Hedging in Chinese Commodity Futures Markets

October 2010

Abstract

Chinese commodity futures markets have become some of the most important derivative markets worldwide. This paper studies the optimal hedge ratios on two popular contracts in China, soybeans and copper, by employing copula functions. Our empirical results suggest that the proposed copula hedging strategy outperforms the simple regression method and dynamic conditional correlation (DCC) method by most appropriately capturing the joint dependence between spot and futures returns. Additionally, the optimal hedging horizon for soybeans is four and five months to maturity given the unique Chinese time-dependent margin rule, not the nearby month as with many other futures contracts.

Keywords: Commodity futures, Time-dependent margin rate, Copula functions.

JEL codes: G13, G15.

INTRODUCTION

In China, commodity futures markets were first established in 1990 with the foundation of the China Zhengzhou Grain Wholesale Market. However, during the 1990s there were serious market squeezes and price manipulations due to the lack of systematic regulations and enforcement (Yao, 1998). From September 1998, rules and regulations were gradually promulgated by China's securities regulated body China Securities Regulatory Commission (CSRC) and in 1999 a dozen futures markets were consolidated and combined into three exchanges in Dalian, Zhengzhou, and Shanghai trading commodity futures products. Since then, commodity futures markets in China have quickly emerged to global significance, with large trading volumes especially for soybeans and copper. Soybean futures contracts are traded in the Dalian Commodity Exchange (DCE), the second largest soybean futures market by trading volumes before 2006 and the largest since 2007. Copper futures contracts are traded in the Shanghai Futures Exchange (SHFE), the second largest copper futures market by volumes before 2009 and the largest in 2009 (Futures Industry Association websites).

The Chinese futures market has a unique margin rule[1]. According to both the settlement rules of the DCE and the SHFE, the margin requirement is calculated as follows:

Today's margin account = Margin account as of yesterday

- (yesterday's position value × yesterday's margin rate)

+ (today's position value × today's margin rate)

- trading costs

[1] A margin is the collateral that the holder of a position in futures contracts has to deposit to cover the credit risk of his or her account.

where margin rates are pre-determined by exchanges. Take the SHFE copper contract as an example. The basic margin rate is 5% of the contract value. It rises to 7% two months before maturity, 10% one and a half months to maturity, 15% one month to maturity, 20% two weeks to maturity, and finally 30% two days before maturity (SHFE margin rates, 2008). Hence the financing required on the margin account increases as the contracts approach maturity. As a result of this time-dependent margin rate, trading peaks at more distant contracts, as noticed in Lien and Yang (2008) and Peck (2008). The rationale behind this time-dependent margin rate is to avoid market squeezes close to maturity, which happened frequently during the 1990s. However, the subsequent shift in the active trading period could have significant side effect on price discovery and hedging.

Hedging is one of the fundamental functions of futures markets, which is trading futures in the opposite position of its underlying products in order to avoid the price fluctuations of the underlying. Hedging strategies have been intensively studied since the 1960s when futures exchanges were still in their early stage. Johnson (1960) introduces the optimal hedging strategy by minimizing the variance of a hedged portfolio. Another early effort is Ederington (1979), who compares the hedging effectiveness over different hedging horizons that refers to the degree of risk reduction of futures trading as compared to the unhedged portfolio for two interest rate futures. He concludes that hedging effectiveness declines with more distant contracts.

The modern method of optimal hedge ratio estimation mainly focuses on modelling the volatility of the spot and futures returns and typically resorts to multivariate

GARCH (generalized autoregressive conditional heteroskedasticity) models. Such estimation result in a time-varying hedge ratio suggesting traders should intertemporally adjust their positions and optimally reduce their risk exposure. Cecchetti et al. (1988), Kroner and Sultan (1993), Park and Switzer (1995), and Gagnon and Lypny (1995) all find that the time-varying hedging method is superior to a constant hedge ratio. More recently, Lee and Yoder (2007) develop a Markov regime-switching time-varying correlation GARCH model and find that it outperforms the traditional GARCH models in reducing the hedged portfolio variance. Switzer and El-Khoury (2007) demonstrate that modelling asymmetries in volatility estimation could improve hedging performance during volatile market conditions.

Despite great interest in this area, the issue of the optimal hedge ratio in Chinese commodity futures markets has only been examined in Lien and Yang (2008). They focus on copper and aluminium contracts traded in the SHFE from January 1996 to December 2004. Comparisons are drawn between a constant hedge ratio, the Ordinary Least Squares (OLS) method, and a series of dynamic conditional correlation (DCC) models. They find that the OLS method and the DCC model with asymmetric basis perform the best.

The mainstream literature discussed thus far invariably adopts the assumption of a joint normal distribution between spot and futures returns despite evidence that this is a very restrictive assumption. As Chen *et al*. (2008) explain, after examining 25 futures contracts with five hedging horizons, none of the joint densities for short-term contracts, and only a very few for longer-term contracts, follow a normal distribution.

Therefore, the first contribution of this study is that we use flexible copula functions to describe the dynamic nonlinear correlation between the spot and futures returns of the Chinese soybeans and copper futures contracts. By relaxing the assumption of a joint normal distribution between the spot and futures returns, we aim to better estimate the optimal hedge ratio for these commodities in order to reduce hedged portfolio variance, increase hedging effectiveness and lower hedging costs.

In the past few years, the copula method has gradually attracted attention in the fields of finance and economics as a flexible econometric tool for constructing joint density distributions (see, inter alios, Patton, 2004, 2006a, 2006b and Cherubini *et al*., 2004). Hsu *et al*. (2008) utilize a number of copula functions to examine the optimal hedge ratios for financial futures. They adopt time-varying Gaussian, Clayton, and Gumbel copula functions, and provide evidence that the Gaussian and Gumbel copula functions out-perform traditional hedging strategies, such as the constant conditional correlation (CCC) GARCH and DCC. However, those copula functions cannot capture upper and lower tail dependence simultaneously (Cherubini *et al*., 2004).

In this paper, we implement copula functions with two-tail dependence to allow both symmetric and asymmetric dependence structure. In particular, the hedge ratio is evaluated using the Gaussian copula (with no tail dependence), the Gumbel copula (with lower tail dependence), the mixture of Gumbel and Gumbel survival copula (with two-tail dependence), the time-varying Gaussian copula (with no tail dependence), the time-varying Gumbel copula (with lower tail dependence), the time-varying Clayton copula (with upper tail dependence), and the time-varying mixture of Gumbel and Gumbel survival copula (with two-tail dependence). We run a horse race between these copula functions and the more conventional OLS method and DCC model, both of which are under the assumption of joint normal distribution.

Our empirical results strongly suggest that the joint density of spot and futures returns is two-tail dependent instead of being normal as has been typically assumed in the literature. The tail dependence is asymmetric for the soybean contracts and symmetric for the copper contracts. We find that the time-varying mixture of Gumbel and Gumbel survival copula function dominates other methods for the 5-month and 7-month soybeans contracts. And the mixture of Gumbel and Gumbel survival copula function outperforms the others for 2-month, 3-month and 4-month copper contracts. Specifically, these copula perform the best in reducing hedged portfolio variance and reducing hedging costs.

Secondly, this paper contributes to the literature by closely examining the performance of the hedging strategies at different horizons. This is especially relevant for the Chinese futures markets due to their unique time-dependent margin rules. Peck (2008) indicates that the most active trading period for the Chinese soybean market from 1999 to 2003 was about 5 to 6 months to maturity, unlike many other exchanges such as the CME. This distinctive trading behaviour may affect the hedging horizon that could maximally reduce the risk of hedged portfolio, which is referred to the optimal hedging horizon in this paper. As no prior literature has provided empirical evidence on this issue, a careful investigation of the optimal hedging horizons becomes important for all market participants and regulators involved in Chinese commodity futures markets.

