The Delivery Option in Credit Default Swaps

R. Jankowitsch[†], R. Pullirsch[‡], T. Veža[††]

Working paper

First version: May 18, 2006
This version: October 18, 2006

Abstract

Under standard assumptions the reduced-form credit risk model is not capable of accurately pricing the two fundamental credit risk instruments – bonds and credit default swaps (CDS) – *simultaneously*. Using a data set of euro-denominated corporate bonds and CDS our paper quantifies this mispricing by calibrating such a model to the bond data and subsequently using it to price CDS, resulting in model CDS spreads up to 50% lower on average than observed in the market. An extended model is presented which includes the delivery option implicit in CDS contracts emerging since a basket of bonds is deliverable in default. By using a constant recovery rate standard models assume equal recoveries for all bonds and hence zero value for the delivery option. Contradicting this common assumption, case studies of Chapter 11 filings presented in the paper show that corporate bonds do *not* trade at equal levels following default. Our extension models the implied expected recovery rate of the cheapest-to-deliver bond and, applied to the data, it largely eliminates the mispricing. The calibrated recovery values lie between 8% and 47% for different obligors, exhibiting strong variation among rating classes and industries. A cross-sectional analysis reveals that the implied recovery parameter depends on proxies for the delivery option, primarily the number of available bonds and the bond pricing errors. No evidence is found for the influence of liquidity proxies.

JEL classification: C13, G12, G13, G15

Keywords: credit risk, default, corporate bond, credit default swap, reduced-form model, recovery rate, delivery option

[†]Corresponding author. Vienna University of Economics and Business Administration, `rainer.jankowitsch@wu-wien.ac.at`

[‡]Bank Austria–Creditanstalt, `rainer.pullirsch@ba-ca.com`

[††]Vienna University of Economics and Business Administration, `tanja.veza@wu-wien.ac.at`

1 Introduction

The pace at which the credit derivatives market has been growing since its inception about ten years ago topped all projections[1], increasingly calling for the development of more and more accurate pricing tools for these products since market reality often reveals that the assumptions underlying the prevalent models are inadequate and misleading.

The instrument this paper focuses on is a credit default swap (CDS). This is a bilateral contract aimed at transferring the credit risk of a (corporate or sovereign) borrower from one market participant (the protection buyer) to another (the protection seller). The CDS buyer pays a periodical premium for the assurance that the CDS seller will compensate him for the loss in case the borrower defaults during the term of the contract. If so, the protection seller pays the notional amount of the contract to the protection buyer as compensation for the loss incurred. The latter, in turn, must deliver obligations (usually bonds) of the defaulted borrower with total principal equal to the notional amount of the CDS contract.

Since the CDS is a derivative instrument based on defaultable debt as the underlying asset, it is natural to enquire about the relation between the prices of credit risk in the bond and derivatives markets charged for resp. to a particular borrower. Such a relation is of crucial importance for pricing and hedging credit exposures. Duffie [8] shows that it is only under highly restrictive and simplifying assumptions that the intuitive equality between the premium on a CDS and the yield spread of a bond over its risk-free counterpart (written on resp. issued by the same corporate borrower) holds. In a static setting, taking merely no-arbitrage arguments into account, the equivalence is valid for par floating-rate notes rather than for par fixed-rate notes. As expected, applying this argument to observable CDS and bond yield spreads, pricing discrepancies are uncovered. The differences do not vanish even if one actually models the credit risk by employing standard pricing models (cf. e.g. Schönbucher [24]) instead of simply replicating cash flows. Not even complex credit risk models are presently able to price in the observed differences. In the market this differential between CDS and bond spreads (of equal maturities, usually 5 years) has become known as the *CDS basis*. Precisely this divergence in the pricing of instruments in the bond and derivatives markets for corporate debt is the topic of our research.

This paper explores the relation between the prices in the bond and derivatives markets on a representative and diverse cross-section of euro-denominated corporate bonds and CDS. Using standard assumptions we quantify the above mentioned mispricing when employing a deterministic reduced-form framework. In an extensive comparison of the pricing properties in the bond market for several parameterizations of the default intensity the Nelson-Siegel specification turns out to be optimal. This parametrization is subsequently used to price CDS, resulting in model CDS spreads up to 50% lower on

[1] The statement is based on a comparison of the figures projected in the BBA Credit Derivatives Survey 2001/2002 and the latest market statistics provided e.g. by ISDA Market Surveys for the global market at `www.isda.org` or by the OCC Bank Derivatives Reports for the US market at `www.occ.treas.gov`.

average than observed in the market. A model extension is therefore proposed which explicitly incorporates the delivery option implicit in CDS contracts: Since in settlement the protection buyer is entitled to choose from a basket of *pari passu* deliverable obligations (bonds), she will prefer to deliver the cheapest bond in the market at default. Our extension thus models the implied recovery value of the cheapest-to-deliver bond. Applying the extension to the data, the new recovery parameter considerably improves the pricing properties in the CDS market, as expected. The average implied recovery rates range from 8% to 47% and strongly vary across obligors and within individual ratings and industries. Analyzing the implied recovery rates a cross-sectional regression reveals a statistically and economically significant dependence on delivery option proxies. Our paper thus points out the necessity for incorporating the random structure of recovery rates into credit risk models in order to accurately price credit-risky instruments.

Considering the academic literature, there are several papers dealing with the pricing difference, as measured by the CDS basis, from a wholly descriptive point of view, i.e. not attempting to model, but simply to present and discuss possible explanatory approaches. Hjort et al. [15] and O'Kane and McAdie [23] distinguish between fundamental and technical (market) factors, describing their likely effects on the relative valuation in the two markets. According to their reasoning, factors such as legal and regulatory risk, new bond issuance, difficulties in shorting corporate bonds, the embedded delivery option for the CDS buyer, the positivity of CDS spreads, and exotic bond features (e.g. coupon step-ups, convertibility) drive CDS spreads higher, whereas funding costs of bonds, counterparty risk, and leveraging opportunities constitute factors reducing CDS spreads, while liquidity is identified as having an ambiguous pricing effect. To the best of our knowledge, due largely to their complexity there have only been very few attempts to include some of the above stated factors in an actual valuation.

The empirical literature on this rather narrow topic is scarce because until recently studies have usually restricted themselves to examining features of just one of the two markets. Rather than fitting a specific credit risk model to their data, Aunon-Nerin et al. [2] and Benkert [4] test for the influence on CDS spreads of theoretical factors motivated by the reduced-form and structural models via linear and semi-logarithmic regressions. In a similar manner Collin-Dufresne, Goldstein and Martin [7] investigate the determinants of corporate bond spreads. The main message of these papers is that CDS spreads react more intensely to firm-specific variables such as (historic or implied) volatility, whereas bond spreads respond more strongly to macroeconomic factors such as interest rates.

There exist two strands of recent empirical literature dealing with the relation between the CDS and bond markets. In the work by Zhu [25] and Blanco, Brennan and Marsh [6] vector time series analysis is applied to investigate the long-term pricing accuracy and the short-term pricing efficiency (dynamic linkages) between the two markets, i.e. these studies test the validity of the theoretical no-arbitrage equality between CDS and bond spreads as deduced by Duffie [8]. Both papers analyze only CDS and bond spreads with a maturity of five years. They find that although credit risk is priced equally in both markets in the long run, there exist substantial mean-reverting discrepancies in the short run. Furthermore, they report that the European and Asian bond markets incorporate new information more quickly than the local CDS markets, contrary to the situation in the US. The reasons they suspect to lie at the heart of these phenomena correspond to the factors specified in the above mentioned heuristic surveys: the costliness of shorting corporate bonds, the delivery option, and liquidity. In addition, they analyze the determinants of the spread differentials, essentially confirming the findings of the separate studies referred to above.

The line of research our work is embedded in are studies relying on the reduced-form model and its extensions. In a work by HOUWELING AND VORST [16], a reduced-form model with a polynomial intensity function and a fixed recovery rate is fitted to bond data and subsequently used to calculate model CDS spreads. The paper points out the differential pricing in the bond and derivatives markets by first directly comparing quoted CDS spreads to bond yield spreads and then to model CDS spreads. Their finding central to our paper is that bond spreads as well as model CDS spreads are lower compared to market CDS spreads. This mispricing is especially pronounced for speculative-grade borrowers, though not equally as clear-cut for investment-grade ones. In the paper the pricing characteristics of a simple reduced-form model specification are examined, but explications for their observations and suggestions for possible model extensions are presented only verbally.

The most popular explanation proposed in the literature for the divergence in the prices of credit risk between the bond and derivatives markets has been liquidity, although there is no consensus about its actual effect on the prices. In a classic paper by JARROW [20] liquidity risk is modelled in a reduced-form framework as a general convenience yield process affecting corporate bond prices. A subsequent empirical paper by JANOSI, JARROW AND YILDIRIM [19] calibrates a concrete specification of this model to corporate bond prices adding an affine function of market variables as the convenience yield. Their data shows that the price fluctuations not captured by interest-rate and credit risk processes are largely idiosyncratic, i.e. do not depend on systematic factors. More importantly, the calibrated convenience yield process changes the sign, which casts doubt on its relation to liquidity. Furthermore, the paper does not test this process against liquidity proxies.

Modelling in a reduced-form framework as well, LONGSTAFF, MITHAL AND NEIS [21] attach a liquidity discount process to corporate cash flows, but, arguing that CDS are the more liquid instrument, do not apply it to CDS spreads. They split the corporate bond spread into a default and a non-default component, inferring the former from the CDS spread. Their non-default component exhibits rapid mean reversion and dependence on market-wide and firm-specific liquidity proxies. As in the HOUWELING AND VORST [16] paper a model-independent comparison between bond and CDS spreads is performed, but surprisingly with the opposite outcome: bond spreads are higher on average than market CDS spreads, and this effect increases (in absolute terms) with lower rating. We suspect that both the sign of the mispricing and the significant dependence of the non-default component on liquidity proxies are a consequence of the specific data set used in the study since the offered liquidity argument would unlikely hold for the data sets analyzed in Houweling and Vorst [16], Blanco, Brennan and Marsh [6] and Janosi, Jarrow and Yildirim [19].

The bottom line is that literature hitherto still leaves open both the actual direction and the determinants of the pricing differences, as well as which explanatory approach should be taken. Applying a standard reduced-form model to our data set results in model CDS spreads which are up to 50% lower than the observed market spreads – a finding qualitatively in line with Houweling and Vorst [16]. Since on average we observe an underpricing of CDS, the liquidity adjustment in Longstaff, Mithal and Neis [21] seems not to be the appropriate choice of a model extension in our case. Therefore, in contrast to the papers discussed above, this paper studies an alternative approach to explaining the divergence in the pricing between the bond and derivatives market: the existence of a delivery option for the protection buyer in a CDS contract with physical delivery. Commonly debt of the same seniority is assumed to trade at the same level following a default, which is reflected by the modelling assumption of identical recovery rates for the defaulted bonds. In contrast to this simplification economically significant price differences routinely persist, as we show in several case studies of recent Chapter 11 filings (cf. Section 3.2). The CDS spread must thus reflect the value of the delivery option at the inception of the CDS contract additionally to capturing the default risk of the borrower. The aim of the present paper is therefore to incorporate the delivery option in the model specification in order to achieve superior pricing across both markets.

