VENTURE CAPITAL AND UNDERPRICING: CAPACITY CONSTRAINTS AND EARLY SALES*

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November 2008

Abstract

I present a new theory that addresses an apparent puzzle of empirical finance literature: why do venture capital (VC) firms take young and poorly developed companies public? The key feature presented to solve this puzzle is human capital capacity constraints. Venture capital firms can take only a limited number of new projects at once, and they must go public with current projects in order to take advantage of new opportunities. The more constrained a VC firm, the earlier it needs to take companies public. This framework also predicts that technological waves and cost-reducing financial innovations reduce time to an IPO and increase expected first-day return, the discrete jump from offer price to first-day market price. Finally, the model endogeneizes the relation between first-day return and time to an IPO by explicitly modeling the process of building competition between potential buyers through time.

Keywords: IPO,Venture Capital, Underpricing, Capacity Constraints.

JEL Codes: G24, G11.

*I would like to thank Jan Eeckhout Ken Burdett and Guido Menzio for several discussions and encouragement during this project. I am also indebted with Andrew Postlewaite, Boyan Jovanovic, Randall Wright, Philipp Kircher, Manju Puri, Ludo Visschers, Chao Fu, Teddy Kim and the participants of Penn Search and Matching workshop and Micro theory club for helpful comments and discussions. I'm also deeply in debt with Cezar Santos for his help with MATLAB. Finally, I'd like to thank Rafael Silveira for lengthy discussions at the beginning of this project. All remaining errors are mine.

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1 Introduction

There is an apparent puzzle in the recent empirical literature on venture capital: Even though initial public offers (IPOs) of young companies are deeply underpriced, venture capital (VC) firms[1] insist on taking these infant companies public. Underprice, the offer price being lower than the price in the first day of public trade, represents a loss for original shareholders. VC-backed companies are usually younger, have lower revenues, and are less likely to be profitable than non-VC-backed companies at their IPOs. This pattern is even clearer if we look at young VC firms: these intermediaries take public companies on average two years younger and raise much less money on the IPO than their more mature counterparts.

In this paper, I present a model that proposes a new explanation for this puzzle. The main trade-off behind the IPO timing decision by a venture capital firm is the opportunity cost of turning down new projects against the benefit of holding on to existing projects. The key to this trade-off is that VC firms are capacity constrained, that is, they are able to handle only a limited number of companies. This means that a VC firm needs to go public with its current project to accept a new partnership. The benefits of taking a project public would then include not only the sale's proceeds but also the opportunity of engaging in new projects. The benefits of waiting to go public come from the increase in the expected return on a given project. This is obtained by improving the project's quality and by increasing the number of potential buyers, thereby increasing the competition for shares offered at the IPO. In this framework, I show that VCs have a higher incentive to take their companies public than entrepreneurs do. VC-backed companies therefore end up going public earlier than non-VC-backed companies in equilibrium. The distinction between young and mature VC firms comes from how binding the capacity constraints are; young firms usually have fewer and less experienced managers and are able to handle less projects at the same time. As a result, they need to go public with younger projects than mature VC firms.

As opposed to previous models, this model also delivers many predictions about market reactions to different technological and financial shocks. According to my model, technological waves reduce the time to an IPO, increase VC-backed companies' underpricing, and increase the average quality of projects looking for a VC. These waves also change VC's relative patience towards better projects. During a technological wave, the minimum quality a VC firm would demand from a new project before going public with a good current project goes up. At the same time, the minimum quality demanded before going public with a low-quality current project goes down. Reductions in new project's starting costs also generate faster IPOs, higher underpricing, and better projects in the VC market. However, in this

[1] I use the abbreviation "VC firms" throughout the paper to stand for venture capital firms. However, I will also use the term "VC" to refer to an individual venture capitalist.

case there is no change in relative patience between good and bad current projects. These predictions are in line with the empirical evidence on hot issues markets, periods where many companies go public in a short interval, suffering large underprices. The model also leads me to conjecture that underpricing signals the arrival of new good projects. Therefore, forward-looking investors invest in VC funds after large underpriced IPOs, establishing a positive relationship between underpricing and fund-raising.

Finally, the model addresses the existence of underpricing itself by endogenizing the expected return obtained through an initial public offer. Following previous empirical and theoretical literature, I assume collusion between underwriter and institutional investors, where investors receive underpriced shares in exchange for future deals with the underwriter. Differently from previous models, I pin down the underpricing's size by looking at the time to an IPO as a process in which competition between institutional investors is built. In a more crowded IPO, less money left on the table is necessary to guarantee future deals for the underwriter. The IPO process creates an S-shaped expected return function. Additional bidders raise expected revenue from competition less and less, hence the concavity. However, there is no competition if one or fewer potential buyers appear. Therefore, the initial convexity derives from the discrete increase in expected revenue from the arrival of the second potential buyer.

I also obtain testable results for the impact of firm age and the time between registration and the IPO on underpricing. This is an important first attempt to evaluate the impact of the road shows on underpricing. My preliminary empirical results seem to agree with the predictions of the model.

The next section will discuss the main features of the VC market. In the third section, I present the setup of the model. The fourth section presents the equilibrium and main results, while the fifth section discusses the possible ways to endogenize the expected return function. The sixth section shows empirical evidence about the correlation between measures of time to IPO and underpricing to support my results from the fifth section. The seventh section concludes the paper. All proofs are presented in the appendix.

2 The Venture Capital Market

A Venture capital firm is a financial intermediary that takes investors' capital and invests it directly in portfolio companies. Its primary goal is to maximize its financial return by exiting investments through a sale or an Initial Public Offering (IPO). Its payment is based on a profit-sharing arrangement, the usual being an 80-20 split: after returning all of the original investment to the external investors, the general partner (VC) keeps 20% of everything else. VC firms are usually small (on average they have 10 senior partners) and handle a restricted amount of resources and few portfolio companies. According to Metrick (2006):

"VCs recognize that most of what they do is not scalable and there are limits on the total number of investments that they can make (...) firms are reluctant to increase fund sizes by very much".

The estimated total committed capital in the industry is US$ 261 billion, which is managed by an estimated 9,239 VC professionals, meaning that the industry is managing about US$ 28 million per investment professional. Even the most famous VC funds usually only manage about US$ 50 million to US$ 100 million per professional. A typical VC fund will invest in portfolio companies and draw down capital over its first five years, which are known as the investment period or commitment period. After the investment period is over, the VC can only continue to invest in its current portfolio companies. However, VC firms usually raise news fund every few years, so that there is always at least one fund in the investment period at all times. Venture capitalists retain extensive control rights, in particular rights to claim control on a contingent basis and the right to fire the founding management team; they keep hard claims in the form of convertible debt or preferred stock, underpinning the right to claim/control and abandon the project; staged financing and the inclusion of explicit performance benchmarks make it possible to fine-tune the abandonment decision. In summary, venture capital firms are financial intermediaries that suffer from capacity constraints (they have limited human capital to manage their portfolio companies), share the profits realized through competitive sale or IPO, have strong control over exit decisions, and keep looking for new opportunities.