We find that in the soybean market, the more distant four to five months period is the optimal hedging horizon as the hedged portfolio variance is the smallest during this period. As active trading and optimal hedging horizon are shifted to the more distant months, the convergence of the spot and futures prices which normally happens towards delivery month has been shifted to further distant futures contracts. For the copper market the nearby two months contract is the best period for hedging. These findings are important to traders, hedgers, and portfolio managers in choosing an appropriate period to lay off their risk exposure. The findings are also relevant for policy makers. The time-dependent margin rules, which were introduced to curb market squeezes a decade ago, clearly have important consequences in terms of price discovery and hedging for the current market which is highly liquid and competitive.

The rest of the paper is organized as follows. In the second section, the OLS method, the DCC method, and the copula functions are introduced together with the measures of comparing the performance of these hedging strategies. In the third section, the data and empirical results are presented, including the in-sample estimation, the out-of-sample forecasts, and the optimal hedging horizon. Finally, section four concludes.

METHODOLOGY

In this section, the functions of hedge ratio estimation and the methods of hedging performance are specified in detail. The hedge ratio functions include the popular OLS and DCC methods, as well as copula functions. The methods of hedging performance contain portfolio variance, hedging effectiveness, statistical significance and economic utility.

OLS hedge ratio

The hedge ratio is defined as the amount of contracts one should hold in the futures market in order to hedge one share of the underlying asset. With the OLS method, the hedge ratio is obtained by the estimation:

$$r_{s,t} = \alpha + \beta r_{f,t} + u_t \qquad (1)$$

where $r_{s,t} = \ln(\frac{P_{s,t}}{P_{s,t-1}}) * 100$, $r_{f,t} = \ln(\frac{P_{f,t}}{P_{f,t-1}}) * 100$, $P_{s,t}$ is the spot closing price on day t, and $P_{f,t}$ is the futures closing price on day t. The value of parameter β is the hedge ratio in equilibrium under the OLS method.

DCC (MV-GARCH) hedge ratio

In the MV-GARCH family, the flexible DCC model is the latest development by Engle (2002). It involves a two-stage estimation. The first stage is the univariate GARCH estimation and the second stage is the covariance matrix estimation. In this paper, the DCC model is employed as a representative of the MV-GARCH family.

Because there is ample empirical evidence of fat-tails in return distributions, a t-GARCH model is used to capture marginal distributions for both spot and futures returns[2]. The same marginal distribution models would be applied in the copula section, so that the difference in the hedging performance between the DCC method and copula functions could only be caused by the difference of the joint dependence between spot and futures returns.

The t-DCC model follows,

$$\begin{aligned} r_{s,t} &= \alpha_s + \beta_{s,1} r_{s,t-1} + \beta_{s,2} r_{f,t-1} + \gamma_s b_{t-1} + u_{s,t} \\ \sigma^2_{s,t} &= \lambda_s + \theta_s u^2_{s,t-1} + \delta_s \sigma^2_{s,t-1} \\ u_{s,t} &\sim St_{\upsilon_s}(0, \sigma^2_{s,t}) \end{aligned} \tag{2}$$

$$\begin{aligned} r_{f,t} &= \alpha_f + \beta_{f,1} r_{f,t-1} + \beta_{f,2} r_{s,t-1} + \gamma_f b_{t-1} + u_{f,t} \\ \sigma^2_{f,t} &= \lambda_f + \theta_f u^2_{f,t-1} + \delta_f \sigma^2_{f,t-1} \\ u_{f,t} &\sim St_{\upsilon_f}(0, \sigma^2_{f,t}) \end{aligned} \tag{3}$$

$$Q_t = (1 - a - b)\overline{Q} + a\varepsilon_{t-1}\varepsilon'_{t-1} + bQ_{t-1} \tag{4}$$

where $b_t = s_t - f_t$, representing the basis to accommodate a long-run equilibrium relationship. $\varepsilon_t = \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix}$, $\varepsilon_{s,t} = \dfrac{u_{s,t}}{\sigma_{s,t}}$ and $\varepsilon_{f,t} = \dfrac{u_{f,t}}{\sigma_{f,t}}$. $\overline{Q} = E[\varepsilon_t \varepsilon'_t]$, and Q_t is the covariance matrix of standardized residual terms.

Let HR_t represent the hedge ratio series. The expected portfolio return becomes,

$$E_t(r_{p,t+1}) = E_t(r_{s,t+1} - HR_t r_{f,t+1}) \tag{5}$$

and the variance of the expected portfolio return is,

$$Var_t(r_{p,t+1}) = Var_t(r_{s,t+1}) + HR_t^2 Var_t(r_{f,t+1}) - 2HR_t \operatorname{cov}_t(r_{s,t+1}, r_{f,t+1}) \tag{6}$$

To minimize the portfolio variance, the optimal hedge ratio follows,

[2] The asymmetric t-GARCH model was employed at the first stage. However, the asymmetric term was eventually removed because it was not significant for any of the data sets.

$$HR_t = \frac{\text{cov}_t(r_{s,t+1}, r_{f,t+1})}{\sigma^2_{f,t+1}} \tag{7}$$

The covariance is obtained from the DCC covariance matrix $Q_t(1,2)$.

Copula hedge ratio

Following Frank (1991), for random variables x and y with marginal distributions F_x and F_y and joint distribution $F_{x,y}$, there is a function $C_{x,y}$ that

$$F_{x,y}(u,v) = C_{x,y}(F_x(u), F_y(v)) \tag{8}$$

where u and v are the cumulative distribution functions (c.d.f.) of x and y.

The joint density function follows,

$$f_{x,y}(u,v) = c_{x,y}(u,v) \times f_x(u) \times f_y(v) \tag{9}$$

where $f_x(u)$ and $f_y(v)$ are the marginal densities of x and y, and $c_{x,y}(u,v)$ is the copula density function. If assume that the marginal densities of spot and futures returns follow t-distribution described by equations (2) and (3), the copula density $c_{x,y}$ would then be the focus.

Firstly, following Hsu et al. (2008), Gaussian copula, Gumbel copula, and Clayton copula are considered. The Gaussian copula is symmetric and it has no tail dependence. Its probability density function (p.d.f.) follows,

$$c_{Gauss}(u,v \mid \rho_G) = \frac{1}{2\pi\sqrt{1-\rho_G^2}} \exp\left(-\frac{1}{2(1-\rho_G^2)}[u^2 + v^2 - 2\rho_G uv]\right) \tag{10}$$

The Gumbel copula has upper tail dependence, and the p.d.f. of Gumbel copula is,

$$c_G(u,v \mid \alpha) = \frac{C_G(u,v)(\ln u \ln v)^{\alpha-1}[(-\ln u)^{\alpha} + (-\ln v)^{\alpha}]^{1/\alpha} + \alpha - 1}{uv[(-\ln u)^{\alpha} + (-\ln v)^{\alpha}]^{2-(1/\alpha)}} \tag{11}$$

where parameter $\alpha \in [1,+\infty)$.

While the Clayton copula has lower tail dependence, and its p.d.f. follows,

$$c_C(u,v \mid \alpha) = \frac{(1+\alpha)(u^{-\alpha} + v^{-\alpha} - 1)^{-(1/\alpha)-2}}{uv^{\alpha+1}} \tag{12}$$

where parameter $\alpha \in [-1,0) \cup (0,+\infty)$.

However, none of the above copula functions can show a two-tail dependence. Therefore, a mixture Gumbel copula is introduced, which contains the Gumbel copula and its survival copula. The p.d.f. of a survival copula follows,

$$\bar{c}_{x,y}(u,v) = c_{x,y}(1-u, 1-v) \tag{13}$$

And the p.d.f. of mixture copula is,

$$c_m(u,v \mid \alpha, \bar{\alpha}, w) = wc(u,v \mid \alpha) + (1-w)\bar{c}(1-u.1-v \mid \bar{\alpha}) \tag{14}$$

where w is the weight and $w \in [0,1]$. The mixture Gumbel copula is two-tail dependence, capturing lower tail and upper tail dependence simultaneously, while still including the possibility of asymmetric distribution.

