# The Valuation of Derivatives on Carbon Emission Certificates - a GARCH Approach

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## Abstract

The introduction of the trade of carbon emission certificates (EUA) has led to the emergence of a variety of derivatives on this underlying. We investigate the dynamics of the ECX December 2008 EUA futures' returns and find excess kurtosis and evidence for heteroscedasticity. The model estimation and the subsequently performance analysis of the models, suggest a GARCH(1,1) model to appropriately reproduce the futures' dynamics. The derivatives are subsequently valued in a risk neutral framework using Monte Carlo simulation.

For short time horizons, the valuation outcomes are quite precise. With an increased time period in the simulation the valuation's accuracy is not outstanding, yet with respect to barrier call options and index trackers quite good result can be obtained. However, regarding barrier put options, there is a certain amount of mispricing in the results. The reason for this outcome is that due to the drift in the unadjusted futures' price simulations, disproportionately many realizations are knocked out.

The comparably small deviation of the valuation results from the observed market prices regarding the participation certificates as well as concerning the call options provide an indication of fair pricing.

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# 1 Introduction

## 1.1 Emission Trading

In January 2005 the EU Emission Trading Scheme (EUTS) came into force. It is comprised of two phases, a three year period from 2005 until 2007 and a five year period from 2008 until 2012. The aim of this emission trading scheme is to cut carbon emission in the European Union in order to meet the emission reduction goals set by the Kyoto Protocol in 1997. Via an annual allocation of emission rights, the $CO_2$ emission should be cut down by 8% compared to the 1990 level. Additionally, the European Union member countries have agreed to reduce emissions by another 12% until the year 2020. As a market-based mechanism emission trading should ensure that emission reduction goals are accomplished at minimal cost. Thus, the cost-benefit ratio should be maximized.

Carbon dioxide has become a new kind of commodity. In order to make $CO_2$ tradeable and to place a price tag on $CO_2$ emissions it had to be commoditised as if it were a barrel of oil or coal (ECX (2008)). This has been achieved by issuing rights to emit $CO_2$ which are referred to as EU Allowances (EUA). Such an EUA equals one ton of $CO_2$ and is tradable in specialized exchanges.

In the EUTS the cap-and-trade approach is the central concept. For every compliance period an overall cap is set which locks in the maximum amount of emissions allowed. Through National Allocation Plans (NAP) emission rights, are allocated among the industries in each of the EU member countries. The sum of those EUAs represents the total amount of $CO_2$ that can be emitted constituting a cap on carbon emissions. Each company is allowed to emit just as much $CO_2$ as it is entitled by its emission certificates. At the end of each period the companies must surrender sufficient EUAs to offset their emissions during the period. If a company fails to cut down on emissions and keeps on emitting too much $CO_2$ it can either buy EUAs on the market or pay a penalty of EUR 40 per additional tonne of $CO_2$ emitted, or EUR 100 at the end of the second compliance period, respectively. On the other hand, companies that have managed to cut down their emissions sufficiently can sell their surplus EUAs on an exchange. Thus, extra profit can be generated through the trading of EUAs.

Several exchanges trade EUAs and derivatives on emission rights. The major ones are the French Bluenext, formerly Powernext, the German European Energy Exchange (EEX), the Nordic Nord Pool Group as well as the British European Climate Exchange (ECX). However, an ongoing consolidation process makes those exchanges cooperating with each other as well as with derivatives exchanges as the EUREX or the NYSE. EUA spot market transactions can be accomplished in the Bluenext exchange, the EEX as well as Nord Pool. Derivatives on EUA, like futures and options for physical delivery on EUAs, can be traded in the ECX as well as in the EEX and Nord Pool. The more the market for emission rights matures the more different products are offered in the exchanges. Most recently, the EEX has launched the trading of options on EUA futures in cooperation with EUREX (ECX (2008)).

The underlying of the futures contracts are 1,000 allowances with annual maturity, adding up to 1,000 tons of $CO_2$. At the moment, there are seven contracts traded with maturity dates from December 2008 up to December 2014. Generally, the EUA futures market is far more liquid than its spot counterpart. For example, in the first quarter of 2008 the total volume traded in the spot market amounted to 653,502 EUAs. In the derivative market the total volume during the same period was 14,391,000 EUAs, as the EEX reported (ECX (2008)). According to Daskalakis, Psychoyios, and Markellos (2007), the same pattern was already observed in 2006, when the spot EUAs were trading with a yearly volume of 50 Mio. compared to an approximate amount of 250 Mio. EUAs in the futures market. According to figures of the ECX, derivatives trades, i.e. futures and options on EUAs, make up 95% of the total volume in the European carbon market compared to only 5% in spot trades (ECX (2008)). The peculiarity of the EUTS can be seen as one possible explanation for the exceptionally high ratio of derivatives in the European carbon market trades. First, the delay of national registries as well as of the final allocations in several EU member states made it hard to ensure the execution and delivery for spot contracts. Second, since the compliance verification has to be provided only at the end of every year, there is no obvious advantage of being long in spot transactions compared to taking a long position in December futures. In general, futures with a shorter time to maturity are more liquid than those with a longer maturity. A reason for this might be heavy emission industries' difficulties to plan a long terms emission allowance strategy. Third, especially in new and volatile markets, derivative instruments are helpful tools to optimize and hedge the emission rights portfolio of each firm (ECX (2008); Uhrig-Homburg and Wagner (2007)).

The benefits of such derivative products on EUA are threefold. First, they may be used for risk management purposes of the participating industries. Hedging strategies can be useful if the actual amount of EUA used for compliance cannot be determined in advance. Moreover, derivatives allow the risk transfer from companies to traders who are willing to accept the risk in order to earn excess profit on their venture capital. In addition, speculation on EUA is desired in order to boost the liquidity of the market (ECX (2008)). Due to the large amount of bid and ask spreads funneled in the market, derivatives markets are often the main source of price discovery for the related commodities. This leads to publicly disseminated prices (Hull (2008); Geman (2006)). Third, there is little correlation between emission allowance price changes and stock markets returns. Thus, diversification aspects are an important reason for incorporating derivatives on carbon emission rights in a portfolio.

Because of its higher liquidity, the EUA futures generally represent the underlying of the derivatives on EUA issued by banks. Therefore, in this article we use the term derivatives on carbon emission certificates referring to retail products such as certificates and leveraged products. There are several banks that offer such certificates. The advantages of such products are manifold. First, there is the previously explained diversification property as well as speculation possibility of an investment in EUAs to be mentioned. Second, the trend to enhanced ecological consciousness may have boosted the demand of such 'green' products by which the investors can contribute to a healthier environment. Third, such certificates offer the possibility to take futures-like positions without the need to access futures markets, which is generally impossible for retail investors because of high contract volumes.

The underlying with respect to all the derivatives presented in this article is the Intercontinental Exchange (ECX) December 2008 EUA futures contract. The reason for choosing the December 2008 futures contract as the underlying for the derivatives on EUA is the fact that it is much more liquid than all the other futures contract in the market as well as the EUA in the spot market. Thus, it can be reasoned that the quality of price discovery is best with respect to the Dec 2008 futures contract. Moreover, high liquidity generally facilitates building up and clearing positions in the underlying, if necessary. The certificates discussed are either participation certificates or leveraged products. In the very recent past, options on futures are introduced by EEX in cooperation with EUREX. As well, knockout options on the December 2009 futures have been issued. Other certificates or structured products basing on the ECX ICE December 2008 EUA futures are not available in the market to our knowledge.

## 1.2 Motivation and Proceeding

Previous papers in the area, such as Daskalakis, Psychoyios, and Markellos (2007), Benz and Trück (2007), and Uhrig-Homburg and Wagner (2007), first and foremost carry out an analysis of the relationship between spot and futures prices. Of particular interest is the analysis by Daskalakis, Psychoyios, and Markellos (2007) where the dynamics of inter- and intraperiod futures are investigated. The separation of the futures according whether their maturity date falls in the first compliance period (intraperiod futures) or the second compliance period (interperiod futures) is appropriate, because they are found to exhibit very different price dynamics. The same authors, as well as Benz and Trück (2007) are additionally comparing the performance of different pricing models. In the case of Benz and Trück (2007) a GARCH model as well as a regime switching model are favored over autoregressive and mean reverting models. Daskalakis, Psychoyios, and Markellos (2007) choose to model the EUA futures price using a two factor equilibrium model based on a jump diffusion process.

However, the data set used in all of the studies mentioned above are not covering spot and futures price data until the end of the fist compliance period. Therefore, the price deterioration of the spot as well as the intraperiod futures was not incorporated fully in the estimation process of the models. In addition, to our knowledge no study on the pricing of certificates on EUA futures has been carried out so far. There may be mispricing in these derivatives which could be either due to the immaturity of the market for EUA or due to the dominant role of a small number of banks offering such certificates. Their property as a market maker is exempting them from competition to a certain extent. Therefore, the main purpose of this paper is to find an appropriate pricing model for derivatives on carbon emission certificates. As a first step, we investigate the price dynamics of the interperiod futures on an enhanced data basis and use the derived results to develop a reliable futures price model. Second, we simulate the particular price dynamics of the EUA and evaluate the pricing of its derivatives.

In the next section we present the data. Section 3 gives the methods for the modeling of the future prices as well as the derivatives pricing. Section 4 presents the results concerning the modeling of the futures price dynamics as well as the valuation of the certificates. Section 5 draws some conclusions and gives a discussion of the results.

# 2 Data

## 2.1 Futures Data

The time series used to model the ECX ICE December 2008 Futures includes observations from April 22, 2005 until April 24, 2008. This set comprises 770 daily settlement prices in total. However, 9 data points are excluded from the basis set used to estimate the process. As it can be seen in figure 1, the returns


Figure 1: EUA Spot
Development of the EUA spot price since the trading started on 05/24/2005 in the French Powernext and later in the Bluenext Exchange, respectively. At the end of the compliance period on 03/20/2008, the trading in this specific contract ceased.

around the market friction in spring 2006 exhibit excessively high volatility. Such a high deviation from the mean was never precedented and have never occurred in later periods to the same extent again. Therefore, it can be concluded that this abnormality was only due to the market friction and does not contain essential information regarding the general behavior of the December 2008 EUA futures price. The composition of the in-sample (IS) and the out-of-sample (OOS) data set is shown in table 1. The in-sample data set contains 590 data points from April 22, 2005 until August 22, 2007. The out-of-sample data set contains approximately one fifth of the total number of data, comprising observations from August 23, 2007 until April 24, 2008.

