainty is priced by taking a look at the significance of each row of the estimated λ_0 and Λ.

I assume a single risk premium factor for each model. This assumption is based on the empirical finding that a single factor is able to explain around 95% of total variations of predicted excess returns. One advantage of this approach is the simplification of estimation procedure. According to the model, the first three columns of Λ can be set to zero. We only need to estimate λ_0 and the last column of Λ since RP is the only factor related to the time-varying market price of risk.

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 & \lambda_{RP} \end{bmatrix} \tag{3.15}$$

It is important to understand the implications of the risk premium factor and λ_{RP}. The risk premium factor is like a price tag of a product. What are priced by the market are uncertainties, and the risk premium factor indicates the bottom line of their pricing. For example, suppose an uncertainty shock to the level factor is priced 5 times higher than a shock to the slope factor. It is plausible since the market would deem a persistent shock more expensive to hedge than a transient one. When the market is less risk averse, it may cost \$5 to hedge a level shock and \$1 for a slope shock. When the market becomes more risk averse, the hedging costs may jump to \$50 and \$10 respectively. The risk premium factor suggests the bottom line pricing relevant to the market sentiment (e.g. \$5 vs. \$50), and λ_{RP} indicates the relative pricing among uncertainties (e.g. \$5 vs. \$1).

This section estimates λ_0 and λ_{RP} by minimizing the sum of squared errors between observed and model-implied bond yields.

$$\{\lambda_0^*, \lambda_{RP}^*\} = \underset{\lambda_0, \lambda_{RP}}{\arg\min} \sum_t \sum_n \left\{ y_t^{(n)} - \hat{y}_t^{(n)} \right\}^2 \tag{3.16}$$

where $\hat{y}_t^{(n)} \equiv -\frac{1}{n}\left(A_n + B_n^\top X_t\right)$ denotes model-implied bond yields. The objective function is optimized numerically. Table 7 shows the estimates of λ_0 and λ_{RP}.

The standard errors are estimated using a bootstrap of bond yields. I estimate a giant system of VAR(1) processes involving all bond yields, and use the system and its residuals to simulate random samples. With the simulated random samples, λ_0 and λ_{RP} are estimated again by optimizing equation (3.16). This process is repeated for 1,000 times. The brackets

Table 7: Pricing Kernel Parameters (λ_0 and Λ)

Each column corresponds to each model. Numbers in brackets are bootstrap confidence intervals with .05 and .95 percentiles. ***, **, and * denote significances at 1%, 5%, and 10% level respectively. SSE denotes the sum of squared errors.

	Model 1 RP = SL	Model 2 RP = CP	Model 3 RP = MX	Model 4 RP = MX2
$\lambda_{0,1}$	-1.937*** [-2.01, -1.79]	-1.112*** [-1.27, -0.85]	-0.631 [-1.21, 0.79]	-1.679*** [-1.81, -1.23]
$\lambda_{0,2}$	2.042*** [0.91, 2.61]	-0.805*** [-1.11, -0.63]	-0.679*** [-0.81, -0.49]	-1.679*** [-0.73, -0.57]
$\lambda_{0,3}$	-0.123*** [-0.16, -0.07]	0.148 [-0.06, 0.27]	-0.092 [-0.80, 0.16]	0.046 [-0.22, 0.26]
$\lambda_{0,4}$	2.487*** [1.37, 3.06]	-0.441** [-1.20, 0.00]	-0.667*** [-1.85, -0.19]	-0.221 [-0.83, 0.09]
$\lambda_{RP,1}$	-0.916*** [-1.02, -0.82]	-1.535*** [-1.67, -1.38]	-3.406*** [-3.89, -2.87]	-4.763*** [-5.59, -4.61]
$\lambda_{RP,2}$	-2.018*** [-2.68, -1.23]	-0.197** [-0.37, -0.02]	-0.563*** [-0.83, -0.43]	-0.684*** [-0.97, -0.66]
$\lambda_{RP,3}$	0.297*** [0.26, 0.33]	0.294*** [0.20, 0.38]	0.597*** [0.45, 0.87]	0.638** [0.03, 0.54]
$\lambda_{RP,4}$	-2.472*** [-3.15, -1.67]	0.477** [0.12, 0.76]	0.359* [-0.07, 0.76]	0.378 [-0.40, 0.62]
SSE	0.232 [0.21, 0.25]	0.032 [0.03, 0.03]	0.028 [0.03, 0.03]	0.025 [0.02, 0.03]

in Table 7 show confidence intervals with 5% and 95% percentiles of those bootstrapped parameter estimates. The design of this bootstrap estimation is basically model-free. Thus, the λs of each model could be estimated under the same circumstance.

To begin with, the table shows that the SSE of Model 1 is almost 7–10 times higher than those of other models. This result supports the argument that the slope factor (SL) is not a good proxy of risk premium. In addition, SSEs monotonically decrease with models. However, based on their confidence intervals, the SSEs of Model 2–4 are not significantly different with each other.

Another interesting result is that the first three elements of λ_{RP} are significant in all models. This result implies that the shocks to shape factors (PC1–PC3) are priced, and their market prices are time-varying and proportional to the risk premium factor. This result can be compared to the previous literature documenting that the level and slope risks are priced by the market.[13]

In contrast, $\lambda_{0,4}$ and $\lambda_{RP,4}$ of Model 4 are not as significant as other elements. The lack of significance implies that the risk of the risk premium factor is not priced by the market. Considering the fact that the half life of a risk premium factor is less than one month, it is reasonable for the market to have little motivation to price the shocks to a risk premium factor. To put it simply, the price tag itself is not priced. This result is also consistent with Lettau and Wachter (2011)'s finding that "shocks to the price of risk are not priced" by the market.

Therefore, the implications of Table 7 can be summarized as follows. First, the slope factor is not a good proxy of a risk premium.[14] Second, the uncertainties to shape factors are priced by the market, and their market prices are proportional to the risk premium factor. Third, however, the uncertainty of the risk premium factor is not priced.

4 Model Implications

4.1 Yield Curve Loadings on State Variables

All of my state variables are estimated from the cross-section of bond yields. Now we can think of the relationship in another way. Since the affine term structure model prescribes that all bond yields are linear functions of state variables, the state variables are supposed to determine the cross-section of model-implied bond yields.

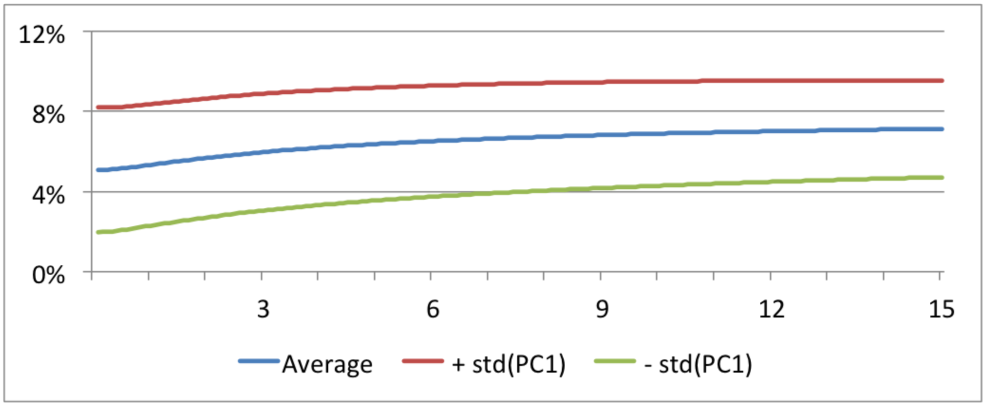
Figure 2 begins with shape factors, PC1–PC3. In each subplot, its corresponding shape factor is deviated by one standard deviation, and the deviation is denoted by red and green lines. For comparison, blue lines denote model-implied bond yields when all state variables

[13] Refer to Joslin, Priebsch, and Singleton (2010) and Duffee (2010).

