

The risk-return tradeoff among equity factors

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Abstract

We examine the risk-return trade-off among equity factors. We obtain a positive in-sample risk-return trade-off for the profitability (RMW) and investment (CMA) factors of Fama and French (2015, 2016), while for the market and momentum factors there is a negative relation. The out-of-sample forecasting power (of factor volatility for factor returns) is economically significant for both RMW and CMA: By constructing a trading strategy that relies on such predictability, we obtain annual Sharpe ratios above one and utility gains above 5% per year. We also find weak evidence that the factor variances are negatively correlated with the aggregate equity premium.

Keywords: Asset pricing, risk-return tradeoff, risk factors, market anomalies, realized volatility, predictability of stock returns, profitability, asset growth

JEL classification: G11; G12; G17

1 Introduction

According to conditional versions of the CAPM of Sharpe (1964) and Lintner (1965) or the ICAPM of Merton (1973), there should be a positive aggregate risk-return trade-off, that is, a positive association between the (conditional) variance of the market return and its (conditional) expected return. This simple prediction has been the focus of an extensive empirical literature. In particular, by using different empirical methods several studies have shown that such positive risk-return relation exists at the aggregate level (e.g., Bollerslev, Engle, and Wooldridge (1988), Scruggs (1998), Harrison and Zhang (1999), Ghysels, Santa-Clara, and Valkanov (2005), Guo and Whitelaw (2006), Lundblad (2007), Pástor, Sinha, and Swaminathan (2008), Bali and Engle (2010), and Hedegaard and Hodrick (2016)).¹ However, most of this literature has focused on examining the market risk-return trade-off, with few studies assessing this trade-off for components of the stock market.² We contribute to filling this gap in the literature by examining the risk-return trade-off among equity factors.

The same way that the market risk-return trade-off is consistent with the conditional CAPM or ICAPM, the factor risk-return relation is consistent with conditional multifactor models widely used in the asset pricing literature. Specifically, we employ conditional versions of the multifactor models of Carhart (1997) and Fama and French (2015, 2016) to motivate our empirical tests. According to those models, there should exist a positive relation between the risk premium and conditional volatility of the size, value, momentum, profitability, and investment risk factors. To proxy for the unobserved conditional factor variances, we compute realized variances based on daily factor observations.

By using monthly data from 1964 to 2015 the results suggest a positive in-sample risk-

¹See also Merton (1980), Turner, Startz, and Nelson (1989), Chan, Karolyi, and Stulz (1992), Glosten, Jagannathan, and Runkle (1993), and Lettau and Ludvigson (2010). In related work, Guo, Wang, and Yang (2013) show that the risk-return relation changes over time. On the other hand, Bali, Demirtas, and Levy (2009) find a positive relation between expected returns and downside risk.

²One exception is the work of Bali (2008), who looks at the effect of conditional (equity portfolio) covariances with the market factor onto portfolio's expected returns. Guo et al. (2009) forecast the value premium based on the conditional covariance with the market and the own conditional variance. On the other hand, Wang, Yan, and Yu (2016) examine the risk-return trade-off among individual stocks.

return trade-off for the profitability and investment factors, while for the market and momentum factors there seems to exist a negative relation. Furthermore, the realized factor variance help to forecast out-of-sample factor returns in the cases of the profitability and investment factors, while the same does not occur for the other factors.

To assess the economic significance of the out-of-sample forecasts from realized variances we construct a trading strategy that relies on such predictability. Specifically, the strategy times each factor by going long (short) the factor whenever the predicted factor risk premium (obtained from the predictive regressions in recursive samples) is positive (negative). This factor exposure comes in addition to a permanent long position in the stock market index, and this dynamic strategy is compared against a simple “buy-hold” strategy on the stock index. The results indicate that the out-of-sample forecasting power (of realized volatility for future returns) is economically significant in the cases of the profitability and investment factors. Specifically, the annual Sharpe ratios are above one in both cases compared to 0.74 for the passive strategy that only invests in the stock market index. Moreover, the utility gains associated with the dynamic strategies corresponding to those two factors are above 5% per year. In comparison, such economic significance does not exist or is not robust for the other equity factors. These results are robust to using different formulations of the trading strategy, namely employing different factor weights or a different in-sample period to obtain the forecasts. By using an alternative trading strategy, which explores only positive factor risk premia, we also obtain economically significant gains in the case of the profitability factor.

In the last part of the paper, we use realized factor variances to forecast the market equity premium. This exercise stems from previous evidence that components of the stock market forecast the aggregate market return (e.g., [Hong, Torous, and Valkanov \(2007\)](#)). The results suggest that some of the factor variances are negatively correlated in-sample with future market returns, although the out-of-sample performance is relatively weak.

The paper proceeds as follows. Section [2](#) provides the theoretical foundations for the

factor risk-return trade-off while Section 3 describes the data and variables. In Section 4, we estimate the risk-return relation among equity factors both in- and out-of-sample. Section 5 evaluates the economic significance of the out-of-sample predictability of factor variances for factor risk premia. In Section 6, we assess whether factor variances can forecast the aggregate equity premium. Finally, Section 7 concludes.

2 Theoretical framework

In this section, we provide the theoretical foundation for the empirical analysis conducted in the following sections.

Consider the stochastic discount factor (SDF) representation of a linear conditional asset pricing model with K factors (see Cochrane (2005)):

$$0 = E_t(M_{t+1}R_{i,t+1}^e), \quad (1)$$

$$M_{t+1} = 1 + \sum_{j=1}^K b_j f_{j,t+1}. \quad (2)$$

In this representation, M denotes the SDF that prices assets, R_i^e represents the excess return on a risky asset i , $f_j, j = 1, \dots, K$ stand for the K risk factors, and $E_t(\cdot)$ represents the conditional expectation at time t .³ We are assuming for simplicity that the SDF coefficients (b_j) are time invariant. Moreover, the risk factors represent excess returns, and thus are tradable.

It is well known that the above representation is equivalent to the following expected return-covariance equation,

$$E_t(R_{i,t+1}^e) = - \sum_{j=1}^K b_j \text{cov}_t(R_{i,t+1}^e, f_{j,t+1}), \quad (3)$$

³The excess return can represent either the difference between a risky return and the risk-free rate or the spread between two risky returns.

where $\text{cov}_t(R_{i,t+1}^e, f_{j,t+1})$ denotes the conditional covariance at time t between the excess return and factor j .