Table 1: In-Sample and Out-of-Sample-Data

| Type | N | Mean | SD | Skewness | Kurtosis | Jarque-Bera |
|---|---|---|---|---|---|---|
| IS | 590 | 8.5860e-004 | 0.0278 | -0.64 | 6.84 | 399.37 |
| OOS | 171 | 0.0016 | 0.0187 | 0.23 | 4.57 | 17.84 |

From the summary statistics it becomes clear that the more mature the market gets the less volatile it becomes. According to the Jarque-Bera test the null hypothesis for both data sets can significantly be rejected at the 5% level, but there might be a trend towards normality in log returns.

As the risk free interest rate, we use a fraction of the 6 and 9 month Euribor rate from the Deutsche Bundesbank.

## 2.2 Derivatives Data

For the time being, two generic types of derivatives on EUA futures contracts are offered: participation certificates and leveraged products.

In order to enable retail investors to invest in the $CO_2$ emission market several banks have offered certificates which let the investor participate on a 100% basis in the development of the EUA ICE Futures 2008. Generally, the underlying of one certificates equals a thousandth part of a future, i.e. one tonne of $CO_2$. Due to the participation rate being 100% the payoff is linear. The certificates in question are all open-end certificates with a yearly rollover procedure. Because of differences in prices between the maturing future and the next nearby future possible losses or gains during the roll over may be incurred. If the futures price of the new contract is higher in comparison with the actual one, as a consequence less items of the new contract can be purchased and a loss has to be faced. Analogously, given the new contract's price is lower than the actual futures price, the rolling over results in a gain.

The investor participates in the development of the futures prices according to the participation rate. In the case of the certificates on EUA Emissions, the rate is generally set equal to 100%. This means that the value of the certificate is derived according to the following formula:

$$F_T^A \cdot PR \cdot 1 \tag{2.1}$$

where $F_T^A$ is the current futures price and $PR$ the participation rate. It has to be noted that because of the above mentioned losses or gains associated with the rolling over of the contracts, the participation rate is liable to change. If the new futures price is less than the current's price the investor will participate more than 100% in the development of the new future and vice versa.

The certificates in question all base on the December 2008 futures. There have been some certificate which base on a future of the first compliance period. Being influenced heavily by the market friction in spring 2006 and the followed price deterioration they all lost most of their value and are therefore not included in the study.

However, the formula 2.1 bases on the current futures rate and therefore no prediction of the possible future price is possible. It would be desirable to model several possible outcomes and to predict the price of such a certificates. Because the value of the certificate always reflects the actual futures price no restriction regarding the payoff exist. Thus, in this paper participation certificates are deemed as having no strike and no barrier.

The second type, leveraged products, generally have lifetimes up to one year and have the same properties as exchange traded futures. By being long or short in such a leveraged product the holder participates in the performance of the underlying in a futures-like manner. The leverage effect comes from the small initial investment requirement. If the underlying moves in the unanticipated direction the leveraged product might be knocked out. In the case of the future there would be a margin call. The knock-out happens when the products' margin is used up which is implicitly made by the initial investment (Wilkens and Stoimenov (2007)).

The payoff of long or index certificates, also known as turbo-certificates, is calculated as the amount from the difference between the underlying quote $S_t$ and the strike price $K$. If $S < K$ the payoff is zero. In addition, a fixed barrier $B$ represents either the knock-in level where the option begins to live, or the knock-out level where the option vanishes if crossed. If $B$ equals $K$, the barrier can be interpreted as the point in time when a margin call would be executed. According to Wilkens and Stoimenov (2007) the knock-out feature of such certificates is mainly due to the fact that it would be impossible to collect a margin call on a OTC traded product. Therefore, those leveraged certificates have a convex payoff structure, because the only losses that can be incurred is the difference between $S - B$. This is contrary to a normal future contract that has a linear payoff structure and therefore unlimited losses can be made if the underlying moves in the unanticipated direction. Because of their properties such turbo-Certificates can be valued like down and out calls or up and out puts. Short certificates are treated analogously, yet the payoff is equal to $K - S$ and the certificate would be knocked out if $S_t > B$.

The data of the certificates to be valued is presented in table 2. The data are daily settlement prices of the certificates retrieved from the EUWAX in Stuttgart for the period from August 23, 2007 until April 24, 2008. This time horizon corresponds to the out-of-sample time period. Like this it is possible to compare the out-of-sample simulated terminal values of the certificates with real data observed in the market and to assess the correctness if the modeled prices. The emission date is not the same in every case. This implies that for the simulation process different numbers of day have to be forecasted as well as the initial futures price changes according to the emission date.

Table 2: Certificates' Data

| Type | Barrier | Strike | Value on Apr 24.2008 | Emission Date | Maturity | Futures Price | Simulation Horizon |
|---|---|---|---|---|---|---|---|
| Knock Out | | | | | | | |
| WKN Calls | | | | | | | |
| DR5C9Z | 7 | 5 | 19.83 | Mar 23.2007 | Dec 03.2008 | 18.81 | 171 |
| DR5C90 | 12 | 10 | 14.83 | Mar 23.2007 | Dec 03.2008 | 18.81 | 171 |
| DROQSR[a] | 20 | 20 | 4.8 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| DROQSS | 18 | 18 | 6.8 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| DROQST | 16 | 16 | 8.8 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| DROQSU | 14 | 14 | 10.8 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| DROQSV | 12 | 12 | 12.8 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| DROQSW | 10 | 10 | 14.8 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| WKN Puts | | | | | | | |
| DR5C9Y | 33 | 35 | 9.87 | Mar 23.2007 | Dec 03.2008 | 18.81 | 171 |
| DR98G7 | 40 | 45 | 20.96 | Oct 11.2006 | Dec 03.2008 | 18.81 | 171 |
| DROQSZ[b] | 26 | 26 | 0.9 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| DROQS0 | 28 | 28 | 2.9 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| DROQS1 | 30 | 30 | 4.9 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| DROQS2 | 32 | 32 | 6.9 | Feb 22.2008 | Dec 03.2008 | 21.48 | 43 |
| WKN Index Tracker | | | | | | | |
| DR98G8 | 0 | 0 | 24.14 | Oct 11.2006 | Dec 03.2008 | 18.81 | 171 |
| DR1WBM | 0 | 0 | 24.4 | Oct 26.2007 | open end | 22.72 | 125 |
| AA0G6VI | 0 | 0 | 25 | Apr 25.2007 | open end | 18.81 | 171 |
| HV2C02 | 0 | 0 | 25.15[c] | Feb 27.2007 | open end | 18.81 | 170 |

[a] the denomination of the following options is to the ratio 1:10. Therefore, the actual price was multiplied by ten.
[b] the denomination of the following options is to the ratio 1:10. Therefore, the actual price was multiplied by ten.
[c] as of Apr 23. 2008

# 3 Methodology

## 3.1 Modeling the Futures' Dynamics

The relationship between spot and futures prices can generally be expressed by the no-arbitrage relationship as in equation 3.1 assuming no income and storage costs (Black (1976))

$$F_t(T) = e^{r(T-t)} S_t \tag{3.1}$$

where $F(T)$ is the forward or futures contract with delivery date at $T$, $S$ the spot price and $r$ the risk free rate. Commodities as consumption assets in contrast to investment assets do not yield any income and are liable to incur storage costs (Hull (2008)). Generally, we assume that EUA are not subject to storage costs and that the great majority of the investments in EUAs are made in order to comply with the regulation imposed by the EUTS. Thus, futures on EUA should yield no income. However, in many commodity market a convenience yield exists, meaning that the holding of a physical commodity incurs not only cost but also yields additional benefits. Such benefits may arise because of the opportunity to bypass shortages in the market, but are not gained by the holder of a futures contract (Hull (2008); Uhrig-Homburg and Wagner (2007)). Assuming a constant flow of benefits, equation 3.1 can be written according to Geman (2006) assuming a cost-and-carry relationship

$$F_t(T) = e^{(r-c)(T-t)} S_t \tag{3.2}$$

where $c$ represents a constant convenience yield without storage costs.

In order to capture the volatility clustering apparently present in the time series of the futures, we apply a GARCH(1,1) model after Bollerslev (1987). The variance $\sigma_n^2$ consists of a long term average variance rate, $V_L$, of the past realization of the return series $y_{n-1}$ and an additional lagged variance term:

$$\sigma_n^2 = \gamma V_L + \alpha y_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{3.3}$$

The respective weights have to sum to unity:

$$\gamma + \alpha + \beta = 1 \tag{3.4}$$

The variance estimated by the GARCH(1,1) model is based on the most recent observation of $y^2$ as well as the most recent observation of the variance rate. Defining $\omega = \gamma\ V_L$ the GARCH(1,1) we can rewrite the model as

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2. \tag{3.5}$$

After having estimated $\omega$, $\alpha$ and $\beta$, we can calculate the long run variance $V_L$ by dividing $\omega$ by $\gamma$, where $\gamma$ is 1 - $\alpha$ - $\beta$. To ensure the stability of the process as well as the long term variance's non negativity, the condition $\alpha + \beta < 1$ has to hold (see e.g. Hull (2008)).

## 3.2 Valuation

In a first step we derive the valuation formula for a futures contract in a risk neutral setting. We model the futures directly and do not rely on the relationship between futures and spot rate, since, as we will show in section 4.1, this relationship was heavily distorted by the market friction and the following price deterioration of the spot price.

The futures price in a risk neutral world has the same behavior as a stock paying a dividend yield at the risk-free rate $r_f$. Therefore, the drift of the futures price in a risk-free world is zero. The assumption for the process followed by a futures price in a risk neutral world where $\sigma$ is constant is

$$\delta F = \sigma F \delta z. \tag{3.6}$$

It follows according to Myers and Hanson (1993) that the important restriction of risk neutral pricing holds, namely that the futures price at $t_0$ is an unbiased predictor of the futures price at maturity. This is consistent with the finding of Uhrig-Homburg and Wagner (2007) that the risk neutral pricing methodology is applicable to the future 2008 contract. They argue that, contrary to the EUA 05/07 spot price, the 2008 futures rates contain all the information required for deriving expectations about future prices due to its maturity date in the second compliance period.