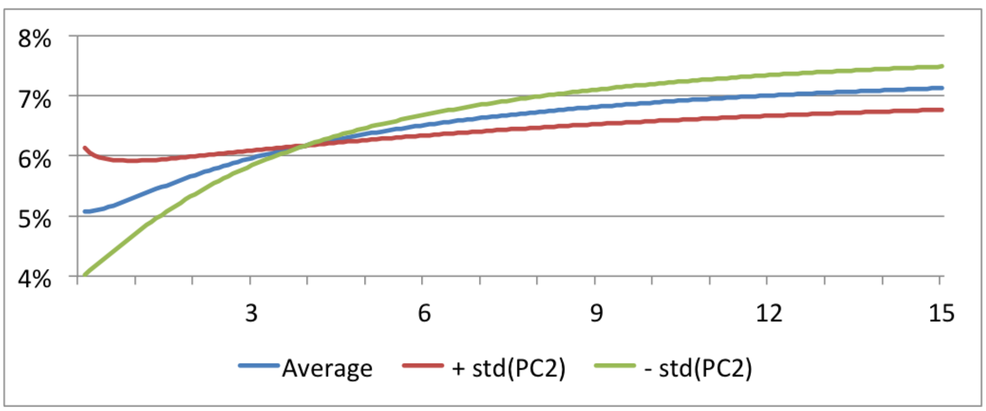
[14] Campbell and Shiller (1991) derive $y_{t+1}^{(n-1)} - y_t^{(n)} = \frac{1}{n-1}\left\{y_t^{(n)} - r_t - e_t^{(n)} - B_{n-1}^{\top}\epsilon_{t+1}\right\}$ where $e_t^{(n)} \equiv E_t\left[exr_{t,t+1}^{(n)}\right]$, and show that the regression coefficient of $y_{t+1}^{(n-1)} - y_t^{(n)}$ on $\left\{y_t^{(n)} - r_t\right\}/(n-1)$ is significantly different from one. This result proves that $e_t^{(n)}$ is not zero, but it does not imply that the slope factor is not a good proxy of a risk premium since $e_t^{(n)}$ can be a multiple of the slope factor. For example, suppose that $e_t^{(n)} = x\left(y_t^{(n)} - r_t\right)$ for some constant x. In this case, the coefficient will equal one if the regression is done on $\frac{1-x}{n-1}\left\{y_t^{(n)} - r_t\right\}$. Thus, it is natural for the original coefficient to become negative if x were bigger than one.

Figure 2: Yield Curve Loadings on Shape Factors

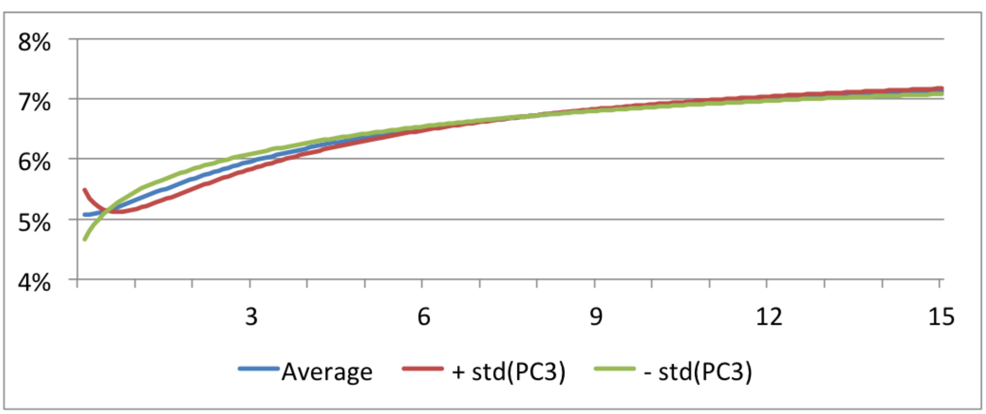
This figure shows how entire yield curves are affected by changing each shape factor by one standard deviation. Each subplot corresponds to each shape factor. The horizontal axis denotes maturities in years, and the vertical axis denotes annualized bond yields. This figure is drawn under the setup of Model 4 in which the risk premium factor is MX2.



(a) PC1 : Level Factor



(b) PC2 : Slope Factor



(c) PC3 : Curvature Factor

are at sample averages. The horizontal axis denotes maturities in years, and this figure is drawn under the setup of Model 4 in which the risk premium factor is MX2.

The figure shows that, as commonly known, the three shape factors respectively represent level, slope and curvature factors. High PC1 implies high level. High PC2 implies low slope. And, high PC3 implies low curvature. These relationships are based on the persistence of each factor. For example, as Table 6 shows, PC1 is almost as persistent as a random walk process. It is supposed to outlive other state variables, and it is how PC1 could move entire yield curves in parallel. In the same spirit, PC2 is the second persistent state variable, which is why its influence gradually and slowly decreases over maturity.

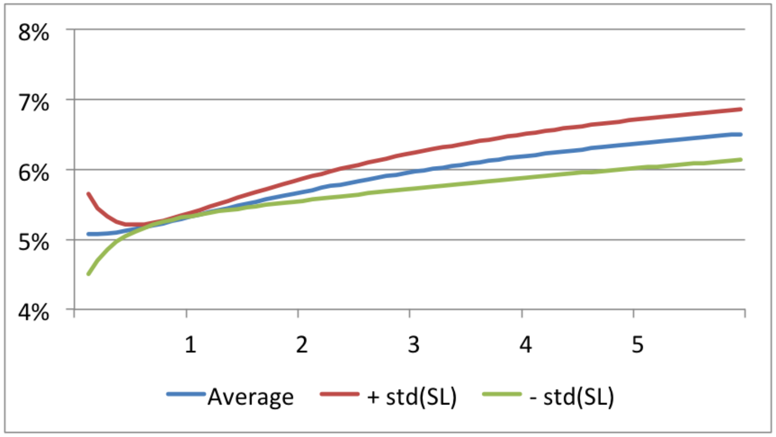
What is interesting is Figure 3, which shows yield curves depending on the values of risk premium factors. This figure visually demonstrates how this paper differs from the previous literature. For example, according to Fama and Bliss (1987) and Campbell and Shiller (1991), a risk premium factor would be visibly revealed as a slope of yield curves as shown by the subplot (a). Duffee (2009b) and Joslin, Priebsch, and Singleton (2010) consider a risk premium factor hidden from the cross-section of bond yields, thus the factor would not show any distinguishable pattern on yield curves. My paper shows that risk premium factors induce a wedge-shaped pattern on T-bill yields, and this pattern dissipates almost completely beyond one-year maturity.

In Figure 3, each subplot corresponds to a different model. Since the effects of risk premium factor are revealed only from the short end of yield curves, the horizontal axis is reduced to 6 years for Model 1 and 2 years for other models. It is because of the weak persistence of risk premium factors. As shown by Table 6, the half lives of risk premium factors are less than 1 month. The scale of vertical axis is also reduced.