In early studies, as Barry at al. (1990), Megginson and Weiss (1991), and Lin and Smith (1998), VC backed IPOs were found to be less underpriced than non VC-backed IPOs. However, more recent papers found clear evidences that VC-backed IPOs are more underpriced than their non VC-backed counterparts, after controlling for selection bias. Looking at a sample of 6,413 IPOs between 1980 and 2000, of which over 37% (2,383) are VC-backed, Lee and Wahal (2004) found out that venture capitalists took smaller, younger firms public. VC-backed companies are on average 7 years old at IPO, compared to 14.7 years old average non VC-backed firms. They also have lower book value (0.76 compared to 6.63) and lower total assets ($104.4 million compared to $543.3). These features imply that VC-backed IPOs are generally smaller (average net proceeds are $40.5 million compared to $58.3), even though they have higher quality underwriters. Controlling for selection bias through matching estimators[2], Lee and Wahal (2004) show that the average difference in first day return between VC-backed firms and comparable non VC-backed ranges from 6.20 to 9.51 percent. Since the average first day return in the full sample is about 18%, a first-day return differential about 9% represents a significant portion of

[2]The instrument variables used in the first stage were: logarithm of net proceeds, 2-digit SIC code dummies, calendar year dummies, headquarter-state dummies, underwriter rank and book value of equity per share. Robustness checks were realized both including other instrumental variables, as logarithm of total assets and firms' age and changing the methodology to endogenous switching regressions. No qualitative change were found from the robustness exercise

average underpricing. Using different econometric approaches and data sets, similar results were found by Francis and Hasan (2001), Loughram and Ritter (2004), Frankze (2004) and Smart and Zutter (2003).

SEE TABLE 1

Some papers proposed theories to answer this apparent puzzle, as Loughran and Ritter (2002), Michelacci and Suarez (2002), Rossetto (2004), and Gompers (1996). Loughran and Ritter (2002) assumes collusion between VCs and underwriters, which empirically just seems to be true from 1999 on, not explaining the long lasting pattern. Both Michellacci and Suarez (2002) and Rossetto (2004) explanations assume scarcity of financial resources and the arrival of new profitable investment opportunities. However, the leading explanation is the grandstanding hypothesis, first proposed by Gompers (1996). According to this theory, since VC firms must periodically raise funds, they need to establish a reputation of being capable of taking portfolio companies public. Therefore, whenever a VC raises a new fund, she has incentives to rush to IPO to signal her high skills. This relation between bringing companies public and fund-raising ability should be stronger for young venture capital firms, since they have less reputation. As a result, they are more willing to bear the cost of greater underpricing.

SEE TABLE 2

Table 2 summarizes some of this empirical evidence: Analyzing a sample of 433 venture-backed initial public offerings (IPOs) from January 1, 1978, through December 31, 1987, Gompers (1996) found that IPO companies financed by young VCs are nearly two years younger and more underpriced when they go public than companies backed by older ones. He also found that young VCs spend on average 14 months less on the IPO company's board of directors and hold smaller percentage equity stakes at the time of IPO than the stakes held by established venture firms. The offerings also differ in magnitude. The equity stake retained by managers and employees after the offering is much larger for firms backed by the less experienced VCs. In addition the dollars raised in the IPOs by firms with seasoned VCs is larger. These results are consistent with Leland and Pyle (1977), who argue that lower quality managers must retain larger equity stakes and raise less money to obtain any external financing.

Testing the grandstanding hypothesis, Lee and Wahal (2004) found that the flow of capital into the lead VC firm is positively related to VC age, the number of previous IPOs done by the VC, and underpricing, implying a benefit to bearing the cost of underpricing. They also found that interaction effects between reputation (proxied by VC age and number of previous IPOs done by the VC firm) and underpricing are negative, supporting grandstanding. The real loss in underpricing for the VC firm is that it transfers wealth from existing shareholders to new share holders. Since venture capitalists owns on average 36% of the firm prior to the IPO and 26.3% immediately after, they also embody a huge loss through underpricing.

However, there are many features in the market that are not answered by the traditional grandstanding model presented by Gompers (1993). The model is silent about what determines and the impact of hot issue markets on VC markets. It also has nothing to say about changes in the VC's cost of entering in a new partnership, which can be reduced by financial innovations or more money available in the market. As argued by many authors including Jovanovic and Rousseau (2001), technological shocks increase the speed at which firms come to an IPO, being considered a main driving forces behind hot IPO markets. Such shocks also seem to increase the fund-raising by VC firms, as empirically confirmed by Gompers and Lerner (1998) and Bouis (2004). Behind the impact of technological shocks in VC fund-raising and faster IPOs lies an important feature missed by grandstanding models: demand-side factors in the VC industry. The demand of capital from entrepreneurs in innovative industries is a major determinant of the amount and allocation of funds. According to Gompers and Lerner(1998) and Poterba (1989), demand-factors actually have a determinant effect on VC fund-raising. As Hellman (1998) says:

"At a theoretical level, it is hard to argue that demand considerations are of no importance. And casual observation suggests that in many countries the obstacles to investing in venture capital are relatively minor, yet there is no active venture capital market, suggesting that supply alone cannot be the problem. Instead, it is frequently argued that the lack of venture capital is due first and foremost to the lack of entrepreneurs."

My goal in this paper is to introduce an alternative explanation that can preserve the results derived from the grandstanding hypothesis while still addressing additional features observed in the market. I will introduce here a model that posits capacity constraints (represented by human capital constraints) associated with the random arrival of new opportunities as the driving forces in this market. In this framework, Gompers's empirical results arise from differences in the strength of these constraints. Younger VC firms have a smaller number of senior partners with less experience and are thus more human-capital-constrained than well established firms. Therefore, they can take part in fewer projects. As a result, when taking on a new project, these companies have to exit younger projects on average than older VC companies. As I show in the fifth section, exiting a young project means lower expected return and higher underpricing.

This model is related to Michelacci and Suarez (2002) and Rossetto (2004) in terms of having capacity constraints as a driving force. However, differently from these models, I consider human capital constraints, which seem more empirically relevant. I am also able to deliver qualitatively different results on the impact of technological waves and cost reducing shocks on VCs' behavior, as well as pin down the quality of projects that look for a VC. These features cannot be incorporated in their models. Finally, I endogenize the expected return of going public and the expected underpricing by modelling the IPO process. This microfoundation establishes a relation between time to an IPO and underpricing which is not present in the current literature.

This model leads me to conjecture that underpricing leads to higher fundraising by signalling a good project has been found. In contrast to the reputational story presented by the literature, my conjecture is that investors are forward-looking. They see the underpricing as a signal of higher expected returns in the future. Since the young VC firms are more capacity-constrained, the effect of underpricing must be higher to them. Finally, the relation between technological shocks, higher fund-raising, and lower average time until the IPO is associated with demand-side explanations. Technological shocks, proxied in the empirical literature by number of patents registered and investment in R&D, can change the inflow, distribution, cost, and return of new projects in the market. These changes induce the VCs to exit earlier from current projects to realize profits and engage in new ventures.