Moreover, the joint dependence would vary over time as the change in both spot and futures markets. Therefore, time-varying copula functions are also introduced by assuming that copula parameters follow a quasi-ARMA(1,1). For the time-varying Gaussian copula,

$$\rho_t = \mu + \eta\rho_{t-1} + \phi(u_{t-1} - 0.5)(v_{t-1} - 0.5) \tag{15}$$

For the time-varying Gumbel copula, the time-varying mixture Gumbel copula and the time-varying Clayton copula, the parameter α and $\overline{\alpha}$ are,

$$\alpha_t = \mu + \eta\alpha_{t-1} + \phi(u_{t-1} - 0.5)(v_{t-1} - 0.5) \tag{16}$$

$$\overline{\alpha}_t = \mu + \eta\overline{\alpha}_{t-1} + \phi[(1-u_{t-1}) - 0.5][(1-v_{t-1}) - 0.5] \tag{17}$$

Eventually, following Hsu et al. (2008), the copula covariance is,

$$\text{cov}_{sf,t} = \sigma_{s,t}\sigma_{f,t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\varepsilon_{s,t}\varepsilon_{f,t}\varphi(\varepsilon_{s,t},\varepsilon_{f,t})drdw \tag{18}$$

where the joint density function is $\varphi_t(\varepsilon_{s,t},\varepsilon_{f,t}) = c_t(u,v)\times f_{s,t}(\varepsilon_{s,t})\times f_{f,t}(\varepsilon_{f,t})$, and $f_{s,t}(\varepsilon_{s,t})$ is the p.d.f. of $\varepsilon_{s,t}$, $f_{f,t}(\varepsilon_{f,t})$ is the p.d.f. of $\varepsilon_{f,t}$. The copula hedge ratio is specified by equation (7).

Hedging performance

The hedging performance is evaluated by portfolio variance, hedging effectiveness, statistical significance test and economic superiority test. The portfolio is constructed as one share of spot and certain shares of futures which are decided by the hedge ratio. The optimal hedging strategy should be the one that can provide the lowest portfolio variance, so as to maximally reduce the risk of the investment.

The hedging effectiveness is defined by Ederington (1979):

$$e = 1 - \frac{Var(P)}{Var(U)} \tag{19}$$

where Var(P) represents the variance of the hedged portfolio, also written as $Var(r_{p,t})$, and Var(U) represents the variance of the unhedged asset, which is the variance of spot returns. Consequently, the hedging effectiveness measures the percentage risk reduction of the hedged portfolio against the unhedged portfolio. The larger the value of equation (19), and the greater the risk reduction, the better the hedging strategy is.

For the statistical significance of the models' hedging performance we resort to White's reality check, which is discussed in White (2000) and employed in Lee et al. (2006) and Lee and Yoder (2007). The null hypothesis of the White reality check is that the selected optimal hedging model does not statistically significantly perform better than the benchmark model, as:

$$H_0 : E(f) \leq 0$$

where

$$f_{t+1} = -(r_{p,t})^2 + (r_{s,t} - HR_{BM,t} r_{f,t})^2 \tag{20}$$

$HR_{BM,t}$ is the hedge ratio of the benchmark model at time t. The test is based on the stationary bootstrap re-sampling method.

Moreover, even if the time-varying hedge ratios can provide better hedging strategy, considering its indication of almost daily position changes, its economic superiority needs to be examined especially with transaction costs. Assuming the trader's expected utility function follows:

$$E_t U(r_{p,t+1}) = E_t(r_{p,t+1}) - \kappa \sigma_t^2 (r_{p,t+1}) \tag{21}$$

where κ is the degree of risk aversion. If the utility value of target hedging strategy is greater than the utility value of benchmark hedging strategy plus transaction costs, then the target strategy is economic superior to the benchmark strategy.

DATA AND EMPIRICAL ANALYSIS

We use soybean contracts traded on the DCE and copper contracts traded on the SHFE. Daily closing prices from 19 November 1993 to 18 May 2007 for the soybeans contracts and from 17 March 1994 to 18 May 2007 for the copper contracts are examined. The data range starts from the very beginning of the futures trading for each product, and covers more recent period comparing with the only Chinese hedge ratio literature Lien and Yang (2008) with the data range from 1 January 1996 to 31 December 2004. As far as we are concerned, this paper investigates the longest period of Chinese futures market in the literature. There are 6 contracts per year for soybeans with maturity months in January, March, May, July, September, and November. There are 12 contracts per year for copper with maturity in every month.

As there is no wholesale market in China to provide a standardized spot price for these commodities, a proxy is needed. Following, inter alia Beck (1994) and Lien and Yang (2008), we employ the front month futures (the nearest contract to maturity) as the spot price.

In most futures markets, distant contracts are usually less actively traded. Due to the distinctive time-dependent margin rule, however, active trading in the Chinese futures markets is typically shifted to the more distant contract period rather than the nearby contract period (Peck, 2008). Therefore the optimal hedging horizon in the Chinese futures market may not be the usual nearby months.

In order to investigate the optimal hedging horizon for this market, three different contract periods are investigated in this study. For soybean contracts, the nearest (to maturity) futures horizon is 2 to 3 months to maturity; followed by the 4 to 5 month to maturity; and the most distant futures horizon is 6 to 7 months to maturity. Together with their corresponding spot series, they are referred to as 3-month, 5-month and 7-month contract series, respectively. Take the 3-month futures series for example. Data are sampled at two months (forty-first trading day) before maturity to three months (sixtieth trading day) before maturity, daily closing prices, on the next-to-nearby futures contract. Because the contract period is two months, the continuous series needs to be rolled over to the next distant contract on the sixty-first day, since the nearby contract is maturing and the next-to-nearby contract is turning to the nearby contract. There are 2758 observations for the 3-month data series. Following Lien and Yang (2008), the spot series is sampled with the rolling over nearby contract, which is from the maturity day to fortieth day before maturity. However, for the copper contracts with monthly delivery, four contract series with progressive maturity are constructed of one month rolling over frequency. The nearest futures horizon is 2 months to maturity, followed by the 3-month, 4-month, and 5-month contracts.

Table I shows the summary statistics of data series. The total variances of returns in the soybean market are lower than those in the copper market. They illustrate greater price fluctuations in the copper market. Within the soybean market, surprisingly, the spot return series and the 4 and 5 months to maturity futures series have the highest variance, while the 2 and 3 months to maturity futures series have the lowest variance. By contrast, within the copper market, the total variance of returns becomes larger with more distant contract series. The Jarque-Bera normality test shows that none of the series are distributed normally. The high values of kurtosis indicate that all the series are fat-tailed. The Ljung-Box statistics suggest the existence of serial autocorrelations. The results in this table indicate that it is reasonable to employ the t-GARCH to model the marginal distributions. In addition, the mean of the basis is negative for both soybean contracts and copper contracts, although copper has a smaller basis on average, implying contango markets overall. Not surprisingly, the basis becomes larger for further distant contracts.

The unit root test also in Table I suggests that all the returns and basis are stationary. The spot and futures prices are cointegrated for all the data sets (in order to save space, the cointegration results are not shown in the tables; however they are available upon requests).

<insert TABLE I>

DCC estimation results

Table II reports the DCC estimation results. Soybean and copper markets exhibit different characteristics. For the soybean market, the degrees of freedom of the t-distribution are all significant with values around 3, suggesting significant fat tails for the marginal distributions. For the copper market, the values of the degree of freedom parameter are larger with more distant contracts. Hence, the returns become more fat-tailed when the contract approaches maturity. Also, the basis does not have significant impacts on the soybean current returns but significant impacts on the copper current returns.

<insert TABLE II>

Copula estimation results

Better fitted copula functions are those which provide high log-likelihood values and low AIC and SBIC, see table III for details. For the soybean 3-month data set, the Gumbel and Gumbel survival time-varying copula has the highest log-likelihood and lowest AIC and SBIC. The values of the weight parameter in the Gumbel and Gumbel survival copula and the Gumbel and Gumbel survival time-varying copula are both around 0.26, suggesting the existence of both lower and upper tail dependence, and the lower tail dependence is stronger, as shown in Figure 1. Therefore the joint dependence is fat tails rather than normally distributed. In general, the lower tail dependence is positive revealing that the possibility of spot and futures decrease together is high, despite occasional negative associations. In the upper tail, the dependence exhibits two semicircles, indicating non-linear associations between those upturn extremes. The largest futures positive returns are not closely linked with the largest spot positive returns, and the highest spots are not likely to happen together with the highest futures but next to the highest. Moreover, because the lower tail dependence is greater than the upper tail dependence, the futures and spot markets are more likely to decrease together, like equity markets.