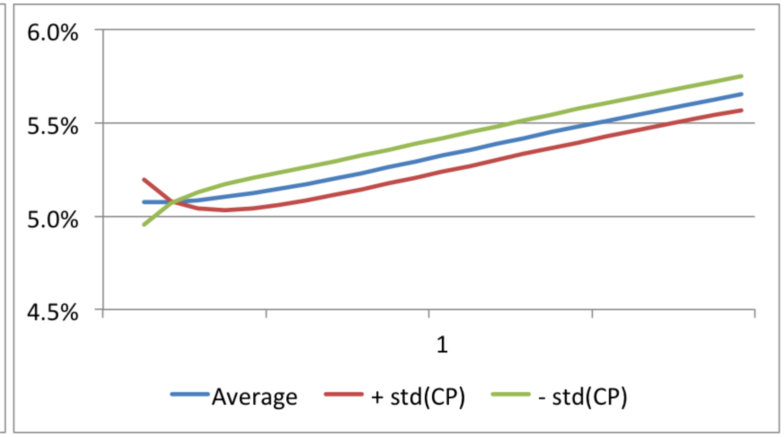
The most important implication of the figure is the wedge shape found at the very left end of all subplots. The wedge shape implies that 3-month yields would decrease compared to 1- and 6-month yields when a risk premium factor is high. This result is consistent with the previous predictability regressions of Table 2 and 5. The previous tables also show that the wedge factor of T-bill yields has significant forecastability. Moreover, according to Figure 3, even the risk premium factors other than MX2 show the same wedge shapes even though T-bill yields are not used for their estimation. Therefore, we can infer that the risk premium factor is visually realized as the wedge factor of T-bill yields.

Figure 3: Yield Curve Loadings on Risk Premium Factors

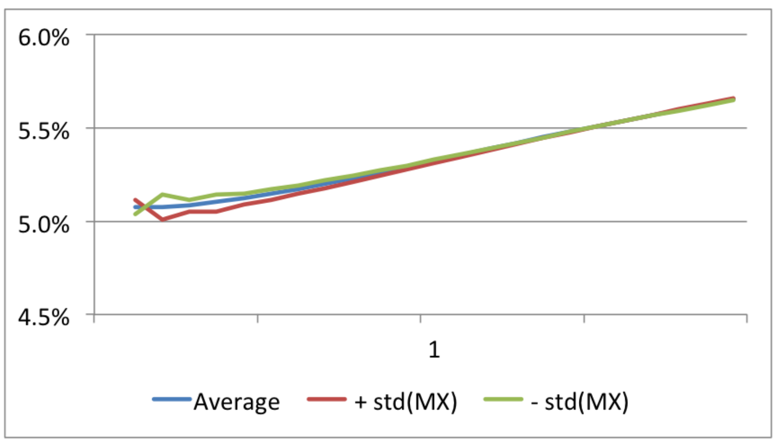
Each subfigure shows how yield curves are affected when a given risk premium factor is deviated by one standard deviation. The horizontal axis denotes maturities in years, and the vertical axis denotes annualized bond yields.



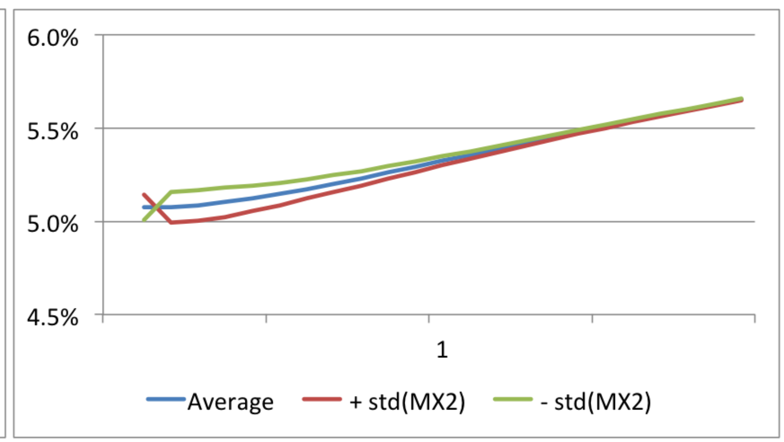
(a) Model 1 : RP = SL



(b) Model 2 : RP = CP



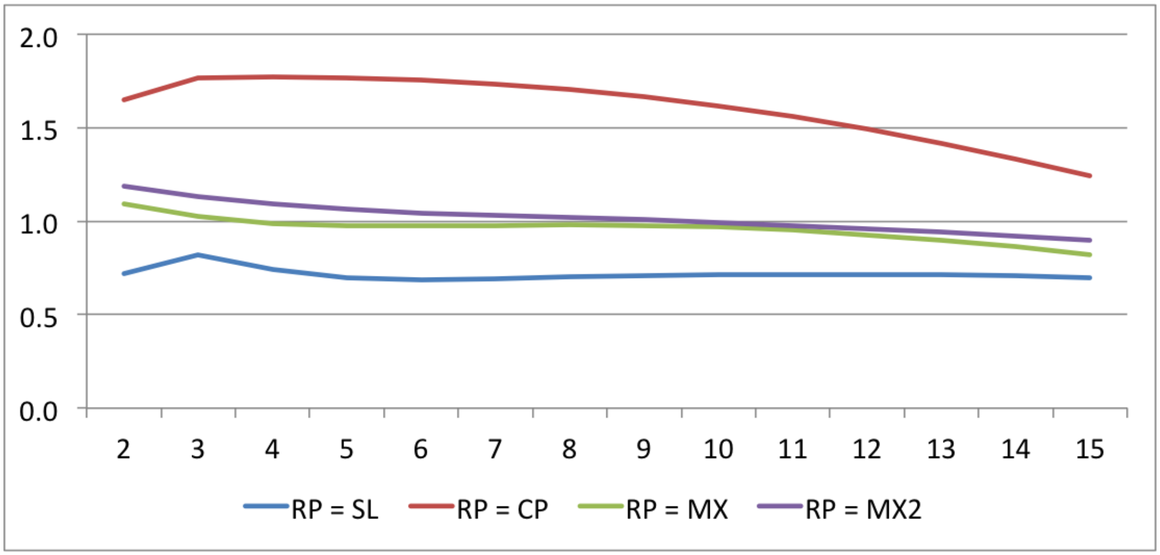
(c) Model 3 : RP = MX



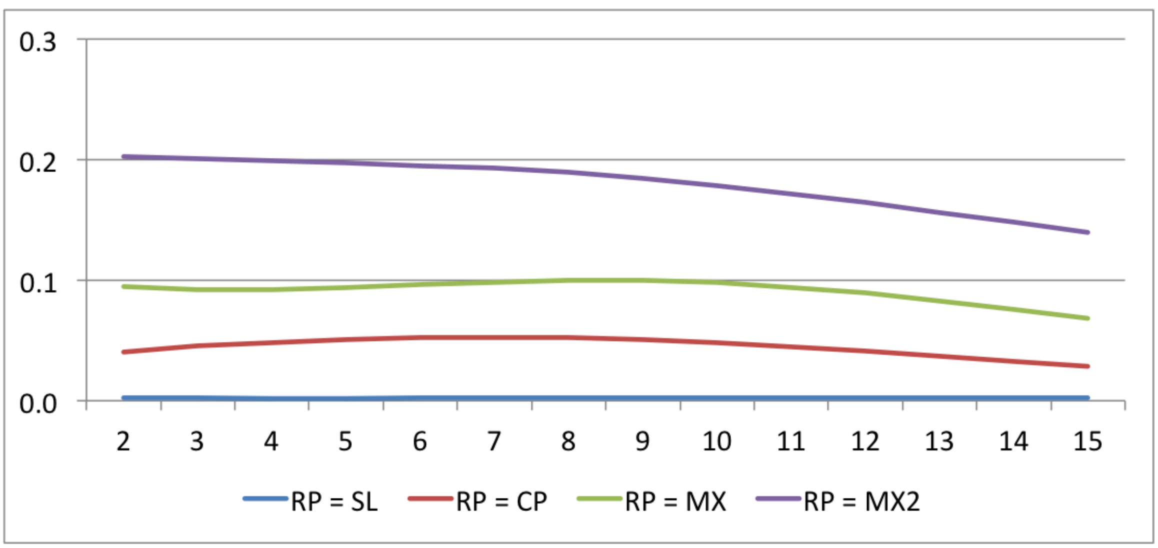
(d) Model 4 : RP = MX2

Figure 4: Dai and Singleton (2002) Test

This figure shows the coefficients and R^2's of regressing $y_{t+1}^{(n-1)} - y_t^{(n)}$ on $\frac{1}{n-1}\left\{y_t^{(n)} - r_t - e_t^{(n)}\right\}$. As predicted by Dai and Singleton (2002), the coefficients are supposed to equal one regardless of maturities. The horizontal axis denotes maturities in years.