3 Setup

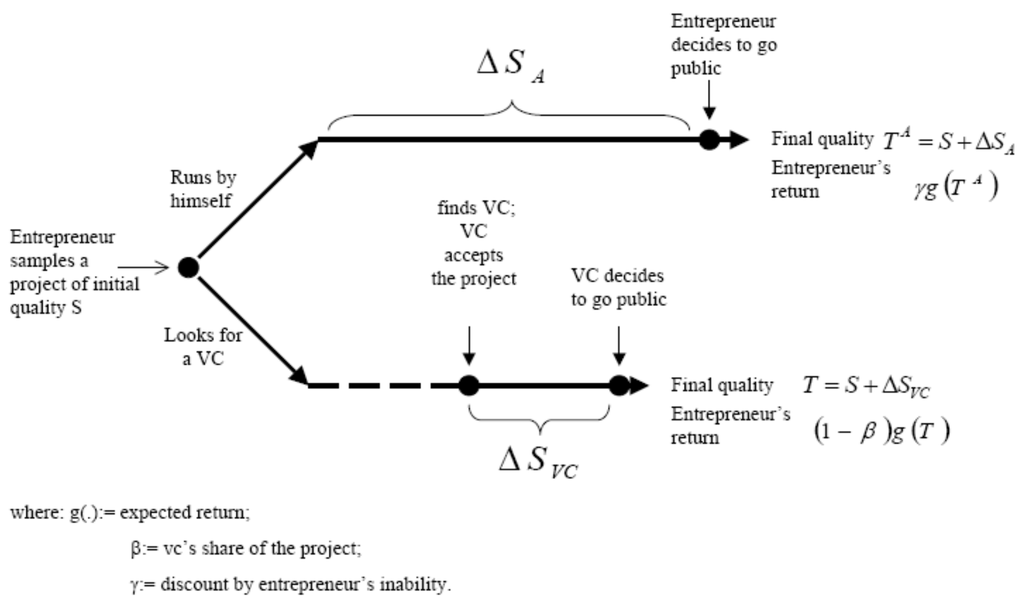
Time is continuous and the horizon is infinite. The economy is populated by four types of agents: VCs, entrepreneurs, underwriters and investors, being all risk neutral and infinitely lived, although I will consider the case in which entrepreneurs leave the market after their project is terminated. All agents discount future utility at rate $r > 0$. These agents are divided between two markets: A decentralized private equity/VC market and a public market. In the VC market, entrepreneurs with new projects and VCs look for partners. The public market is where investors negotiate projects previously took public by VCs with the help of underwriters. I assume that there are two types of investors: institutional investors, that collude with the underwriter and are allowed to participate in initial public offers (IPOs) and regular investors, which are only allowed to buy seasoned shares.

At each instant, a measure n of entrepreneur receive a new project. The project's initial quality S is a draw from a distribution H with support $\left[0, T_{\emptyset}^{*}\right]$ [3]. This quality improves as time is spent running it. A project with initial quality S developed during an interval of time ΔT has quality $T = S + \Delta T$ at the end of the interval. All projects will be considered identical except for the initial quality, i.e., any project can become as good as any other if enough time and effort is spent improving it. I also assume

[3] The use of $T_{\emptyset}^{*}$ is only a simplification that does not generate qualitative changes in the model that could possibly change my explanations on the main features presented by the model.

that the expected return from selling a project depends only on the project's current quality and VC's partnership.

Once he draws his project, an entrepreneur must decide if he tries to find a VC or run the project by himself. If he runs by himself, he decides when to take the company public and therefore, its quality at the IPO. The expected proceeds follow the deterministic rule $g(T)$, where T is the project's quality at the time of the sale. I assume $g'(\cdot) > 0$ and $\frac{g''(T)}{g'(T)} < r$, for reasons that will be clear later. Since an entrepreneur is less skillful marketing his project, he obtains $\gamma g(T)$, where γ is the discount factor that determines the reduction on the proceeds. If he chooses to look for a VC, the project's quality is constant until he finds a VC . If he finds a VC, which happens with probability $\eta > 0$ and the VC accepts his project, they start running it together. Once the project is sold, the entrepreneur will receive $(1-\beta)g(T)$, where $(1-\beta) > \gamma$ is the fraction of the sales revenue that belongs to the entrepreneur and T is the project's quality at the IPO. In this case, however, I assume that the VC determines when the project will be sold[4]. Her decision depends on the outside options that she faces, i.e., the new projects that are randomly offered to her. Once a entrepreneur is in a partnership, he cannot find another VC and change partners. I summarize this structure in the picture below.



Given these features, the only choice made by the entrepreneur is to enter or not to enter in the VC market. Once he entered, he would accept any VC, since all VCs are homogeneous conditional

[4]We could consider here that both parts need to agree to keep the partnership, otherwise the project is sold and the partnership dissolved. However, it would make no difference since we show that the VC always have a higher incentive to walk away.

on accepting the project. Therefore, entrepreneurs' decisions are summarized by VC market projects' distribution F, with support possibly on $\left[0, T_{\emptyset}^{*}\right]$.

There is a measure v of VCs in the private equity/ VC market. Each VC has capacity constraints, being able to handle one or two projects at the same time. I assume that VCs can hold only one project, unless mentioned differently. A VC receives new offers with probability λ, which will be endogenized in equilibrium. If the VC is currently in a partnership and decides to enter in a new one, she needs to take her current project public. A project switch generates the benefit of realizing current project's profits, represented by $(1-\beta)\, g(T)$. It also embodies two costs. One is a sunk cost c of entering in a new project. The second cost is losing the opportunity of keeping improving the current project, represented by the value $V(T)$. The decision rule obtained from the VC's problem, $T_c^*(T)$, gives the new project's minimum quality that would induce the VC to go public with a current project of quality T. The decisions about which would be the minimum acceptable quality for an unmatched VC, R and the quality at which a VC would take the project public and open a vacancy, $T_{\emptyset}^{*}$ complete the VC's decision set. The definition of private equity market equilibrium is given below:

Definition 1 An Equilibrium is this economy is a vector $\left\{T_c^*(\cdot), R, T_{\emptyset}^{*}, T^{\bigstar}(\cdot), F(T)\right\}$ such that:

- Given $F(T)$; Venture Capitalists are choosing optimally their termination rule $T_c^*(\cdot)$;
- Give $F(T)$; unmatched Venture Capitalist accept projects with initial quality equal or higher than R;
- Given $F(T)$; a Venture Capitalist matched with a project with current quality equal or higher than $T_{\emptyset}^{*}$ goes public even without a new project;
- Given $F(T)$ and $T_c^*(\cdot)$, entrepreneurs choose optimally their entrance rule $T^{\bigstar}(c)$;
- Given $T_c^*(\cdot)$ and $T^{\bigstar}(c)$, the distribution of initial projects' quality in the market is given by $F(T)$.

Notice that this is a partial equilibrium, since the expected return of an IPO, given by $g(\cdot)$ is taken as given and it needs to be determined in the public market, which I describe in the next paragraph. I also endogenize the probabilities of finding a VC and finding a new project, given by η and λ, respectively, by introducing a matching function with constant returns to scale, $m(u,v)$, where u is the measure of unmatched entrepreneurs in the VC market. Therefore, $\eta = \frac{m(u,v)}{u}$ and $\lambda = \frac{m(u,v)}{v}$.

The public market is populated by investors. As mentioned before, I consider that only institutional investors are able to participate in IPOs. Following previous papers in the literature, as Jovanovic and Szentes (2007), I assume that there is a collusion between underwriter and institutional investors. Once receiving underpriced shares today, investors oblige themselves to a future contract with the underwriter. However, the more institutional investors interested in the current project, the lower the underpriced necessary to commit investors to future business with the underwriter. Assuming that the longer a project is marketed the higher the number of institutional investors interested in it, underpricing is decreasing on marketing time. The first day return occurs because regular investors are able to extract a signal of the quality of the firm through the IPO price and adjust their reserve price accordingly. The more crowded an IPO, the better is the signal quality, the lower the underprice. A more detailed description of the IPO process is given in section 5.

4 Benchmark model

I present the model following these steps: First, I will address the VC problem, taken as given the decisions by entrepreneurs and investors, embodied in the distribution $F(\cdot)$ and the expected return function $g(\cdot)$, respectively. Then, taking as given the VC's optimal choices $\left\{R, T_c^*(\cdot), T_\emptyset^*\right\}$ and investors' decisions, I address the entrepreneur's choice between entering the VC/private equity market and autarky. Finally, I address the investors' problem and give a microfoundation to the expected return function $g(\cdot)$.