Since the factors are traded, the pricing equation also applies for each factor (e.g., Cochrane (2005), Lewellen, Nagel, and Shanken (2010), and Peñaranda and Sentana (2015)),

$$\mathbb{E}_t(f_{j,t+1}) = -b_j \text{var}_t(f_{j,t+1}) - \sum_{k=1, k \neq j}^K b_k \text{cov}_t(f_{j,t+1}, f_{k,t+1}), j = 1, \dots, K. \quad (4)$$

By assuming that the own conditional variance is significantly more important in driving risk premia than the conditional covariances with the other factors, we have the following approximation to the conditional model:

$$\mathbb{E}_t(f_{j,t+1}) \approx -b_j \text{var}_t(f_{j,t+1}), j = 1, \dots, K. \quad (5)$$

This equation represents the risk-return tradeoff for each factor: if $b_j < 0$ (that is, a positive innovation in the factor translates into a lower realization of the SDF, that is, lower marginal utility) we have that an increase in risk (as measured by a rise in the factor's conditional variance) translates into higher conditional factor risk premia. Assuming that we have an empirical proxy for the conditional variance of each factor, the risk-return trade-off can be tested empirically through the following predictive regression (for each factor),

$$f_{j,t+1} = \alpha_j + \gamma_j \text{var}_t(f_{j,t+1}) + \varepsilon_{j,t+1}, \quad (6)$$

where ε_j represents a zero-mean forecasting error. Given the relation, $\gamma_j = -b_j$, the SDF coefficients can be retrieved from the estimates of γ_j .

Under the CAPM of Sharpe (1964) and Lintner (1965), we have

$$M_{t+1} = 1 + b_M RM_{t+1}, \quad (7)$$

where RM represents the excess market return. This implies the following risk-return trade-off relation,

$$RM_{t+1} = \alpha_M + \gamma_M \text{var}_t(RM_{t+1}) + \varepsilon_{M,t+1}, \quad (8)$$

which has been the focus of the empirical risk-return tradeoff literature. In the equation above, $\gamma_M = -b_M$ represents an estimate of the average relative risk aversion coefficient in the economy. This equation is also compatible with the ICAPM of [Merton \(1973\)](#) if we assume that the average investor has log utility and/or investment opportunities are time-invariant (and hence, the “hedging” factors are not priced in equilibrium).

Under the four-factor model of [Carhart \(1997\)](#), we have,

$$M_{t+1} = 1 + b_M RM_{t+1} + b_{SMB} SMB_{t+1} + b_{HML} HML_{t+1} + b_{UMD} UMD_{t+1}, \quad (9)$$

where SMB , HML , and UMD represent the (zero-cost) size, value, and momentum factors, respectively. Consequently, we have the following risk-return relations for each factor:

$$RM_{t+1} = \alpha_M + \gamma_M \text{var}_t(RM_{t+1}) + \varepsilon_{M,t+1}, \quad (10)$$

$$SMB_{t+1} = \alpha_{SMB} + \gamma_{SMB} \text{var}_t(SMB_{t+1}) + \varepsilon_{SMB,t+1}, \quad (11)$$

$$HML_{t+1} = \alpha_{HML} + \gamma_{HML} \text{var}_t(HML_{t+1}) + \varepsilon_{HML,t+1}, \quad (12)$$

$$UMD_{t+1} = \alpha_{UMD} + \gamma_{UMD} \text{var}_t(UMD_{t+1}) + \varepsilon_{UMD,t+1}. \quad (13)$$

Under the five-factor model of Fama and French (2015, 2016), the SDF is given by

$$M_{t+1} = 1 + b_M RM_{t+1} + b_{SMB} SMB_{t+1} + b_{HML} HML_{t+1} + b_{RMW} RMW_{t+1} + b_{CMA} CMA_{t+1}, \quad (14)$$

where RMW and CMA denote the profitability and investment factors, respectively. This

model leads to the following risk-return equations:

$$RM_{t+1} = \alpha_M + \gamma_M \text{var}_t(RM_{t+1}) + \varepsilon_{M,t+1}, \quad (15)$$

$$SMB_{t+1} = \alpha_{SMB} + \gamma_{SMB} \text{var}_t(SMB_{t+1}) + \varepsilon_{SMB,t+1}, \quad (16)$$

$$HML_{t+1} = \alpha_{HML} + \gamma_{HML} \text{var}_t(HML_{t+1}) + \varepsilon_{HML,t+1}, \quad (17)$$

$$RMW_{t+1} = \alpha_{RMW} + \gamma_{RMW} \text{var}_t(RMW_{t+1}) + \varepsilon_{RMW,t+1}, \quad (18)$$

$$CMA_{t+1} = \alpha_{CMA} + \gamma_{CMA} \text{var}_t(CMA_{t+1}) + \varepsilon_{CMA,t+1}. \quad (19)$$

All the factors above are designed in a way to deliver positive risk premiums. Hence, we expect that the estimated slopes from the regressions above are positive in all cases.⁴ In the following sections, we aim to test empirically these risk-return relations.

3 Data and variables

In this section, we describe the data and variables employed in the following sections.

To estimate the risk-return tradeoff for each equity factor we need an empirical proxy for the unobserved conditional factor variances. Following [Haugen, Talmor, and Torous \(1991\)](#), [Goyal and Santa-Clara \(2003\)](#), [Bali et al. \(2005\)](#), [Guo \(2006\)](#), [Guo and Savickas \(2006\)](#), [Barroso and Santa-Clara \(2015\)](#), among others, we use the realized variance in month t ,

$$RV_t = \sum_{j=0}^{20} f_{d_t-j}^2, \quad (20)$$

as an estimate of the conditional variance of a given factor f , ($\text{var}_t(f_{t+1})$). In the above expression, f_{d_t-j} denotes each of the last 21 daily realizations of f and $d_t, t = 1, \dots, T$ rep-

⁴The theoretical background for these “empirical” multifactor models is not as clear as the cases of the CAPM or ICAPM models. One possible explanation is that the empirical models are consistent with the APT framework of [Ross \(1976\)](#) to the extent that the factors in those models explain a large share of the time-series variation in the cross-section of stock returns. Another plausible explanation is that the empirical models are consistent with the ICAPM to the extent that state variables associated with the (non-market) factors in those models forecast future aggregate investment opportunities (see [Maio and Santa-Clara \(2012\)](#) and [Cooper and Maio \(2016\)](#), among others).

resents the time-series of the dates of the last trading sessions of each month. The data on the daily factors are obtained from Kenneth French’s data library. The sample is 1964:01 to 2015:12.

Table 1 presents the descriptive statistics for the realized variances while Figure 1 plots the respective time-series. The realized volatility of *UMD* is by far the most volatile (standard deviation of 1% per month), followed by the realized market variance (0.41%). The other factors exhibit significantly less volatility of volatility (around or lower than 0.10%). The most remarkable spikes in market volatility are centered around the 1987 stock market crash and the recent (2007-09) financial crisis whereas the size premium volatility also had a major rise in the 1987 crash. On the other hand, the variances of the profitability and investment factors show a sharp increase around the correction of the NASDAQ bubble (in early 2000s). The value factor was also especially volatility during that period and in the most recent bear stock market. Regarding momentum, the largest spike in volatility occurs in the recent crisis. These results indicate that several of the factor realized variances are only weakly correlated.