(a) Dai and Singleton (2002) Test : Coefficient



(b) Dai and Singleton (2002) Test : R^2

4.2 Dai and Singleton (2002) Test

Based on Campbell and Shiller (1991)'s work, Dai and Singleton (2002) suggest a way to test affine term structure models. Their test method is based on the intuition that the change of bond yields can be predicted by the combination of their slopes and expected excess returns. This intuition can be derived as

$$\begin{aligned} y_{t+1}^{(n-1)} - y_t^{(n)} &= -\frac{1}{n-1}\left\{A_{n-1} + B_{n-1}^\top X_{t+1}\right\} + \frac{1}{n}\left\{A_n + B_n^\top X_t\right\} \\ &= \frac{1}{n-1}\left\{y_t^{(n)} - r_t\right\} - \frac{1}{n-1}\left\{B_{n-1}^\top \Omega\left(\lambda_0 + \Lambda X_t\right) - \frac{1}{2}B_{n-1}^\top \Omega B_{n-1} + B_{n-1}^\top \epsilon_{t+1}\right\} \\ &= \frac{1}{n-1}\left\{y_t^{(n)} - r_t - e_t^{(n)} - B_{n-1}^\top \epsilon_{t+1}\right\} \qquad (4.1) \end{aligned}$$

where $e_t^{(n)} \equiv E_t\left[exr_{t,t+1}^{(n)}\right]$. Therefore, the regression of $y_{t+1}^{(n-1)} - y_t^{(n)}$ on $\frac{1}{n-1}\left\{y_t^{(n)} - r_t - e_t^{(n)}\right\}$ is always supposed to deliver the coefficient of unity. This holds true regardless of maturities.

Figure 4 runs this regression for each model and compare their coefficients and R^2's. The dependent variable is the change of bond yields over one month, which could have been estimated thanks to the Svensson curve approximation. The expected excess return on the right-hand side, $e_t^{(n)}$, is estimated using the equation (3.9). Note that $e_t^{(n)}$ is estimated differently for each model since it is a linear function of a risk premium factor only. Shape factors are not involved in the expected excess returns. Thus, all differences from the Dai and Singleton (2002)'s test results are due to the different estimation of risk premium factors.

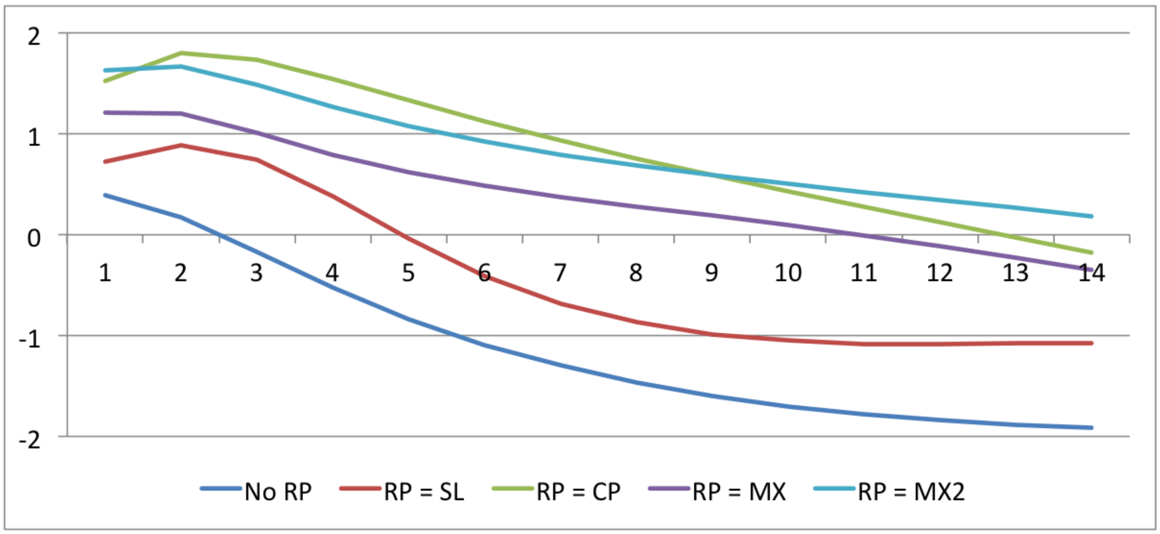
Panel (a) of Figure 4 compares the coefficients first. We can see that the coefficients of MX and MX2 models are virtually equal to one for all maturities, as predicted by the test design. In contrast, the coefficients of SL model are persistently below one, and those of CP model are above one. Thus, this figure implies that the estimated excess returns of SL and CP models are biased while those of MX and MX2 are not.

Panel (b) of the figure compares the R^2's of the regressions. If the coefficients were about a bias, these R^2's are about forecasting power. As shown by this panel, the MX2 factor shows clear-cut dominance over other risk premium factors, followed by MX and CP. The R^2's of SL factor are nearly zero, implying no forecastability at all. Note that these results are also closely related to Table 4, which regresses excess returns directly on the risk premium factors. The scales of R^2's in Table 4 are very close to the R^2's in this figure too.

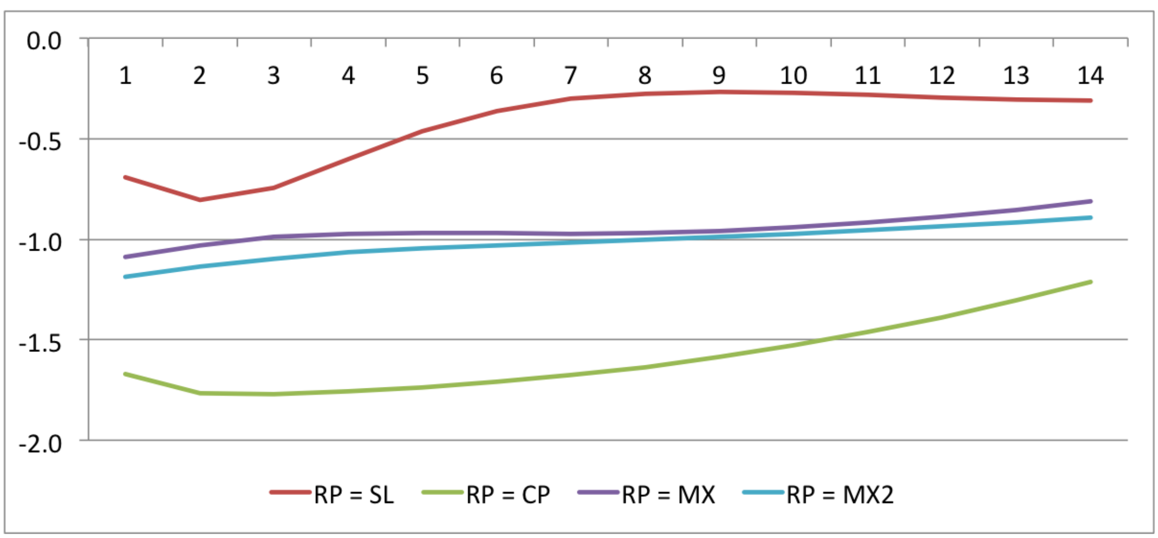
Figure 5 is also based on Dai and Singleton (2002)'s test, but it regresses the changes

Figure 5: Dai and Singleton (2002) Test with the Separation of Slopes and Expected Excess Returns

This figure shows the coefficients of regressing $y_{t+1}^{(n-1)} - y_t^{(n)}$ on slopes $\left(\left\{y_t^{(n)} - r_t\right\}/(n-1)\right)$ and expected excess returns $\left(e_t^{(n)}/(n-1)\right)$ separately. Dai and Singleton (2002) predicts that their coefficients are supposed to equal 1 and -1 respectively. The horizontal axis denotes maturities in years.