Bond prices at default enter the valuation of CDS through the expected recovery rate of the cheapest-to-deliver bond. Since the delivery option essentially depends on the minimum bond price at default, it must be related to the recovery value expected by market participants at inception of the CDS contract. Therefore we include this (risk-neutral) implied recovery value of the cheapest-to-deliver bond in the model and extract it from CDS data as an indicator for the implicit value of the delivery option. The implied recovery parameter considerably improves the pricing properties in the CDS market and strongly vary across obligors and within individual ratings and industries. Using regression analysis we explore the driving factors of the implied recovery rates. A cross-sectional regression reveals a statistically and economically significant dependence on delivery option proxies. In order to test whether liquidity possesses any explanatory power, the implied recovery rates are regressed against liquidity proxies, but they prove to be unambiguously insignificant. In summary, our paper provides solid evidence that the documented differences in pricing between the bond and CDS market can be attributed to the effect of the delivery option the CDS buyer has at the time of default.

The paper is structured as follows: Section 2 presents the standard reduced-form model and evidences its weakness in the simultaneous pricing of bonds and CDS. In Section 3 we motivate, introduce and empirically examine an extension to the standard setup based on the delivery option. Finally, Section 4 summarizes our findings.

2 Credit Risk Modelling

In this section standard reduced-form credit risk models are applied to bond and CDS data in order to analyze their performance when pricing simultaneously in these two markets. Since the existing literature lacks a systematic comparison of available model specifications, the pricing ability of several parameterizations is examined by calibrating them to the bond market. Subsequently, the pricing accuracy in the CDS market is examined for the best performing model specification. Given that previous studies report both over- and undervaluations of CDS contracts, as outlined in Section 1, further insight is thus provided into the direction of the mispricings. Since an accurate estimate of the mispricing is crucial for the choice of a model extension, details of CDS contracts neglected in other studies are precisely taken into account, especially the exact maturity and accrual payments.

2.1 Bond Valuation

In line with standard reduced-form modelling, as e.g. presented in Schönbucher [24], an arbitrage-free market without transaction costs is assumed, where uncertainty is modelled by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{Q})$. The measure $\mathbb{Q}$ denotes the pricing measure associated with the (riskless) money market account. In this market riskless and defaultable zero-coupon bonds and defaultable coupon bonds are traded. Denote by $P(t,T)$ the time-t value of a riskless zero-coupon bond with maturity T, and by τ the random default time, implicitly independent of the riskless term structure under the pricing measure. Let $Q(t,T) = \mathbb{E}_t^{\mathbb{Q}}[1_{\{\tau > T\}}]$ be the risk-neutral survival probability over the time period $\langle t, T]$.

Consider a defaultable coupon bond with outstanding coupon payments c at times $t_1 < t_2 < \ldots < t_N$, maturity t_N and a face value normalized to 1. Denote by $\delta(t_{n-1}, t_n)$ the fraction of the year between the payment dates t_{n-1} and t_n taking into account the relevant day count convention. Under the recovery of face value assumption (cf. Section 2.3.2), i.e. a fixed fraction π of face value being paid at the default time τ, the time-t price $C(t, \{t_n\}, c, \pi)$ of this coupon bond is obtained by applying the risk-neutral valuation principle to the coupon, face value and recovery cash flows:

$$\begin{aligned} C(t, \{t_n\}, c, \pi) &= \sum_{n=1}^{N} c\,\delta(t_{n-1}, t_n)\, P(t, t_n)\, \mathbb{E}_t^{\mathbb{Q}}\left[1_{\{\tau > t_n\}}\right] + P(t, t_N)\, \mathbb{E}_t^{\mathbb{Q}}\left[1_{\{\tau > t_N\}}\right] + \\ &\quad + \mathbb{E}_t^{\mathbb{Q}}\left[\pi\, P(t,\tau)\, 1_{\{\tau \le t_N\}}\right] \\ &= \sum_{n=1}^{N} c\,\delta(t_{n-1}, t_n)\, P(t, t_n)\, Q(t, t_n) + P(t, t_N)\, Q(t, t_N) + \\ &\quad + \pi \int_t^{t_N} P(t,s)\, f_\tau(t,s)\, ds\,, \end{aligned} \tag{1}$$

where $t_0 = t$ and $f_\tau(t,s)$ denotes the probability density function of the default time τ given information at time t. The density exists if the survival probability function $Q(t,T)$ is differentiable from the right in T and in that case it can be expressed as:

$$f_\tau(t,s) = -\frac{\partial}{\partial s}\, Q(t,s)\,.$$

The required differentiability is ensured in all our model specifications (cf. Section 2.3.1). The integral in Eq. (1) is therefore numerically approximated via differences over a time grid $t = s_0 < s_1 < \ldots < s_M = t_N$:

$$\int_t^{t_N} P(t,s)\, f_\tau(t,s)\, ds \;\approx\; \sum_{m=1}^{M} P(t,s_m)\big(Q(t,s_{m-1}) - Q(t,s_m)\big)\,. \tag{2}$$

Effectively, the mesh of the time grid corresponds to the time step in our observations whether default has yet occurred, the underlying simplifying assumption being that recovery is paid at the observation time immediately following default. Basically, the accuracy of the discretization increases by raising the default observation frequency. In the empirical evaluations monthly time steps are used since higher frequencies, e.g. weekly time steps, result in practically identical prices.

2.2 CDS Valuation

There are two sides to a CDS contract: the fixed leg, comprising of the regular payments by the protection buyer, and the default leg, containing the contingent payment by the protection seller. The exact cash flow structure of the fixed leg in a standard ISDA contract (cf. 2003 ISDA Credit Derivatives Definitions [18]) is specified as follows: Premium payment dates are fixed and do not depend on the specific contract date. They are quarterly and happen on the 20th of March, June, September and December. Thus, if a CDS is contracted between those dates, the first period is not a full quarter and the first premium payment is adjusted accordingly. In addition, we account for the now variable maturity of CDS contracts: As a result of fixing the premium payment dates, the length of the protection period varies and depends on the contract date since the quoted CDS maturity begins on the first premium payment date. Furthermore, the accrued premium in case of default must be taken into account. Lastly, the day count convention used in CDS contracts is *actual/360*.

Consider a CDS with outstanding premium payments p at times $t_1 < t_2 < \ldots < t_N$, maturity t_N and notional normalized to 1. The same recovery assumption as for corporate bonds is employed. Denoting the time-t value of the fixed leg by $V^{\text{fix}}(t,\{t_n\},p)$ and the time-t value of the default leg by $V^{\text{def}}(t,t_N,\pi)$, then the time-$t$ value of the CDS contract to the buyer is $V^{\text{def}}(t,t_N,\pi) - V^{\text{fix}}(t,\{t_n\},p)$.

If default happens within the protection period, the protection buyer has made $I(\tau) =$

$\max\{1 \leq n \leq N : t_n \leq \tau\}$ premium payments, the remaining ones $I(\tau) + 1, \ldots, N$ being no longer due, except for an accrual payment of $p\,\delta(t_{I(\tau)}, \tau)$ at time τ. Hence, the time-t value of the fixed leg is given by

$$
\begin{aligned}
V^{\text{fix}}(t, \{t_n\}, p) &= \sum_{n=1}^{N} p\,\delta(t_{n-1}, t_n)\, P(t, t_n)\, \mathbb{E}_t^{\mathbb{Q}}\left[1_{\{\tau > t_n\}}\right] + \\
&\qquad + \mathbb{E}_t^{\mathbb{Q}}\left[p\,\delta(t_{I(\tau)}, \tau)\, P(t, \tau)\, 1_{\{\tau \leq t_N\}}\right] \\
&= \sum_{n=1}^{N} p\,\delta(t_{n-1}, t_n)\, P(t, t_n)\, Q(t, t_n) + \\
&\qquad + p \int_t^{t_N} \delta(t_{I(s)}, s)\, P(t, s)\, f_\tau(t, s)\, ds\,,
\end{aligned}
$$

where $t_0 = t$. On the other hand, the time-t value of the default leg is given by

$$
V^{\text{def}}(t, t_N, \pi) = \mathbb{E}_t^{\mathbb{Q}}\left[(1-\pi)\, P(t, \tau)\, 1_{\{\tau \leq t_n\}}\right] = (1-\pi) \int_t^{t_N} P(t, s)\, f_\tau(t, s)\, ds\,. \tag{3}
$$

In both valuation formulas the integral is approximated in the same manner as in Eq. (2).

At initiation of a CDS the premium $p(t, \{t_n\}, \pi)$ is chosen such that the contract value to both parties is zero, and since the value of the fixed leg is homogeneous of degree 1 in p, it follows that

$$
p(t, \{t_n\}, \pi) = \frac{V^{\text{def}}(t, t_N, \pi)}{V^{\text{fix}}(t, \{t_n\}, 1)}\,. \tag{4}
$$

2.3 Model Specification

In order to complete the valuation a precise model parametrization must be chosen for the intensity function, the recovery rate, and the riskless interest rate to be used in the empirical study.

2.3.1 Intensity Function

The fundamental choice when modelling the default intensity is whether it should be stochastic or deterministic. We consider the deterministic model to be entirely adequate, as already argued by Houweling and Vorst [16] and Malherbe [22]. A stochastic representation may appear more realistic though, the more so as dependencies with other risk factors (e.g. interest rate risk and recovery risk) can be implemented in this case. Surprisingly, the theoretically additional flexibility of stochastic models with dependent risk factors does not substantially improve the model fit (as documented e.g. in Duffie, Pedersen and Singleton [9]), so independent risk factors are mostly assumed (as in all previously mentioned studies), thereby reducing the main advantage of stochastic modelling. Moreover, the scarcity of data in the corporate bond market poses serious restrictions on the number of model parameters to be estimated (cf. Section 2.4).

Assuming the existence of a non-negative bounded deterministic function $\lambda(t)$ representing the intensity of the default time τ under the pricing measure $\mathbb{Q}$, the risk-neutral survival probability can be expressed as

$$Q(t,T) = \exp\left\{ -\int_t^T \lambda(s)\,ds \right\}.$$

To the best of our knowledge, the academic literature lacks a comprehensive comparison between the pricing abilities of the available parameterizations for the intensity function. There exists a tradeoff when choosing a specific functional form since on the one hand the intensity should reproduce market prices as accurately as possible, and on the other hand it should be specified as parsimoniously as possible to cope with data restrictions. This study examines six functional forms commonly encountered in the literature to find the one optimal for our corporate credit risk data. The following specifications are employed (cf. Table 1): polynomials up to order 3 (as in Houweling and Vorst [16]), a log-linear function, and the Nelson-Siegel and Svensson functions.