Table 2 shows the estimates of an AR(1) process for each realized variance. The results show that, apart from *SMB*, all factors have realized variances that are somewhat (but not very) persistent over time as indicated by the autoregressive coefficients close to 0.80 (which are strongly significant) and R^2 estimates above 50%. In comparison, the market variance is significantly less persistent than these four factors with a AR(1) slope of 0.56 and an R^2 of 32%.

The descriptive statistics for the monthly equity factors, which are obtained from Kenneth French’s web page, are presented in Table 3. The factor with the largest mean is clearly *UMD* (1.35% per month), followed by the aggregate equity premium (0.49%). On the other hand, the factors with the lowest average are *SMB* and *RMW* (around 0.25% per month). The most volatile factors are the equity premium and (especially) the momentum factor, with standard deviations above 4% per month. At the other end of the spectrum, the investment

and profitability factors are the least volatile with standard deviations around 2% per month. These values imply that the highest (annualized) Sharpe ratio is for the momentum factor (0.67), followed by *CMA* (0.53). *UMD* shows negative skewness (−1.42), which combined with very high kurtosis (10.84), implies significant downside risk. This is confirmed by the large cumulative loss (maximum drawdown) of −80% for the momentum factor, which is significantly larger (in magnitude) than the cumulated losses estimated for the other factors (below 56% in magnitude). Among these, *CMA* has the smallest maximum drawdown (−18%), followed by *RMW* (−39%). The profitability factor has large kurtosis (14.55), but the negative skewness is smaller than for the momentum and market factors, thus resulting in lower downside risk.

4 Estimating the risk-return trade-off

In this section, we evaluate empirically the risk-return trade-off for each of the equity factors presented in the last section.

4.1 In-sample predictability

We start by evaluating the risk-return trade-off for each equity factor in-sample.⁵ Specifically, we estimate by OLS the following (one-month ahead) predictive regression:

$$f_{t+1} = \alpha + \gamma RV_t + \varepsilon_{t+1}. \quad (21)$$

The statistical significance of the estimated slope ($\hat{\gamma}$) is assessed by using heteroskedasticity-robust *t*-ratios (White (1980)).

The results displayed in Table 4 indicate a negative risk-return trade-off for the market factor with marginal significance (10% level). This finding is in line with part of the related

⁵Most of the literature analyze return predictability for the market portfolio. An incomplete list of papers that look at predictability for stock portfolio returns (based on different predictors) includes Kong et al. (2011), Maio and Santa-Clara (2015), and Maio (2014a, 2014b).

literature that shows a negative aggregate risk-return relation (e.g., [Campbell \(1987\)](#), [Nelson \(1991\)](#), [Glosten, Jagannathan, and Runkle \(1993\)](#), [Whitelaw \(1994\)](#), [Brandt and Kang \(2004\)](#), among others).⁶ The estimated risk-return trade-off for *UMD* is also negative (significant at the 10% level), but the fit of the relation is significantly larger than for the equity premium (R^2 above 4% compare to 0.93% for *RM*).

The estimated slopes for *SMB* and *HML* are negative and positive, respectively. Yet, in both cases there is no statistical significance for those estimates and the R^2 estimates are around zero. On the other hand, the results for both *RMW* and *CMA* are somewhat different. The estimated slopes in the regressions corresponding to these factors are positive and marginally significant (10% level). Moreover, the explanatory ratios (close to 3%) are clearly above the fit obtained for the market risk-return relation. In sum, the results from [Table 4](#) suggest a positive in-sample risk-return trade-off for the profitability and investment factors, while for the market and momentum factors there seems to exist a negative relation.

4.2 Out-of-sample predictability

Next, we estimate the out-of-sample risk-return trade-off for each factor. This allows one to assess the parameter instability in the forecasting regressions over time by using recursive samples. Moreover, it allows to mimic the behavior of a forecaster in real time.⁷ The downside of the out-of-sample analysis relies on the low statistical power of “out-of-sample” regressions as a result of the small sample size, especially for the first sub-samples within the evaluation period (see [Inoue and Kilian \(2004\)](#) and [Cochrane \(2008\)](#) for a discussion).

To assess the out-of-sample (OS) predictability of realized volatility, the null (or restricted) model is a regression containing only a constant in which the best forecast of the

⁶On the other hand, several studies do not find a significant positive market risk-return trade-off (e.g., [French, Schwert, and Stambaugh \(1987\)](#), [Baillie and DeGennaro \(1990\)](#), [Campbell and Hentschel \(1992\)](#), [Bollerslev and Zhou \(2006\)](#), among others).

⁷An incomplete list of papers that analyze the out-of-sample predictability of realized stock volatility for the market return includes [Guo \(2006\)](#), [Guo and Savickas \(2006\)](#), [Goyal and Welch \(2008\)](#), [Rapach, Strauss, and Zhou \(2010\)](#), and [Maio \(2014b, 2016\)](#).

factor is the corresponding historical average,

$$\begin{aligned} H_0 : f_{t+1} &= a + u_{t+1}, \\ H_a : f_{t+1} &= a + bRV_t + v_{t+1}, \end{aligned} \tag{22}$$

where H_a corresponds to the alternative or unrestricted model, which represents the predictive regression associated with realized factor variance, RV_t .

The first measure to assess predictive performance is the out-of-sample coefficient of determination,

$$R_{OS}^2 = 1 - \frac{MSE_U}{MSE_R}, \tag{23}$$

where $MSE_U = \frac{1}{T_{OS}} \sum_{t=1}^{T_{OS}} \hat{v}_t^2$ denotes the mean-squared forecast error associated with the unrestricted model, and MSE_R represents the same for the restricted model. T_{OS} is the number of observations for the evaluation (out-of-sample) period. The out-of-sample R^2 is positive if $MSE_U < MSE_R$, that is, the squared forecast errors associated with the unrestricted model are lower than those associated with the restricted model.

The second evaluation measure is the F -test of [McCracken \(2007\)](#),

$$MSEF = T_{OS} \frac{MSE_R - MSE_U}{MSE_U}, \tag{24}$$

where the null hypothesis is that the MSE associated with the restricted model is less than the corresponding value from the unrestricted model.

The third statistic is the encompassing test proposed by [Harvey, Leybourne, and Newbold \(1998\)](#) and [Clark and McCracken \(2001\)](#),

$$ENC = \frac{\sum_{t=1}^{T_{OS}} (\hat{u}_t^2 - \hat{u}_t \hat{v}_t)}{MSE_U}, \tag{25}$$

in which the null hypothesis assumes that the restricted model encompasses the unrestricted model, that is, the unrestricted model cannot improve the forecast from the restricted model.