<insert TABLE III>

For the soybean 5-month data set, the Gumbel and Gumbel survival copula suits the data the best. The weights in the Gumbel and Gumbel survival copula and the Gumbel and Gumbel survival time-varying copula are both around 0.37, indicating stronger lower tail dependence. Compare Figure 2 with Figure 1; the association with the 5-month horizon is less noisy and the possibility of negative dependence is rare.

The Gumbel and Gumbel survival time-varying copula demonstrate the data better than the other copulas for the soybean 7-month data set. The weights of the Gumbel and Gumbel survival copula and the Gumbel and Gumbel survival time-varying copula are around 0.33, indicating that the lower tail dependence is stronger than the upper tail dependence.

<insert Figure 1>

<insert Figure 2>

In the copper market, the best fitted copula function is the Gumbel and Gumbel survival time-varying copula for all the four data sets. It shows the rejection of the joint normal distribution. However, the weight parameters in the Gumbel family are all around 0.5, suggesting an almost symmetric tail dependence in copper markets which is different from the asymmetric dependence in soybean markets, see figure 3 the 2-month data set for example. Therefore the possibility of upturns in both spot and futures markets is nearly even with the possibility of market downturns together.

<insert Figure 3>

Overall, the best fitted copula models are the Gumbel and Gumbel survival copula and the Gumbel and Gumbel survival time-varying copula. It indicates three points. Firstly, the assumption of normal distribution for the joint dependence is not suitable for those markets. Secondly, the dependence is not linear but multi-dimensional between the spot and futures returns. Figures 1 to 3 demonstrate that the dependence is far more complicated than the traditional distributions could describe. Thirdly, it is two-tail dependent rather than no-tail (time-varying Gaussian copula) or one-tail (time-varying Gumbel copula or time-varying Clayton copula) dependent employed in Hsu et al. (2008).

Moreover, our results are the first to show that the joint return dependence is asymmetric in the Chinese soybean markets but almost symmetric in the Chinese copper markets. The greater lower tail in the soybean market suggests that the possibility of dependence in bear markets is higher than in bullish markets. It illustrates that the futures market responds better to the spot market in bear markets. In futures markets, traders in the long position would lose money if the price decreases, and consequently they are more sensitive with market falls. Therefore the greater lower tail dependence demonstrates that the traders in the long position are more dominant than those in the short position. But in the copper market not much difference is seen between the lower tail and the upper tail dependence, showing a well balanced trading power of the long position and the short position.

In-sample hedging performance

The concentration is on the comparison of the hedging performance among the OLS method, the DCC method and the copula functions.

In soybean markets, the Gumbel and Gumbel survival time-varying copula provides the best hedging strategy with the lowest portfolio variance and the highest hedging effectiveness among all the copula functions over the three contract horizons.

Comparing the copula method with the OLS method and the DCC model, the OLS method provides the lowest portfolio variance with the 3-month data set, followed by the DCC method and the optimal copula function respectively. With the 5-month data set, the optimal copula model exhibits the lowest portfolio variance. The DCC out-performs the OLS method and ranks the second. The optimal copula model also performs the best with the 7-month data set. The second is the DCC method and the OLS strategy is the last.

<insert TABLE IV>

In the copper market, the Gumbel and Gumbel survival copula provides the best hedging performance among the copula functions with the 2-month, 3-month and 4-month data sets, and the Gumbel time-varying copula is the best with the 5-month data set. The optimal copula function also out-performs the OLS method and the DCC method with the 2-month, 3-month and 4-month data sets. Surprisingly, the DCC method performs the worst. However for the copper 5-month data set, the OLS method provides the lowest portfolio variance, with the optimal copula function the second and the DCC method the last.

Overall, the optimal copula model exhibits superior in-sample hedging performance for both soybean and copper markets.

In order to investigate whether the superiority of the optimal copula model significantly out-performs the others, following Lee et al. (2006) and Lee and Yoder (2007), the White reality check is employed to test the statistical significance of the optimal copula hedging performance, with the OLS model as the benchmark model.

The test results show that the optimal copula model statistically out-performs the OLS model with the 2-month, 3-month and 4-month copper data sets. Their bootstrap P-values are 0.40%, 2.84% and 4.75% respectively. However, for the 3-month soybean data set and the 5-month copper data set, where the OLS out-performs the optimal copula model, the OLS model is switched as the tested model with the optimal copula model as the benchmark model. The White reality check reveals that the OLS model is not significantly superior to the optimal copula models in the two data sets either, with P-values 26.97% and 100% for soybean 3-month and copper 5-month accordingly.

Although the time-varying hedging strategies largely reduce the portfolio risks, it is still uncertain if they are worth being used given the requirement of everyday transaction costs. Therefore, the investigation of economic utility can provide more convinced suggestion on the performance of hedging strategies. The results of economic superiority test are shown in table IV. Following Lee at el. (2006) and Coakley et al. (2008), the expected return is assumed zero and the coefficient of risk aversion $\kappa = 4$ from equation (21). The OLS hedging strategy is treated as the benchmark. In 5 out of 7 cases, copula models show larger economic utility than the OLS method. However, transaction costs need to be considered. For the soybean contracts, the transaction cost is 4 RMB/contract. Its average daily price during the sample period is about 2591 RMB/ton. Because it is 10 ton/contract, the approximate transaction cost is 0.15%. It is much less than the utility difference between the optimal copula strategy and the OLS strategy with 5-month and 7-month data sets (see table IV). While for the copper contracts, the transaction cost is 0.02% per contract. It is also much less than the utility difference between the optimal copula functions and the OLS method with 2-month, 3-month and 4-month data sets. Hence, the copula hedging strategy does not only maximally reduce the portfolio risk, but is also economic superior when transaction cost is considered.

Out-of-sample performance

Within the in-sample hedging performance copula functions outperform the other hedging strategies in 6 out of 8 cases. However, in order to investigate whether this superior hedging performance is caused by data snooping, this section aims to examine the out-of-sample hedging performance under those strategies. In the out-of-sample study, 2500 observations are included as the in-sample estimation, and then a one-step rolling over forecast is used for the OLS method, the DCC method, and copula functions.

<insert TABLE V>

In soybean markets, the DCC method out-performs the others in the 3-month out-of-sample hedging. The OLS method is the second best, with the optimal copula function which is the time-varying Gaussian copula the third. With the 5-month data set, the optimal copula, the Gumbel and Gumbel survival time-varying copula, provides the best hedging strategy compared with the OLS method and the DCC model which rank the second and the third respectively. With the 7-month data set, the Gumbel copula is the optimal and provides the lowest out-of-sample portfolio variance. The DCC model ranks the second, with OLS the last.

In copper markets, the Gumbel and Gumbel survival copula shows the best out-of-sample hedging performance for 2-month, 3-month and 4-month horizons, the OLS method and the DCC model as second and third respectively. However with the 5-month horizon, the OLS method provides the most effective hedging performance.

However, the test of White reality check indicates that no hedging strategy significantly out-perform the others statistically in all the out-of-sample hedging. While in term of the economic superiority test, again the copula function out-perform the OLS method in 5 out of 7 cases considering the transaction costs.

From both in-sample and out-of-sample hedging performance, optimal copula method provides the best hedging strategy in 10 out of 14 cases. This could be explained by the better selection of the joint density between spot and futures returns. By contrast, a wrong selection of the joint density, like the commonly assumed normality, is likely to result in low hedging effectiveness. Moreover, under an incorrect assumption of the joint dependence, a time-varying hedging strategy may not only indicate an inappropriate average hedge ratio, but may also wrongly suggest a pattern in the ratio series. This could explain why constant hedge ratio methods, like the OLS method, can sometimes out-perform the time-varying methods like the DCC model. The superiority of copula functions in hedging performance is consistent with Hsu et al. (2008) who reveal that the hedging strategy by copula functions out-performs those by the OLS model, the MV-GARCH models. However, extending their copula functions from no tail dependence, like time-varying Gaussian copula, and one-tail dependence, as time-varying Gumbel copula and time-varying Clayton copula, to two-tail dependence, as the Gumbel and Gumbel survival copula and the corresponding time-varying copulas, our results demonstrate that our two-tail dependent copulas fit the data better and out-perform their no-tail and one-tail dependent copulas.