2.3.2 Recovery Rate

As stated in Sections 2.1 and 2.2 on valuation, the recovery payoff in default is expressed in the *recovery of face value* (also called *recovery of par*) formulation, where a fraction π of the contract's notional amount is paid back in default. The idea underlying this recovery formulation is a liquidation of the defaulted obligor's assets by a bankruptcy court, in which case all claims are only on the notional (e.g. bond coupons are disregarded) and relative priority of claims is respected. Thus, in default the investor receives a fraction of the face value of an asset depending on its seniority. We opted for this formulation because it coincides with the definition of the default payment in CDS contracts, where only the face value of debt is protected.

As common in academic literature and practical applications, the recovery parameter π is assumed constant. In our calculations it is set equal to 40% in both markets (as assumed e.g. by Malherbe [22] and in standard pricing tools in Bloomberg as well). In analyses not reported here we employ recovery rates in a range between 20% and 60%, but the effect on prices is negligible since the default intensity adjusts accordingly.

2.3.3 Riskless Rate

For the valuation of bonds and CDS one additionally needs a term structure of riskless interest rates. Although a natural choice is offered by interest rates derived from government bonds, lately it has repeatedly been evidenced that investors have shifted to use (plain vanilla) interest rate swaps as the reference riskless curve instead of government bonds (as reported e.g. in Houweling and Vorst [16] and Hull, Predescu and White [17]). This shift could have originated from several factors, for instance from the introduction of the euro, which caused the bonds of the member countries to trade at different interest rate levels in the same currency making a definite choice impossible (cf. Geyer, Kossmeier and Pichler [12]). A further drawback of government securities is their illiquidity in comparison to interest rate swaps arising from the fact that in nature bonds are in limited supply, whereas the notional in an interest rate swap can be contracted almost arbitrarily large.

A disadvantage of the swap rate is that it actually entails credit risk from two sources, namely counterparty risk and the underlying floating payments being indexed to a defaultable short-term interbank rate (cf. Feldhütter and Lando [11]). Nevertheless, interest rate swaps are the most liquidly traded interest rate product and reflect the current term structure of riskless interest rates most accurately. For this reason we employ riskless zero-coupon term structures derived from swap rates.

2.4 Data

The data set underlying our study consists of daily price quotations for euro-denominated bonds and CDS of a broad cross-section of corporate borrowers. The data span two years from January 2003 to January 2005. We use senior unsecured plain vanilla coupon bonds without any optional features and CDS on senior obligations with specified physical delivery and the 'modified modified restructuring' clause, which is common in Europe. All quotes are snapshots taken from Reuters at 15:00 GMT/BST with a time window of plus/minus 90 minutes. The riskless term structure of interest rates is constructed from synchronous money market and swap rates.

Since the reliability of data is a critical issue in corporate credit markets, in order to ensure the quality of quotations, all bond quotes entering the analysis represent averages over at least three quotes stemming from different contributors, with an upper bound for the respective bid-ask spreads and for the discrepancy between the pairs of quotes. Nevertheless, the data set contains merely quotations and not actual trade prices. However, these quotations are used by practitioners in daily business and typically hold for a contract size in the order of magnitude of 10 million euro, with most transactions taking place within the quoted bid-ask spread. Similar quality checks are also applied to the CDS quotes, which are additionally compared to actual trade data from brokers to discard quotations potentially far from actual prices. In our analyses we use the mid of the bid-ask quotations.

Regarding the maturity structure of the data, there exists a high concentration of CDS quotes at the five-year maturity, which led most studies dealing with corporate credit data to focus on this one specific maturity where data is available for a larger number of borrowers (as e.g. in [6], [21] and [25]). This approach has the obvious disadvantage that credit risk effects are only observed for one single point on the term structure. In order for our study to provide a more detailed insight, we select borrowers with enough data to estimate a complete term structure of the default intensity on a daily basis. Though this requirement significantly reduces the number of eligible borrowers, it provides the opportunity to observe credit risk effects for the whole maturity spectrum.

We choose obligors for which both bond and CDS quotes are available for at least two maturities on approximately 75% of all trading days in the two-year period, additionally requiring that the bond and CDS maturity ranges sufficiently overlap: Bonds with maturities longer than ten years are excluded since longer-dated CDS are seldom traded. On each day, only CDS with maturities not shorter than the shortest bond maturity and not longer than one year after the longest bond maturity are included in the analysis.

Based on these criteria twelve corporate borrowers, presented in Table 2, are singled out. Although we only have a small sample available, it consists of high quality data and enables us to carry out term structure estimations. The selected companies cover a wide range of industries and rating classes therefore constituting a representative sample. For each obligor there are on average 3.8 bonds and 3.1 CDS available for estimation on each day. The maturity range spanned by the bonds and CDS is roughly five years, concentrated in the maturities between three and seven years.

2.5 Methodology

Using the presented data set we analyze to which extent common deterministic reduced-form models are able to simultaneously price bonds and CDS by first calibrating the models to bond data and subsequently examining their pricing ability in the CDS market.

All six models are calibrated on a daily basis to the bond quotations for each of the issuers. Denote by θ the parameter vector and by Θ the space of admissible parameters for the respective model. Suppressing the issuer and model indices, on every day t there are I_t bonds available with observable market quotes $C_{i,t}^{\text{mkt}}$, $1 \leq i \leq I_t$. The calibration is carried out by minimizing the mean absolute bond pricing errors of the respective model:

$$\theta_t^* = \arg\min_{\theta \in \Theta} \sum_{i=1}^{I_t} \left| C_{i,t}^{\text{mkt}} - C_i(t, \{t_n^i\}, c_i, \pi; \theta) \right| .$$

The estimation is implemented via non-linear optimization.

The various parameterizations of the intensity are compared on the basis of resulting bond pricing errors to find the optimal functional form exhibiting an acceptable mean absolute error and a parsimonious number of free parameters. To analyze the pricing performance in the CDS market, on each day model-implied CDS spreads are calculated for maturities lying within the bond maturity range, employing the best performing intensity function.

2.6 Results

We identify the optimal parametrization of the intensity function for our bond data by comparing the model and market bond prices. Table 3 displays the resulting mean absolute pricing errors (MAE) for the various specifications.

Overall, the Svensson function provides the lowest MAE of 9.02 bp based on six free parameters. Comparing the two functional forms with four parameters, the Nelson-Siegel function exhibits a 9.62 bp MAE, which is lower than the 13.94 bp MAE for the cubic function. Considering three-parameter families, the log-linear function (MAE 14.92 bp) performs better than the quadratic function (MAE 25.51 bp). Finally, the linear function, being a model with only two parameters, has the highest MAE of 41.42 bp. The pricing performance of most models thus lies within an average bid-ask spread of roughly 30 bp usually encountered in the corporate bond market.

Although the pricing accuracy of a model is the dominant criterion, a parsimonious representation is almost equally important since the scarcity of data presents a considerable modelling constraint in corporate debt markets. Judging by the MAE, polynomial functions as employed in Houweling and Vorst [16] seem to be an unsatisfactory choice for the intensity because there exist alternative functional forms with an equal number of parameters providing lower MAE, namely the Svensson and Nelson-Siegel parameterizations. A further advantage of the latter two functional forms over the log-linear and polynomial functions is the convergence of the intensity to a long-term limiting value, which is especially useful for extrapolations. Taking the scarcity of data into account, the Nelson-Siegel function appears to represent an acceptable tradeoff: Its pricing accuracy is nearly as high as for the Svensson function, but the number of parameters is considerably lower (four vs. six). We therefore choose the Nelson-Siegel specification for all following analyses.

Having identified the functional form for the intensity with the least feasible number of free parameters reproducing observed bond prices sufficiently closely, we examine its pricing performance in the CDS market by comparing the observed to the model-implied CDS spreads. Table 4 presents the CDS pricing errors for each obligor.

On average the mean absolute pricing error (MAE) is 24.30 bp or 23.92% expressed in relation to the spread size, which constitutes a considerable mispricing. The mean relative absolute errors (MRAE) lie within a minimum of 10.25% and a maximum of 57.53% of the spread. Moreover, the mean pricing errors (ME) are biased: For eight obligors the model CDS spread is lower on average then the market CDS spread, and higher for four obligors. The mean difference between model and market CDS spreads is thus negative and tends to increase with lower rating grades when measured in basis points, but not when measured as a percentage of the market spread. Figure 1 shows the time series of CDS pricing errors for the different maturities for DaimlerChrysler as a representative example.

The findings are qualitatively in line with Houweling and Vorst [16] and Blanco, Brennan and Marsh [6], but opposed to the CDS basis reported in Longstaff, Mithal and Neis [21], where the model-independent CDS spread, proxied by the bond spread, turns out higher than the market CDS spread, with the difference getting more pronounced for lower rating grades. The lack of pricing accuracy in the CDS market thus obviously necessitates an extension of the standard setup to obtain a credit risk model able to accurately price CDS and bonds at the same time. In the next section we therefore discuss potential extensions and motivate our decision to model the delivery option of the CDS contract.

3 Modelling Extension

The literature proposes a multitude of potential factors for explaining the origin of the differences in the pricing between the bond and CDS markets. As mentioned in the introductory section, these are put forward e.g. in studies by Hjort et al. [15] and O'Kane and McAdie [23], though in a purely descriptive manner. The explanatory factors most often cited in the literature are liquidity and the delivery option, the rest being even less tangible.

3.1 Liquidity vs Delivery Option

The traditionally popular explanation for real-world market imperfections – liquidity – has hitherto already been thoroughly analyzed in the literature, yet with conflicting conclusions. The studies fail to unambiguously answer several crucial questions: First of all, it is still unclear whether liquidity actually presents a valid explanation in the first place since the arguments produced could affect spreads either way, i.e. which market should be more liquid than the other and why. Even after accepting liquidity as the driver of the mispricings, it remains unresolved whether the difference in liquidity is perhaps between CDS and bonds of single maturities rather than between the markets as a whole, whether the level of (relative) liquidity alternates with time between the two markets and/or between instruments, or whether the bond and CDS markets for similar issuers (e.g. of a particular rating) must exhibit similar relative liquidity properties.