The alternative hypothesis is that the unrestricted model has additional information that can improve the performance of the restricted model. The statistical inference associated with the *MSEF* and *ENC* statistics is based on the critical values derived in [McCracken \(2007\)](#) and [Clark and McCracken \(2001\)](#), respectively, which are obtained from Monte-Carlo simulations.⁸ The first recursive regression (in-sample period) uses data from 1964:01 to 1973:12 (120 months) so that the evaluation period starts in 1974:01.

The fourth OS metric is the constrained out-of-sample coefficient of determination, denoted by R_{COS}^2 , which is proposed by [Campbell and Thompson \(2008\)](#). This measure is based on forecasting residuals from constrained regressions, that is, whenever the unrestricted model (OS regression) forecasts a negative factor realization, this estimate is truncated to zero. Thus, the OS regressions rule out negative factor risk premia.

The results for the OS evaluation metrics are presented in Table 5. We can see that the R_{OS}^2 estimates are negative for most factors. The sole exception is *CMA* (0.33%), and despite this small fit it turns out that the null hypothesis (that the historical average beats the volatility-based forecast) is clearly rejected for the investment factor (5% level). When we impose the restriction of positive fitted factor premia the explanatory ratio becomes positive in the case of *RMW* (2.48%) and does not change in the case of *CMA*. Hence, the restriction of positive forecasted factor returns is clearly binding in the case of the profitability factor. Regarding the other four factors, imposing the positivity constraint on the regression-based forecasts does not improve significantly the forecasts associated with realized volatility as the explanatory ratios are negative in all cases (below -1%).⁹

In sum, the results from Table 5 indicate that realized factor variance helps to forecast (out-of-sample) factor returns for *RMW* and *CMA*, while the same does not occur for the other factors. In particular, the estimated in-sample risk-return trade-off of the momentum factor does not subsist out-of-sample, thus showing that the in-sample relation documented

⁸Specifically, the 90% and 95% critical values associated with the *MSEF*-statistic are 0.616 and 1.518, respectively, whereas for the *ENC*-statistic these values are 1.442 and 2.374, respectively.

⁹The results are qualitatively similar if we use an in-sample period of 240 months (so that the first forecast starts in 1984:01).

above is unstable over time. A similar pattern holds for the market risk-return trade-off. These results are partially in line with existing evidence showing that it is significantly more difficult to forecast stock returns out-of-sample than in-sample (e.g., [Goyal and Welch \(2008\)](#)).

5 Economic significance

In this section, we evaluate the economic significance of the out-of-sample estimated risk-return trade-off for each factor.

5.1 Methodology

We construct binary trading strategies based on the out-of-sample factor predictability, in line with [Breen, Glosten, and Jagannathan \(1989\)](#), [Goyal and Santa-Clara \(2003\)](#), [Maio \(2014b, 2016\)](#), among others.

Specifically, at each time t , we estimate the following forecasting regression using the information available up to that period,

$$f_s = a + bRV_{s-1} + u_s, s = 1, \dots, t, \quad (26)$$

The forecasted factor return for next period is calculated as $\hat{f}_{t+1} = \hat{a} + \hat{b}RV_t$, where \hat{a} and \hat{b} denote the estimated coefficients from the regression above.

The benchmark trading strategy (denoted by Strategy 1) invests 100% in the stock index plus a dynamic exposure to the factor. The strategy goes long the factor (with a weight of 150%) if the forecasted return is positive, otherwise it shorts the factor (with a weight of -150%). In symbols, the trading strategy can be represented as

$$\omega_t = \begin{cases} 1.5 & \text{if } \hat{f}_{t+1} \geq 0 \\ -1.5 & \text{if } \hat{f}_{t+1} < 0 \end{cases}, \quad (27)$$

where ω_t denotes the exposure to the factor.

At time $t + 1$, the realized return for the trading strategy is given by

$$R_{p,t+1} = \omega_t f_{t+1} + R_{m,t+1}, \quad (28)$$

where $R_{m,t+1}$ denotes the market return and f_{t+1} is the factor realization at $t+1$. By iterating this process forward, we create a time-series of realized returns for the active strategy.

The benchmark passive strategy is a passive “buy-hold” strategy that simply allocates 100% to the stock index:

$$\tilde{R}_{p,t+1} = R_{m,t+1}. \quad (29)$$

Strategy 1 is suitable for an investor that is skeptical about investing in the factor strategies. Thus, in average he holds the market portfolio and is willing to time the factors (in both directions) according to the signals of the predictive regressions.

Following [Campbell and Thompson \(2008\)](#), [Ferreira and Santa-Clara \(2011\)](#), [Maio \(2013, 2014b\)](#), among others, to evaluate the economic significance of the out-of-sample risk-return trade-off, we compute the change in average utility

$$\Delta U = E(R_{p,t+1}) - E(\tilde{R}_{p,t+1}) + \frac{\gamma}{2} \left[\text{var}(\tilde{R}_{p,t+1}) - \text{var}(R_{p,t+1}) \right]. \quad (30)$$

This measure assumes a simple mean-variance utility function,

$$U(R_{p,t+1}) = E(R_{p,t+1}) - \frac{\gamma}{2} \text{var}(R_{p,t+1}), \quad (31)$$

where γ represents the level of relative risk aversion. ΔU can be interpreted as the annual “fee” that an investor is willing to pay to invest into a trading strategy instead of holding the corresponding passive strategy. To assess the sensitivity of the results to γ , we calibrate three different values of risk aversion (three, five, and ten).

We also compute standard performance evaluation measures for each strategy: mean

return, standard deviation of the return, annualized Sharpe ratio, skewness, kurtosis, and maximum drawdown. This last metric represents the maximum cumulative loss observed during the lifetime of the strategy. We also report the fraction of months in which the dynamic strategy takes a long position in the factor.

5.2 Results

The results associated with Strategy 1 are presented in Table 6. The passive strategy produces an average return of 0.98% per month, which combined with a volatility of 4.57% per month, yields an annual Sharpe ratio of 0.74. With the exception of *SMB* all factor trading strategies produce higher Sharpe ratios than the passive strategy. This comes from higher mean returns that more than compensate for the higher volatilities (in most cases) relative to the buy-hold rule. In particular, the active strategies corresponding to *RMW* and *CMA* have annual Sharpe ratio above one. Among these, the strategy associated with *CMA* clearly dominates the passive strategy as it conveys both a higher mean return (1.48%) and a slightly lower volatility (4.35%).