The optimal hedging horizon

According to Table IV, the lowest portfolio variances are 0.8450, 0.7368, and 0.7699 from their optimal hedging strategies for the 3-month, 5-month and 7-month data sets respectively; and the variance effectiveness (the variance reduction) is 28.55%, 32.77% and 31.57% accordingly. Therefore we conclude that the 5-month data set, in which the futures contract is 4-months and 5-months to maturity, is the most suited trading period that could maximally reduce the portfolio risk. The 7-month data set is the second best, in which the futures contract is 6-months and 7-months to maturity. And the 3-month data set, in which the futures contract is 2–months and 3-months to maturity, the shortest trading period for soybean futures in this paper, shows the poorest capability of risk reduction.

In the copper market, the portfolio variances provided by the optimal hedging strategy are 0.1179, 0.1404, 0.1699 and 0.2294 with the 2-month, 3-month, 4-month and 5-month data sets respectively; and the variance reductions, shown in table 4.9, are 89.78%, 87.83%, 85.30% and 81.52% accordingly. Based on the same spot asset, the portfolio which can maximally reduce the risk is the one which involves the 2-month futures contract. The ability of matching the spot, and therefore reducing the portfolio variance, declines as the contract moves further to maturity. Hence to choose a copper futures contract in order to hedge the spot, the best period would be 2-months to maturity, then 3-months to maturity, 4-months to maturity, and lastly 5-months to maturity.

In addition, the optimal hedging horizon is investigated for out-of-sample hedging by comparing portfolio variance and hedging effectiveness, shown in table V. For the 4 to 5 months futures contracts, the variance of the hedged portfolio is the lowest, with 74.61% of risk reduction compared with the unhedged portfolio. The next choice for the traders who want to minimize their risk exposure would be the 6 to 7 months futures contracts, with 59.32% of risk reduction. And the worst are the 2 to 3 months futures contracts which could only reduce 36.16% of the unhedged portfolio risk.

Comparing hedging performance among the four trading periods, the 2-month horizon is the best hedging period in order to reduce the portfolio variance, with the 3-month horizon being the second, the 4-month horizon the third, and the 5-month horizon the last; their maximum risk reductions are 93.13%, 91.93%, 90.49%, and 84.27% respectively.

Overall, the best hedging horizon is a 4 to 5 month contract period for soybeans and a 2 month contract period for copper, both in-sample and out-of-sample. The literature has generally acknowledged the nearby contract as the optimal hedging horizon (such as Ederington (1979), Lee et al. (1987), Bryant and Haigh (2005), Switzer and El-Khoury (2007) and many others), however, our empirical results suggest that it would not be a good hedging period in the Chinese soybean market. The soybean 3-month data set includes 2 to 3 months contract, when the margin rate has started to increase. Because more money is needed in order to trade during this period, certain trading should be shifted to more distant contracts. The lack of sufficient liquidity may affect its function performance as the futures, and therefore impair its hedging effectiveness. By contrast, the contract in the soybean 5-month data set is under the lowest margin rate, and therefore would be most actively traded as shown in Peck (2008). It would enhance the functional performance in hedging, and the results show that 4 to 5 months are the best hedging period. In copper markets, however, the findings are consistent with Ederington (1979) that the hedging performance increases as the contract approaching maturity. The best hedging period is the nearest 2-month horizon, and the closer to maturity the better, although 2-month and 3-month contracts are under high margin regimes. It indicates that the time-dependent margin rate has a significant impact on the optimal hedging horizons for soybeans but has no effect for copper.

Conclusion

Optimal hedge ratios have been intensively studied in the literature, yet mainly with the assumption of joint normal distribution. Employing copula functions, this paper overcomes the restriction of normal dependence. In addition, due to the time-dependent margin rule in the Chinese futures market, an optimal hedging period is investigated for the Chinese commodity market.

The empirical results show that the assumption of joint normal dependence does not apply for the Chinese soybean or copper markets. The dependence would be best described by the Gumbel and Gumbel survival time-varying copula for both soybeans and copper. It indicates that the spot and futures returns have stronger (than normal) associations during volatile periods; and the values of the weight parameters suggest that the linkage during market downturn is tighter than during market upturn in the soybean market, but almost symmetric in the copper market. The hedging strategies under the copula functions provide greater risk reductions than those strategies under joint normal dependence. Therefore the choice of an appropriate joint density is crucial for hedging performance.

By investigating the optimal hedging horizon for the Chinese commodity markets, where the margin rule would have impact on active trading periods, it suggests that different from the literature, the more distant contract (4 to 5 months to maturity), can maximally reduce the hedged portfolio risk in the soybean market. And hedging with nearby contracts is not recommended. However in the copper market the hedging performance has not been much affected by the margin rule and hedging becomes more effective as the contract approaching maturity.

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TABLE I Summary Statistics

Variable	Mean	Std.dev.	Skewness	Kurtosis	J-B	Q(20)	$Q^2(20)$	Unit root
Soybean 3-month spot	-0.0175	1.0875	-0.3792	12.408	10238**	86.97**	651.9**	-22.74**
Soybean 3-month futures	-0.0076	0.9381	0.2327	8.0287	2930.9**	42.30**	66.85**	-34.35**
Soybean 3-month basis	-0.7129	4.7287	1.8000	8.1979	4594.2**	2.8e5**	1.8e5**	-6.770**
Soybean 5-month spot	-0.0146	1.0469	-0.0913	9.7669	5105.6**	48.01**	159.8**	-35.19**
Soybean 5-month futures	-0.0119	0.9921	0.1047	5.3871	639.74**	48.52**	137.4**	-33.77**
Soybean 5-month basis	-1.3433	6.5934	0.6915	3.8861	300.59**	3.4e5**	1.5e5**	-5.552**
Soybean 7-month spot	-0.0156	1.0607	-0.0582	9.6961	4948.5**	45.75**	178.8**	-35.51**
Soybean 7-month futures	-0.0143	0.9760	0.1278	5.6583	786.89**	48.93**	162.3**	-33.84**
Soybean 7-month basis	-1.7246	7.4618	0.5055	2.6144	129.17**	3.6e5**	2.4e5**	-5.328**
Copper 2-month spot	0.0286	1.0740	-0.2501	6.8446	1837.5**	77.02**	3400**	-33.85**
Copper 2-month futures	0.0187	1.1927	-0.1615	5.4747	761.40**	43.12**	4014**	-35.48**
Copper 2-month basis	-0.0144	2.4570	1.6133	5.6747	2147.2**	4.7e5**	3.7e5**	-4.667**
Copper 3-month spot	0.0287	1.0739	-0.2501	6.8469	1840.4**	76.93**	3403**	-33.86**
Copper 3-month futures	0.0199	1.2122	-0.1291	5.2886	648.66**	39.92**	3944**	-35.45**
Copper 3-month basis	-0.0990	3.3845	1.5548	5.3723	1870.7**	5.0e5**	4107**	-4.152**
Copper 4-month spot	0.0278	1.0751	-0.2528	6.8405	1825.0**	75.75**	3377**	-33.82**
Copper 4-month futures	0.0171	1.2079	-0.1341	5.2023	598.63**	44.67**	3486**	-34.94**
Copper 4-month basis	-0.1939	4.1579	1.4965	5.1054	1628.7**	5.1e5**	5.1e5**	-3.691**
Copper 5-month spot	0.0181	1.1141	-0.2456	6.5031	1373.8**	70.68**	2971**	-32.12**
Copper 5-month futures	0.0094	1.2232	-0.0792	4.9039	400.72**	46.49**	3449**	-32.73**
Copper 5-month basis	-0.1895	4.9753	1.4161	4.6641	1184.7**	4.7e5**	4.1e5**	-3.245**

The spot and futures series refer to spot return and futures return. The column of J-B represents the statistics of Jarque-Bera normality test. The column of Q(20) represents the statistics of Ljung-Box test up to 20 lags. The column of $Q^2(20)$ represents the statistics of Ljung-Box test of each squared series up to 20 lags. The last column represents the ADF unit root test. ** refers to 5% significant level.