The incorporation of time-varying volatility does not violate the restriction placed upon risk neutral pricing. Only the growth rate of the variables changes if we move from the real to the risk neutral world, However, the volatilities of the variables remain the same. In this article, the future price is assumed to follow the process as in 3.7. This GARCH process is only dependent on the volatility, the conditional mean equation is a simple constant and does not contain a drift term:

$$\Delta F = C + \varepsilon_t \tag{3.7}$$

with

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \tag{3.8}$$

where $C$ and $\omega$ are constants and $h$ is the conditional volatility, both estimated in the GARCH Model.

The current value $V_f$ of a futures contract which matures in $T$ is $V_f = e^{-r(T-t)}[F_t^T - F_0^T]$, according to Geman (2006). Assuming time-varying variance, the formula results in:

$$\Delta V_f = e^{-r(T-t)}[(C + \varepsilon_t) - (C + \varepsilon_0)] \tag{3.9}$$

with

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \tag{3.10}$$

In the present case that the future returns exhibit ARCH effects as well as excess kurtosis, Myers and Hanson (1993) propose to generalize the probability model of futures log return distribution of Black (1976) in order to account for those requirements, as in the equations 3.11 to 3.13 for the option valuation:

$$\Delta f_t = \mu + \varepsilon_t \tag{3.11}$$

with

$$\varepsilon|\Omega_{t-1} \sim t(0, h, v) \tag{3.12}$$

and

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{3.13}$$

where $\Omega$ is the distribution of the innovations, $h$ is the conditional variance of futures price changes which is estimated by the GARCH model and $v$ the degrees of freedom of the t distribution. According to Myers and Hanson (1993), the risk neutral option valuation formula can be rewritten:

$$P_t = e^{-r_t(T-t)} \int_k^\infty [e^{f_T} - K] g(f_t) df_t \tag{3.14}$$

where $f_T = ln(F(t))$, $K$ = strike price and $g(\cdot)$ the density of $f_t$ conditional on $\Omega$ (which includes $f$).

Under the assumption that the process followed by $f_t$ can be expressed by a GARCH model, $f_T$ equals $f_t$ and a sum of weakly dependent and heterogeneously distributed GARCH innovations. Thus, although each innovation is drawn from an i.i.d normal random sample, the property that the GARCH model allows for autocorrelation in the innovations, $g(\cdot)$ cannot be assumed to be normal (Engle (1982)). Yet, this does not imply that the unconditional distribution of the innovations is normal as well. As mentioned above it can be assumed that the distribution has fatter tails than the normal distribution. Bollerslev (1987) proposed a student-t distribution for the innovations, but, if the number of the degrees of freedom is high, it converges to a normal distribution. However, according to Myers and Hanson (1993) it can be shown that there is no closed form solution for g($\cdot$) and thus there is also no closed form solution for the integral in equation 3.14. Nonetheless, using numerical procedures the option can be priced.

## 3.3 Monte Carlo Simulation

The first step is to simulate the futures price at the maturity of the option given today's futures price $F_t$. In the following example the simulation of the futures' log returns assuming t-distributed innovations is presented. In case of i.i.d. innovations, the approach can be used analogously. As Myers and Hanson (1993) point out, the realization has to satisfy the unbiased futures market assumption, i.e. $F_t$=$E[F_T]$. Following Myers and Hanson (1993) the value of the future return at period $t+1$ is calculated as:

$$y_{t+1}^i = y_t + \sqrt{\hat{\sigma}_{t+1}(\hat{v}-2)/\hat{v}} e_{t+1}^i \tag{3.15}$$

where $e^i_{t+1}$ is a random draw from a standardized t-distribution with $\hat{v}$ degrees of freedom which can be generated as in equation 3.16.

$$e^i_{t+1} = x_i / \sqrt{\sum_{j+2}^{\hat{v}+1} x_j^2 / \hat{v}} \tag{3.16}$$

where $x_i$ is a draw out of $\hat{v}$ +1 i.i.d. standard normal variables.

An estimate $\hat{\sigma}_{t+1}$ of $f_{t+1}$'s variance conditional on $\Omega_t$ is estimated in section 4.2 by the t-GARCH(1,1) model.

$$\hat{\sigma}^i_{t+1} = \hat{\omega} + \hat{\alpha}\hat{\varepsilon}_t^2 + \hat{\beta}\hat{h}_t \tag{3.17}$$

The conditional variance of $\varepsilon^i_{t+1} = \sqrt{\sigma \hat{_{t+1}} (\hat{v} - 2)/\hat{v}} e^i_{t+1}$ simplifies to $\sigma_{t+1}$ due to the variance of the t variate of $e_{t+1}$ being $\hat{v}/(\hat{v} - 2)$ by construction. The conditional variance of the second period can then be calculated as

$$\hat{\sigma}^i_{t+2} = \hat{\omega} + \hat{\alpha}\sigma \hat{_{t+1}}((\hat{v} - 2)/\hat{v})(e^i_{t+1})^2 + \hat{\beta}\hat{h}_{t+1} \tag{3.18}$$

This second period variance is then used to simulate $f_{t+2}$ analogously to equation 3.15. This process continues $n$ times, where $n$ is the number of days until maturity. At the maturity date, the simulated returns are converted to the terminal futures price using the antilogs. The same procedure is repeated $m$ times in order to get a representative set of sample terminal futures prices with $m$ as the number of simulation runs (Myers and Hanson (1993); Hull (2008)). Since we have to deal with barrier products, the fixed knock-out barrier $B$ is tested during the simulation process.

However, as Myers and Hanson (1993) mention, given the fact that the simulation does not impose any drift in the expected terminal futures price value, the expected value will exhibit some drift as a consequence. According to the findings of section 3.2 this in not in accordance with risk neutral valuation. As remedy Myers and Hanson (1993) suggest to adjust each realization. The adjustment is necessary to ensure that the average value of the terminal futures prices equals the initial futures price while satisfying the risk neutrality condition. This necessary adjustment is done by multiplying each realization by the initial futures price and subsequently dividing it by the average value of the terminal futures price. The last step in the valuation process is then to calculate the mean over all $m$ realizations and discounting it back at the risk free rate in order to get an estimate of the option price at time $t$ (Hull (2008)).

# 4 Results

## 4.1 Dynamics of the EUA Spot and Futures Markets

As the derivatives' performance is dependent on the underlying's price as well as on the underlying's volatility, it is crucial to examine the price dynamics of the underlying under scrutiny. It has to be checked if a general no-arbitrage pricing assumption describes well the relationship between spot and futures price. Furthermore, empirical evidence has shown that the distribution of the proportional changes in commodity futures prices tend not to be lognormal (Hull (2008); Geman (2006)).

The EUA 05/07 spot prices range from 0.01 cents at the end of the trading period to EUR 30 in April 2006 with a mean of EUR 10.43 as figure 1 shows. As mentioned above, spot prices soared high just before the first verified reports about each EU member states' emissions during the first year of the compliance period were published. The market turned out to be not as short as it was assumed to be since many of the member states have overallocated the allowance to their industries (Daskalakis, Psychoyios, and Markellos (2007)). Due to this plunge in the spot price, the market value of EUA was halved in just a few days. Moreover, the overallocation happened to such a great extent that the spot price has never recovered and lost value ever since. This market friction and the following price development let the volatility of the annualized daily log returns increase to almost 90%[3].

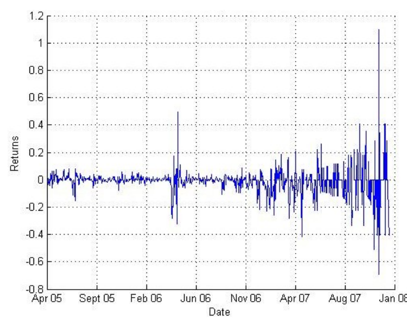


Figure 2: EUA Spot Returns
Log returns of the EUA spot prices for the first allowance period.

In figure 2 the existence of volatility clusters can easily be spotted. Especially, during the last part of the first compliance period the price development is very unstable. For the first compliance period, we calculate a kurtosis of 15.35 and a skewness of $-1.41$.

In the futures market, yearly maturities are available, with the nearest contract being the most liquid. Two futures were previously traded at the ECX and the EEX but they matured in December 2006 or December 2007, respectively. In order to analyze the behavior of the futures prices on EUA in general, the development of those futures should nevertheless be compared to the price process of the futures still traded. In figure 3 the two futures maturing within the first

[3]In order to calculate the volatility of the spot price of the first allowance period, the data set is limited to 450 observations, i.e. from April 22, 2006 until end of April 2007. After this date the EUAs were traded for less than one Euro. The fact that proportional changes were sometimes as big as 40% on a daily basis, distorted the overall volatility estimate and therefore the estimate has been corrected for such extreme values.

compliance period and two nearest future maturing in the second compliance period together with the spot quotes are plotted.


Figure 3: Development of the EUA Spot and Futures Prices
The 2007 contract is referred to as 'intraperiod future' because it matures when the spot is still traded. The 2008 contract is the 'interperiod future', since its maturity is longer than the first compliance period. The 2006 and the 2009 contract are left aside for simplicity.

From visual inspection of the different graphs it is obvious that the interperiod futures closely follow the spot at least in the very first part of the compliance period, i.e. until February 2006. After this date the futures switched from being traded at carry to being in contango. This prevailed until both futures matured implying that there was a positive convenience yield. The December 2008 is first in a backwardation situation and switched after the market correction to being in contango. After all, the interperiod futures seem not to have suffered to the same extent from the plunge in the spot price caused by the market friction as the intraperiod futures did. Moreover, the interperiod futures prices are traded much higher after the market correction compared with the intra period futures.

As in the case of the EUA spot the December 2007 EUA future exhibits a very high volatility of 90% with respect to a volatilit of 48% of the December 2008 future. The correlation analysis supports the perviously made findings. In general, there is a quite big correlation between spot and intraperiod futures prices. In contrast the December 2008 and December 2009 futures contract show very little correlation with the spot as well as the intraperiod futures.