The change in certainty equivalent estimates confirm that the strategies associated with the investment and profitability factors have the best overall performance: the annualized fees are above 5% in all cases, which indicates large economic significance. While in the case of *RMW* these estimates decrease slightly with the level of risk aversion (from 6.84% for $\gamma = 3$ to 5.34% for $\gamma = 10$), we observe an opposite pattern for *CMA* (from 6.37% for $\gamma = 3$ to 7.18% for $\gamma = 10$). Among the other factors, only in the case of *HML* do we observe positive ΔU estimates, although the gain in utility is quite modest for high levels of risk aversion (0.61% for $\gamma = 10$). We can also verify that the strategy associated with the momentum factor yields negative ΔU estimates, which become quite extreme for high levels of risk aversion (−47% for $\gamma = 10$). This stems from the very large volatility of the active momentum strategy (11.54% per month).

Regarding the other evaluation metrics, it turns out that the strategies corresponding to

SMB, *HML*, and *UMD* all have negative skewness (around -0.25) combined with large Kurtosis (above 6). This indicates significant downside risk, which is especially relevant in the case of the momentum factor (see, for example, Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016)). The strategy associated with *CMA* also exhibits negative Skewness, but there is less kurtosis in comparison to the three factors referred above. The estimates for MDD are largely consistent with these estimates for the third and four empirical moments. Indeed, the maximum cumulative loss is obtained for *UMD* (-92%), followed by *HML*, while *CMA* (-51%) and *RMW* (-40%) show the lowest cumulated losses. Hence, the trading strategies corresponding to the investment and profitability factors have lower downside risk than the strategies associated with the other equity factors. However, the two factor dynamic strategies differ significantly in the factor exposures over time: while in the case of *CMA* the dynamic rule goes long the factor in all months, in the case of *RMW* the dynamic strategy goes long only about 83% of the time (the lowest percentage among all factors). Figure 2 shows that these negative factor weights occur mainly in the first half of the sample.

We conduct a couple of robustness checks to the results associated with the benchmark strategy. First, we use an evaluation period of 20 years so that the first forecast occurs for 1984:01. The results are shown in Table 7. As in the benchmark case, the Sharpe ratios for the dynamic strategies associated with *RMW* and *CMA* are above one. Moreover, the utility gains are above 7.5% in the case of *RMW* and above 5% in the case of *CMA*. All the other factor strategies generate negative utility gains, with the exception of *HML* for low and moderate risk aversion levels ($\gamma = 3, 5$).

Second, we define an alternative version of Strategy 1,

$$\omega_t = \begin{cases} 2 & \text{if } \hat{f}_{t+1} \geq 0 \\ -2 & \text{if } \hat{f}_{t+1} < 0 \end{cases}, \quad (32)$$

which imposes a more levered position on the factor (2 versus 1.5). The results presented in

Table 8 are similar to the benchmark results. Specifically, the Sharpe ratios associated with the trading strategies for *RMW* and *CMA* are above one in both cases. Further, the utility gains for these two strategies are above 7% in most cases, the exception being the case of *RMW* when $\gamma = 10$ (around 4%).

5.3 Alternative strategy

We define an alternative dynamic strategy (denoted by Strategy 2) that is similar to Strategy 1, except that if the forecasted return is negative the investor does not short the factor:

$$\omega_t = \begin{cases} 1.5 \text{ if } \hat{f}_{t+1} \geq 0 \\ 0 \text{ if } \hat{f}_{t+1} < 0 \end{cases}. \quad (33)$$

The corresponding passive strategy invests 100% in the stock index and takes a permanent positive weight on the factor,

$$\tilde{R}_{p,t+1} = R_{m,t+1} + 1.5f_{t+1}. \quad (34)$$

Notice that, in contrast with Strategy 1, each factor has a different passive strategy. Strategy 2 applies for an investor who has a positive prior on each of the factors, that is, he wants to maintain a positive average factor exposure. Hence, the investor wants to evaluate the benefit of timing the positive factor exposure against a “buy-hold” factor exposure.

The results associated with Strategy 2 are presented in Table 9. The active strategies associated with *RMW* (1.16) and *UMD* (0.90) generate higher Sharpe ratios than the corresponding factor buy-hold strategies. Consequently, the utility gains associated with these two dynamic strategies are positive and economically significant at all levels of risk aversion, ranging from 1.78% to 3.56% in the case of *RMW* and from 2.35% to 9.40% in the case of *UMD*. The trading strategy associated with *CMA* coincides with the corresponding passive rule as the former goes long in the factor in every period. The dynamic rule associated

with *HML* also generates positive utility gains, but the economic significance is modest (0.52% when $\gamma = 10$), in line with the evidence obtained for Strategy 1. In sum, the results from Table 9 indicate that an investor who takes a permanent long position in the factors can benefit from timing this exposition (based on the forecasting power of realized factor variances) in the cases of the profitability and momentum factors.

In sum, the results of this section show that the out-of-sample forecasting power (of realized volatility for future returns) is economically significant in the cases of the profitability and investment factors. In comparison, such economic significance does not exist or is not robust for the other equity factors.

6 Forecasting the equity premium with factor variances

In this section, we use realized factor variances to forecast the market equity premium,

$$RM_{t+1} = \alpha + \gamma RV_t + \varepsilon_{t+1}, \quad (35)$$

where RV denotes here one of the variances of *SMB*, *HML*, *UMD*, *RMW*, and *CMA*.

This exercise stems from previous evidence that components of the stock market forecast the aggregate market return (e.g., [Hong, Torous, and Valkanov \(2007\)](#)). Hence, it could be that some of the factor realized variances have greater forecasting power for the equity premium than the own market volatility. This analysis can also shed light on which segments of the stock market (e.g., factors) are responsible for the negative aggregate relation estimated in Section 4.

The results for the in-sample regressions are presented in Table 10. The estimated slopes are negative in all cases, in line with the negative aggregate relation estimated in Section 4. Yet, there is only statistical significance for the realized variances associated with *SMB*,

RMW, and *CMA* (in this latter case, marginally so). The largest fit is obtained for the variances associated with *RMW* and *CMA*, with R^2 estimates around 1%. In the case of *RMW*, this represents a marginally higher fit than for the aggregate risk-return trade-off. The results from Table 10 also suggest that the negative market risk-return relation stems from the negative correlation between the realized variances of small, high-profitability stocks, and low-asset growth stocks with the future market return.

The out-of-sample predictability results are displayed in Table 11. We can see that the OS explanatory ratios are negative in all cases. Yet, when we impose the restriction of positive fitted aggregate risk premium it follows that the OS R^2 estimates become positive in the cases of *HML*, *UMD*, and *CMA*. Hence, when we use the realized variances of *SMB* and *RMW* as predictors it turns out that the forecasting performance is negative. This finding indicates that there is high instability in the corresponding in-sample predictive relations described above. If anything, the results from this section suggest that some of the factor variances are negatively correlated in-sample with future market returns, although the out-of-sample performance is relatively weak.