4.1 Venture Capitalist's Problem

The VC needs to take 3 decisions: First, which projects to accept if she currently has an open spot; Second, once she is in a partnership with project's quality T and a new opportunity is offered, which new projects would generate an IPO and partnership change; Finally, which current project's quality would generate an IPO even though no new partnership is at sight. These decisions will be represented by $\left\{R, T_c^*(\cdot), T_\emptyset^*\right\}$, respectively, where $T_c^*(\cdot)$ is a function of the current project's quality T.

Since $T_c^*(\cdot)$ and $T_\emptyset^*$ have similar reasonings, I will consider both at first. Therefore, let's consider a VC in a partnership with a project with current quality T. The value of being in this partnership is given by:

$$
\begin{aligned}
(1+rdT) V(T) &= \lambda dT E_{\widetilde{S}} \max\left\{V\left(\widetilde{S}\right) - c + \beta g(T+dT), V(T+dT)\right\} + \\
&\quad (1-\lambda dT) \max\left\{\beta g(T+dT) + V^0, V(T+dT)\right\}.
\end{aligned}
$$

The value function above presents choices and features of the VC's problem: there is no flow benefit of keeping the project, since all profits are realized through exit. Once a new partnership is offered, which happens with probability λ, the VC needs to decide if she accepts the new partnership or she turns down the new offer to keep marketing the current project. If she takes the current project public, she realizes her profits $\beta g(T+dT)$, and pays the sunk cost of starting a new venture c. Since partner are found at random and there are projects with different initial qualities looking for VC to start a new venture, I assume the quality of the project offered is a draw from the market distribution of initial qualities $F\left(\widetilde{S}\right)$. Finally, even if no new project is offered, A VC has to decide if she goes public with the current project, obtaining $\beta g(T+dT)$ and opening a vacancy, or she stays in the partnership.

Notice that this value function clearly presents the interaction between timing and capacity constraints, with the costs and benefits of waiting to go public: The cost of waiting is postponing profits and turning down partnerships offered in the meanwhile. The benefit is improving the project's quality and marketing it better, which generates a higher expected return when it goes public. Starting with the decision of keeping the current project or going public if no new venture is offered. As I show in the appendix, this decision is represented by a threshold $T_{\emptyset}^{*}$, which is given by:

$$\frac{g'\left(T_{\emptyset}^{*}\right)}{g\left(T_{\emptyset}^{*}\right)}=r$$

Notice that $T_{\emptyset}^{*}$ only depends on g and r. Since the probability a VC receives a new offer is independent of being in a current project or not, there is no additional benefit of going public than realizing the payoff $\beta g(T)$. If no new project appears, the VC will hold the project until the cost of postponing consumption is equal to the benefit of holding it. For any $T<T_{\emptyset}^{*}$, I have $V(T+dT)>\beta g(T+dT)+V^{0}$. Then, taking $T\in\left[0,T_{\emptyset}^{*}\right]$ and $dT\rightarrow 0$, I obtain:

$$rV(T)=\lambda E_{\widetilde{S}}\max\left\{V\left(\widetilde{S}\right)-c+\beta g(T)-V(T),0\right\}+\frac{\partial V(T)}{\partial T} \tag{1}$$

Given the nature of this problem, there is a threshold $T_{c}^{*}(T)$ in which the VC is indifferent between keeping the current project of size T or ending it to engage in a new project of size $T_{c}^{*}(T)$. This threshold is defined by:

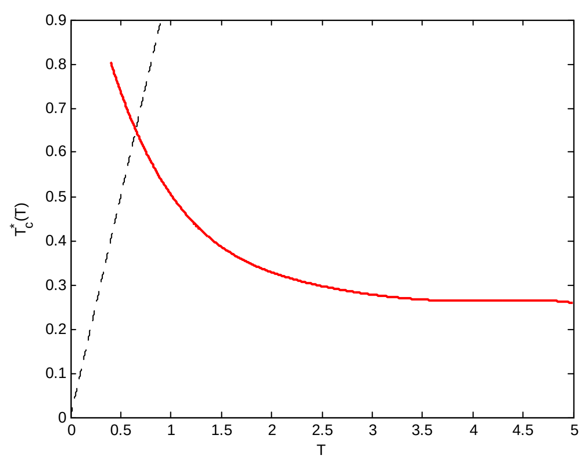
$$V\left(T_{c}^{*}(T)\right)+\beta g(T)-c=V(T) \tag{2}$$

According to this equation, a VC would finish the current project of quality T to enter in a new project with starting quality $T_{c}^{*}(T)$ if the gain from the termination of the current project, $\beta g(T)$ plus the value of the new project minus the initial sunk cost, $V\left(T_{c}^{*}(T)\right)-c$ compensates the loss of the value of the current project $V(T)$. Comparing this to the investment literature, this is simply telling us that the VC will compare Net Present Values. After several manipulations, I obtain the following implicit expression for $T_c^*(T)$:

$$\beta g\left(T_c^*(T)\right) - c + \int_T^{T_c^*(T)} \int_S^{T_\emptyset^*} e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega} \beta \left[g''(z) - rg'(z)\right] dz dS = 0 \tag{3}$$

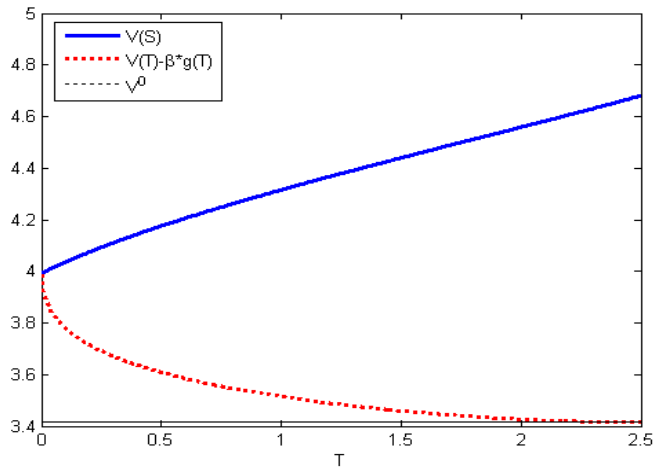
Using this expression, I can evaluate the impact of changes in the parameters of the model - c, β, and r - on $T_c^*(T)$. This exercise gives us an intuition of how changes in interest rates, sunk costs and bargaining power affect VC's decision rules. Since $F(\cdot)$ depends on the entrepreneurs' decision and these are also affected by the parameters, this is just a preliminary study which takes the market distribution of projects' qualities as exogenously given. This would be the case if VCs are cash constrained and unable to obtain loans from banks. Next section endogenizes $F(\cdot)$ and the probabilities λ and η by studying entrepreneurs' decision problem and introducing a matching function.