The paper by Janosi, Jarrow and Yildirim [19] reveals the difficulty: Their calibrated convenience yield process exhibits a varying sign, which raises doubts on its relation to liquidity. Longstaff, Mithal and Neis [21] indeed report a dependence on liquidity proxies of their residual yield spread obtained by calibrating an extra discount process to corporate bonds; since in addition they report an overestimation of CDS spreads before adjusting for liquidity – which is in contrast to most other studies – our conjecture is that both the observed direction of the mispricings and the liquidity dependence are in all likelihood attributable to the specific data set used. Their argument would unlikely hold for the data sets analyzed in Houweling and Vorst [16], Blanco, Brennan and Marsh [6] and Janosi, Jarrow and Yildirim [19]. The data set at our disposal displays mispricings in line with the ones reported in Houweling and Vorst [16] and Blanco, Brennan and Marsh [6], and furthermore it turns out that liquidity proxies do not possess any explanatory power (cf. Section 3.4).

For the stated reasons this paper considers the delivery option as an alternative explanation for the mispricing between the bond and CDS market stemming from the manner in which a CDS is usually settled in default. Since the form of settlement prevailing in the CDS market by far is physical delivery of defaulted assets (in contrast to cash settlement), one must examine its implications for CDS pricing. Namely, a CDS contract commonly refers not to one single deliverable obligation only, but to a basket of deliverable obligations satisfying certain conditions, the crucial one being the seniority of the debt delivered. As illustrated by event studies in Section 3.2, contrary to the common modelling assumption of equal bond prices in default, the differences between post-default prices of deliverable bonds cannot be ignored.

One conceivable origin of differing bond values in and after default is put forth in a recent theoretical paper by Guo, Jarrow and Zeng [14] for instance, who develop a reduced-form model based on the idea that a default does not need to immediately lead to bankruptcy. According to their definition, the issuer continues to operate after default depending on whether she is solvent or not. As a result, the issuer's bonds continue to exist as well, and trade at different levels depending on their characteristics (coupon and maturity). Studies by Guha [13] or Duffie and Singleton [10] also suggest that bond prices at default could reflect market expectations whether the obligor will continue operating after the credit event or rather be liquidated straight away. Another possible origin could be deduced for example from particular supply and demand considerations in default, e.g. when one market participant is accumulating debt of a defaulted borrower to influence the outcome of the bond settlement process. A third origin of differing bond prices in and after default could arise from trading frictions (e.g. high transaction costs) and market imperfections (e.g. the impossibility of shorting), which induce individual bonds to trade away from their supposedly fair values. Bond pricing errors as calculated in Section 2.6 are a possible indicator of such deviations.

The protection buyer thus possesses an option to deliver the cheapest bond(s) upon default. Obviously, the spread at the inception of the CDS contract must reflect the uncertain recovery values (i.e. bond prices) in default, additionally to capturing the default risk of the borrower. The common modelling assumption of a recovery value which is constant and identical across both markets has several consequences: For one, it implies that bond prices are equal in default, making the delivery option worthless. Moreover, such a recovery rate forces the CDS spread to be driven exclusively by the default risk of the underlying, as can be clearly discerned from Eq. (3), potentially causing unnatural fluctuations in the implied default intensity. Lastly, from a modelling point of view, plugging in constant recovery rates precludes the analysis of their mutual dependence on the default intensities.

As observed in Section 2.6, mispricing arises when *equal* recovery values are used in the bond and CDS markets. The aim of the present paper is to analyze whether the inclusion of variable CDS recovery values representing the cheapest-to-deliver bond price in default can bring about pricing effects and explain the emergence of the CDS basis.

3.2 Case Studies of Defaults

For the purpose of verifying the conjecture that the delivery option potentially possesses value, we inspect the behavior of bond prices during the time period immediately before and after default for three companies which filed for Chapter 11 bankruptcy protection during 2005. The obligors are Delta Air Lines, Inc. and Northwest Airlines, Inc., which filed for Chapter 11 on Wednesday, September 14$^{\text{th}}$, 2005, and Delphi Corp., which filed for Chapter 11 on Saturday, October 8$^{\text{th}}$, 2005. Price quotations from Bloomberg are at our disposal for four (senior unsecured) bonds issued by Delta and Northwest Airlines each and for three (senior unsecured) bonds of Delphi. The price information is available for the whole month in which the respective defaults occurred and consists of the mid-quotations of the daily low, high and closing prices for each bond.

Figure 2 shows the daily average closing prices for each company in its month of default. The default event had a clear impact on the bond prices of Northwest and Delphi, whereas the default of Delta apparently happened as no surprise to the market since it had no noticeable effect on bond prices. Comparing the average closing prices at default, we observe that the price levels of the three obligors are quite dispersed: Delta had the lowest price level with an average bond price of 15.92 (per 100 of face value), followed by Northwest with 26.81 and Delphi with 57.92. These figures point at high variations in the recovery between different corporate defaults.

Since the delivery option in a CDS is worthless in default if bond prices of the company coincide, our main interest lies in comparing the individual bond prices of each obligor to detect their potential discrepancies. In the ideal case one seeks to compare price information for different bonds obtained at exactly the same points in time. Since only the daily low, high and closing prices for each bond are at our disposal, we are merely able to infer certain *bounds* on the contemporaneous maximum price differences, as described below. Closing prices provide some indication of contemporaneous price differences, though with two drawbacks: First, closing prices might not be contemporaneous since the end-of-day values could stem from different points in time during the trading day, and second, intra-day price deviations might be both lower and higher than suggested by closing prices. Information on the intra-day price differences is therefore inferred by looking at the daily low and high of each bond price.

Let us assume for a moment that an obligor has only two bonds outstanding on default day, bond A and bond B. We have at our disposal the high and low for both bonds on default day. Without loss of generality, let bond A have the smaller low price. In the period during this trading day when bond A was at its low, bond B was by definition trading at a price greater or equal to its own low. It follows that if the difference between these lows is non-zero, there must have existed a period during the day when the price difference between these bonds was at least that much. For this reason we term this value the lower bound of the contemporaneous (maximum) price differences. On the other hand, bond B traded at most at its own high price on this day, and especially in the period when bond A was at its low. The difference between these values is thus an upper bound on the contemporaneous price differences. This is the highest possible price difference which might have been realized on this day. The case with more than two bonds outstanding at the time of default is analogous. In order to infer the lower bound we compare the maximum and the minimum of the bonds' low prices, and for the upper bound we consider the maximum of the highs and the minimum of the lows.

Table 5 contains the bounds for the contemporaneous price difference for each obligor on its day of default. For Delta Airlines the values are in the range between 4.42 and 8.67, and the dispersion of closing prices amounts to 1.81. For Delphi the price differences are of comparable magnitude with 1.50 between closing prices, a lower bound of 3.00, and an upper bound of 8.50. For Northwest Airlines the differences are even higher with a 6.00 difference between closing prices, a 3.39 lower bound, and a 12.40 upper bound. Overall, we find substantial contemporaneous price deviations in the range of 3 to 12, strongly indicating that the delivery option is valuable and requires further consideration.

Since on the one hand the default event is anticipated by the market for some obligors, e.g. Delta Airlines, and on the other hand the settlement period for CDS contracts lasts 30 days following a default, price differences before and after default are of additional interest. Figure 3 shows the daily lower and upper bounds of the price differences in the month of default for each company. The price differences before and after default are quite similar for Delta Airlines and Delphi, whereas for Northwest Airlines significantly higher differences are observed before default possibly because it happened as a surprise to the market. After default, the lower bounds are in the range of 1 to 8 and the upper bounds are between 3 and 14 for all companies, strengthening the argument against equal default prices. Furthermore, bond prices vary strongly over time after the default event: Comparing the maximum high and the minimum low of bond prices over the post-default period (cf. Table 5) yields 9.6 vs. 22 for Delta Airlines, 19.5 vs. 32 for Northwest Airlines, and 49.2 vs. 70.5 for Delphi.

The findings described above are in line with the ones in a more comprehensive study of corporate defaults by Guha [13]. Upon closer inspection of Table VIII in the cited paper, which shows the bond-price ranges of all obligors in the sample on default day, one finds that obligors with a price range wider than one dollar are almost as numerous as the ones with prices converging to approximately the same value (i.e. price range within one dollar). Incomprehensibly, the author claims that "in the *vast majority* of cases bonds of the same issuer and seniority are valued equally or within one dollar by the market, irrespective of their time to maturity" [13, p. 21, emph. added]. This claim obviously contradicts his observations, the more so as bond price differences in default are likely to be even higher when focusing only on the subset of obligors which are actively traded in the CDS market and taking into account not just their day of default, but the whole period up to the CDS settlement day.

In this case study we have uncovered substantial bond price differences both contemporaneously and over time, showing the complex stochastic nature of recovery rates. The findings indicate that the delivery option is potentially valuable and thus needs to be explicitly accounted for in credit risk models. One drawback of our case study is that the data only include mid-prices, but the deviations of the individual bonds are nevertheless obvious and should hold when the bid-ask spread is included.

3.3 Extended Methodology

As discussed in the preceding sections, within the reduced-form framework little has been written on the implementation of recovery rate models although they are equally as significant to the accuracy of a credit risk model as is the default likelihood. Hence, since the aim of this paper is to analyze the influence of the delivery option, we are compelled to go beyond the customary model, which implicitly ignores its existence.

The prevailing intensity-based model values bonds and CDS with the same underlying credit risk using a constant recovery parameter equal for both markets, usually inferred from surveys of historically realized recovery rates such as Altman, Resti and Sironi [1]. This is exactly the setup we adopted in our basic analysis in Section 2, finding out that it is inadequate for simultaneous pricing in both the underlying and derivatives market.

Going beyond the usual model means that we need to induce uncertainty in the recovery rates of bonds at the time of default. Formally, a straightforward way of achieving this goal within the reduced-form framework is augmenting the Poisson (one-point) process modelling the survival and default of an obligor by a vector of random markers representing the recovery rates of bonds and drawn at the time of default. In this respect our setup builds on Schönbucher [24]. Based on a general model exhibiting random recovery rates as a motivation for our approach, we deduce and justify the assumptions adopted in the subsequent empirical analysis in order to reduce the parameters to a computationally tractable number.

Taking one step back, not only are realized recovery rates among bonds in default different, but there is another source of randomness driving the correct recovery parameter in the pricing of CDS – the number of deliverable bonds outstanding at the time of default. Bühler and Düllmann [5] face a similar problem when developing a conversion factor system for a multi-issuer bond futures contract. The paper conveniently assumes that the clearing house pledges to substitute defaulted bonds by bonds of similar characteristics in order to avoid dealing with a random number of deliverable bonds. Analogously, we also make the assumption that the number of deliverable bonds, K, is constant and known at inception of the CDS contract. It can be argued that the firms in the sample are mature enough to have reached a balanced number of outstanding debt instruments over time, so though some debt may mature before the maturity of the CDS, in all likelihood new debt will be issued instead. Based on this assumption we next present our methodology.