7 Conclusion

We contribute to the risk-return trade-off literature by examining the risk-return relation among equity factors. The same way that the market risk-return trade-off is consistent with the conditional CAPM or ICAPM, the factor risk-return relation is consistent with conditional multifactor models widely used in the asset pricing literature. Specifically, we employ conditional versions of the multifactor models of Carhart (1997) and Fama and French (2015, 2016) to motivate our empirical tests. According to those models, there should exist a positive relation between the risk premium and conditional volatility of the size, value, momentum, profitability, and investment risk factors. To proxy for the unobserved conditional factor variances, we compute realized variances based on daily factor observations.

By using monthly data from 1964 to 2015 the results suggest a positive in-sample risk-return trade-off for the profitability and investment factors, while for the market and momentum factors there seems to exist a negative relation. Furthermore, the realized factor variance help to forecast out-of-sample factor returns in the cases of the profitability and investment factors, while the same does not occur for the other factors.

To assess the economic significance of the out-of-sample forecasts from realized variances we construct a trading strategy that relies on such predictability. Specifically, the strategy times each factor by going long (short) the factor whenever the predicted factor risk premium (obtained from the predictive regressions in recursive samples) is positive (negative). The results indicate that the out-of-sample forecasting power (of realized volatility for future returns) is economically significant in the cases of the profitability and investment factors. Specifically, the annual Sharpe ratios are above one in both cases compared to 0.74 for the passive strategy that only invests in the stock market index. Moreover, the utility gains associated with the dynamic strategies corresponding to those two factors are above 5% per year. In comparison, such economic significance does not exist or is not robust for the other equity factors.

In the last part of the paper, we use realized factor variances to forecast the market equity premium. This exercise stems from previous evidence that components of the stock market forecast the aggregate market return (e.g., [Hong, Torous, and Valkanov \(2007\)](#)). The results suggest that some of the factor variances are negatively correlated in-sample with future market returns, although the out-of-sample performance is relatively weak.

Table 1: Descriptive statistics for realized variances

This table reports descriptive statistics for the realized variance of each factor. RM , SMB , HML , UMD , RMW , and CMA denote the market, size, value-growth, momentum, profitability, and investment factors, respectively. The sample is 1964:01–2015:12.

	Mean(%)	SD(%)	Min.(%)	Max.(%)
RM	0.22	0.41	0.01	5.48
SMB	0.06	0.11	0.00	2.14
HML	0.05	0.09	0.00	0.74
UMD	0.39	1.01	0.01	13.56
RMW	0.03	0.06	0.00	0.79
CMA	0.03	0.05	0.00	0.59

Table 2: Persistence of realized variances

This table reports the results for AR(1) process for the realized variance of each factor. RM , SMB , HML , UMD , RMW , and CMA denote the market, size, value-growth, momentum, profitability, and investment factors, respectively. ϕ denotes the predictive slope, while t represents the corresponding GMM-based t -ratio. R^2 denotes the coefficient of determination. t -ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. The sample is 1964:01–2015:12.

	ϕ	t	$R^2(\%)$
RM	0.56	3.99***	31.78
SMB	0.36	2.13**	12.75
HML	0.75	10.25***	55.84
UMD	0.77	6.59***	59.90
RMW	0.75	3.77***	55.52
CMA	0.71	4.88***	50.23

Table 3: Descriptive statistics for equity factors

This table reports descriptive statistics for the equity factors. RM , SMB , HML , UMD , RMW , and CMA denote the market, size, value-growth, momentum, profitability, and investment factors, respectively. The statistics are the average return (Mean), standard deviation (SD), minimum return (Min.), maximum return (Max.), annualized Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). The sample is 1964:01–2015:12.

	Mean(%)	SD(%)	Min.(%)	Max.(%)	Sharpe	Skew.	Kurt.	MDD(%)
RM	0.49	4.46	−23.24	16.10	0.38	−0.52	4.90	−55.71
SMB	0.24	3.11	−16.70	22.32	0.27	0.53	8.64	−52.82
HML	0.34	2.87	−13.11	13.91	0.41	0.01	5.61	−45.21
UMD	1.35	7.00	−45.79	26.16	0.67	−1.42	10.84	−80.36
RMW	0.25	2.13	−17.57	12.19	0.40	−0.40	14.55	−39.17
CMA	0.31	2.01	−6.81	9.51	0.53	0.29	4.65	−17.62

Table 4: Risk-return tradeoff in-sample

This table reports the results for regressions of each factor on its lagged realized variance. RM , SMB , HML , UMD , RMW , and CMA denote the market, size, value-growth, momentum, profitability, and investment factors, respectively. γ denotes the predictive slope, while t represents the corresponding GMM-based t -ratio. R^2 denotes the coefficient of determination. t -ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. The sample is 1964:01–2015:12.

	γ	t	$R^2(\%)$
RM	−1.04	−1.91*	0.93
SMB	−0.45	−0.26	0.03
HML	1.62	0.69	0.28
UMD	−1.41	−1.75*	4.14
RMW	6.08	1.69*	2.52
CMA	7.01	1.82*	2.76

Table 5: Out-of-sample predictability

This table presents out-of-sample evaluation statistics for the risk-return tradeoff associated with each factor. RM , SMB , HML , UMD , RMW , and CMA denote the market, size, value-growth, momentum, profitability, and investment factors, respectively. R_{OS}^2 denotes the out-of-sample coefficient of determination (in %), $MSEF$ is the McCracken (2007) F-statistic, and ENC stands for the encompassing test proposed by Clark and McCracken (2001). R_{COS}^2 is the constrained out-of-sample R^2 . The total sample is 1964:01–2015:12 and the out-of-sample evaluation period starts in 1974:01. The numbers marked with * and ** indicate that the null hypothesis associated with $MSE - F$ or $ENC - NEW$ is rejected at the 10% and 5% levels, respectively.