TABLE II DCC model estimations

	Soybeans						Copper							
	3-month		5-month		7-month		2-month		3-month		4-month		5-month	
	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures
Mean Eq.														
α	0.0062 (0.485)	-0.0053 (-0.377)	0.0117 (0.846)	-0.0186 (-1.234)	0.0105 (0.747)	-0.0327** (-2.168)	0.0310** (2.321)	0.0292* (1.799)	0.0332** (2.487)	0.0289* (1.731)	0.0350** (2.605)	0.0298* (1.796)	0.0263* (1.688)	0.0191 (0.979)
β_1	-0.0794** (-3.415)	-0.0974** (-4.297)	-0.0654** (-2.774)	-0.0792** (-3.389)	-0.0630** (-2.705)	-0.0669** (-2.846)	0.0674* (1.672)	-0.1892** (-4.313)	0.0651* (1.769)	-0.1394** (-3.335)	0.0601 (1.618)	-0.1002** (-2.470)	0.0684* (1.861)	-0.1241** (-2.938)
β_2	0.0360* (1.649)	0.0538** (3.623)	0.0131 (0.731)	0.0295 (1.591)	0.0028 (0.156)	-0.0010 (-0.059)	-0.0930** (-2.830)	0.1444** (2.888)	-0.0894** (-3.064)	0.0852* (1.757)	-0.0886** (-3.015)	0.0441 (0.933)	-0.1102** (-3.607)	0.0735 (1.566)
γ	0.0018 (0.668)	0.0025 (0.822)	0.0031 (1.518)	0.0006 (0.237)	0.0021 (1.131)	-0.0010 (-0.496)	0.0353** (5.554)	0.0469** (5.644)	0.0252** (5.574)	0.0298** (5.130)	0.0201** (5.496)	0.0216** (4.594)	0.0166** (4.994)	0.0183** (4.260)
Variance Eq.														
λ	0.4102** (4.311)	0.2362** (3.918)	0.4401** (4.572)	0.0467* (1.869)	0.4906** (4.130)	0.0533** (2.057)	0.0118** (2.629)	0.0102** (2.240)	0.0115** (2.766)	0.0119** (2.463)	0.0114** (2.821)	0.0127** (2.487)	0.0147** (2.804)	0.0112** (2.155)
θ	0.6362** (4.621)	0.2749** (4.840)	0.6236** (4.478)	0.1109** (3.558)	0.6545** (4.337)	0.1165** (3.516)	0.1263** (6.795)	0.0961** (6.968)	0.1256** (6.853)	0.0947** (6.996)	0.1246** (6.956)	0.0942** (6.589)	0.1295** (6.177)	0.0873** (6.922)
δ	0.4397** (7.668)	0.5746** (7.586)	0.3955** (6.458)	0.8735** (21.88)	0.3563** (4.632)	0.8520** (18.08)	0.8844** (53.75)	0.9076** (69.55)	0.8852** (56.36)	0.9059** (69.56)	0.8863** (61.11)	0.9059** (66.63)	0.8817** (52.64)	0.9120** (71.21)
ν	2.5907** (15.79)	3.1312** (14.21)	2.6462** (15.53)	3.2763** (12.81)	2.6477** (15.05)	3.6246** (11.86)	3.9111** (12.33)	4.9012** (10.17)	3.9142** (12.27)	5.4987** (9.362)	3.9087** (12.16)	5.5078** (8.873)	3.8571** (11.53)	6.4728** (7.661)
Covariance Eq.														
a	0.0962** (19.29)		0.0731** (17.81)		0.0794** (18.36)		0.0426** (17.77)		0.0653** (75.06)		0.0672** (69.62)		0.0894** (25.40)	
b	0.8525** (119.2)		0.8910** (154.6)		0.8705** (123.5)		0.9510** (334.3)		0.9263** (1.6E5)		0.9235** (74776)		0.8882** (203.9)	

The estimated models are equation (2), (3) and (4). The numbers are the coefficient values; the numbers in parenthesis are the corresponding t-statistics.
** at 5% significant level; * at 10% significant level.

TABLE III Copula model criterions

	Soybeans			Copper			
Samples	3-month	5-month	7-month	2-month	3-month	4-month	5-month
Weight parameters in survival copulas							
Survival copula	0.2633** (5.745)	0.3661** (6.634)	0.3327** (6.290)	0.5726** (14.25)	0.5323** (13.21)	0.4889** (11.71)	0.5303** (7.111)
Time-varying survival copula	0.2588** (12.54)	0.3660** (15.19)	0.3328** (12.66)	0.5725** (18.41)	0.5251** (12.79)	0.4380** (15.48)	0.5299** (15.53)
Log-likelihood							
Gaussian copula	977.1	773.5	657.1	2917.8	2721.2	2580.2	2130.4
Gumbel copula	1003.6	804.4	652.8	2939.6	2721.1	2574.6	2146.2
Survival copula	1110.4	882.9	729.5	3052.3	2842.4	2698.5	2249.1
Time-varying Gaussian copula	979.9	773.7	657.7	2879.0	2721.2	2580.2	2131.8
Time-varying Gumbel copula	1009.5	837.4	627.0	2973.0	2753.6	2532.7	2120.7
Time-varying Clayton	869.7	805.0	352.9	2472.1	2293.3	2195.9	1806.4
Time-varying survival copula	1111.7	882.8	729.6	3052.3	2845.1	2713.9	2249.3
AIC							
Gaussian copula	-1954.1	-1547.1	-1314.2	-5835.7	-5442.5	-5160.3	-4260.8
Gumbel copula	-2007.1	-1608.8	-1305.5	-5879.3	-5442.3	-5149.2	-4292.5
Survival copula	-2220.7	-1765.8	-1459.0	-6104.6	-5684.9	-5396.9	-4498.2
Time-varying Gaussian copula	-1959.9	-1547.4	-1315.3	-5837.9	-5442.5	-5160.3	-4263.5
Time-varying Gumbel copula	-2019.0	-1609.9	-1254.0	-5758.0	-5255.0	-5065.4	-4241.3
Time-varying Clayton	-1739.4	-1325.0	-750.9	-4944.1	-4586.6	-4391.9	-3612.9
Time-varying survival copula	-2223.4	-1765.6	-1459.3	-6104.5	-5690.2	-5427.9	-4498.5
SBIC							
Gaussian copula	-1954.1	-1547.1	-1314.2	-5835.7	-5442.4	-5160.3	-4260.8
Gumbel copula	-2007.1	-1608.8	-1305.5	-5879.3	-5442.3	-5149.2	-4292.5
Survival copula	-2220.7	-1765.8	-1458.9	-6104.6	-5684.9	-5396.9	-4498.2
Time-varying Gaussian copula	-1959.9	-1547.4	-1315.3	-5837.9	-5442.5	-5160.3	-4263.5
Time-varying Gumbel copula	-2019.0	-1609.9	-1254.0	-5758.0	-5255.0	-5065.4	-4241.3
Time-varying Clayton	-1739.4	-1325.0	-750.9	-4944.1	-4586.6	-4391.9	-3612.9
Time-varying survival copula	-2223.4	-1765.5	-1459.2	-6104.5	-5690.2	-5427.8	-4498.5

The “Survival copula” refers to the Gumbel and Gumbel survival copula; and the “Time-varying survival copula” refers to the time-varying Gumbel and Gumbel survival copula.
** significant at 10% level.