Example 1 Let's consider the case in which $R = 0.3$, $c = 0.4$, $r = 0.1$, $\lambda = 0.15$, $g = \sqrt{T}$, $\beta = 0.5$ and $F(\cdot)$ is a uniform distribution with support $\left[R, \frac{1}{2r}\right]$. Then, numerically computing $T_c^*(\cdot)$ using the above expression, I have:

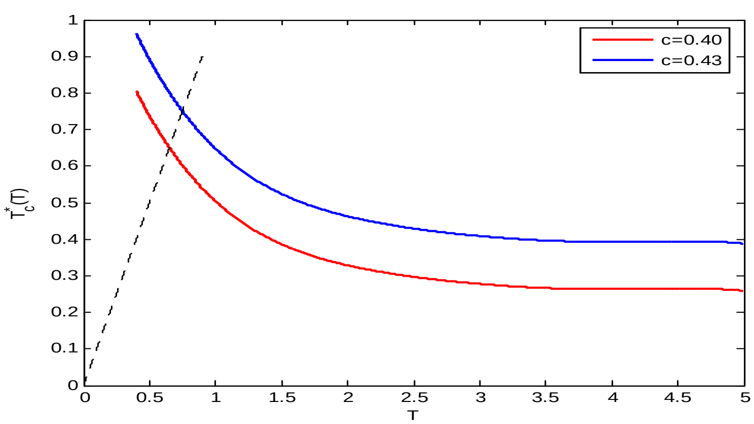


where the dashed line represents the 45^o line.□

First of all, let's consider the case in which there is no sunk cost to enter in a new project. In this case, from eq. (2), I can easily show that $T_c^*(T) = 0$, $\forall T \in \left[R, T_\emptyset^*\right]$. Graphically:



Therefore, regardless the quality of the current project, a venture capitalist has an incentive to enter in a new partnership whenever this is offered to her, taking the current project public. The intuition is simple, since entering in a new partnership is always costless, the VC has an incentive to realize profits whenever a new offer appears. This intuition is extended for changes in c, when there is a positive sunk cost. For example, consider a reduction of the initial sunk cost. From eq. (3) I can show that $\frac{dT_c^*(T)}{dc} > 0$, $\forall T \in \left[R, T_\emptyset^*\right]$. Graphically, following the parameters presented in example 1, I have:



I can notice that an increase in the sunk cost raises the threshold everywhere, i.e., a change in sunk costs does not affect relative propensity of taking a project public given its current project's quality - a VC becomes pickier does not matter if he has a high or a low quality project in hands. This result is an indication in favor of a explanation proposed by the literature that changes in technology that would generate a lower initial investment and a higher expected return would reduce the time to an

IPO. A leading example of this theory is the work by Jovanovic and Rousseau (2001) on the impact of Information Technology in reducing the time until IPO in the last 30 years.

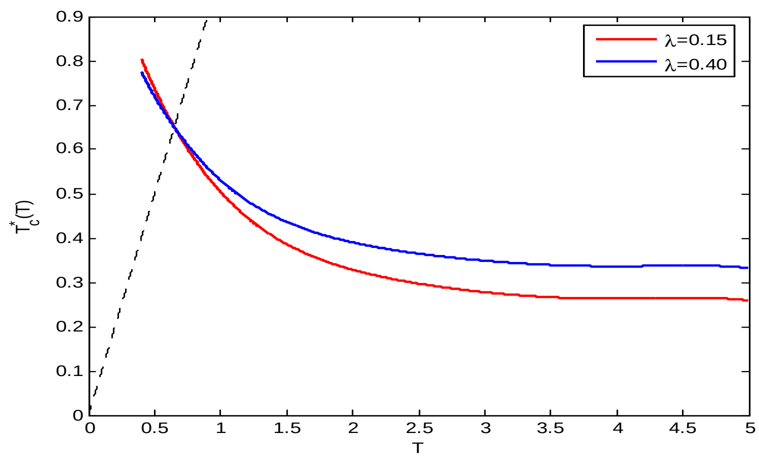
Considering a change in market tightness, the ratio between the industry's total capacity of marketing projects and the number of projects, we have strikingly different results. A decrease in market tightness implies that the probability a VC finds a new project goes up. Through eq. (3), I show that this generates a change in relative propensity to take a given project public, i.e.:

$$\frac{\partial T_c^*(T)}{\partial \lambda} < 0 \text{ if } T < T_c^*(T)$$

and

$$\frac{\partial T_c^*(T)}{\partial \lambda} > 0 \text{ if } T > T_c^*(T)$$

where $T = T_c^*(T) \Rightarrow \beta g(T) = c$. Graphically:



This means that an increase in market tightness in the VC/private equity market makes the VC relatively more prone to spend time with high quality projects. The reason is a shift in the costs and benefits of holding a project. In particular, the reduction in the risk of staying to long with a high quality project and staying unmatched for a long while. Since the probability of receiving an offer of a new partnership is high, the risk of staying unmatched for a long period is low. Therefore, the VC does not need to accept a partnership with initial quality very low for insurance purposes.

It's easy to show that changes in interest rate have the inverse effect. A increase in interest rates make the VC more impatient, being relatively more prone to take public high quality projects. In this sense, the decision rule $T_c^*(\cdot)$ would turn in favor of less developed projects.

The proposition below summarizes the results from comparative statics. I would like to reinforce that these are just preliminary results, since they take entrepreneurs' decisions as given. A full development of these results will be considered in the next subsection, once I take into account the entrepreneur's problem. As mentioned before, all proofs are in the appendix.

Proposition 1 From the VC's problem, taking entrepreneurs' decisions as given, I obtain the following:

- If there are no sunk cost to enter in a new project, VCs will take a project public whenever a new opportunity appears.
- If sunk costs are positive, VC's acceptance rule of a new partnership is given by a threshold, which decreases the higher is the current project's quality;
- As the sunk cost of entering in a new project increases, VC demands higher quality to enter in a new project and bring the current public .
- If the market tightness of the market increases, VCs become relatively more prone to keep longer better developed projects. If interest rate goes up, less developed projects are relatively more probable to be kept.

Finally, the VC needs to decide, once she has a vacancy and needs to pay the sunk cost to start a new partnership. Therefore, a VC would accept a new project of initial quality $\widetilde{T}$ if and only if:

$$V\left(\widetilde{T}\right) - c \geq V^0$$

where V^0 is the value of being unmatched. I can easily show that there is a initial quality R such that:

$$V(R) - c = V^0$$

again, by manipulating these expressions, I obtain:

$$\beta g(R) = c + \int_R^{T_c^*(R)} \int_S^{T_\emptyset^*} e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega} \beta \left[g''(z) - rg'(z)\right] dzdS$$

with together with the expressions for $T_c^*(\cdot)$ and $T_\emptyset^*$ pins down the optimal choices for the VC.

The next step I need to consider is endogenizing the distribution of projects looking for VCs, $F(\cdot)$, by looking at the entrepreneurs' decision problem.

4.2 The entrepreneurs' Entry Decision

An entrepreneur has two potential decisions: First, if he looks for a VC or runs the project by himself; Second, if he decides to run the project in autarky, he needs to choose to optimal time to take it public

Let's start with the second choice. If he decides to go to autarky (undertaking the project by himself), he faces the following optimal stopping problem:

$$\max_T e^{-r(T-S)} \left[\gamma g\left(T\right)\right]$$

with solution:

$$\frac{g'\left(T^A\right)}{g\left(T^A\right)} = r \tag{4}$$

Notice that $T^*_{\emptyset} = T^A$. Therefore, if no new project is offered to the VC running a current project, both VC and entrepreneur agree when the project should be finished. As mentioned before, this result comes from the fact of "on-the-project" search and search while unmatched are equally efficient, such that there is no benefit in opening a vacancy.

Now, let's consider the decision of entering or not the VC market. Since this decision is way more involving, I start presenting the simplest case in which $c = 0$, i.e., the VC has no sunk cost in entering a new partnership. As I showed before, in this case the VC finishes the current partnership whenever a new project is found. Later, I generalize my results to the case in which $c > 0$.