	RM	SMB	HML	UMD	RMW	CMA
$R_{OS}^2(\%)$	−3.00	−1.79	−2.00	−0.19	−11.35	0.33
$MSEF$	−14.67	−8.85	−9.88	−0.98	−51.38	1.65**
ENC	−3.23	0.70	−2.20	4.26**	−4.52	7.23**
$R_{COS}^2(\%)$	−2.00	−1.78	−1.54	−1.02	2.48	0.33

Table 6: Benchmark trading strategy

This table reports the performance evaluation measures associated with Strategy 1, which is based on the out-of-sample forecasting power of the realized variance for the return of each factor. *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the active strategies for the size, value-growth, momentum, profitability, and investment factors, respectively. *P* denotes the passive strategy. The statistics are the average return (Mean), standard deviation (SD), annualized Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). “Long” denotes the fraction of months in which the active strategy goes long in the factor. ΔU , $\gamma = 3$, ΔU , $\gamma = 5$, and ΔU , $\gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1964:01–2015:12 and the out-of-sample evaluation period starts in 1974:01.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
<i>P</i>	0.98	4.57	0.74							
<i>SMB</i>	1.34	7.25	0.64	-0.24	6.07	-57.28	99.80	-1.38	-5.19	-14.70
<i>HML</i>	1.43	5.37	0.92	-0.26	7.00	-67.90	99.40	3.96	3.00	0.61
<i>UMD</i>	2.66	11.54	0.80	-0.24	9.67	-91.87	96.63	-0.08	-13.55	-47.25
<i>RMW</i>	1.60	4.95	1.12	0.05	6.26	-40.02	82.74	6.84	6.41	5.34
<i>CMA</i>	1.48	4.35	1.18	-0.38	4.76	-51.25	100.00	6.37	6.60	7.18

Table 7: Benchmark trading strategy: alternative in-sample period

This table reports the performance evaluation measures associated with Strategy 1, which is based on the out-of-sample forecasting power of the realized variance for the return of each factor. *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the active strategies for the size, value-growth, momentum, profitability, and investment factors, respectively. *P* denotes the passive strategy. The statistics are the average return (Mean), standard deviation (SD), annualized Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). “Long” denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1964:01–2015:12 and the out-of-sample evaluation period starts in 1984:01.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
<i>P</i>	0.95	4.45	0.74							
<i>SMB</i>	0.99	7.11	0.48	-0.26	6.37	-57.28	99.74	-5.01	-8.70	-17.94
<i>HML</i>	1.26	5.40	0.81	-0.71	6.20	-67.90	99.22	2.01	0.90	-1.90
<i>UMD</i>	2.59	11.83	0.76	-0.16	10.43	-91.87	95.57	-1.90	-16.31	-52.35
<i>RMW</i>	1.74	4.79	1.26	0.08	7.39	-29.11	92.45	8.89	8.51	7.58
<i>CMA</i>	1.35	4.31	1.09	-0.56	4.81	-51.25	100.00	5.08	5.22	5.59

Table 8: Benchmark trading strategy: alternative weights

This table reports the performance evaluation measures associated with Strategy 1 with alternative weights, which is based on the out-of-sample forecasting power of the realized variance for the return of each factor. *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the active strategies for the size, value-growth, momentum, profitability, and investment factors, respectively. *P* denotes the passive strategy. The statistics are the average return (Mean), standard deviation (SD), annualized Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). “Long” denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1964:01–2015:12 and the out-of-sample evaluation period starts in 1974:01.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
<i>P</i>	0.98	4.57	0.74							
<i>SMB</i>	1.46	8.52	0.59	-0.06	6.55	-60.45	99.80	-3.54	-9.76	-25.29
<i>HML</i>	1.58	6.36	0.86	-0.10	6.90	-72.88	99.40	3.67	1.32	-4.56
<i>UMD</i>	3.22	14.95	0.75	-0.35	10.09	-98.24	96.63	-9.62	-33.95	-94.75
<i>RMW</i>	1.81	5.53	1.13	0.29	7.85	-42.56	82.74	8.24	7.07	4.16
<i>CMA</i>	1.65	4.71	1.21	-0.25	4.20	-51.97	100.00	7.79	7.63	7.23

Table 9: Alternative trading strategy

This table reports the performance evaluation measures associated with Strategy 2, which is based on the out-of-sample forecasting power of the realized variance for the return of each factor. *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the active strategies for the size, value-growth, momentum, profitability, and investment factors, respectively. The second row associated with each factor denotes the passive strategy, which is factor specific. The statistics are the average return (Mean), standard deviation (SD), annualized Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). “Long” denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1964:01–2015:12 and the out-of-sample evaluation period starts in 1974:01.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
<i>SMB</i>	1.35	7.25	0.65	-0.24	6.08	-54.60	99.80	-0.13	-0.12	-0.09
	1.36	7.25	0.65							
<i>HML</i>	1.44	5.21	0.96	-0.29	6.84	-67.90	99.40	0.05	0.19	0.52
	1.45	5.31	0.95							
<i>UMD</i>	2.71	10.40	0.90	-0.90	7.79	-73.97	96.63	2.35	4.37	9.40
	2.77	11.18	0.86							
<i>RMW</i>	1.52	4.54	1.16	-0.30	4.52	-36.04	82.74	1.78	2.29	3.56
	1.43	4.98	1.00							
<i>CMA</i>	1.48	4.35	1.18	-0.38	4.76	-51.25	100.00	0.00	0.00	0.00
	1.48	4.35	1.18							

Table 10: Forecasting the equity premium with factor variances

This table reports the results for regressions of the market factor onto lagged factor realized variances. *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the variances associated with the size, value-growth, momentum, profitability, and investment factors, respectively. γ denotes the predictive slope, while t represents the corresponding GMM-based t -ratio. R^2 denotes the coefficient of determination. t -ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. The sample is 1964:01–2015:12.

	γ	t	$R^2(\%)$
<i>SMB</i>	−3.44	−2.75***	0.79
<i>HML</i>	−4.28	−1.33	0.81
<i>UMD</i>	−0.25	−0.66	0.32
<i>RMW</i>	−8.33	−2.45**	1.09
<i>CMA</i>	−9.01	−1.80*	0.94

Table 11: Forecasting out-of-sample the equity premium with factor variances

This table presents out-of-sample evaluation statistics for forecasting the market return with lagged factor variances. *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the variances associated with the size, value-growth, momentum, profitability, and investment factors, respectively. R_{OS}^2 denotes the out-of-sample coefficient of determination (in %), *MSEF* is the McCracken (2007) F-statistic, and *ENC* stands for the encompassing test proposed by Clark and McCracken (2001). R_{COS}^2 is the constrained out-of-sample R^2 . The total sample is 1964:01–2015:12 and the out-of-sample evaluation period starts in 1974:01. The numbers marked with * and ** indicate that the null hypothesis associated with *MSE* – *F* or *ENC* – *NEW* is rejected at the 10% and 5% levels, respectively.