Table 3: Correlogram

| | Spot | Dec 2006 | Dec 2007 | Dec 2008 | Dec 2009 |
|---|---|---|---|---|---|
| Spot | 1 | | | | |
| Dec 2006 | 0.97 | 1 | | | |
| Dec 2007 | 0.99 | 0.99 | 1 | | |
| Dec 2008 | 0.45 | 0.81 | 0.46 | 1 | |
| Dec 2009 | 0.41 | 0.77 | 0.42 | 0.90 | 1 |

As Daskalakis, Psychoyios, and Markellos (2007) point out, the correlation between the spot and the futures decreases with increasing maturity of the contracts with exception of the correlation between spot and future of December 2007. However, the correlation between the futures maturing in the same period is very high.

In figure 3 it is clearly visible that the cost-and-carry relationship with a no-income-no-storage cost assumption holds at least in the very first trading periods for the intra period futures. The futures price is equal to the spot price. Around January 2006 the market for intraperiod futures switched to be in contango. At the other hand, the interperiod futures prices are in backwardation until the market friction occurs. After this date, all futures prices are in contango. Especially, in the case of the interperiod futures the difference between the spot and futures rates was increasing more and more after the market disruption. Following Uhrig-Homburg and Wagner (2007) the only reason for the EUA futures rate to differ from the respective spot price is the presence of a convenience yield. This is because the only storage cost incurred is the foregone interest rate and interest rates are not assumed to be stochastic. The question remains why there is such a big difference in the convenience yields of intra- and interperiod futures. Daskalakis, Psychoyios, and Markellos (2007) suggest that one explanation might be the banking prohibition[4] from 2007 to 2008, because the contracts are equally specified only with exception of the date of maturity. Such a policy distorts the pricing of EUA futures because different pricing mechanisms have to be applied on contracts according their maturity. Further, another reason for this substantial convenience yield in the relationship between spot and interperiod futures can be the absence of information about the planned allocations during the second compliance period. Because banking is not allowed, there is a uncertainty about the extent of the future availability of EUA 2008-2012. Hence, the convenience yield can be interpreted as the risk premium required by investors as well as speculators in order to compensate this uncertainty about future market development and the possible risk of failing compliance (Daskalakis, Psychoyios, and Markellos (2007).

Regarding the interperiod futures, Uhrig-Homburg and Wagner (2007) as well as Daskalakis, Psychoyios, and Markellos (2007), conclude that those prices should be sufficiently explained by the cost of carry approach as stated in equation 3.2. However, in the case of the interperiod futures, a cash and carry arbitrage is not possible, because the first period's spot certificate could not be transferred to the second period. Thus, different assets, i.e. either EUA 2005-2007 or EUA 2008-2012, underly the futures contracts respective to their different maturity. In addition, the price of the EUAs of the second compliance period is influenced by factors that did not determine the first period's EUA prices, i.e. expectations about the EU's future decision about the allocation in the second period (Uhrig-Homburg and Wagner (2007).

Daskalakis, Psychoyios, and Markellos (2007) as well as Uhrig-Homburg and Wagner (2007) reason that standard non-arbitrage pricing models assuming a constant convenience yield cannot be applied to value the interperiod futures.

[4]According to the EU ETS Directive, banking any allowances exceeding surrendered amount from the first compliance period to the next one is forbidden (ECX (2008)).

Instead, Daskalakis, Psychoyios, and Markellos (2007) suggest the use of an equilibrium model.

Since February 26th, 2008 the trading of the second period's EUA has started and therefore new information is available. It seems that the expectations about the value of the allowance of the second compliance period have been priced quickly in the rates of the futures maturing in the second compliance period. In addition, it can be reckoned that a no-arbitrage based relationship now can summarize the relationship between futures prices and spot price correctly. The December 2008 EUA futures is converging to the spot price after having been in contango. The more mature the market gets, the more stable the price development appears to be. This is also consistent with the observation of the in-sample and out-of-sample data set in table 1.

However, the data history of the EUA 2008-2012 rates is too small to derive reliable conclusions about the future development. Therefore, the data used to estimate a reliable futures price model is taken from the much longer 2008 futures' data history starting at April 22, 2006, when the second period EUA's were not traded yet. In order to model the future prices from the outset of the emissions trading onwards, Daskalakis, Psychoyios, and Markellos (2007) suggest to use a two factor equilibrium model with a stochastic convenience yield as a second factor. In their study they tested several continuous time models with the result that a geometric Brownian motion with an additional jump-diffusion component is favored. Despite the fact that the model is quite cumbersome and therefore is in conflict with the requirement of parsimony, Benz and Trück (2007) found that it is outperformed in a comparison either by regime switching or non-constant variance models. Uhrig-Homburg and Wagner (2007) found in their research study that the valuation of derivatives on EUA's should not be based on the spot price of the EUA 05-07. This is due to the fact that it does not reflect all the necessary information in order to build reliable expectations about the future spot price during the second compliance period. Contrary, the futures maturing in the second compliance period do reflect the necessary information.

## 4.2 Model Estimation

Generally, in order to deploy statistical inference on time series, the process is required to be of weak stationarity. To investigate the stationary properties of the log returns we perform an Augmented Dickey-Fuller test (Dickey and Fuller (1979)). With a test statistic of -19.19, the null-hypothesis of a unit root can be rejected at all significance levels for the log returns as well as for the squared log returns. The test was performed with and without a trend estimation. It could be shown that the series have no trend.

The autocorrelation function (ACF) of the December 2008 futures' log returns on the left panel in figure 4 show that most of the autocorrelation coefficients are not significantly different from zero at a 95% confidence level with exception of the lags 1, 10 and 16. The very similar picture shows the partial autocorrelation function (PACF) in the right panel of the same figure.

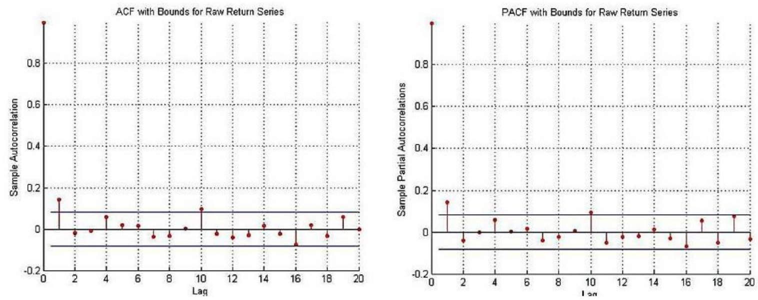


Figure 4: ACF and PACF of the returns.
Autocorrelation function and partial autocorrelation function of the December 2008 futures' returns. Bounds show the 95% confidence interval.

Although the errors themselves seem not to be heavily correlated, the squared errors in figure 5 show some autocorrelation up to lag 12. Therefore, we can infer that there is some serial dependence in the second moments meaning that the assumption of constant variance cannot be made. This is consistent with financial market observations when the returns have a leptokurtic distribution.

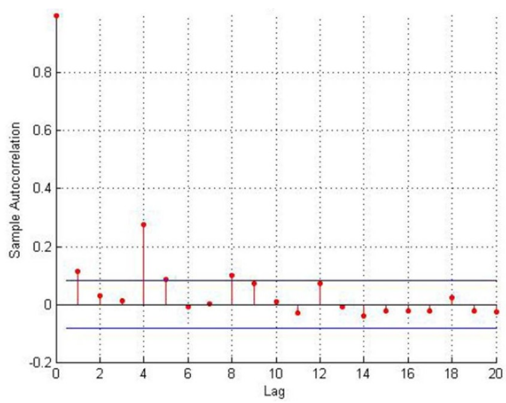


Figure 5: ACF of the squared returns.
Autocorrelation function of the squared December 2008 futures' returns.
Bounds show the 95% confidence interval.

In addition, if the correlation is quantified by deploying the Ljung-Box-Pierce Q-test (LBPQ) (Box, Jenkins, and Reinsel (1994)) in table 4, the null-hypothesis of no serial correlation in the innovation has to be rejected at the 5% level of significance only regarding the first 10 lags in the case of the log returns. However, the squared returns are heavily autocorrelated in every case. This finding is also supported by the results of the Engle's ARCH Test (Engle

(1982))[5]. The test results in table 5 clearly reject the null-hypothesis.

Table 4: LBPQ Test of Innovations and (squared Innovations)

| Lags | p Value | Statistics | Critical Value |
|---|---|---|---|
| 10 | 0.0197 (0.0000) | 21.19 (67.66) | 18.30 |
| 15 | 0.0743 (0.0000) | 23.48 (72.93) | 24.99 |
| 20 | 0.0731 (0.0000) | 29.80 (74.62) | 31.41 |

Table 5: ARCH Test

| Lags | p Value | Statistics | Critical Value |
|---|---|---|---|
| 10 | 0.0000 | 54.62 | 18.30 |
| 15 | 0.0000 | 58.71 | 24.99 |
| 20 | 0.0000 | 62.27 | 31.41 |

A conditional mean model is generally considered as adequate if the error terms show no autocorrelation and if the normal distribution hypothesis cannot be rejected. The visual inspection of the December 2008 Futures log returns graph, implies a presence of volatility clusters. In addition, as the Jarque-Bera test showed, the null hypothesis of a normal distribution can clearly be rejected. The distribution of the log returns has fat tails and excess kurtosis. These findings imply evidence of the presence of GARCH effects in the time series. It was therefore necessary to test for the presence of conditional heteroscedasticity before estimating an GARCH Model. Such GARCH effects are present, if the normal and partial autocorrelations of the squared innovations (i.e. the residuals) are different from zero (Gourieroux and Jasiak (2001)). The results from the above analysis, together with the findings from the Engle's ARCH Test cause significant evidence of GARCH effects in the innovations of the log returns.

Given the findings above which showed strong evidence of heteroscedasticity, the deployment of an autoregressive heteroscedastic Model (ARCH) is appropriate. Benz and Trück (2007) found in their study that a GARCH approach as well as a regime-switching process results in reasonable simulations of the $CO_2$ allowance spot prices. Daskalakis, Psychoyios, and Markellos (2007) used a constant variance jump diffusion process, but failed to catch the dynamics of the underlying to the same extent compared to a GARCH approach (Benz and Trück (2007)). In addition, a study by Liu and Enders (2003) investigating the fitting of nonlinear models to economic time series, found that both the in-sample and out-of-sample measures of fit favor the nonlinear GARCH functional form. Consequently, fitting a GARCH functional form to the observed data is deemed appropriate.