TABLE IV In sample hedging performance

	Soybeans			Copper			
Samples	3-month	5-month	7-month	2-month	3-month	4-month	5-month
Hedging performance $Var(r_{s,t} - HR_t r_{f,t})$							
OLS	0.8450	0.8059	0.8563	0.1494	0.1735	0.1913	0.2294
DCC	0.8515	0.7686	0.7984	0.5200	0.4954	0.5004	0.5698
Gaussian copula	0.8734	0.7588	0.785	0.7887	0.7665	0.7332	0.7090
Gumbel copula	0.9906	0.8157	0.8522	0.2733	0.2561	0.2456	0.2555
Survival copula	1.0285	0.8966	0.7716	0.1179	0.1404	0.1699	0.2632
Time-varying Gaussian copula	0.887	0.8289	0.9152	0.8538	0.7759	0.7249	0.5793
Time-varying Gumbel copula	1.0381	0.8306	0.8865	0.2572	0.2457	0.2329	0.2446
Time-varying Clayton copula	1.1348	1.0560	1.0949	0.1929	0.2245	0.2386	1.1929
Time-varying survival copula	0.8616	0.7368	0.7699	0.1527	0.1405	0.2579	0.4439
Hedging effectiveness $e = 1 - \frac{Var(P)}{Var(U)}$							
Unhedged variance	1.1827	1.0960	1.1251	1.1535	1.1532	1.1559	1.2411
OLS	0.2855	0.2647	0.2389	0.8705	0.8496	0.8345	0.8152
DCC	0.2800	0.2987	0.2904	0.5492	0.5705	0.5671	0.5409
Gaussian copula	0.2615	0.3077	0.3023	0.3163	0.3355	0.3657	0.4287
Gumbel copula	0.1624	0.2557	0.2426	0.7631	0.7780	0.7875	0.7941
Survival copula	0.1304	0.1819	0.3142	0.8978	0.8783	0.8530	0.7879
Time-varying Gaussian copula	0.2500	0.2437	0.1866	0.2598	0.3274	0.3729	0.5332
Time-varying Gumbel copula	0.1223	0.2422	0.2121	0.7770	0.7870	0.7985	0.8029
Time-varying Clayton copula	0.0405	0.0365	0.0268	0.8328	0.8054	0.7936	0.0388
Time-varying survival copula	0.2715	0.3277	0.3157	0.8676	0.8782	0.7769	0.6423
Utility $E_t U(r_{p,t+1}) = E_t(r_{p,t+1}) - \kappa\sigma_t^2(r_{p,t+1})$							
OLS	-3.3800	-3.2236	-3.4252	-0.5976	-0.6940	-0.7652	-0.9176
DCC	-3.4060	-3.0744	-3.1936	-2.0800	-1.9816	-2.0016	-2.2792
Gaussian copula	-3.4936	-3.0352	-3.1400	-3.1548	-3.0660	-2.9328	-2.8360
Gumbel copula	-3.9624	-3.2628	-3.4088	-1.0932	-1.0244	-0.9824	-1.0220
Survival copula	-4.1140	-3.5864	-3.0864	-0.4716	-0.5616	-0.6796	-1.0528
Time-varying Gaussian copula	-3.5480	-3.3156	-3.6608	-3.4152	-3.1036	-2.8996	-2.3172
Time-varying Gumbel copula	-4.1524	-3.3224	-3.5460	-1.0288	-0.9828	-0.9316	-0.9784
Time-varying Clayton copula	-4.5392	-4.2240	-4.3796	-0.7716	-0.8980	-0.9544	-4.7716
Time-varying survival copula	-3.4464	-2.9472	-3.0796	-0.6108	-0.5620	-1.0316	-1.7756
Utility improvement under OLS $E_t U' - E_t U_{OLS}$							
DCC	-0.0260	0.1492	0.2316	-1.4824	-1.2876	-1.2364	-1.3616
Gaussian copula	-0.1136	0.1884	0.2852	-2.5572	-2.3720	-2.1676	-1.9184
Gumbel copula	-0.5824	-0.0392	0.0164	-0.4956	-0.3304	-0.2172	-0.1044
Survival copula	-0.7340	-0.3628	0.3388	0.1260	0.1324	0.0856	-0.1352
Time-varying Gaussian copula	-0.1680	-0.0920	-0.2356	-2.8176	-2.4096	-2.1344	-1.3996
Time-varying Gumbel copula	-0.7724	-0.0988	-0.1208	-0.4312	-0.2888	-0.1664	-0.0608
Time-varying Clayton copula	-1.1592	-1.0004	-0.9544	-0.1740	-0.2040	-0.1892	-3.8540
Time-varying survival copula	-0.0664	0.2764	0.3456	-0.0132	0.1320	-0.2664	-0.8580

TABLE V Out-of-sample hedging performance

	Soybeans			Copper			
Samples	3-month	5-month	7-month	2-month	3-month	4-month	5-month
Hedging performance $Var(r_{s,t} - HR_t r_{f,t})$							
OLS	0.5360	0.2159	0.4401	0.2760	0.3213	0.3606	0.5302
DCC	0.5340	0.2267	0.4314	1.9440	1.8980	1.9379	1.7290
Gaussian copula	0.5688	0.2453	0.4124	2.2179	2.1694	2.0705	1.8812
Gumbel copula	0.6129	0.1833	0.3546	0.7159	0.6559	0.6051	0.6754
Survival copula	0.6382	0.2276	0.3745	0.2265	0.2657	0.3179	0.6101
Time-varying Gaussian copula	0.5504	0.3462	0.6183	1.4610	2.1970	2.0460	1.5418
Time-varying Gumbel copula	0.6447	0.1907	0.3720	0.6681	0.6241	0.5657	0.6449
Time-varying Clayton copula	0.8213	0.6174	0.8523	0.4744	0.5590	0.5833	3.1347
Time-varying survival copula	0.5517	0.1672	0.3635	0.2987	0.2659	0.5397	1.0259
Hedging effectiveness $e = 1 - \frac{Var(P)}{Var(U)}$							
Unhedged variance	0.8621	0.6584	0.8936	3.2961	3.2910	3.3430	3.3697
OLS	0.3783	0.6721	0.5075	0.9163	0.9024	0.8921	0.8427
DCC	0.3806	0.6557	0.5172	0.4102	0.4233	0.4203	0.4869
Gaussian copula	0.3402	0.6275	0.5385	0.3271	0.3408	0.3806	0.4417
Gumbel copula	0.2891	0.7216	0.6032	0.7828	0.8007	0.8190	0.7996
Survival copula	0.2597	0.6544	0.5809	0.9313	0.9193	0.9049	0.8189
Time-varying Gaussian copula	0.3616	0.4743	0.3081	0.5567	0.3324	0.3880	0.5425
Time-varying Gumbel copula	0.2522	0.7104	0.5838	0.7973	0.8104	0.8308	0.8086
Time-varying Clayton copula	0.0473	0.0624	0.0462	0.8561	0.8301	0.8255	0.0697
Time-varying survival copula	0.3600	0.7461	0.5932	0.9094	0.9192	0.8386	0.6956
Utility $E_t U(r_{p,t+1}) = E_t(r_{p,t+1}) - \kappa\sigma_t^2(r_{p,t+1})$							
OLS	-2.1440	-0.8636	-1.7604	-1.1040	-1.2852	-1.4424	-2.1208
DCC	-2.1360	-0.9068	-1.7256	-7.7760	-7.5920	-7.7516	-6.9160
Gaussian copula	-2.2752	-0.9812	-1.6496	-8.8716	-8.6776	-8.2820	-7.5248
Gumbel copula	-2.4516	-0.7332	-1.4184	-2.8636	-2.6236	-2.4204	-2.7016
Survival copula	-2.5528	-0.9104	-1.4980	-0.9060	-1.0628	-1.2716	-2.4404
Time-varying Gaussian copula	-2.2016	-1.3848	-2.4732	-5.8440	-8.7880	-8.1840	-6.1672
Time-varying Gumbel copula	-2.5788	-0.7628	-1.4880	-2.6724	-2.4964	-2.2628	-2.5796
Time-varying Clayton copula	-3.2852	-2.4696	-3.4092	-1.8976	-2.2360	-2.3332	-12.5388
Time-varying survival copula	-2.2068	-0.6688	-1.4540	-1.1948	-1.0636	-2.1588	-4.1036
Utility improvement under OLS $E_t U' - E_t U_{OLS}$							
DCC	0.0080	-0.0432	0.0348	-6.6720	-6.3068	-6.3092	-4.7952
Gaussian copula	-0.1312	-0.1176	0.1108	-7.7676	-7.3924	-6.8396	-5.4040
Gumbel copula	-0.3076	0.1304	0.3420	-1.7596	-1.3384	-0.9780	-0.5808
Survival copula	-0.4088	-0.0468	0.2624	0.1980	0.2224	0.1708	-0.3196
Time-varying Gaussian copula	-0.0576	-0.5212	-0.7128	-4.7400	-7.5028	-6.7416	-4.0464
Time-varying Gumbel copula	-0.4348	0.1008	0.2724	-1.5684	-1.2112	-0.8204	-0.4588
Time-varying Clayton copula	-1.1412	-1.6060	-1.6488	-0.7936	-0.9508	-0.8908	-10.4180
Time-varying survival copula	-0.0628	0.1948	0.3064	-0.0908	0.2216	-0.7164	-1.9828