(a) Slopes : $\left\{y_t^{(n)} - r_t\right\}/(n-1)$



(b) Expected Excess Returns : $e_t^{(n)}/(n-1)$

of bond yields separately on slopes and expected excess returns. According to the theory, their coefficients are supposed to equal 1 and -1 respectively regardless of maturities.

Panel (a) shows the coefficients on slopes. As documented by Campbell and Shiller (1991), the solid blue line of "No RP" model shows increasingly negative coefficients.[15] In comparison, the coefficients of MX2 and CP are still close to unity. Their average values are 0.845 and 0.856 respectively. The coefficients of MX are somewhat diverted with the average of 0.397. None of their coefficients (CP, MX and MX2) are statistically different from one.

Panel (b) shows the regression coefficients on expected excess returns. The coefficients of MX and MX2 are very close to -1. Their average values are -0.952 and -1.017 respectively. In comparison, the coefficients of SL and MX show large diversion even though their coefficients are not statistically rejected.

In sum, Dai and Singleton (2002)'s test leads to two implications. First, MX and MX2 are overall the least biased risk premium factors among the four candidates. Second, MX2 outperforms MX when the regressions are done separately on slopes and expected excess returns. Third, the forecasting power of MX2 is about twice as high as the power of MX, which is again twice higher than that of CP.

4.3 Average Excess Return and Sharpe Ratio

One interesting fact in the bond market is that short-maturity bonds bask in higher Sharpe ratios than long-maturity bonds. In general, long-maturity bonds earn higher average excess returns, but the standard deviations of their excess returns increase with maturity faster than the averages. In contrast, short-maturity bonds earn moderate excess returns on average, but their excess returns are much less volatile than those of long-maturity bonds.

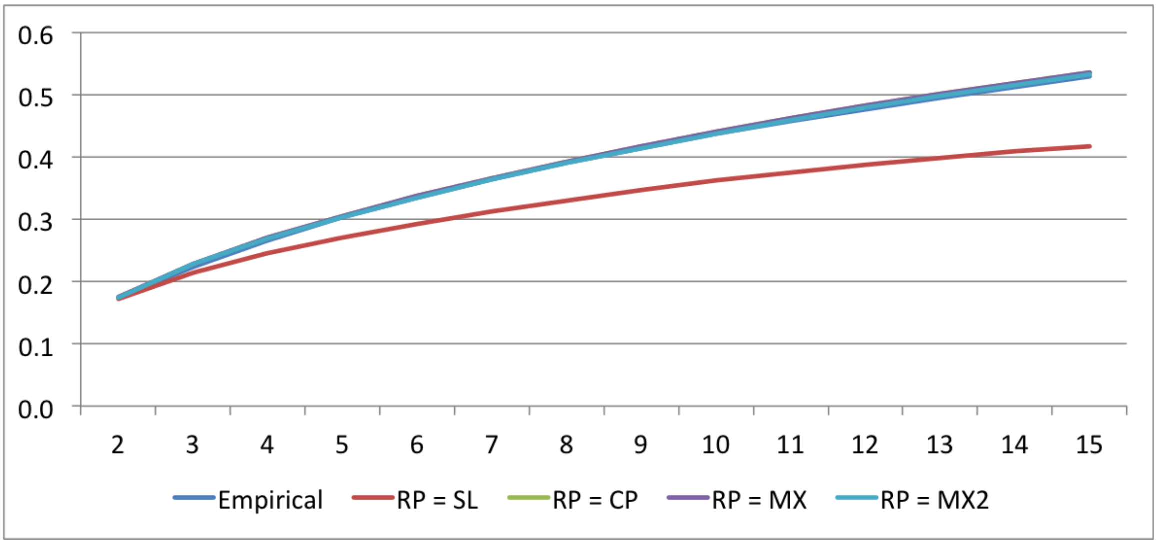
This section is designed to test if the models are able to match the first two moments– average and standard deviation– of empirical excess returns. The excess returns are estimated on a monthly basis. The model-implied average returns and Sharpe ratios are computed using equation (3.9) and the unconditional moments of state variables.

First, the subplot (a) of Figure 6 shows that the average excess returns increase monotonically with maturities. For example, the average excess return from holding 2-year bonds

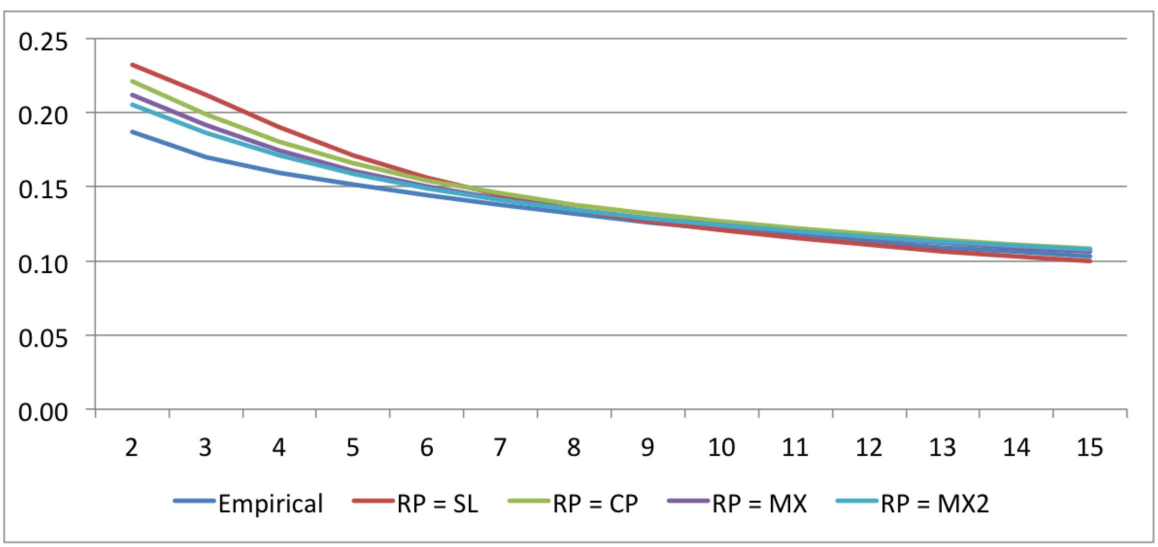
[15] The negative coefficients are significantly different from one, which imply the rejection of expectations hypothesis.

Figure 6: Average Excess Return and Sharpe Ratio

This figure compares empirical average excess returns and Sharpe ratios in the bond market with those implied by models. The horizontal axes in both subplots denote maturities in years. The vertical axis in subplot (a) denotes returns in percentage, and the vertical axis in subplot (b) denotes the ratio of average monthly excess returns and their standard deviations.



(a) Average Monthly Excess Returns



(b) Sharpe Ratios

is 0.174% per month, and the average excess return gradually increases to 0.529% for holding 15-year bonds. All models exhibit the same upward trend. However, Model 1 with SL factor consistently underestimates average excess returns, and this underestimation gets widened over maturities. Except for the Model 1, however, all other models manage to fit the average returns almost perfectly.