Usually, a simple GARCH(1,1) model is adequate for the empiric modeling of financial market data. Therefore, the first model estimation is based on a

[5]Each of the tests extracts the sample mean from the actual returns. The innovations' process is e(t)=y(t)-C, and C is the mean of y(t).

constant mean model with conditional variance (GARCH):

$$y_t = C + \varepsilon_t \tag{4.1}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{4.2}$$

with the constraints $\alpha$ >0, $\beta$ >0, $\omega$>0, $\alpha$ + $\beta$ <1 According to the conditional mean model in equation 4.1, the returns $y_t$ consist of a simple constant plus an uncorrelated white noise disturbance $\varepsilon_t$. Most financial returns series do not require an ARMAX model. This is consistent with the estimation performed in table 6. The conditional variance model in equation 4.2 consists of a constant plus a weighted average of the last period's forecast and last period's disturbance. Although simplistic, the parsimony of this model, should ensure the correct forecasting of financial data. According to Hamilton (1994), more complex model can better track the data over the historical period, but then fail to perform well in the out-of-sample forecasting. In the course of the modeling process this model is compared to a GARCH Model with higher lags. The conditional probability distribution is Gaussian.

We estimate the parameter listed in table 6 with maximum likelihood.

Table 6: GARCH (1,1) Parameters

| Parameter | Value | Std. Error | t-Statisitc |
|---|---|---|---|
| $C$ | 0.0014831 | 0.0011357 | 1.30 |
| $\omega$ | 9.9413e-005 | 1.999e-005 | 4.97 |
| GARCH (1) | 0.69467 | 0.043441 | 15.99 |
| ARCH(1) | 0.18842 | 0.031444 | 5.99 |

The values of the t-statistic indicate that all estimated parameters are significantly different from zero, with exception of the value of the constant $C$. The log-likelihood value is 1306.5.

The estimated model equation is therefore:

$$y_t = 0.0014831 + \varepsilon_t \tag{4.3}$$

$$\sigma_t^2 = 9.9413e - 005 + 0.18842\varepsilon_{t-1}^2 + 0.69467\sigma_{t-1}^2 \tag{4.4}$$

The modeling results are now plotted against the raw return data in order to compare the two (cf. figure 6).

The sum of $\alpha$ and $\beta$ represents the integrated non-stationary boundary given in the constraints in equation 4.2. In empirical research, it is often found to be close to one (Gourieroux and Jasiak (2001)). However, in the case of the GARCH(1,1) model the sum of $\alpha$ and $\beta$ amounts to 0.88309 which should ensure the stationarity of the model. The model's unconditional variance $V_L$ can be calculated from the values of the parameters $\alpha$ and $\beta$ as well as $\omega$. It represents the longterm expectation of the model's variance. Therefore, $V_L$ is equal to $\frac{V_L}{\gamma}$ = 0.029160553, where $\gamma$ = 1-$\alpha$-$\beta$.

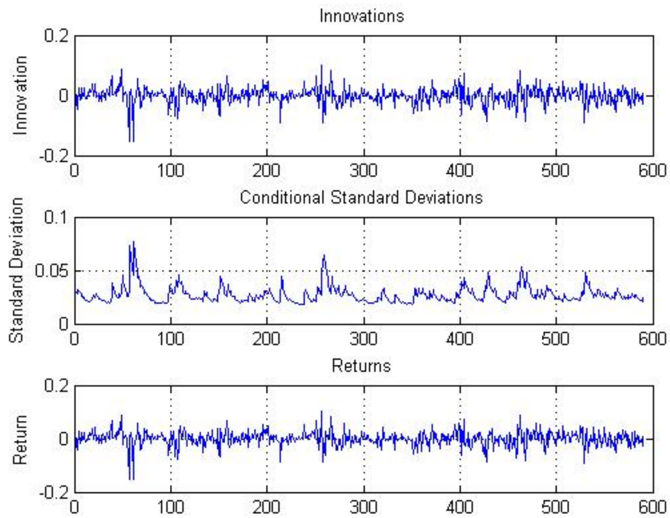


Figure 6: GARCH Plot

The derived innovations are standardized by dividing them by their standard deviation. In figure 7 it can be seen that there are less volatility clusters in the plotted standardized innovation in comparison to the raw returns (cf. figure 2). Moreover, the ACF of the standardized innovations show less autocorrelation (cf. figure 4.

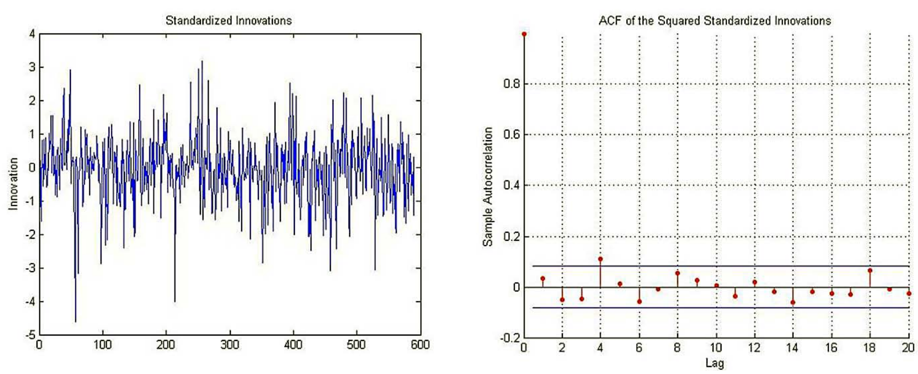


Figure 7: Standardized Innovations and their Autocorrelation

Comparing the correlations of the standardized innovations to the results of the pre-estimation analysis it is apparent that there is no autocorrelation in the standardized innovations with exception of lag 4. However, according the test results, the null hypothesis of no serial correlation cannot be rejected anymore. The respective p values support the explanatory power of the estimated model. This finding is supported as well by the ARCH test's results which show that there are no ARCH effects in the estimated innovations anymore (cf. tables 7 and 8).

Table 7: LBPQ Test of Standardized Squared Innovations

| Lags | p Value | Statistics | Critical Value |
|---|---|---|---|
| 10 | 0.1367 | 14.8747 | 18.3070 |
| 15 | 0.2477 | 18.2901 | 24.9958 |
| 20 | 0.3190 | 22.4039 | 31.4104 |

Table 8: ARCH Test Standardized Innovations

| Lags | p Value | Statistics | Critical Value |
|---|---|---|---|
| 10 | 0.2234 | 13.0050 | 18.3070 |
| 15 | 0.3860 | 15.9405 | 24.9958 |
| 20 | 0.3740 | 21.3988 | 31.4104 |

The results support the use of a constant mean/GARCH Model to model the time series. In the next section an Akaike (AIC) as well as a Bayesian Information Criterion (BIC) test is performed in order to see if there is evidence to use a higher lagged GARCH model (Akaike (1974); Schwarz (1978)). In addition, several t-GARCH models, where $\varepsilon\sim$ t(0,h,v), are compared to the GARCH models. The incorporation of t-distributed innovations has been suggested by Bollerslev (1987)).

We compare the previously estimated model to other GARCH Models with higher lags in $p$ and $q$, as well as to several t-GARCH models. The analysis is performed using the Akaike and Bayesian Information Criteria. Table 9 summarizes the calculated AIC and BIC values. Contrary to AIC, the BIC favors the parsimony of models and therefore penalizes the use of more parameters (Hamilton (1994)).

Table 9: AIC and BIC Values of the Estimated Models

| Model | AIC ($*1.0e{+}003$) | BIC ($*1.0e{+}003$) |
|---|---|---|
| GARCH(1,1) | -2.6049 | -2.5874 |
| t-GARCH(1,1) | -2.6506 | -2.6287 |
| GARCH(2,1) | -2.6071 | -2.5852 |
| t-GARCH(2,1) | -2.6492 | -2.6229 |
| GARCH(2,2) | 2.6051e | -2.5788e |
| GARCH(3,1) | -2.6126 | -2.5863 |
| t-GARCH(3,1) | -2.6496 | -2.6190 |

The main result of the model comparison is that in general t-GARCH models perform better than GARCH models with the assumption of normality in the distribution of the innovations in modeling the December 2008 EUA futures' returns (cf. table 9). As far as the number of lags are concerned, the t-GARCH(1,1) model is favored over a t-GARCH(3,1) model by the relative BIC value. The incorporation of a second ARCH term was evaluated, but there was no statistical significance found in the estimation. Therefore, adding a second ARCH term in GARCH(2,2) is not favored over GARCH(2,1) by the AIC as well as the BIC value. Thus, we concluded that a higher lagged ARCH term does not improve the fit of the model. The comparison of the relative BIC value indicates that the best fit should be reached deploying a simple t-GARCH(1,1) model, although it is not favored over an t-GARCH(3,1) model by the AIC value. However, empirical evidence suggests that the number of coefficients correlates negatively with the precision of the model when it comes to forecast volatility (Hamilton (1994)). Therefore, for the selection of the model the BIC value which penalizes the use of additional coefficients is taken as a benchmark.

The estimated parameters of the specified t-GARCH Model with $p = 1$ and $q = 1$ are presented in table 10. Again the t-statistics show that the parameters

Table 10: t-GARCH(1,1) Parameters

| Parameter | Value | Std. Error | t-statisitc |
|---|---|---|---|
| $C$ | 0.0019947 | 0.00094195 | 2.11 |
| $\omega$ | 9.5277e-005 | 4.0123e-005 | 2.37 |
| GARCH (1) | 0.72677 | 0.075566 | 9.61 |
| ARCH(1) | 0.17902 | 0.059341 | 3.01 |
| DoF | 4.0846 | 0.91871 | 4.44 |

are significantly different from zero. The newly estimated model therefore can be written as:

$$y_t = 0.0019947 + \varepsilon_t \tag{4.5}$$

$$\sigma_t^2 = 9.5277e - 005 + 0.17902\varepsilon_{t-1}^2 + 0.72677\sigma_{t-1}^2 \tag{4.6}$$

where

$$\varepsilon_t \sim t(0, h, v) \tag{4.7}$$

where $h$ is the variance and $v$ the degrees of freedom (DoF). The daily long term unconditional variance of the innovations $V_L$ can be calculated by the equation $V_L = \frac{\omega}{\gamma}$ (Hull (2008)). Because $\gamma$ = 1-$\alpha$-$\beta$= 0.09421, it follows that $V_L$ = 0.001011326. This corresponds to a daily volatility of 0.031801348.