Denote by $G(t, d\pi)$ the K-dimensional distribution (under the martingale measure $\mathbb{Q}$) on $[0,1]^K$ of the random vector π of recovery rates conditional on default happening in the infinitesimal time interval $\langle t, t+dt]$. The randomness is introduced *ad hoc* because the present literature does not yet agree about which fundamental factors (such as the firm's asset value, bond maturity or coupon amount) influence the bond value at default and how. For this reason we deliberately leave aside the potential origin of the differences in recovery rates wanting to focus rather on their consequences for now. The only technical requirement placed on this distribution is integrability with respect to the martingale measure $\mathbb{Q}$.

The only part of the bond valuation formula (1) affected by these considerations is the recovery payment in default. In full generality, the time-t value of the now random recovered amount $\pi_k(\tau)$ on bond k, $1 \le k \le K$, with maturity T^k is expressed as:

$$\begin{aligned}\mathbb{E}_t^{\mathbb{Q}}\left[\pi_k(\tau)\, P(t,\tau)\, 1_{\{t<\tau\le T^k\}}\right] &= \int_t^{T^k}\int_{[0,1]^K} \pi_k(s)\, P(t,s)\, Q(t,s)\, G(s,d\pi)\, \lambda(s)\, ds \\ &= \int_t^{T^k} \pi_k^e(s)\, P(t,s)\, Q(t,s)\, \lambda(s)\, ds\,,\end{aligned}$$

where $\pi_k^e(t) := \int_{[0,1]^K} \pi_k(t)\, G(t,d\pi)$ denotes the locally (i.e. time-t) expected recovery rate for bond k.

In general, it is justified to use the locally expected recovery rate when pricing a corporate bond because its payoff depends linearly on the recovery rate – as is conveniently the case with both bonds and CDS, but is violated e.g. by recovery swaps.

The modeler is now free to choose the (deterministic) locally expected recovery function she deems appropriate, though data availability poses considerable constraints, rendering impossible a calibration of both the intensity function and an elaborate recovery function (potentially even separately for each of the bonds), which leads us to adopt the following simplifying assumption when pricing bonds:

Assumption 1. (bonds)
All one-dimensional marginal distributions of the vector of recovery rates have identical expectations regardless of the timing of default:

$$\pi_k^e(t) = \pi^e \in [0,1] \quad \text{for all} \quad 1 \le k \le K \quad \text{and} \quad t \ge 0.$$

Since we are only looking at bonds of a single seniority this assumption makes sense economically nevertheless: Though bonds in the same class are *expected* to recover identical amounts in the event of default, the actual realizations need not be equal. Note also that the assumption corresponds to our recovery specification in the basic model (cf. Section 2).

Having dealt with bond valuation under random recovery, we next turn to CDS pricing. The default-contingent payoff of a CDS contract with physical delivery depends on the value of the cheapest-to-deliver bond, i.e. on the minimum recovery rate over all deliverable obligations at the time of default:

$$\pi_{\min}(\tau) = \min_{1 \le k \le K} \pi_k(\tau).$$

Analogously to the above, only the default-contingent loss payment in a CDS is affected by these considerations. In full generality, the present value of the now random loss compensation $1 - \pi_{\min}(\tau)$ in a CDS with maturity T is expressed as

$$\begin{aligned} \mathbb{E}_t^{\mathbb{Q}}\left[(1-\pi_{\min}(\tau))\, P(t,\tau)\, 1_{\{t<\tau\le T\}}\right] &= \int_t^T \int_{[0,1]^K} (1-\pi_{\min}(s))\, P(t,s)\, Q(t,s)\, G(s,d\pi)\, \lambda(s)\, ds \\ &= \int_t^T (1-\pi^e_{\min}(s))\, P(t,s)\, Q(t,s)\, \lambda(s)\, ds\,, \end{aligned}$$

where $\pi^e_{\min}(t) := \int_{[0,1]^K} \pi_{\min}(t)\, G(t,d\pi)$ denotes the locally (i.e. time-t) expected minimum recovery rate.

Once more, the modeler is now free to choose an appropriate (deterministic) locally expected minimum recovery function, but the data pose restrictions again. Therefore, we abstain from calibrating a complex minimum recovery function, but adopt the following simplifying assumption when pricing CDS in the extended setting:

Assumption 2. (CDS)
The distribution of the locally expected minimum recovery rate has identical expectations regardless of the timing of default:

$$\pi^e_{\min}(t) = \pi^e_{\min} \in [0,1] \quad \text{for all} \quad t \ge 0.$$

$\pi^e_{\min}$ therefore represents the expected value of the cheapest-to-deliver bond, identical for all possible default times. Thus, the main consequence of our extension is that the expected recovery rate for each bond π^e and the expected minimum recovery rate $\pi^e_{\min}$ in the CDS are now allowed to differ.

In the implementation, we fix the recovery rate for each individual bond (at 40%) as in standard credit risk models. Additionally the implied minimum recovery parameter is calibrated to CDS data. For the purpose of measuring the influence of the delivery option this parameterization is sufficient, since the implicit value of the delivery option is reflected by the difference in the recoveries.

We calibrate the implied minimum recovery parameter every day to CDS data of each issuer. Suppressing the issuer and day indices, let there be J CDS available with observable market quotes p_j^{mkt}, $1 \le j \le J$. The implied expected minimum recovery parameter is obtained by minimizing the mean absolute CDS pricing errors of the model:

$$\pi^e_{\text{min}} = \arg \min_{\pi \in [0,1]} \frac{1}{J} \sum_{j=1}^{J} \left| p_j^{\text{mkt}} - p_j(\cdot, \{t_n^j\}, \pi; \theta^*) \right| ,$$

where θ^* is the parameter vector of the Nelson-Siegel model already calibrated to bond prices (cf. Section 2.5). As previously, the estimation is implemented via non-linear optimization.

3.4 Results

The average implied minimum recovery lies in the range between 8.87% and 46.34% (cf. Table 6) and exhibits substantial dispersion among the analyzed obligors. It strongly varies even within an individual rating and industry class, thereby strengthening our argument for a firm-specific delivery option. The average standard deviation of the implied recovery values is approximately 10%, indicating significant fluctuation over time. Figure 4 shows the time series of the implied minimum recovery rate estimated for DaimlerChrysler and is representative of the time-series properties for the whole sample. The implied recovery for DaimlerChrysler averages 11.5% and exhibits a seemingly cyclical or mean-reverting behavior over time.

The findings indicate that the effect of the delivery option on the CDS spread is strong enough to result in plausible implied recovery values, i.e. one does not observe a dominance of boundary solutions (0% or 100%) which would suggest that the delivery option cannot explain the CDS pricing errors and thus does not drive CDS spreads. Considering the lowest and highest implied minimum recovery in the time series per obligor (cf. Table 6), the value of 100% is never estimated as the optimal parameter, whereas for half of the obligors the value of 0% is estimated at least once, which could be deemed too low and could indicate that effects other than the delivery option may be at work, which are also reflected in the calibrated recovery parameter.

Basically, these additional effects could have been introduced by the simplifying assumptions on the recovery parameter. One effect may be a dependence of the minimum recovery on the CDS maturity because for CDS with longer maturities not all bonds might be available for delivery in default as some may mature without being replaced. Furthermore, the default intensity and the recovery rate could be correlated in reality. Finally, the implied recovery parameter could be driven by other factors, such as liquidity. All these effects potentially influence the calibrated implied recoveries. For this reason, we conduct a cross-sectional analysis of the implied recoveries to demonstrate that this parameter is primarily driven by the value of the delivery option.

Modelling the delivery option by introducing the implied minimum recovery rate we expect a significant improvement in pricing ability for the CDS market. Table 7 documents the pricing performance of the extended model on CDS. The average MAE is reduced from

24.3 bp to 7.9 bp, resp. from 23.6% to 11.1% expressed as relative error. Compared to the range of bid-ask spreads of 3 to 10 bp (with the exception of Fiat, where bid-ask spreads are up to 40 bp), these pricing errors seem acceptable, which is a crucial improvement with respect to the initial model. From a technical point of view pricing errors are naturally expected to decrease by adding a further parameter to the model, so as a next step it is essential to demonstrate that the additional parameter possesses an economic justification.

To this end, we investigate whether the implied recovery rates are linked to factors driving the value of the delivery option by taking a regression-based approach. Liquidity proxies are taken into account as control variables in order to test whether liquidity affects the estimated recovery rates. The following list presents the presumptive proxies for the value of the delivery option and our hypotheses for their influence on the implied minimum recovery:

- number of bonds:
  The more bonds available for delivery, the lower the expected minimum price in default, which is exactly the recovery of the cheapest-to-deliver bond. In our data set the average number of available bonds per obligor is in the range from two to seven bonds.
- maximum bond price difference:
  The maximum bond price difference is defined as the difference between the highest and the lowest market bond price. If the difference persists in default, corporates with higher price differences will exhibit lower implied recovery rates. In our sample the average maximum bond price differences lie in the range from 2.56 to 15.74.
- absolute bond pricing error:
  In principle, pricing errors indicate that there exist bonds whose market values deviate from their model prices. If the magnitude of the deviations persists in default, corporates with higher bond absolute pricing errors will exhibit lower implied recovery rates. As displayed in Table 7, in our sample the bond MAE is in the range from 0.24 bp to 39.06 bp.
- minimum bond pricing error:
  The minimum bond pricing error is defined as the pricing error of the bond with the largest deviation below its model price. If the magnitude of the deviations persists in default, corporates with higher minimum bond pricing errors will exhibit lower implied recovery rates. In our sample the average minimum bond pricing errors fall in the range from 0.003 bp to 63.6 bp.

The following liquidity proxies are also included in the regressions: the average bid-ask spread (between 18.07 bp and 78.03 bp), the notional amount outstanding (in the range 0.6 bn to 2.5 bn euro), the average bond coupon (4.34% to 6.27%) and the rating (cf. Table 2), all employed in Longstaff, Mithal and Neis [21] as well.

Time averages of the implied minimum recovery values and all explanatory variables are determined for each obligor and the implied recovery rate is cross-sectionally regressed separately against each proxy. Table 8 presents the regression statistics. All four variables representing the delivery option are both statistically and economically significant, and the hypothesized sign of the respective coefficients is confirmed. Based on the R^2, the number of bonds and the minimum bond pricing errors have the highest explanatory power. Interpreting the influence of these two factors, it follows that the implied minimum recovery decreases by 6% on average if an additional bond is available, and by 5.4% on average if the minimum bond pricing error increases by 10 bp. Due to the small cross-section of only twelve obligors, one needs to examine whether the significance of the parameters in the regressions possibly stems from outliers. To this end, we plot the data sets and lay the respective regression lines on top, but no indication of such misfitting is found. As a representative example the scatter plot of the average implied recovery rate against the number of bonds is included in Figure 5.