Subplot (b) of the figure compares Sharpe ratios. As explained above, the ratios are shown to decline over maturities. The Sharpe ratios of holding 2-year and 15-year bonds are respectively 0.187 and 0.103. The Sharpe ratio of 2-year bonds is almost twice as high as that of 15-year bonds. As before, all models display decreasing trends of Sharpe ratios, but they match the Sharpe ratios with different precisions. Model 4 with MX2 shows the closest proximity to the empirical Sharpe ratios, followed by MX, CP and SL in sequence.

4.4 Out-of-Sample Prediction of Future Bond Yields

The most vexing problem of term structure literature is the lack of out-of-sample forecastability. For a model to be deemed practical, it needs to forecast at least better than a random walk. However, as shown by Duffee (2002), many sophisticated term structure models actually fail to outperform a random walk process, the simplest benchmark. Moreover, Duffee (2009a) and Joslin, Singleton, and Zhu (2011) show that the imposition of no-arbitrage restriction does not improve forecastability.

However, every cloud has a silver lining. Even though the no-arbitrage restriction does not help, the forecastability is shown to be improved by select variables. For example, Duffee (2009b) points out the existence of a hidden factor with substantial forecasting power. Furthermore, Ludvigson and Ng (2009) and Joslin, Priebsch, and Singleton (2010) show that macroeconomic variables are also able to improve the forecastability of future bond yields.

My finding is also in the same vein as the literature. I find that the risk premium factor MX2 improves the forecastability. However, this finding is also new to the literature since the risk premium factor is neither a hidden factor nor a macroeconomic variable, although it is correlated to both of them. Moreover, I show that the improvement of forecastability has nothing to do with the no-arbitrage restriction. The improvement is all thanks to the variable alone.

To test this argument, I estimate four models and compare their forecastability. The models are briefly described by following subsections.

4.4.1 Random Walk

Random walk is the simplest benchmark. However, it does never shy away in terms of forecasting power. The out-of-sample forecast of bond yields in the next month is made as

$$\hat{y}_{t+1}^{(n)} \equiv E_t \left[y_{t+1}^{(n)} \right] = y_t^{(n)} \tag{4.2}$$

4.4.2 Principal Components (PC)

This model uses the three shape factors–level, slope and curvature– for forecast. First, the model estimates the shape factors by applying the principal component analysis to the bond yields available up to time t. Second, it runs cross-section regressions of concurrent bond yields on the three shape factors.

$$y_s^{(n)} = \beta_0^{(n)} + \beta^{(n)} X_s + \epsilon_s, \qquad s = 1, \cdots, t \tag{4.3}$$

where $X_s \equiv \begin{bmatrix} PC1_s & PC2_s & PC3_s \end{bmatrix}^\top$. Third, the model estimates the time-series parameters of X_t using the following VAR(1) process.

$$\begin{aligned} PC1_{s+1} &= \mu_1 + \phi_{1,1}\, PC1_s + u_{1,s+1} \\ PC2_{s+1} &= \mu_2 + \phi_{2,2}\, PC2_s + \phi_{2,3}\, PC3_s + u_{2,s+1} \\ PC3_{s+1} &= \mu_3 + \phi_{3,3}\, PC3_s + u_{3,s+1} \end{aligned}$$

for $s = 1, \cdots, t-1$. Note that this VAR(1) specification is based on the results of Table 6. The shape factors behave as if independent AR(1) processes except that the slope factor ($PC2_{s+1}$) is also affected by the lagged curvature factor ($PC3_s$). For a robustness check, I regress shape factors on their lagged values for subsamples and find the same pattern.

Lastly, the out-of-sample forecasts are made by combining the cross-section and time-series estimations.

$$\begin{aligned} \hat{X}_{t+1} &= \mu + \Phi X_t \\ \hat{y}_{t+1}^{(n)} &= \hat{\beta}_0^{(n)} + \hat{\beta}^{(n)} \hat{X}_{t+1} \end{aligned}$$

Note that the out-of-sample forecasts could have been simplified by directly regressing future bond yields on shape factors. However, the simplified version would make it difficult to compare with the affine term structure model. According to the affine model, the no-arbitrage restriction imposes limitations only on the cross-sectional relationships of bond yields. It is why this benchmark model separates cross-section and time-series estimations. For robustness, the simplified version is also tested and found to show virtually the same forecastability as the current version.

4.4.3 Principal Components + Risk Premium Factor (PC + RP)

This model uses the risk premium factor in addition to the shape factors. The problem is that the risk premium factor is estimated by predictability regressions on bond yields. However, bond yields are highly correlated to each other, and the correlation among explanatory variables is a poison pill to out-of-sample forecast. Thus, an alternative strategy is devised to estimate the risk premium factor (RP).

First, the first six principal components are estimated from the bond yields up to t.

$$PC \equiv \begin{bmatrix} PC1 & PC2 & PC3 & PC4 & PC5 & PC6 \end{bmatrix}^\top \tag{4.4}$$

Second, monthly excess returns are regressed on PC.

$$exr_{s+1}^{(n)} = \gamma_0^{(n)} + \gamma^{(n)} PC_s + e_{s+1}^{(n)}, \qquad s = 1, \cdots, t-1 \tag{4.5}$$

Third, RP is estimated as the first principal component of the predicted excess returns ($\widehat{exr}_{s+1}^{(n)}$) for $s = 1, \cdots, t$. Note that PC_t is not used by the second step but by this third step. Thus, RP_t is estimated without the information of excess returns at $t+1$.

Fourth, estimate the cross-sectional relationships as

$$y_s^{(n)} = \beta_0^{(n)} + \beta^{(n)} X_s + \epsilon_s, \qquad s = 1, \cdots, t \tag{4.6}$$

where $X_s \equiv \begin{bmatrix} PC1_s & PC2_s & PC3_s & RP_s \end{bmatrix}^\top$.

Fifth, the time-series parameters are estimated as

$$\begin{aligned}
PC1_{s+1} &= \mu_1 + \phi_{1,1}\, PC1_s + \phi_{1,4}\, RP_s + u_{1,s+1} \\
PC2_{s+1} &= \mu_2 + \phi_{2,2}\, PC2_s + \phi_{2,3}\, PC3_s + u_{2,s+1} \\
PC3_{s+1} &= \mu_3 + \phi_{3,3}\, PC3_s + u_{3,s+1} \\
RP_{s+1} &= \mu_4 + \phi_{4,4}\, RP_s + u_{4,s+1}
\end{aligned}$$

for $s = 1, \cdots, t-1$. This VAR(1) specification is also based on the results of Table 6. Note that the only differences from the previous model are the addition of RP as an independent AR(1) process and the forecast of RP for the future level factor.

Lastly, the out-of-sample forecasts are again made by combining the cross-section and time-series estimations.

$$\begin{aligned}
\hat{X}_{t+1} &= \mu + \Phi X_t \\
\hat{y}_{t+1}^{(n)} &= \hat{\beta}_0^{(n)} + \hat{\beta}^{(n)} \hat{X}_{t+1}
\end{aligned}$$

4.4.4 PC + RP with no-arbitrage restriction (PC + RP, affine)

The state variables and the time-series parameters of this model are estimated by the same way as in the previous one. The only difference is how to estimate the cross-sectional relationship of bond yields. The previous two models directly regress bond yields on state variables. In this model, the pricing kernel parameteres, λ_0 and λ_{RP}, are estiamted by minimizing the sum of squared errors between observed and model-implied bond yields up to time t.

$$\{\lambda_0^*,\, \lambda_{RP}^*\} = \underset{\lambda_0,\, \lambda_{RP}}{\arg\min} \sum_{s=1}^{t} \sum_{n} \left\{ y_s^{(n)} - \hat{y}_s^{(n)} \right\}^2 \tag{4.7}$$