As in the case of the GARCH(1,1) Model, the dynamics of the GARCH process are modeled quite good by the conditional variance. Yet, volatility clusters in the innovations and the return can be spotted. However, the sum of the coefficients of the conditional and unconditional variance is still below 1. Moreover, with $\alpha_1 + \beta_1 = 0.90579$, it is less close to the boundary condition than in the case of the formerly estimated model. The log-likelihood value is

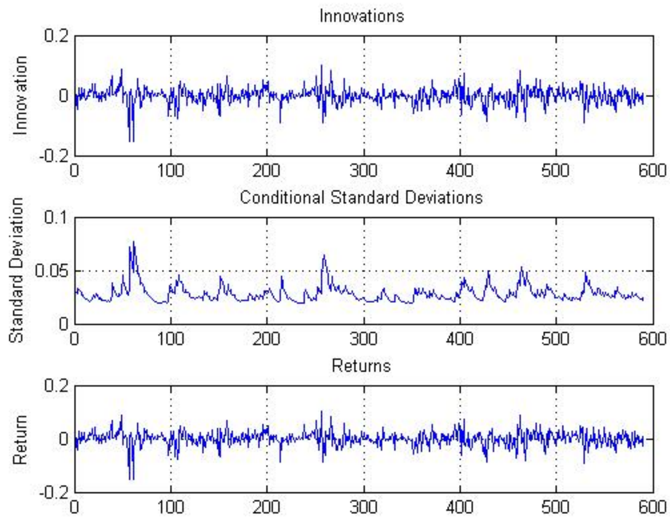


Figure 8: t-GARCH Plot

1330.3, which is higher than in the case of the previously estimated GARCH(1,1) model.

The highly significant and rather large GARCH parameter in the model described above suggests that the persistency of the variance is quite high. The effect of this persistency in the variance is the observed volatility clustering. Thus, the bigger the amount of yesterday's $(t-1)$ conditional volatility with respect to the unconditional variance, the larger the contribution of yesterday's $(t_0)$ variance term to the value of today's variance. The value of the $\alpha$ parameter describes the reaction of the variance on shocks in the log returns. The value of the $\alpha$ in the model is rather small. This suggests that the effect of a shock in yesterday's realization on today's return is not very significant.

Based on the newly estimated model the values of the conditional variances and the innovations are derived. Even though not clearly visible in the correlation graph, the autocorrelation was reduced in comparison to the previously estimated GARCH(1,1) model. According to the test results, t-GARCH(1,1) performs better in terms of the LBPQ-Test as well as with respect to the ARCH-Test. The $p$ values are significant at the 5% level (cf. table 11 and table12). There is neither serial dependence in the innovations nor ARCH effects in the standardized innovations. Investigating the autocorrelation function of the GARCH, it can be inferred that no autocorrelation in the standardized innovation exists, with exception of lag four. The comparison of the results of the LBPQ and ARCH test show that there is no rejection of the null hypothesis of serial correlation as well as no ARCH effects at a significance level of 5%. In the pre-estimation analysis of the raw returns both null-hypotheses had to be

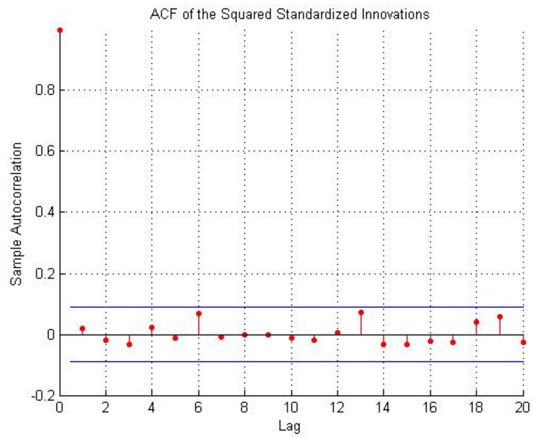


Figure 9: ACF of the Standardized Innovations

rejected significantly. All things considered, it can therefore be concluded that the model sufficiently explains the heteroscedasticity in the raw returns. This proves the explanatory power of the derived model.

Table 11: LBPQ Test of Standardized Squared Innovations (t-GARCH)

| Lags | p Value | Statistics | Critical Value |
|---|---|---|---|
| 10 | 0.1415 | 14.7477 | 18.3070 |
| 15 | 0.2326 | 18.5969 | 24.9958 |
| 20 | 0.3052 | 22.6718 | 31.4104 |

Table 12: ARCH Test Standardized Innovations (t-GARCH)

| Lags | p Value | Statistics | Critical Value |
|---|---|---|---|
| 10 | 0.2219 | 13.0310 | 18.3070 |
| 15 | 0.3710 | 16.1680 | 24.9958 |
| 20 | 0.3596 | 21.6537 | 31.4104 |

## 4.3 Forecasting

A times series over 170 days is forecasted using the estimated t-GARCH(1,1) and GARCH(1,1) model. The purpose of this comparison is to estimate the performance of both models that performed best by their respective AIC values, but differ only in the assumption regarding the distribution of the innovations. After having forecasted this time series, the results are compared with their counterparts derived by the Monte Carlo simulation. The returns have been estimated using $m = 20,000$ runs.

With an increasing forecasting horizon the conditional variances converge to the long term unconditional variance. Figure10 shows that this is the fact also in the case of the estimated model. The asymptotic behavior of the conditional variance can be clearly spotted. The standard deviation of the innovations is approaching the level of the long term unconditional variance, which was found to be 0.031801348 or 0.029160553 in the previous section. The minimum mean squared error (MMSE) forecast lies in the middle of the standard deviation of the innovations derived by the Monte Carlo simulation. Especially, in the very short run the simulated and the forecasted volatility are equal. With respect to the t-GARCH(1,1) model, the period after day 70 exhibits a greater fluctuation in the simulated volatility. In the long run, the simulated sigmas converge towards the long run variance. In case of the GARCH(1,1) model, the convergence of the simulated and forecasted realization is better.

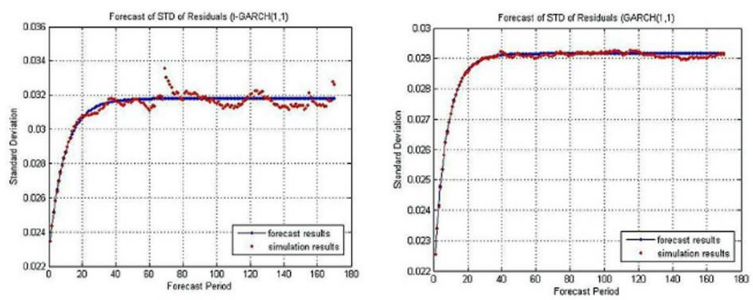


Figure 10: Forecast of the Model's Standard Deviation of the Residuals
The left panel shows the forecast of the t-GARCH(1,1) model with respect to the simulation results. The right panned gives the same data for the GARCH(1,1) model.

Figure 11 shows that the forecasted conditional return is always 0.0019947 or 0.0014831, respectively, because the expected value of the $\varepsilon_t$ is zero. The simulated returns are evenly distributed around the mean forecast.

In figure 12 the root mean squared errors (RMSE) of the forecasted returns are plotted with the standard deviation of the simulated returns. In general, with respect to both models, the volatility measures converge quite well. However, regarding the t-GARCH model there are a few outliers, caused by the assumed student t distribution in the residuals. Furthermore, around lag 120 increased volatility can be spotted.

In the following, we compare the Monte Carlo simulation output the in-sample and out-of-sample data in order to asses the predicting power of the model using the observed data in the market. In table 13 the distribution moments of the simulation as well as the sample figures are listed. To get an estimate of the distribution moments regarding the in-sample period, the returns

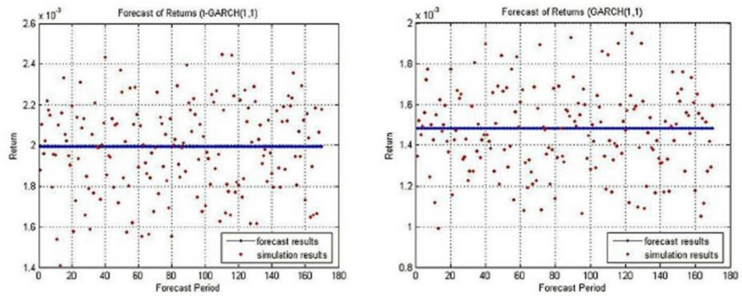


Figure 11: Forecasted Mean Returns and Simulated Returns
The left panel shows the forecast of the t-GARCH(1,1) model with respect to the simulation results. The right panned gives the same data for the GARCH(1,1) model.

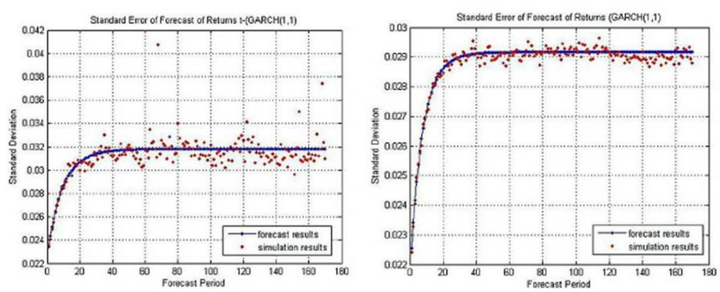


Figure 12: Standard Errors of Forecast
Standard errors of forecast of the returns for the t-GARCH(1,1) model in the left panel and for the GARCH(1,1) model in the right panel.

and variances have been simulated over a 590 day period. In the case of the out-of sample period, the same data as in the previous section are used.