4.2.1 Case 1: $c = 0$

In this case, the entrepreneur knows that the VC will terminate the partnership whenever a new project with quality $T^*_c\left(T\right)$ or higher is offered to her. Therefore, his expected value of the partnership is:

$$rP\left(T\right) = \lambda\left\{\left(1-\beta\right)g\left(T\right) - P\left(T\right)\right\} + \frac{dP\left(T\right)}{dT}$$

Then, let's consider the value function of a entrepreneur in the VC's market searching for a partner with a project with initial quality S. Since the entrepreneur would accept any VC and would also be accepted by any of them, I would have:

$$S\left(S\right) = \frac{\eta}{\left(r+\eta\right)} P\left(S\right) \tag{5}$$

where η is the meeting rate for an entrepreneur. Then, an entrepreneur would enter VC's market after getting a project of initial quality S, if and only if:

$$S(S) \geq A(S)$$

Lemma 1 There exists a $T^{\bigstar}$ in which any project larger than $T^{\bigstar}$ enters the VC market.

Therefore, I have that the better projects will enter the market, being terminated earlier than the worse projects that chose autarky, even though those ones give a higher expected value for their entrepreneurs. Finally, notice that a measure $n\left[1-H\left(T^{\bigstar}\right)\right]$ of entrepreneurs will enter the market every instant.

To close the equilibrium in the VC/private equity market, I need obtain $F(\cdot)$ and u^{*}, which are the distribution of initial qualities in the VC market and the measure of entrepreneurs looking for a VC in steady state. In order to obtain these values, let's introduce a matching function $m(u,v)$, where u is the measure of entrepreneurs looking for a VC and v is the measure of VCs in the VC/private equity market. I assume that the matching function has constant returns to scale and satisfy Inada Conditions[5]. Then, η and λ will be defined as $\frac{m(u,v)}{u}$ and $\frac{m(u,v)}{v}$, respectively. All my analysis here will be in steady state. Let's start with u^{*}. In steady state, since $\eta = \frac{m(u^{*},v)}{u^{*}}$, I must have:

$$n\left[1-H\left(T^{\bigstar}\right)\right] = m\left(u^{*},v\right)$$

Since I am assuming that v is a constant, I can define $q(u) \equiv m(u;v)$. Doing this, I can express $u^{*} = q^{-1}\left(n\left[1-H\left(T^{\bigstar}\right)\right]\right)$. Before I analyze the consequences of this result, let's pin down $F(\cdot)$. In steady state, defining $F'(T) \equiv f(T)$, I obtain:

$$f(T) = \frac{n}{\eta} h(T)\, k$$

where k is a constant term that allows $f(\cdot)$ to integrate 1. In this case one can clearly see that $k = \frac{\eta}{n}\left[1-H\left(T^{\bigstar}\right)\right]$. Therefore, the pdf of project qualities in the market looking for a VC is given by:

$$f(T) = \frac{h(T)}{\left[1-H\left(T^{\bigstar}\right)\right]}.$$

Now, let's study the impact of changes in the number of entrepreneurs that obtain a new project, n. This can be seen as a technological boom, in which many new ideas are appearing and start-ups need money and expertise. I show in the appendix that $\frac{\partial T^{\bigstar}}{dn} > 0$; $\frac{\partial u^{*}}{\partial n} > 0$ and consequently that $\frac{\partial \eta}{\partial n} < 0$ and $\frac{\partial \lambda}{\partial n} > 0$. The next proposition summarizes and interpret these results.

Proposition 2 In a technological boom (increase in n):

[5] $\lim_{u\to 0} \frac{\partial m(u,v)}{\partial u} = \infty;\ \lim_{u\to\infty} \frac{\partial m(u,v)}{\partial u} = 0;\ \lim_{v\to 0} \frac{\partial m(u,v)}{\partial v} = \infty;\ \ \lim_{v\to\infty} \frac{\partial m(u,v)}{\partial u} = 0$

- The average quality of the pool of projects that look for a VC increase;
- The expected time a VC stays in a given partnership decreases;
- It takes longer for a given project to find a VC.

Now, let's introduce a positive sunk cost. As expected, this will make calculations more involving and the I will need to pin down the distribution of quality of current projects to analyze the market.

4.2.2 Case 2: $c > 0$.

In this case, VC's decision of going public with her current project and starting a new partnership is given by the downward sloping function $T_c^*(T)$. Therefore, the value for an entrepreneur of now being in a partnership with project with current quality T is:

$$rP(T) = \lambda \left[1 - F\left(T_c^*(T)\right)\right]\left\{(1-\beta)\, g(T) - P(T)\right\} + \frac{dP(T)}{dT}$$

Then, let's consider the value function of a entrepreneur in the VC market searching for a partner with a project with initial quality S. In this case, the value of looking for a VC is given by:

$$S(S) = \frac{\eta\left\{\left[1 - J\left(T_c^{*-1}(S)\right)\right]\left(1 - \frac{v_v}{v}\right) + \frac{v_v}{v}\right\}}{r + \eta\left\{\left[1 - J\left(T_c^{*-1}(S)\right)\right]\left(1 - \frac{v_v}{v}\right) + \frac{v_v}{v}\right\}} P(S)$$

where $J(\cdot)$ is the distribution of projects in current partnerships and v_v the measure of unmatched VCs.. A entrepreneur decides to enter the VC/Private Equity market if:

$$S(S) > A(S)$$

From this decision problem, I obtain the following lemma:

Lemma 2 There exists a $T^{\star}(c)$ in which any project larger than $T^{\star}(c)$ enters the VC market.

Therefore, the existence of a threshold of quality in projects entering the VC market is robust to the introduction of an initial sunk cost. However now the acceptance/termination rule depends on the size/quality of the project (better developed projects have a higher probability of being accepted and terminated than less developed ones) which would induce a steady state distribution of projects in the market that has a higher weight on smaller projects than the one observed in the inflow of projects. To be able to address these questions, I need to pin down, $F(\cdot)$, u^* and this time I also need to determine v_v and $J(\cdot)$.

Let's start taking a look at v_v. In steady state:

$$\frac{v_v}{v} = \frac{j\left(T_{\emptyset}^{*}\right)}{\lambda + j\left(T_{\emptyset}^{*}\right)}.$$

Now, let's look at $F(\cdot)$, the distribution of projects looking for a partnership. In steady state, I obtain:

$$F'(T) \equiv f(T) = \frac{knh(T)}{\eta\left[\frac{v_v}{v} + \left(1 - \frac{v_v}{v}\right)\left[1 - J\left(T_c^{*-1}(T)\right)\right]\right]}$$

where k is a constant that guarantees that the density integrates 1. Let's now obtain a formula to $J(\cdot)$. In steady state, taking $J'(T) = j(T)$:

$$j'(T) = \lambda\left[1 - F\left(T_c^{*}(T)\right)\right] j(T) + \frac{u^{*}}{v} nh(T) k$$

Finally, let's obtain an expression for u^{*}. In steady state:

$$n\left[1 - H\left(T^{\star}(c)\right)\right] = \eta u^{*} \int_{T^{\star}(c)}^{T_{\emptyset}^{*}} \left\{\frac{v_v}{v} + \left(1 - \frac{v_v}{v}\right)\left[1 - J\left(T_c^{*-1}(S)\right)\right]\right\} f(S)\, dS$$

substituting $f(S)$ and manipulating it:

$$u^{*} = \frac{1}{k}.$$

which can be substituted in the definitions of $f(T)$ and $j'(T)$. These equations pin down the distribution of projects in partnerships and looking for VCs, up to a constant. However, given that the system is quite involving, and $T_c^{*}(\cdot)$ is only determined implicitly, which impossibilities full theoretical analysis. However, a numerical analysis is still possible and will be done in future work.