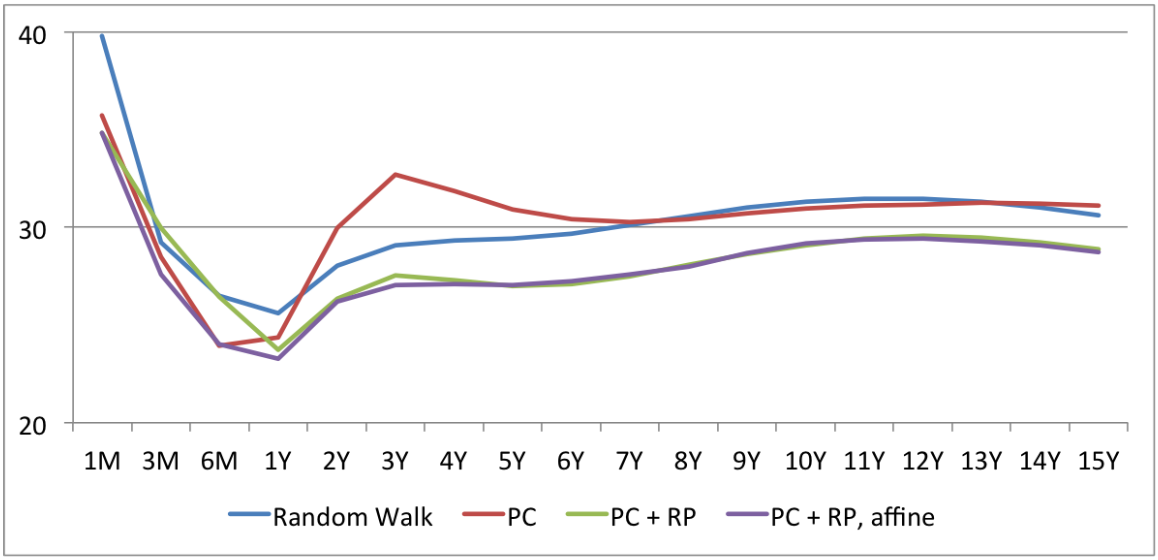
where $\hat{y}_t^{(n)} \equiv -\frac{1}{n}\left(A_n + B_n^\top X_t\right)$ denotes model-implied bond yields. Section 3.5 explains the estimation procedure in detail.

4.4.5 Comparison of Their Out-of-Sample Forecasting Power

Figure 7 compares the RMSEs of out-of-sample forecasts from the aforementioned four models. The dependent variable is bond yields in one month. The forecast horizon is from Apr 2006 to Mar 2011, the last five years of my sample. The vertical axis denotes RMSEs in basis points, and the horizontal axis denotes maturities.

Figure 7: Out-of-Sample Prediction of Future Bond Yields

This figure compares the out-of-sample forecastability of four models, which are described in Section 4.4. The dependent variable is the bond yields in one month. The vertical axis denotes RMSEs in basis points, and the horizontal axis denotes maturities. The out-of-sample forecast horizon is from Apr 2006 to Mar 2011, the last five years of my sample.



First and foremost, the figure shows that the PC+RP affine model shows the best forecasting power. It outperforms the random walk model, the first benchmark, by 1.63–4.98 basis points. Its outperformance is pervasive over all maturities. Moreover, even though the mere comparison of RMSEs seems to be humble, its implication in the financial market is quite substantial. For example, the PC+RP affine model forecasts 15-year bond yields better than random walk by 2 basis points. Considering its duration, the price of 15-year bonds can be predicted better by 30 basis points per month, and this outperformance can improve performance returns by 3.6% per year.

The question is how the affine model could achieve such a high out-of-sample forecastability. We can find its answer by comparing with two other benchmark models: PC and PC+RP.

The figure shows that the PC+RP model outperforms the PC model in the forecast of bond yields whose maturities are longer than or equal to one year. It is because the RP factor has a significant forecastability of the level factor. As shown by Table 6, high RP predicts low level in the next month with statistical significance. Note that the level factor is the primary principal component of bond yields. Especially, long-maturity yields are virtually driven by the level factor only. Thus, better forecast of level factor implies better forecast of bond yields over all maturities. It is how the RP factor could give an advantage to the forecast.

The contribution of no-arbitrage restriction, i.e. the affine term structure model, can be found from the forecast of short-maturity yields. Particularly, the difference between PC+RP and PC+RP affine models is conspicuous from the forecasts of 3- and 6-month yields. It is because the RP factor is very volatile as shown by Figure 1, and the presence of the RP factor is visible only through short-maturity yields as by Figure 3. Thus, the cross-section coefficients of the PC+RP model could have been compromised by errors for short maturities. However, the errors are minimized by the affine model since it estimates the cross-section relationship by using all bond yields simultaneously.

For another robustness check, I run the out-of-sample forecasts without the sample of the Volcker period since, as shown by Figure 1, the risk premium factors are contaminated with a great deal of noises in the early 80s. It is because the Svensson curve approximation is not good enough to fit the yield curves of the period. As a result, the forecasts could be further improved by several basis points. However these results are not reported in this paper since I err on the conservative side. The results are available on request.

To summarize, this section suggests two approaches how to improve the out-of-sample forecast of bond yields. First, the RP factor improves time-series forecast. Second, the affine term structure model improves cross-section estimation. In other words, the RP factor and the affine model specification work in harmony in two different dimensions.

5 Relation to Macroeconomic Variables

5.1 Industrial Production Growth

The previous section shows that the risk premium factor can improve the forecasts of bond yields. This section is intended to test if the factor can also predict macroeconomic activities.

Table 8 regresses the growth rate of industrial production on lagged risk premium factors. The factors are averaged over the past three months to reduce noises. Panel A corresponds to the total index of industrial production, and Panel B uses its sub-index for consumer goods.

The slope of yield curves (SL) shows positive predictability of industrial production growth. This finding is consistent with the literature; high slope is known to precede high

Table 8: Forecasts of Industrial Production Growth

The dependent variable is the growth rate of industrial production in the next month, $(IP_{t+1} - IP_t)/IP_t \times 100$. Risk premium factors in the place of explanatory variables are averaged over the past three months to reduce noises. Numbers in parentheses are Newey-West t statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

	(1)	(2)	(3)	(4)	(5)
Panel A. Industrial Production : Total Index					
SL	0.121**				0.176***
	(2.330)				(3.025)
CP		-0.002			
		(-0.043)			
MX			-0.152*		
			(-1.962)		
MX2				-0.208**	-0.272***
				(-2.247)	(-2.909)
obs	372	372	372	372	372
R^2	0.027	0.000	0.029	0.046	0.100
Panel B. Industrial Production : Consumer Goods					
SL	0.125***				0.167***
	(3.335)				(4.155)
CP		0.025			
		(0.599)			
MX			-0.074		
			(-1.266)		
MX2				-0.143**	-0.203***
				(-2.093)	(-3.035)
obs	372	372	372	372	372
R^2	0.023	0.001	0.005	0.017	0.055

economic growth. For example, Stambaugh (1988) explains that "inverted term structures precede recessions and upward-sloping structures precede recoveries." Ang, Piazzesi, and Wei (2006) also document that "every recession after the mid-1960s was predicted by a negative slope–an inverted yield curve–within 6 quarters of the impending recession. Moreover, there has been only one 'false positive' (an instance of an inverted yield curve that was not followed by a recession) during this time period."

However, all other risk premium factors exhibit negative predictability, and their significances become stronger as we move from CP in column (2) to MX2 in column (4).

Moreover, in column (5), SL and MX2 show striking contrasts with each other. They both are significant at 1% confidence level with opposite signs.