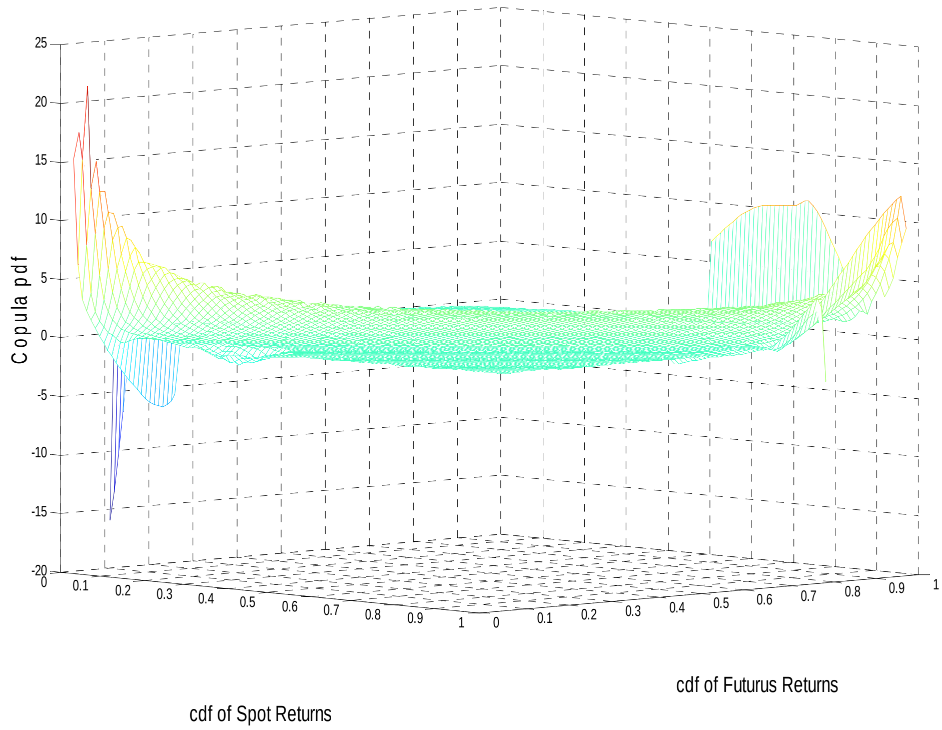

FIGURE 1 Gumbel and Gumbel survival time-varying copula with soybean 3-month data set.

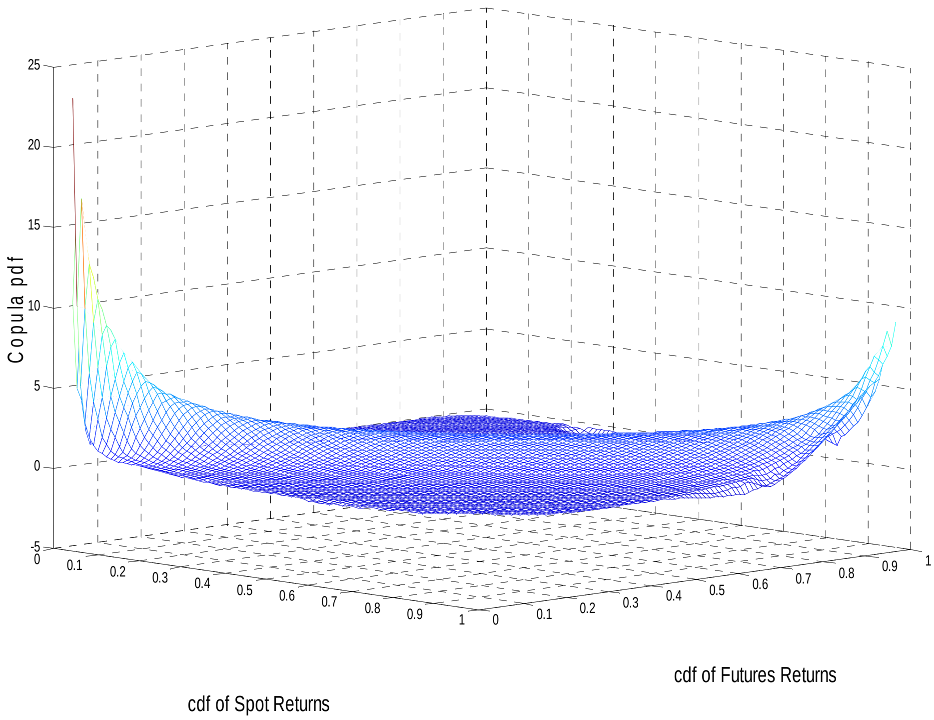

FIGURE 2 Gumbel and Gumbel survival time-varying copula with soybean 5-month data set.

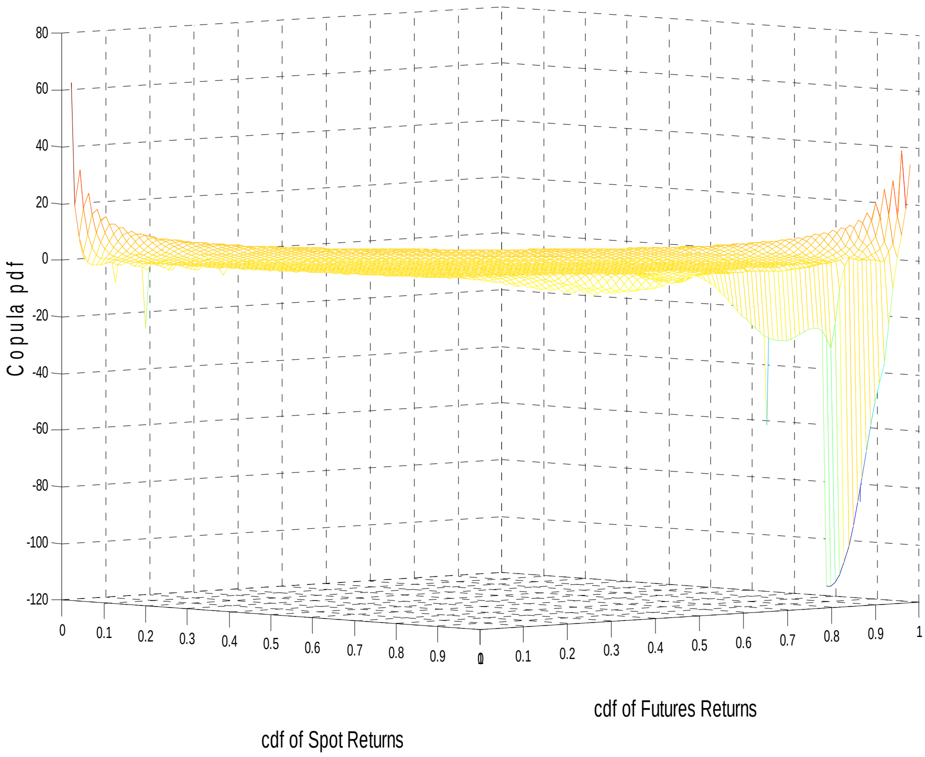

FIGURE 3 Gumbel and Gumbel survival time-varying copula with copper 2-month data set.