On the other hand, all four variables representing liquidity are statistically insignificant and not even close to the 90% confidence interval, indicating that in our sample there is no serious influence of liquidity on the estimated implied recovery rates – at least not the way we measure it. The importance of including the delivery option into credit pricing models is thereby further strengthened. As a consistency check the time series is split into two parts (2003 and 2004), and the cross-sectional regression analysis repeated on both subperiods, but the results stay virtually the same (details not reported).

Finally, a time series analysis is performed for each obligor to explore the effect of the implied recovery rate on the CDS spread. Since the CDS spread is driven both by default risk and by recovery risk, the implied recovery rate is expected to account for part of the observed variation in CDS spreads over time, i.e. an increase in the recovery rate should result in a decrease of the CDS spread. To explore this relation we specify a univariate time series model for the CDS spread and include the recovery rate as an explanatory variable. The autocorrelation structure of CDS spreads is taken into account by modelling first differences as an MA(1) process. First differences of the implied recovery rate are added to this model setup as an explanatory variable. Table 9 contains the main regression statistics. The coefficients of the implied recovery rate exhibit the expected negative sign for all obligors, meaning that a decrease in the implied recovery rate induces an increase in the CDS spread, and for ten out of twelve obligors the implied recovery rate is significant for explaining the CDS spread (even after accounting for autocorrelation effects, which itself could be caused by recovery risk). Lastly, one notices that there exist differences in the relative importance of default risk and recovery rate risk for the individual obligors, as indicated by the differing R^2 levels.

4 Conclusion

The tremendous growth rates and the increasing diversity of products in the credit risk market are continuously calling for the development of more and more sophisticated pricing methods. As shown in recent academic studies, under standard assumptions reduced-form credit risk models are not fully capable of accurately pricing the two fundamental credit risk instruments – bonds and credit default swaps (CDS) – *simultaneously*.

Using a data set of euro-denominated corporate bonds and CDS this paper quantifies the mispricing observed when employing a deterministic reduced-form framework. In an extensive comparison of the pricing properties in the bond market for several parameterizations of the default intensity the Nelson-Siegel specification turns out to be optimal. This parametrization is subsequently used to price CDS, resulting in model CDS spreads up to 50% lower on average than observed in the market.

The traditionally popular explanation for real-world imperfections in credit markets – liquidity – has already been thoroughly analyzed in academic literature, yet with conflicting conclusions. The studies fail to unambiguously answer questions such as which market is more liquid or whether liquidity is priced at all. In this paper an alternative extension is therefore presented which models the delivery option implicit in CDS contracts: Since in default a basket of bonds is deliverable, the effect of the cheapest-to-deliver bond price needs to be accounted for in CDS valuation. By using a constant recovery rate standard credit risk models assume equal recoveries for all bonds, and hence implicitly assume zero value for the delivery option.

Contradicting this common modelling assumption, case studies of recent Chapter 11 filings presented in this paper illustrate that bonds of a defaulted obligor do *not* trade at equal levels following default. Our extension therefore models the implied expected recovery rate of the cheapest-to-deliver bond and, applied to the data, it largely eliminates the mispricing. The calibrated recovery values lie in the range between 8% and 47% for the different obligors, exhibiting strong variation among rating classes and industries. A cross-sectional analysis reveals that the implied recovery parameter depends on proxies for the delivery option: The variables with the highest explanatory power are the number of available bonds and the bond pricing errors. No evidence is found for the influence of liquidity proxies.

Our paper thus points out the necessity for incorporating the random structure of recovery rates at default into credit risk models in order to accurately price credit-risky instruments.

References

[1] E. Altman, A. Resti, A. Sironi, *Default Recovery rates in Credit Risk Modelling: A Review of the Literature and Empirical Evidence*, Economic Notes by Banca Monte dei Paschi di Siena SpA, Vol. 33 (2004), No. 2.

[2] D. Aunon-Nerin, D. Cossin, T. Hricko, Z. Huang, *Exploring for the Determinants of Credit Risk in Credit Default Swap Transaction Data: Is Fixed-Income Markets' Information Sufficient to Evaluate Credit Risk?*, FAME Research Paper No. 65 (2002).

[3] G. Bakshi, D. Madan, F. Zhang, *Understanding the Role of Recovery in Default Risk Models: Empirical Comparisons and Implied Recovery Rates*, Working Paper (2001).

[4] C. Benkert, *Explaining Credit Default Swap Premia*, Journal of Futures Markets, Vol. 24 (2004), No. 1.

[5] W. Bühler, K. Düllmann, *Conversion Factor Systems and the Delivery Option of a Multi-Issuer Bond Future*, Working Paper (2005).

[6] R. Blanco, S. Brennan, I.W. Marsh, *An Empirical Analysis of the Dynamic Relation between Investment-Grade Bonds and Credit Default Swaps*, Journal of Finance, Vol. 60 (2005), No. 5.

[7] P. Collin-Dufresne, R.S. Goldstein, J.S. Martin, *The Determinants of Credit Spread Changes*, Journal of Finance, Vol. 56 (2001), No. 6.

[8] D. Duffie, *Credit Swap Valuation*, Financial Analysts Journal, Jan./Feb. Issue (1999).

[9] D. Duffie, L.H. Pedersen, K.J. Singleton, *Modeling Sovereign Yield Spreads: A Case Study of Russian Debt*, Journal of Finance, Vol. 53 (2003), No. 1.

[10] D. Duffie, K.J. Singleton, *Modeling Term Structures of Defaultable Bonds*, Review of Financial Studies, Vol. 12 (1999), No. 4.

[11] P. Feldhütter, D. Lando, *Decomposing Swap Spreads*, Working Paper (2005).

[12] A. Geyer, S. Kossmeier, S. Pichler, *Measuring Systematic Risk in EMU Government Yield Spreads*, Review of Finance, Vol. 8 (2004), No. 2.

[13] R. Guha, *Recovery of Face Value at Default: Theory and Empirical Evidence*, Working Paper (Dec. 2002).

[14] X. Guo, R.A. Jarrow, Y. Zeng, *Modeling the Recovery Rate in a Reduced Form Model*, Working Paper (Sep. 2005).

[15] V. Hjort, S. Dulake, M. Engineer, N.A. McLeish, *The High Beta Market – Exploring the Default Swap Basis*, Morgan Stanley Credit Derivative Strategy Special Report (2002).

[16] P. Houweling, T. Vorst, *Pricing Default Swaps: Empirical Evidence*, Journal of International Money and Finance, Vol. 24 (2005), No. 8.

[17] J.C. Hull, M. Predescu, A. White, *Bond Prices, Default Probabilities and Risk Premiums*, Journal of Credit Risk, Vol. 1 (2005), No. 2.

[18] International Swaps and Derivatives Association, Inc., *2003 ISDA Credit Derivatives Definitions.*

[19] T. Janosi, R. Jarrow, Y. Yildirim, *Estimating Expected Losses and Liquidity Discounts Implicit in Debt Prices*, Working Paper (2002).

[20] R. Jarrow, *Default Parameter Estimation Using Market Prices*, Financial Analysts Journal, Sep./Oct. Issue (2001).

[21] F.A. Longstaff, S. Mithal, E. Neis, *Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market*, Journal of Finance, Vol. 60 (2005), No. 5.

[22] E. de Malherbe, *A Simple Probabilistic Approach to the Pricing of Credit Default Swap Covenants*, Journal of Risk, Vol. 8 (2006), No. 3.

[23] D. O'Kane, R. McAdie, *Explaining the Basis: Cash versus Default Swap*, Lehman Brothers Structured Credit Research (2001).

[24] P.J. Schönbucher, *Credit Derivatives Pricing Models: Models, Pricing and Implementation.*, John Wiley & Sons, 2003.

[25] H. Zhu, *An Empirical Comparison of Credit Spreads Between the Bond Market and the Credit Default Swap Market*, BIS Working Paper No. 160 (2004).

functional forms		free parameters
linear	$\alpha_0 + \alpha_1 t$	2
quadratic	$\alpha_0 + \alpha_1 t + \alpha_2 t^2$	3
cubic	$\alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3$	4
log-linear	$\alpha_0 + \alpha_1 t + \frac{\kappa_1}{1+t}$	3
Nelson-Siegel	$\beta_0 + \beta_1 e^{-\frac{t}{\kappa_1}} + \beta_2 \frac{t}{\kappa_1} e^{-\frac{t}{\kappa_1}}$	4
Svensson	$\beta_0 + \beta_1 e^{-\frac{t}{\kappa_1}} + \beta_2 \frac{t}{\kappa_1} e^{-\frac{t}{\kappa_1}} + \beta_3 \frac{t}{\kappa_2} e^{-\frac{t}{\kappa_2}}$	6

Table 1: Specifications of the intensity function examined. α_i, β_i and κ_i denote the model parameters satisfying the usual constraints for the Nelson-Siegel and Svensson parameterizations. The term *log-linear* applies here to the integrated intensity function.

obligor	Moody's rating	industry
Rabobank	Aaa	Banking
ABN AMRO	Aa3	Banking
Siemens	Aa3	Electrical Equipment
Aventis	A1	Pharmaceuticals
British American Tobacco (BAT)	A2	Tobacco
Commerzbank	A2	Banking
Bayer	A3	Pharmaceuticals
DaimlerChrysler	A3	Automobiles
France Telecom	A3	Telecom
Philips	Baa1	Electronics
Telecom Italia	Baa2	Telecom
Fiat	Ba3	Automobiles

Table 2: Obligors selected for the study with corresponding rating and industry.

obligor	linear	quadratic	cubic	log-linear	Nelson-Siegel	Svensson
Rabobank	16.32 bp	15.01 bp	15.76 bp	17.39 bp	15.35 bp	15.58 bp
ABN AMRO	7.27 bp	4.47 bp	7.37 bp	5.01 bp	9.59 bp	5.81 bp
Siemens	3.64 bp	1.97 bp	5.52 bp	0.14 bp	1.22 bp	3.62 bp
Aventis	4.47 bp	4.22 bp	4.12 bp	0.14 bp	0.24 bp	1.30 bp
BAT	47.08 bp	17.40 bp	8.84 bp	11.19 bp	5.65 bp	2.01 bp
Commerzbank	9.05 bp	7.27 bp	8.77 bp	9.70 bp	13.16 bp	11.63 bp
Bayer	35.58 bp	17.39 bp	15.08 bp	8.36 bp	1.70 bp	3.92 bp
DaimlerChrysler	56.13 bp	36.66 bp	12.83 bp	10.50 bp	9.37 bp	7.62 bp
France Telecom	32.04 bp	16.10 bp	16.24 bp	46.88 bp	15.23 bp	16.67 bp
Philips	26.00 bp	10.98 bp	9.78 bp	7.17 bp	2.15 bp	3.10 bp
Telecom Italia	43.04 bp	22.13 bp	18.76 bp	13.87 bp	2.78 bp	3.69 bp
Fiat	216.43 bp	152.49 bp	44.18 bp	48.64 bp	39.06 bp	33.25 bp
overall	41.42 bp	25.51 bp	13.94 bp	14.92 bp	9.62 bp	9.02 bp