Now the question is why the risk premium factors forecast macroeconomic activities in a negative direction. The answer is related to the fact that the factors forecast the decrease of level factor, as Table 6 shows. The level factor tends to decline during recessions since the Federal Reserve lowers the federal funds rate to rein in business cycles. Moreover, market participants also flock toward government bonds during recessions in the search of safe havens, thereby leading to the decline of overall yield curves. This is essentially the key mechanism explaining how the risk premium factors can forecast excess returns from holding bonds; lower level factor implies higher excess returns.

As macroeconomic activities worsen, investors would expect yield curves to decline in the near future. This expectation might be reflected in Treasury bill yields, and the risk premium factors may capture this reflection. All in all, macroeconomic activities, excess returns and yield curves are shown to be intimately connected.

5.2 Surveys of Professional Forecasters

Recall from Table 6 that all risk premium factors predict a decrease in the level factor. This predictability of the level factor is the key mechanism how they forecast excess returns; lower level implies an overall increase of bond prices. This section is intended to test whether the level factor predictability is taken into account by professional forecasters.

The Federal Reserve Bank of Philadelphia surveys professional market participants for a wide range of forecasts on a quarterly basis. The FRB sends out its survey form to professionals in the end of January (April/July/October), and the professionals submit their forecasts by the middle of February (May/August/November). The FRB then publishes its results by the end of February. Thus, the first survey forecasts are supposed to be made for the values in one month, i.e. at the end of the current quarter. These data are freely accessible from the website of the Federal Reserve Bank of Philadelphia.[16]

Table 9 regresses forecasted and realized changes of 10-year T-bond yields on two risk premium factors, SL and MX2. The forecasts of T-bond yields have been conducted since 1992Q1. FC1 and FC2 of the table denote the forecasted changes of T-bond yields in the current and next quarters, $FC1 \equiv F_t\left[y_{t+1m}^{(10y)}\right] - y_t^{(10y)}$ and $FC2 \equiv F_t\left[y_{t+4m}^{(10y)}\right] - y_t^{(10y)}$. $F_t[\,\cdot\,]$

[16] http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/

Table 9: Survey Forecasts and Realized Changes of T-bond Yields

FC1 and FC2 denote the forecasted changes of T-bond yields in the current and next quarters, $FC1 \equiv F_t\left[y_{t+1m}^{(10y)}\right] - y_t^{(10y)}$ and $FC2 \equiv F_t\left[y_{t+4m}^{(10y)}\right] - y_t^{(10y)}$. $F_t[\cdot]$ operator denotes survey forecasts. RC1 and RC2 denote their realized changes, $RC1 \equiv y_{t+1m}^{(10y)} - y_t^{(10y)}$ and $RC2 \equiv y_{t+4m}^{(10y)} - y_t^{(10y)}$. Numbers in parentheses are Newey-West t statistics with 4 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

dep. var.	FC1	FC2	RC1	RC2	RC1	RC2
SL	-0.174***	-0.183***	-0.007	-0.057	-0.009	-0.097
	(-7.810)	(-7.022)	(-0.253)	(-1.081)	(-0.311)	(-1.645)
MX2	0.052	0.056	-0.103**	-0.115**	-0.102**	-0.103*
	(1.641)	(1.447)	(-2.600)	(-2.046)	(-2.416)	(-1.703)
FC1					-0.010	
					(-0.102)	
FC2						-0.213
						(-1.057)
obs	77	77	77	76	77	76
R^2	0.290	0.264	0.108	0.047	0.108	0.063

operator denotes survey forecasts. RC1 and RC2 denote their realized changes, $RC1 \equiv y_{t+1m}^{(10y)} - y_t^{(10y)}$ and $RC2 \equiv y_{t+4m}^{(10y)} - y_t^{(10y)}$.

First, the first two columns show that professionals forecast a decrease in 10-year yields when the slope factor is high. At first glance, this relationship looks very puzzling since high slope implies that one-period riskfree rates are expected to increase in risk-neutral probability measure. Of course, the survey forecasts are made for market expectations in physical probability measure, and the two probability measures are different. However, it is still interesting that the market expectations under the physical and risk-neutral dynamics are opposite to each other.

The negative correlation of SL with the forecasted changes can be understood by considering the business cycle. As explained in the previous section, flat yield curves often coincide with asset bubbles. During a bubble period, the market may expect the Fed to raise interest rates to pop up the bubble. In this way, we can explain the negative correlation between SL and forecasted 10-year yield changes.

However, the survey forecasts are not vindicated by the realized changes of 10-year bond yields in following periods. The third and fourth columns show that SL loses its

Table 10: Survey Forecasts of GDP and Industrial Production Growth

X_t denotes the level of macroeconomic indicators: real GDP in Panel A and industrial production in Panel B. The dependent variable is the forecasted growth of macroeconomic activities in the first and second next quarters. $F_t[\cdot]$ operator denotes survey forecasts. Numbers in parentheses are Newey-West t statistics with 4 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

dep. var.	Growth in the Next Quarter $F_t\left[X_{t+4m}/X_{t+1m}\right]$			Growth in Two Quarters $F_t\left[X_{t+7m}/X_{t+4m}\right]$		
			Panel A. X_t : Real GDP			
SL	0.071		0.078*	0.062**		0.072**
	(1.576)		(1.670)	(2.120)		(2.454)
MX2		-0.038	-0.050		-0.056*	-0.067**
		(-0.878)	(-1.193)		(-1.967)	(-2.532)
obs	125	125	125	125	125	125
R^2	0.043	0.012	0.063	0.069	0.054	0.145
			Panel B. X_t : Industrial Production			
SL	0.134**		0.149**	0.139**		0.161**
	(2.284)		(2.381)	(2.068)		(2.505)
MX2		-0.076	-0.099		-0.125**	-0.150**
		(-1.021)	(-1.369)		(-1.998)	(-2.511)
obs	125	125	125	125	125	125
R^2	0.047	0.014	0.071	0.078	0.060	0.163

forecastability with regard to realized yield changes. Moreover, as shown by the last two columns, even the survey forecasts fail to predict the realized changes even though it is what survey forecasts are supposed to do.

In contrast, MX2 shows qualitative differences from SL. First, MX2 does not show any significant correlation with forecasted changes, implying that professional forecasters do not take into account the information of MX2. However, MX2 still shows strong forecastability for realized yield changes. This result is consistent with the previous finding that high MX2 predicts a decrease in the level factor. Also, as shown by the last two columns, MX2 outperforms even the survey forecasts.

Table 10 tests how SL and MX2 are correlated to the survey forecasts of real GDP and industrial production growths. The first and last three columns respectively denote the forecasted growths in the first and second next quarters. Being consistent with the previous section, high SL predicts large economic growth and MX2 does the opposite. One interesting observation is that MX2 is now correlated to survey forecasts. It is probably because MX2 is related to some economic indicators that professionals take into account for the forecasts of economic growth but not for the forecasts of bond yields.

6 Conclusion

This paper is founded on a simple idea that the hidden factor may not be completely hidden. Given that the information of risk premiums is placed beyond the first three principal components of yield curves, it is believed to be true that the risk premium factor has a very short half-life. However, it also seems to be too restrictive to assume a completely hidden factor since it restricts the risk premium factor to have exactly zero persistence under the risk-neutral probability measure. I believe in between–weak but non-zero persistence–and try to search for unique risk premium information from Treasury bill yields.

As expected, T-bill yields are found to be significantly informative of risk premiums. The information from T-bill yields substantially improve the forecasting power of the risk premium factor. Moreover, the implications of the risk premium factor are also consistent with the perspective of the business cycle. It predicts low economic growth, low level of term structure, and high excess returns.

Now the new question is where the risk premium factor comes from. It is too volatile to be attributed to macroeconomic activities. As Shiller (1981) and Poterba and Summers (1986) suggest, macroeconomic activities are not volatile enough to explain the high volatilities in the financial market. According to survey forecasts, market participants do not seem to be aware of the risk premium factor either.

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