In the next subsection, I analyze two extensions of this benchmark model. The first one addresses the case in which VCs are only allowed to search whenever they have a vacancy. This constraint will generate as a result a smaller total time a VC would spend with a given project and a knife edge result on the VC market, in which or all entrepreneurs look for VCs or none of them does. The second extension considers the possibility of a VC holding two projects at the same time. This extension addresses the distinction between young and old VCs as a difference in capacity constraints: it shows that the expected quality that a project goes public increases as capacity constraints are ease. Therefore, the results on grandstanding are addressed by this simple extension.

4.2.3 Extensions

No "on-the-project" search This section considers the case in which only unmatched VCs will be able to enter in a new partnership. This case shows why it is important that, differently from Michelacci and Suarez (2002), I consider a better structured set up to understand some of the stylized facts obtained in the empirical literature.

Consider that each unmatched VC finds a new project with probability λ. Then, the optimal termination time is the solution for the following problem:

$$\max_{T} e^{-r(T-S)} \left[\beta g(T) + V^0\right]$$

Notice that V^0 now embodies the benefit of being to look for new opportunities. At the optimal selling time[6] :

$$g'(T^*) - rg(T^*) = \frac{rV^0}{\beta} \tag{6}$$

Because the constraint on $g(T)$, the LHS is decreasing on T. Since the RHS is a constant on T, it guarantees that there is a unique T^* that satisfies the above equation. It can seen that, as expected, the higher the value of a vacancy V^0, the lower the waiting time to finish the project. Notice that V^0 is given by:

$$(1 + rdT) V^0 = \lambda dT \int \max\left\{V\left(\widetilde{S}\right), V^0\right\} dF\left(\widetilde{S}\right) + (1 - \lambda dT) V^0$$

Therefore, the value of being able to look for a new project $\left(V^0\right)$ is given by the expected return of the arrival of a new opportunity with initial quality $\widetilde{S}$, given by a draw from the distribution of initial qualities in the market F .

But note that since you can always get the project and finish it immediately, receiving $\beta g(T)$, where $T \geq 0$, the VC will always accept it. Then, manipulating and taking $dT \to 0$:

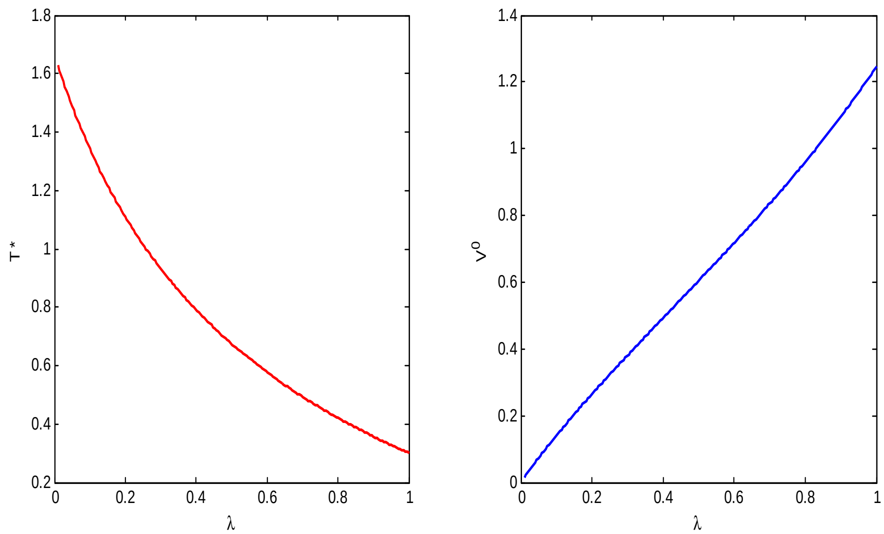
$$V^0 = \frac{\lambda}{r+\lambda} \int_0^{\overline{T}} V\left(\widetilde{S}\right) dF\left(\widetilde{S}\right) \tag{7}$$

Substituting these expressions in my previous results:

$$\frac{g'(T^*)}{g(T^*)} = \frac{(r+\lambda) r}{r + \lambda\left(1 - e^{-rT^*} \int_0^{\overline{T}} e^{r\widetilde{S}} dF\left(\widetilde{S}\right)\right)} \tag{8}$$

[6] We could have corner solutions in which the VC immediately sells the project without compromising the model. However, given our assumptions, no VC would keep a project forever.

The above expression implicitly defines the optimal time to go public, as a function of the distribution of initial qualities in the market and parameters of the model. Once I have T^*, I can easily obtain V^0. From these expressions, it can be shown that $\frac{dT^*}{d\lambda} < 0$ and $\frac{dV^0}{d\lambda} > 0$, which intuitively means that, if the market has too many projects relatively to the number of VCs, each project would be kept for a shorter period of time, since the value of looking for and starting a new project goes up. The graph below presents an example where $g(T) = T^{\frac{1}{2}}$ and uniform distribution.



These results still capture the hot issue market behavior, in which we observe a wave of many underpriced IPOs occurring at the same time. My explanation to these behavior is that in these occasions there are too many new opportunities entering in the market, such that keeping a project longer would incur a very high opportunity cost. To close this model as an equilibrium model, I need to consider the entrepreneurs' decision. If a entrepreneur decides to run the project by himself, he would solve the following problem:

$$\max_T e^{-r(T-S)} [\gamma g(T)]$$

with solution:

$$\frac{g'(T^A)}{g(T^A)} = r \tag{9}$$

This implies $T^A > T^*$. It is easy to see why: In this set up, notice that the Venture Capitalist has two gains whenever she goes public with her current project: First, she realizes the gains, obtaining $\beta g(T)$. But she also can now receive offers from new projects. So, in this framework the vacancy by itself is valuable for a VC. However, this difference in potential sources of earnings would create a clear distinction for the entrepreneur: He knows that if he looks for a Venture Capitalist, the company will necessarily be sold before its optimal time, while, if he decides to run the company by himself, he would choose the optimal time to sell it, however he would not be able to usufruct all the expertise that a VC has (therefore, obtains $\gamma << 1-\beta$). If a entrepreneur with a project with initial quality S enters the VC market, he has a expected value of:

$$\frac{\eta}{\eta+r}e^{-r(T^{*}-S)}\left(1-\beta\right)g\left(T^{*}\right)$$

Therefore, he would choose to look for a VC if and only if:

$$\frac{\eta}{\eta+r}e^{-rT^{*}}\left(1-\beta\right)g\left(T^{*}\right)>e^{-rT^{A}}\gamma g\left(T^{A}\right)$$

Therefore, the initial quality S does not affect the decision of looking for a VC or not. This implies that all qualities would join the market or it would simply shut down, with all entrepreneurs going to autarky. Finally, this framework cannot address the question why young VCs would go public earlier than tenured ones: Once VCs are only allowed to search for new opportunities if they have an opening, capacity constraints are not binding in this framework. Another point we should emphasize for future references is that the introduction of an initial sunk cost generates an increase in T^{*} and a decrease in the value of a vacancy.

The proposition below summarizes my findings on this section.

Proposition 3 In an economy with no "on-the-project search":

- All qualities of projects enter the VC market or not;
- All projects would go public at same age/quality;
- Hot issue markets would be occur whenever the market is tight (λ is close to 1);
- There would have no differentiation between the behavior of young and old VC firms.
- If there is a initial sunk cost ($c>0$), T^{*} will be increasing in c, while V^{0} will be decreasing on it.