Table 3: Mean absolute pricing errors (MAE) for the bonds of each obligor over the period examined for the six specifications of the intensity function.

	bonds		CDS			
obligor	ME	MAE	ME	MRE	MAE	MRAE
Rabobank	−0.98 bp	15.35 bp	−5.03 bp	−52.48%	5.51 bp	57.53%
ABN AMRO	0.52 bp	9.59 bp	−2.12 bp	−12.25%	3.02 bp	17.41%
Siemens	−1.22 bp	1.22 bp	−5.73 bp	−19.78%	8.04 bp	27.73%
Aventis	−0.24 bp	0.24 bp	4.95 bp	26.90%	5.11 bp	27.74%
BAT	−2.06 bp	5.65 bp	6.85 bp	9.53%	7.37 bp	10.25%
Commerzbank	−2.18 bp	13.16 bp	−8.62 bp	−26.31%	9.17 bp	28.02%
Bayer	−1.69 bp	1.70 bp	6.36 bp	10.60%	7.26 bp	12.10%
DaimlerChrysler	−1.69 bp	9.37 bp	−26.83 bp	−28.80%	27.27 bp	29.27%
France Telecom	−1.55 bp	15.23 bp	−13.62 bp	−16.30%	14.52 bp	17.37%
Philips	−1.55 bp	2.15 bp	0.56 bp	1.01%	6.23 bp	11.21%
Telecom Italia	−1.11 bp	2.78 bp	−4.80 bp	−5.93%	8.43 bp	10.40%
Fiat	−5.38 bp	39.06 bp	−186.80 bp	−37.38%	189.63 bp	37.95%
overall		9.62 bp			24.30 bp	23.92%

Table 4: Pricing properties over the period examined for the Nelson-Siegel specification of the intensity function. Reported are the mean pricing errors (ME) and the mean absolute pricing errors (MAE) of both bonds and CDS, as well as the mean relative pricing errors (MRE) and the mean relative absolute pricing errors (MRAE) for CDS. A negative value means that the model price resp. spread lies below the one observed in the market.

	Delta	Northwest	Delphi
Chapter 11 filing	14.09.2005	14.09.2005	08.10.2005
Day of default	14.09.2005	14.09.2005	10.10.2005
Observation period	09/2005	09/2005	10/2005
Contemporaneous (maximum) price differences on default day:			
Closing price	1.81	6.00	1.50
Lower bound	4.42	3.39	3.00
Upper bound	8.87	12.40	8.50
Prices in the post-default period:			
Minimum price	9.58	19.50	49.20
Maximum price	22.00	32.00	70.50

Table 5: Summarized details of case studies of defaults (differences resp. prices per 100 of face value).

	mean	std. dev.	min	max
Rabobank	8.87%	18.72%	0.00%	73.61%
ABN AMRO	35.34%	12.88%	0.00%	61.54%
Siemens	31.86%	9.86%	0.00%	57.68%
Aventis	46.11%	6.91%	24.50%	62.98%
BAT	45.93%	4.37%	24.88%	74.86%
Commerzbank	27.14%	13.32%	0.00%	58.87%
Bayer	46.34%	4.97%	29.92%	71.11%
DaimlerChrysler	11.48%	9.55%	0.00%	33.62%
France Telecom	29.97%	6.78%	0.00%	73.10%
Philips	43.91%	6.85%	31.34%	81.83%
Telecom Italia	33.37%	8.14%	15.50%	52.65%
Fiat	13.74%	13.42%	0.00%	53.19%

Table 6: Descriptive statistics of the implied minimum recovery rate.

	bonds		CDS			
obligor	ME	MAE	ME	MRE	MAE	MRAE
Rabobank	−0.98 bp	15.35 bp	−3.23 bp	−33.74%	3.55 bp	37.02%
ABN AMRO	0.52 bp	9.59 bp	−1.24 bp	−7.15%	1.76 bp	10.15%
Siemens	−1.22 bp	1.22 bp	−2.50 bp	−8.64%	5.30 bp	18.27%
Aventis	−0.24 bp	0.24 bp	2.45 bp	13.31%	3.17 bp	17.23%
BAT	−2.06 bp	5.65 bp	−0.71 bp	−0.99%	0.75 bp	1.04%
Commerzbank	−2.18 bp	13.16 bp	−3.18 bp	−9.72%	3.35 bp	10.22%
Bayer	−1.69 bp	1.70 bp	0.55 bp	0.91%	1.78 bp	2.97%
DaimlerChrysler	−1.69 bp	9.37 bp	2.38 bp	2.56%	6.84 bp	7.34%
France Telecom	−1.55 bp	15.23 bp	−3.57 bp	−4.27%	6.75 bp	8.08%
Philips	−1.55 bp	2.15 bp	−3.30 bp	−5.94%	3.42 bp	6.16%
Telecom Italia	−1.11 bp	2.78 bp	1.54 bp	1.90%	2.63 bp	3.25%
Fiat	−5.38 bp	39.06 bp	−46.96 bp	−9.40%	55.04 bp	11.02%
overall		9.62 bp			7.86 bp	11.06%

Table 7: Pricing properties over the period examined of the extended model. Reported are the mean pricing errors (ME) and the mean absolute pricing errors (MAE) of both bonds and CDS, as well as the mean relative pricing errors (MRE) and the mean relative absolute pricing errors (MRAE) for CDS. A negative value means that the model price resp. spread lies below the one observed in the market.

	constant	coefficient	p-value	R^2
nr of bonds	0.5481	−0.0634	0.0004	73.25%
max. price difference	0.4865	−0.0237	0.0041	57.73%
abs. bond error	0.3936	−0.0085	0.0168	45.07%
min. bond error	0.4017	−0.0054	0.0025	61.47%
bid-ask spread	0.3637	−0.0016	0.5732	3.28%
principal	0.1702	0.0001	0.1693	17.99%
coupon	0.2803	0.0060	0.9393	0.06%
rating	0.3176	−0.0009	0.9514	0.04%

Table 8: Results of the univariate cross-sectional regressions of the implied minimum recovery rate on proxies of the delivery option and liquidity.

obligor	constant	recovery	MA(1)	R^2
Rabobank	−0.0182	−0.0453***	−0.6001***	30.76%
ABN AMRO	−0.0440	−0.0099	−0.2683***	6.91%
Siemens	−0.1231***	−0.0575***	−0.3214***	18.56%
Aventis	−0.0108	−0.1693***	−0.3004***	32.74%
BAT	−0.0618	−0.6468***	−0.1806***	13.81%
Commerzbank	−0.2229	−0.0200*	0.1147*	1.67%
Bayer	−0.1580	−0.5343***	0.2068***	9.92%
DaimlerChrysler	−0.1814	−0.2717***	−0.3805***	16.39%
France Telecom	−0.6065***	−0.1965***	−0.2595***	10.40%
Philips	−0.2009**	−0.0330	−0.0604	0.93%
Telecom Italia	−0.1696	−0.4968***	−0.1260**	23.04%
Fiat	−1.0284	−1.4342***	−0.1160**	6.93%

Table 9: Results of the time-series regressions of the first differences of CDS spreads on an MA(1) term and first differences of the implied minimum recovery time series. The asterisk indicates significance at the 90% (*), 95% (**) and 99% (***) level.

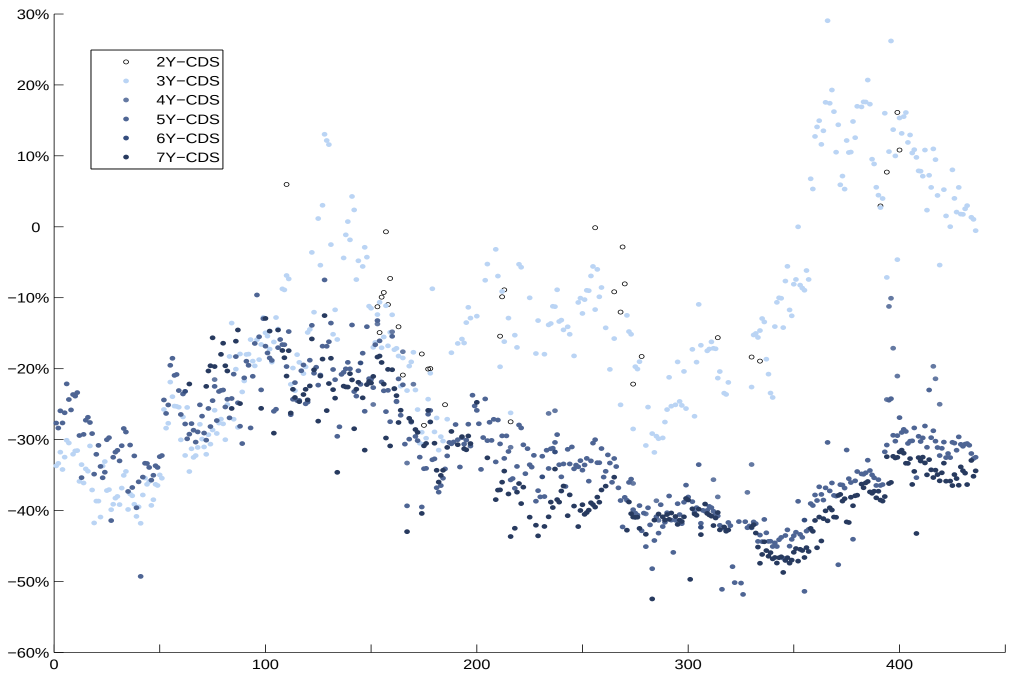

Figure 1: Time series of CDS relative errors for DaimlerChrysler, where a negative value means that the model spread lies below the one observed in the market.