Table 13: Distribution Moments

| | mean | p StD | Skewness | Kurtosis |
|---|---|---|---|---|
| IS data | 0.0008586 | 0.0278 | -0.6406 | 6.8458 |
| OOS data | 0.0016 | 0.0187 | 0.2283 | 4.5856 |
| Simulated GARCH IS | 0.0014 | 0.0290 | 0.0037 | 4.7462 |
| Simulated t-GARCH IS | 0.0020 | 0.0308 | -0.6761 | 32.2073 |
| Simulated GARCH OOS | 0.0016 | 0.0288 | 0.0135 | 4.1272 |
| Simulated t-GARCH OOS | 0.0022 | 0.0310 | 0.7320 | 26.1105 |

The t-GARCH(1,1) model seems to have too many realizations around the mean, leading to a excessively high kurtosis. Moreover, there are also a few outliers as observed in the RMSE comparison. In addition, the simulation outcomes are not stable. The moments of the distribution differ highly, when the number of simulation runs is changed. This might be because of the outliers due to the fat tails of the distribution. Contrary, the GARCH(1,1) model seems to have better simulation results.

Having compared the distribution moments of the in-sample and out-of-sample simulation with the sample data, it is quite astonishing that the in-sample simulation outcomes of both models are quite modest. Especially, the amount of the in-sample data's kurtosis could not be reproduced by the model. In this respect the t-GARCH model performs quite badly. As mentioned previously, the model seems to lack stability. However, regarding the GARCH model the outcomes are stable even if the time horizon of the simulation is changed. In addition, in terms of the out-of-sample modeling results, the GARCH model perform well.

Based on the findings, it can be reckoned that incorporating t distributed residuals in the model has not the intended effect regarding the fit of the model, even though the t-GARCH model performs better with respect to the Akaike and Bayesian Information Criteria.

Following Wooldridge (2003) an additional out-of-sample comparison is made. The RMSE is calculated according the formula $\sqrt{\frac{1}{N}\Sigma_{t=1}^{N}\hat{e}^2}$. Where $\hat{e}$ is the deviation of the observed futures rate from the forecasted mean return. Again, 20,000 different paths have been simulated, with the effect that 20,000 RMSE could be obtained. In table 14 the mean over all 20,000 RMSEs is presented. In addition to the RMSE comparison, the mean of the relative absolute errors (MAE) of the terminal simulated errors is listed in table 14. The relative MAE can be calculated as $\frac{1}{N}\Sigma_{t=1}^{N}|\hat{e}|$. The errors are $\hat{e} = \frac{\hat{y}_T - y_T}{y_T}$. Contrary to the RMSE where the mean of the forecasted returns is compared to the observed returns over the whole time horizon, in the case of the relative MAE calculation only the simulated terminal values of the futures contracts are compared to the very last observation of the OOS data set. By doing this it is possible to get an estimate about the relative errors in the last realizations of the simulated futures prices, which are used to calculated the derivatives' payoff.

Table 14: RMSE between the Simulated and observed Returns

| Model | RMSE | MAE |
|---|---|---|
| GARCH | 5.6210 | 0.3662 |
| t-GARCH | 6.4849 | 0.4639 |

According to the relative comparison of the RMSE as well as the MAE of the two models, the GARCH(1,1) performs better than the t-GARCH(1,1) model. In the last resort and having taken into account all the findings of this section, it can be reckoned that the simplistic GARCH(1,1) model performs best in terms of capturing the dynamics of the December 2008 futures returns.

## 4.4 Valuation Results

In this section we carry out the valuation of the certificates according to equation 3.14. The futures rate has been simulated over the out-of-sample period from August 23, 2007 until April 24, 2008, which is equal to 170 days. During the simulation process the futures prices are tested against the knock-out barrier. This has the effect that all the price realizations which have hit the barrier are not incorporated in the calculation of the payoff. This procedure is repeated $m = 20,000$ times in order to get an reliable estimate about the derivative's payoff. At the end of the simulation process the average value of the payoffs is discounted back to $t_0$ at the risk free rate which is approximated by the Euribor adjusted for 8.16 months). The outcome of this procedure is an estimate of the derivative's price at the beginning of the out-of-sample period or at the beginning of the life of the option, respectively. The reason for evaluating the derivatives over the out-of-sample period is that it allows to compare the valuation error to the errors of the simulated futures prices declared in table 14.

It is important to note that the payoffs have to be adjusted in order to satisfy the risk neutral valuation conditions as mentioned in section 3. The risk neutral valuation approach places a restriction on the development of the futures price. The futures price at $t_0$ is deemed to be an unbiased predictor of the futures rate at time $T$, meaning that the drift is equal to zero. In the GARCH(1,1) models' specification, the returns are simulated by a constant plus a random error term. As a result the simulations of the futures returns have a mean that is different from zero as in table 13). Thus, it is reasonable that there is a drift in the Monte Carlo Simulation, as well. Myers and Hanson (1993) suggest to adjust each terminal realization of the futures price simulation by multiplying the rate by the initial futures price and subsequently dividing it by the terminal realizations' average value. This adjustment has the effect (as shown by the column (Mean $\hat{F}_T$) in table 15), that the mean of the terminal futures price realizations just equals the initial futures price. Thus, satisfying risk neutral valuation which requires the initial futures price to be an unbiased predictor of the futures rate at maturity.

### 4.4.1 Out-of-sample Valuation

In table 15 we present the valuation results. To measure the model's performance, the MSE as well as the relative MAE is calculated. The MAE values show that, with exception of a few derivatives, the deviation of most of the simulated prices from the market rates range between less than one and two percent. The average value of the absolute errors amounts to 5.5%. In the case of only two of the options, the deviation is more than 10%. If those single cases are excluded from the calculation, the relative MAE is more than halved to less than 2.5%. Either way, this good performance of the model is a very surprising result, taken into account that the MAE of the *unadjusted* terminal futures prices is roughly 37% (cf. table 14). The outcomes from the valuation of the derivatives provide strong evidence that risk neutral valuation is applicable to situations when heteroscedasticity is present in the returns.

However, the pricing performance of the model is not beyond doubt. In case of the down-and-out call options, the model tend to overprice calls with a small intrinsic value. There is a pricing error of almost 40% in the case of the call with a strike price of 20 and a respective futures price of 21.45. This outcome could be due to the large number of knocked-out simulation runs which differs to a great extent from the other derivatives' knock-out figures. Too many low realizations of the futures rate might not be considered in the payoff calculation. However, the fact that the barrier is close to the actual futures rate makes this outcome reasonable. If the barrier is not set close to the current futures price, e.g. deep-in-the-money options, this effect has not a such big weight and the valuation yields good results. However, the number of the available derivatives with the EUA futures 2008 as a underlying is too small to draw a final and valid conclusion about which type of option is priced wrongly by the model.

With respect to the up-and-out puts the situation is reversed. The model is inclined to misprice deep-in-the-money puts in comparison to puts with a strike closer to the actual futures rate. Here, the specific pricing errors are not as large as in the case of the call analyzed previously but yet substantial. It is quite astonishing that in the case of the certificate DR5C9Y, with $K = 35$ and $H = 33$, the amount of times that the barrier has been crossed differs to such a large extent, when compared to the certificate with the barrier set to 33. The most likely explanation for this can be found in the specification of the valuation process. As stated above, the options labeled with two stars were issued February 22, 2008. Because of this, the futures rate was only modeled for a period of 43 days. Consequently, the modeling was initiated at a different rate, i.e. EUR 21.48, instead of EUR 18.81. Because of the longer time horizon, more simulated futures crossed the barrier and their rate was not included in the payoff calculation which lead to a underpricing of the derivative. However, this explanation does not hold true in any case. Regarding the calls such effects could not be observed.

Table 15: Out of Sample Certificate Valuation Results

| Type | $H^a$ | $X^a$ | Market Value [b] | Simulated Value | $e^2$ | relative $\|e\|$ | $H$ Crossed | Mean $\hat{F}_T$ |
|---|---|---|---|---|---|---|---|---|
| KNOCK OUT | | | | | | | | |
| Calls | | | | | | | | |
| DR5C9Z* | 7 | 5 | 13.55 | 13.3574 | 0.03709 | 0.01421 | 193 | 18.81 |
| DR5C90* | 12 | 10 | 8.55 | 8.4726 | 0.00599 | 0.00905 | 3302 | 18.81 |
| DROQSR** | 20 | 20 | 1.3 | 1.9056 | 0.36675 | 0.46585 | 10688 | 21.48 |
| DROQSS** | 18 | 18 | 3.3 | 3.6608 | 0.13017 | 0.10933 | 4432 | 21.48 |
| DROQST** | 16 | 16 | 5.3 | 5.4983 | 0.03932 | 0.03742 | 1365 | 21.48 |
| DROQSU** | 14 | 14 | 7.3 | 7.4316 | 0.01732 | 0.01803 | 318 | 21.48 |
| DROQSV** | 12 | 12 | 9.3 | 9.4041 | 0.01084 | 0.01119 | 47 | 21.48 |
| DROQSW** | 10 | 10 | 11.3 | 11.3865 | 0.00748 | 0.00765 | 7 | 21.48 |
| MSE/MAE Calls | | | | | 0.07687 | 0.08409 | | |
| Puts | | | | | | | | |
| DR5C9Y* | 33 | 35 | 15.95 | 13.5761 | 5.63540 | 0.14883 | 5694 | 18.81 |
| DR98G7* | 40 | 45 | 26 | 23.7758 | 4.94706 | 0.08555 | 2553 | 18.81 |
| DROQSZ** | 26 | 26 | 4.2 | 4.2476 | 0.00227 | 0.01133 | 6768 | 21.48 |
| DROQS0** | 28 | 28 | 6.3 | 6.3067 | 4.49E-05 | 0.00106 | 3593 | 21.48 |
| DROQS1** | 30 | 30 | 8.3 | 8.3535 | 0.00286 | 0.00645 | 1820 | 21.48 |
| DROQS2** | 32 | 32 | 10.3 | 10.3796 | 0.00634 | 0.00773 | 913 | 21.48 |
| MSE/MAE Puts | | | | | 0.79758 | 0.04349 | | |
| INDEX TRACKER | | | | | | | | |
| DR98G8* | 0 | 0 | 18.28 | 18.2073 | 0.00529 | 0.00398 | | 18.81 |
| DR1WBM*** | 0 | 0 | 22.65 | 22.1855 | 0.19758 | 0.01964 | | 22.72 |
| AA0G6VI* | 0 | 0 | 18.55 | 18.2037 | 0.11744 | 0.01847 | | 18.81 |
| HV2C02* | 0 | 0 | 18.47 | 18.2073 | 0.06901 | 0.01422 | | 18.81 |
| MSE/MAE Trackers | | | | | 0.09733 | 0.01408 | | |
| MSE/MAE | | | | | 0.64435 | 0.05500 | | |