	<i>SMB</i>	<i>HML</i>	<i>UMD</i>	<i>RMW</i>	<i>CMA</i>
$R_{OS}^2(\%)$	−5.83	−0.75	−3.26	−0.27	−0.10
<i>MSEF</i>	−27.76	−3.75	−15.89	−1.38	−0.52
<i>ENC</i>	−4.96	0.84	1.10	1.60*	0.48
$R_{COS}^2(\%)$	−5.69	0.12	0.48	−0.11	0.15

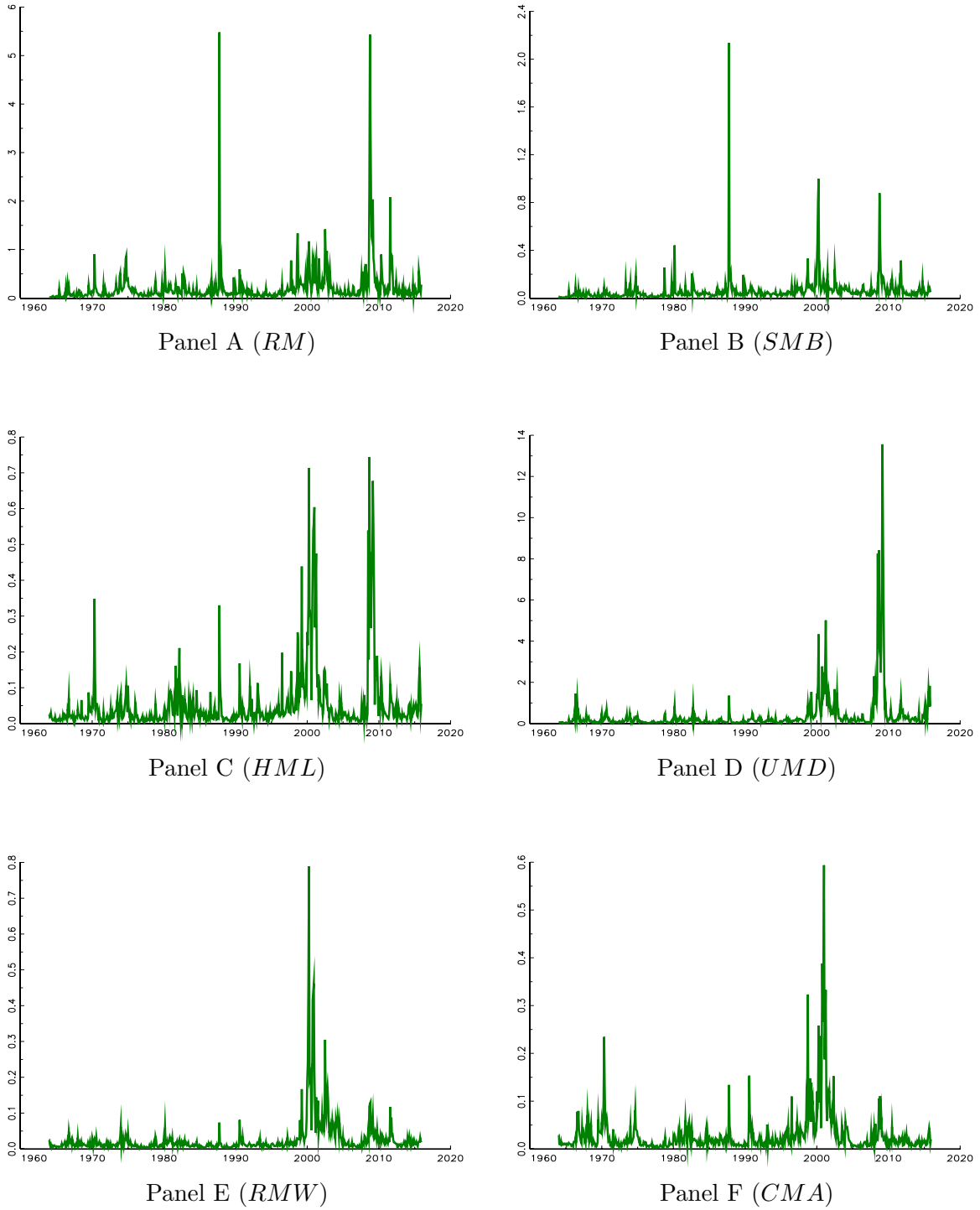
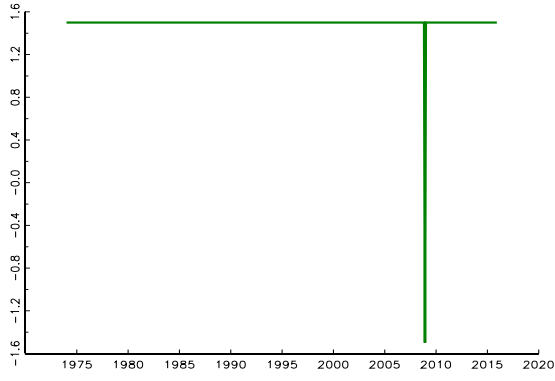


Figure 1: Realized variances

This figure plots the realized variances (in %) of each equity factor. RM , SMB , HML , UMD , RMW , and CMA denote the market, size, value-growth, momentum, profitability, and investment factors, respectively. The sample is 1964:01–2015:12.



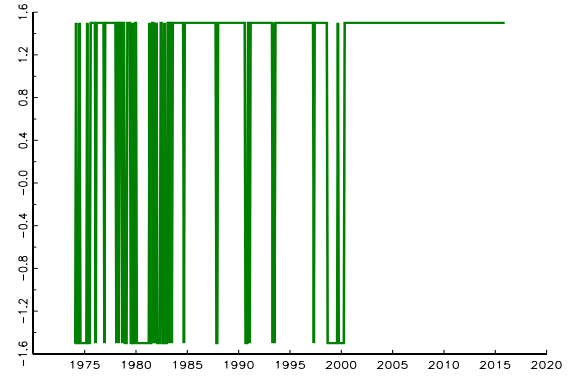
Panel A (*SMB*)



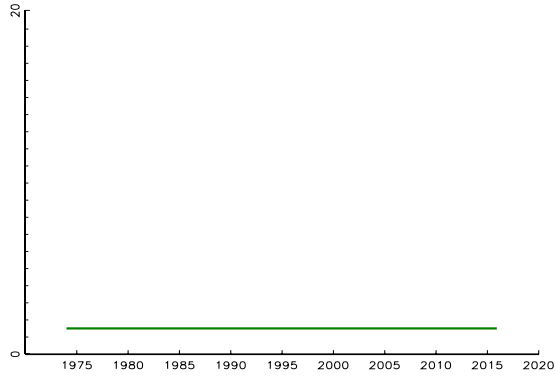
Panel B (*HML*)



Panel C (*UMD*)



Panel D (*RMW*)



Panel E (*CMA*)

Figure 2: Factor weights

This figure plots the weights in the factors in the case of dynamic Strategy 1. *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the size, value-growth, momentum, profitability, and investment factors, respectively. The sample is 1974:01–2015:12.

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