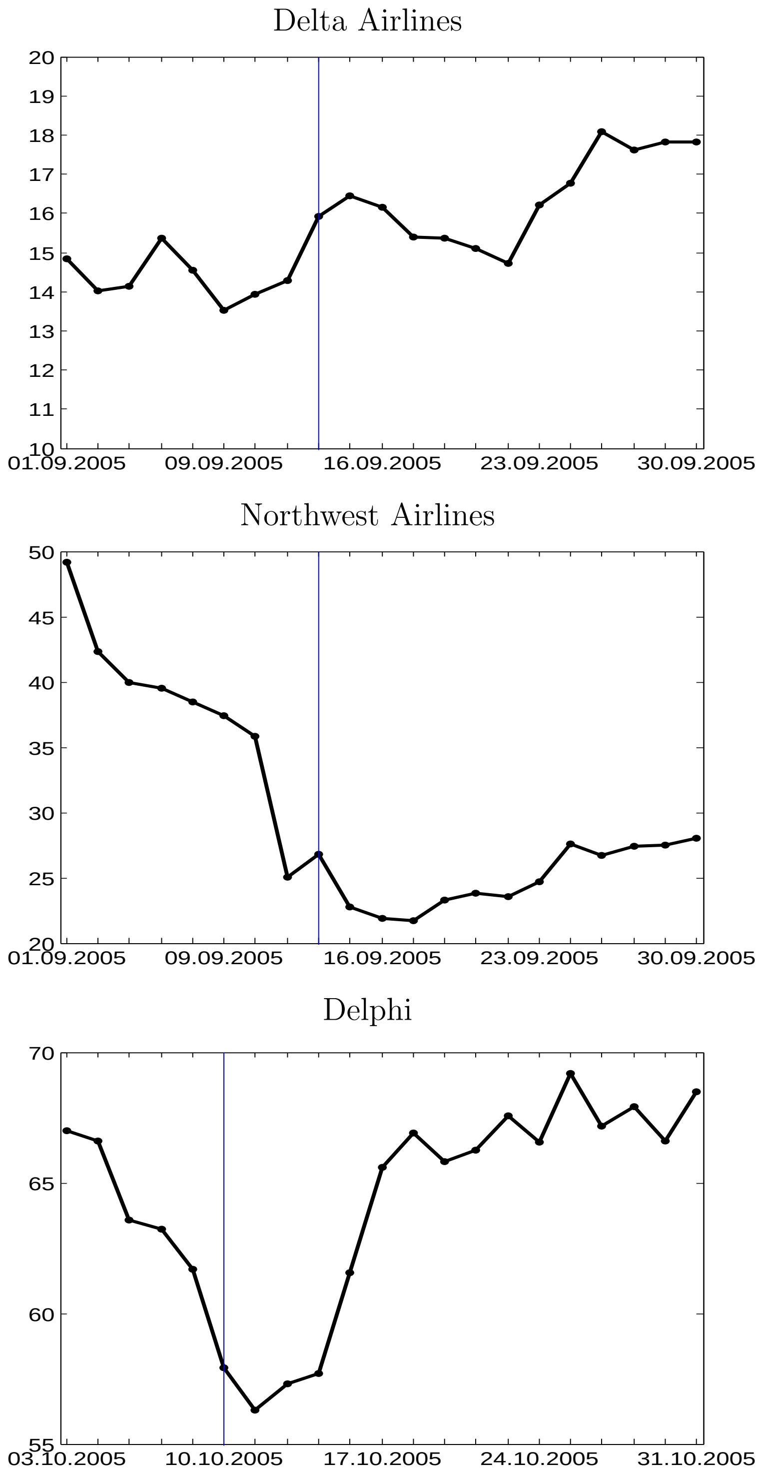

Figure 2: Daily average closing prices for bonds in the month of default. The vertical line indicates the day of the Chapter 11 filing.

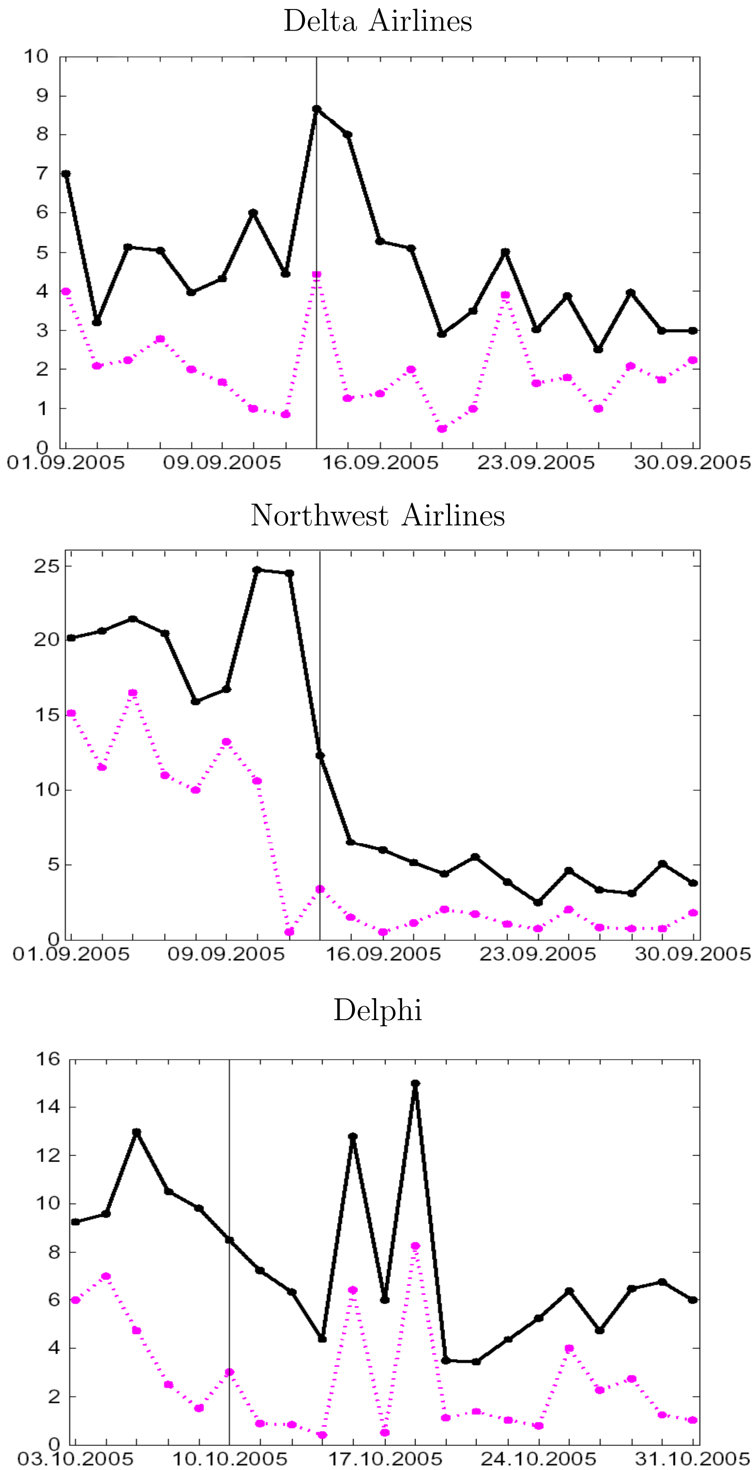

Figure 3: Daily lower and upper bound for the contemporaneous maximum bond price differences in the month of default. The vertical line indicates the day of the Chapter 11 filing.

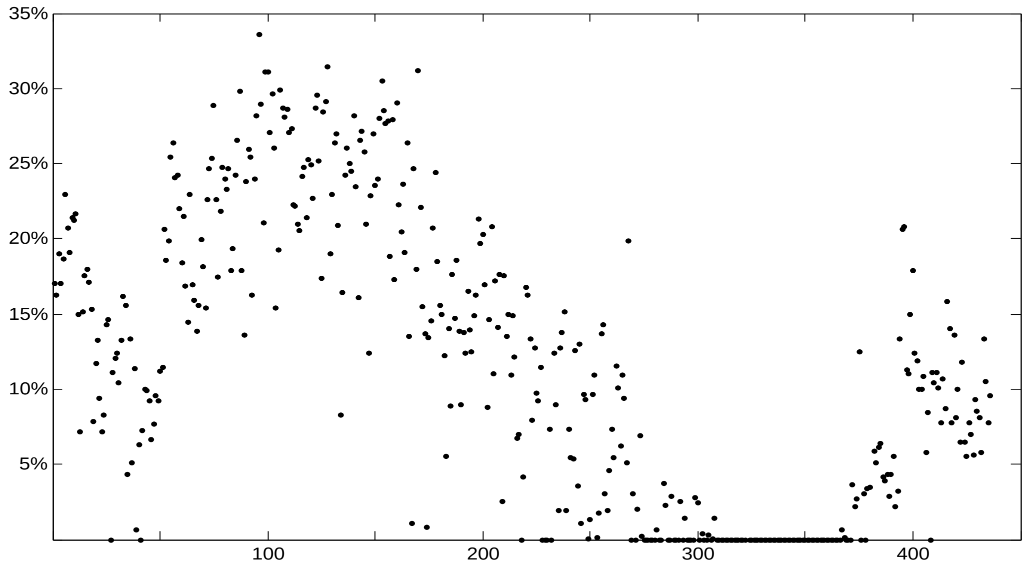

Figure 4: Time series of the implied minimum recovery rate for DaimlerChrysler.

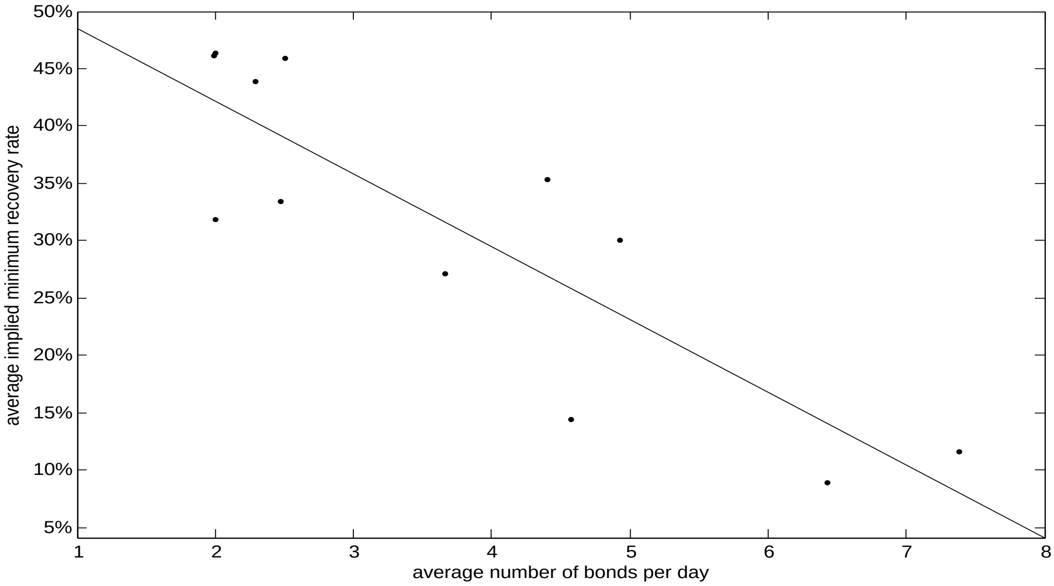

Figure 5: Cross-sectional scatter plot of the average implied minimum recovery rate against the average number of bonds per day.