[a] $H$=Barrier, $X$=Strike
[b] * as of Aug 23, 2007, **as of Feb 22, 2008, *** as of Oct 26, 2007. The corresponding futures prices were EUR 18.81, 21.48, 22.72, respectively

### 4.4.2 Valuation up to Maturity

One possible corrective action would be to model the underlying of the short maturity option, labeled with ** in table 15, as well with a time horizon of 170 days starting on August 23, 2007. Thus, the number of knock-outs would be equal to the long maturity options. Because of the big simulation errors of the model,the associated problem would be that the futures price on February 22, 2008, when the option began to live, were inclined to be different from the simulated futures price. Therefore, the option would be priced wrongly even on the first day, resulting from the simulation errors in the model. However, the results found in the previous analysis suggest that the number of knock-out events might influence the correctness of the value's estimate. Therefore, a second valuation is made, this time the options are valued up to their maturity with a start date on April 24, 2008. This equals a time period of 160 days. As the risk free rate the Euribor rate adjusted for 7.68 months is used. The results of the valuation are listed in table 16.

For the down-and-out call prices, setting time horizon equal to 160 days for all simulations in the valuation of the derivatives, qualifies the very good results of the previously made valuation at first sight. Referring to the MSE figures the deviation of the model price from the market prices grew substantially. As a consequence, the relative absolute pricing error is now slightly above 11% for the period from April 24, 2008 until December 3, 2008 which is the maturity date of all the knock-out options. Yet, inference about the valuation power of the used model may be derived based on the comparison of the outcomes when different time horizons are simulated. Analyzing the valuation outcomes under scrutiny, it can be reckoned the relatively large total MAE is mainly due to the increase of the put option price simulation's MAE. Moreover, with respect to the calls, using a simulation period of 160 days for all options, reduces the respective MAE figures by more than 3%. This is due to the fact that the pricing errors are now more homogeneously distributed with respect to every strike price, meaning that the range of the deviations could be reduced compared to the perviously made valuation as can be seen in table 3.2. The absolute errors were generally smaller, but some big pricing errors distorted the total of MSE and MAE. No general valid inference can be made anymore which links the pricing errors to the moneyness of the calls.

The pricing performance of the valuation regarding the up-and-out puts is very bad with an MAE of almost 25%. In the analysis of the first valuation run, it was hypothesized that the pricing error is dependent on how many times that the barrier has been crossed by the simulated futures terminal realizations. At first sight, this hypothesis holds true, because the number of knock-outs of the puts is much larger than the respective figures of the calls which are priced more accurately. However, the only put that is priced with a high accuracy is the one with the highest knock-out figures. Thus, for the time being, this hypothesis cannot be corroborated. However, this fact needs clarification.

The pricing performance of the model regarding the index trackers in the first as well as in the second valuation run is very good. All results differ from

Table 16: Certificate Valuation Results up to Maturity

| Type | H[a] | $X^a$ | Market Value [b] | Simulated Value | $e^2$ | relative $\|e\|$ | H Crossed | Mean $\hat{F}_T$ |
|---|---|---|---|---|---|---|---|---|
| KNOCK OUT | | | | | | | | |
| | | | | | | | | |
| Calls | | | | | | | | |
| DR5C9Z | 7 | 5 | 19.83 | 18.9088 | 0.84860 | 0.04645 | 26 | 24.51 |
| DR5C90 | 12 | 10 | 14.83 | 14.0432 | 0.61905 | 0.053055 | 789 | 24.51 |
| DROQSR | 20 | 20 | 4.8 | 5.3494 | 0.30184 | 0.11446 | 8713 | 24.51 |
| DROQSS | 18 | 18 | 6.8 | 6.9566 | 0.02452 | 0.02303 | 5727 | 24.51 |
| DROQST | 16 | 16 | 8.8 | 8.6126 | 0.03512 | 0.0213 | 3412 | 24.51 |
| DROQSU | 14 | 14 | 10.8 | 10.3469 | 0.2053 | 0.0412 | 1832 | 24.51 |
| DROQSV | 12 | 12 | 12.8 | 12.182 | 0.38192 | 0.04828 | 798 | 24.51 |
| DROQSW | 10 | 10 | 14.8 | 14.0791 | 0.5197 | 0.04871 | 259 | 24.51 |
| MSE/MAE Calls | | | | | 0.36700 | 0.04965 | | |
| | | | | | | | | |
| Puts | | | | | | | | |
| DR5C9Y | 33 | 35 | 9.87 | 6.864 | 9.03604 | 0.30456 | 12041 | 24.51 |
| DR98G7 | 40 | 45 | 20.96 | 16.2256 | 22.41454 | 0.22588 | 6806 | 24.51 |
| DROQSZ | 26 | 26 | 0.9 | 0.8218 | 0.00612 | 0.08689 | 18592 | 24.51 |
| DROQS0 | 28 | 28 | 2.9 | 1.9876 | 0.83247 | 0.31462 | 16861 | 24.51 |
| DROQS1 | 30 | 30 | 4.9 | 3.4608 | 2.07129 | 0.29371 | 14916 | 24.51 |
| DROQS2 | 32 | 32 | 6.9 | 5.1665 | 3.00502 | 0.25123 | 12988 | 24.51 |
| MSE/MAE Puts | | | | | 6.22758 | 0.24615 | | |
| | | | | | | | | |
| INDEX TRACKER | | | | | | | | |
| | | | | | | | | |
| DR98G8 | 0 | 0 | 24.14 | 23.7562 | 0.1473 | 0.0159 | | 24.51 |
| DR1WBM | 0 | 0 | 24.4 | 23.7562 | 0.41448 | 0.02639 | | 24.51 |
| AA0G6VI | 0 | 0 | 25 | 23.7562 | 1.54704 | 0.04975 | | 24.51 |
| HV2C02 | 0 | 0 | 25.15 | 23.7562 | 1.94268 | 0.05545 | | 24.51 |
| MSE/MAE Trackers | | | | | 1.01287 | 0.03686 | | |
| | | | | | | | | |
| Total MSE/MAE | | | | | 2.46406 | 0.11231 | | |

[a] H=Barrier, X=Strike
[b] as of Apr 24 2008. The corresponding futures prices was EUR 24.51

the observed rates by not more than 2%. This pricing error may be well to the margins of the investment banks that have emitted theses products. Two of the four certificates were issued by Dresdner Bank with either no maturity (DR1WBM) or with maturity date on December 3, 2008 (DR98G8), one open-end certificate by ABN Amro and one open-end certificate by Hypo Vereinsbank. Assuming that the modeled price reflects the true value of the futures, then the index tracker DR98G8 can be seen as being priced fairly by the bank, with a deviation in its price of roughly 1.5%. This deviations might only be due to the simulation error. In addition, this certificate had by far the lowest error in the previous valuation run with 0.5%, as well, while the others deviated by at least 1.5%. The other three certificates' prices differ in the second valuation run from the simulated price by more than 2.5%. Theoretically, there should be no difference in the prices because all of the four certificates track the future 2008 on a 100% basis. However, it can be reasoned that in case of the three open-end certificates, the rolling-over process should be reimbursed but this is generally done by paying the commission when purchasing the product. However, due to the limited amount of index tracking certificates available in the market, it is not possible to significantly assess if the price differences among those products are technically justifiable or intended.

# 5 Summary and Conclusion

The aim of this article was first and foremost to find an appropriate valuation procedure for derivatives on EUAs. Second, the pricing of those certificates and leveraged products should be assessed with respect to its fairness. To achieve the first objective an investigation of the EUA futures' price dynamics was undertaken. The ECX December 2008 EUA futures returns were found to exhibit excess kurtosis and evidence of heteroscedasticity. The model estimation and the subsequently made performance analysis of the models, suggested a GARCH(1,1) model to appropriately reproduce the futures dynamics. In a next step, it had to be detected if an appraisal of the respective derivatives in a risk neutral framework by deploying a Monte Carlo simulation would lead to the right outcomes.

For short time horizons, the valuation outcomes were quite precise. With an increased time period in the simulation the valuation's precision is not outstanding, yet with respect to the calls and the index trackers quite good result could have been obtained. It can therefore be inferred that risk neutral pricing is applicable to derivatives on EUA futures even in situations when heteroscedasticity is present. The small deviations of the simulated futures prices from the market rates are rather to be attributed to errors in the simulation of the underlying, than to be interpreted as evidence of the not applicability of risk neutral valuation. However, it could be shown in section 4.4 that the drift in the unadjusted simulation is responsible for the mispricing of path-dependent options. This drift is influencing all but the last realizations with the effect that disproportionately many realizations are knocked out.

Drawing a conclusion about the fairness of the pricing of derivatives on EUA by the issuing banks is quite hard. However, the small deviation of the valuation results from the observed market prices regarding the participation certificates as well as concerning the call options should provide an indication of fair pricing. The fact that the simulated prices are generally smaller in value in comparison to the market data can be rather attributed to errors in the simulation process than to an intended overpricing of the options and certificates by the issuing banks. However, it is astonishing to detect deviations of the market value of the certificates in comparison to the futures price observed in the market. Theoretically, the certificates should mirror the futures price on a 100% basis. In this respect the certificates issued by Dresdner Bank are priced fairly. The respective products' prices issued by ABN Amro and Hypo Vereinsbank show evidence of overpricing. Yet, due to the number of those certificates included in the study, a generally valid and statistically significant conclusion about the fairness of the pricing cannot be made.

The main findings of this study are in the first place, that it could be show that risk neutral valuation is applicable in situations where heteroscedasticity is present in the underlying's returns. Second, even tough the model is inclined to underestimate the value of the options it can be assumed that the options are generally priced fairly.

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