Two project VCs As mentioned before, I consider that young and mature VC firms are distinct in terms of how binding capacity constraints are. To address this point, I extend the model presented here to the case in which a VC can hold two projects at the same time. In this case, it can be shown that whenever a VC has to go public with a given project to enter in a new venture, she chooses the older/better quality one. Then, in the case in which the entrepreneur still accepts any VC[7], the expected time until an IPO is larger for a mature (two spots) VC firm. Therefore, my model can obtain the same result obtained by the grandstanding hypothesis literature.

Therefore, the expected value for a VC of holding projects of quality T_1 and T_2 is given by:

$$
\begin{aligned}
rV(T_1,T_2) &= \lambda E_{\widetilde{T}} \max \left\{ \begin{array}{c} \left[\begin{array}{c} V\left(\widetilde{T},T_2\right)+\beta g(T_1) \\ -c-V(T_1,T_2) \end{array} \right], \\ \left[\begin{array}{c} V\left(T_1,\widetilde{T}\right)+\beta g(T_2) \\ -c-V(T_1,T_2) \end{array} \right], \\ 0 \end{array} \right\} \\
&\quad + \left[\frac{\partial V}{\partial T_1}(T_1,T_2) + \frac{\partial V}{\partial T_2}(T_1,T_2) \right]
\end{aligned}
$$

Then, looking at the cut off rule, let's analyze the conditions in which:

$$
V\left(\widetilde{T},T_2\right)+\beta g(T_1)-c-V(T_1,T_2) \geq V\left(T_1,\widetilde{T}\right)+\beta g(T_2)-c-V(T_1,T_2)
$$

Simplifying, and assuming symmetry:

$$
V\left(\widetilde{T},T_2\right)+\beta g(T_1) \geq V\left(T_1,\widetilde{T}\right)+\beta g(T_2)
$$

First of all, consider a symmetry condition:

$$
[\beta g(T_1)-\beta g(T_2)] - \left[V\left(T_1,\widetilde{T}\right)-V\left(T_2,\widetilde{T}\right)\right] \geq 0
$$

Manipulating this expression, I show in Appendix B the following result:

Lemma 3 Whenever a VC has two current projects and decides to go public to join a new venture, she goes public with the better developed one.

As a corollary of this result:

Corollary 1 The Expected time of a partnership increases the easier is the capacity constraint faced by a VC.

[7] The presence of young and mature VCs can also change the distribution of projects in the market. However, whenever both VCs are accepted by the same projects, they face the same distribution.

5 A Microfoundations for the IPO procedure: Endogenizing $g(T)$

In this section, I present one way in which the return from the exit in a venture investment can be endogenized. Since the most successful exits are through initial public offers (IPOs), endogenizing $g(T)$ necessarily involves a discussion about IPOs and their main players and features.

The main feature that we observe in IPOs is the presence of underpricing: According to Jenkinson and Ljungqvist (2001), the first day returns are positive in virtually all country. They typically average more than 15% in industrialized countries and around 60% in emerging markets. Such returns are viewed as anomalies: In efficient markets, competition between investors would necessarily exhaust all possible gains from private information, as presented by Grossman (1976), and no money would be "left on the table".

Many theories were developed to address this issue, all with limited success. The traditional explanations are based on asymmetric information. According to Rock (1986), there is asymmetric information between potential buyers, which generates a lemon's problem for the uninformed buyer. In Benveniste and Spindt (1989), the seller that is actually trying to extract information from institutional investors about the value of the company being sold. The main problem with these theories is that they take the sale's procedure as given. Rock (1986) takes as given firm commitment offers, avoiding the transmission of information from informed to uninformed buyers through price changes. Benveniste and Spindt (1989) assume the existence of a pre-market in which only regular investors participate. They assume that "cost of conducting an all-inclusive pre-market is prohibitive". Another theoretical explanation for underpricing presented by the literature is signaling. In this case, the seller knows the quality of the firm being sold, while underpricing would be a way to signal better quality. This explanation, although theoretically elegant suffers from empirical flaws (some assumptions used in these models are wrong and the data don't corroborate their results) and it also seems susceptible of collusion between the seller and some buyers or even the seller can "create" false investors, as discussed lately in the auction literature.

The explanation I am going to present here is related to the one defended by Jovanovic and Szentes (2007) and empirically discussed by Loughram and Ritter (2004). Jovanovic and Szentes (2007) claim that underprice is created by an agreement between institutional investors and underwriters. Investment bankers allocate underpriced stocks to institutional investors in the hope of winning their future investment banking business. This practice, called 'spinning', is well-documented. The most famous case the $100 million fine that Credit Suisse First Boston received because of these activities. Firm's original owners accept this practice because IPOs disclosure information to the market.

I introduce a step further, considering the competition between institutional investor for underpriced shares and how this impacts the amount of information released and therefore, the size of the first day return. Given this effect of competition, I consider how the increase in competition is related with time. I claim that one important role performed by Venture Capitalists and underwriters is to publicize the firm to be marketed. The more institutional investors that get information about the firm, higher the expected competition for its shares and therefore, lower underprice. One simple way that I can see this effort on publicizing an IPO firm and its impact on competition is looking at the length of the road show, that I will proxy looking at the number of days in registration. The Road Shows are tours taken by IPO firms' top managers and investment bankers to visit groups of invited institutional investors, publicizing the firm and also to elicit bids from investors. Although these bids don't have legal tender, there is a strong presumption that investors should be prepared to honour their bids. Looking at the data for more than 1500 IPOs in the period between 1984 and 2004 (I used data from the SDC Platinum), I can show a negative correlation between the number of days in registration and the first day return. This result is robust to different specifications of my multivariate linear regressions or even multivariate fractional polynomial models. Although the lack of data with respect to the number of bidders doesn't allow me look for deeper empirical relations, I imagine that this is a clear indication that timing and its impact on building up competition is an important factor to understand the size of the underpricing. In the next section, I give details on my empirical results.

I agree that this is one of many ways in which I could endogenize the expected return. However, I believe that my explanation not only adds in matching some empirical evidence that was not considered before, but it also gives a clear theoretical foundation for some hypothesis presented by the empirical literature on firm managers behavior. In addition, my specifications are not in disagreement with other conjectures, as the idea that Venture Capitalists not only publicize the project but also increase its quality (as it can be seen below, all results are kept if I imagine that the real value of the venture ω is an increasing function of time spent in the partnership).

5.1 Basic Framework

In this basic framework, I model IPOs as first price auctions in which only institutional investors participate. Even though the bookbuilding process is not an auction, the road show has a structure that can be approximated to one. I initially consider that there are N potential buyers participating in this auction. Later, I show ways to endogenize N and therefore evaluate the expected return on exiting as time passes. I assume that the value of the company ω is known by institutional investors and/or it can be credible communicated by the underwriter. I also assume that the underwriter and the venture capitalist knows ω but other players in the market don't . However, from my claim about the agreement between underwriter and investors, the investor that wins the auction obliges himself to a future contract with the underwriter. Consider that this future contract between investor and investment banker generates a private cost to the investor. This cost is i.i.d. draw ε from a distribution Z with support on $[0,\omega]$ [8]. Therefore, investor i's gain in winning the auction is given by $\omega - \varepsilon_i - p$ when he bids p and this is the highest bid. Therefore, I have the following payoff function:

$$\Pi_i = \begin{cases} \omega - \varepsilon_i - p_i & \text{if} \quad p_i > \max_{j \neq i} p_j \\ 0 & \text{if} \quad p_i < \max_{j \neq i} p_j \end{cases}$$

Then, the problem of investor i is:

$$\max_{p \geq 0} \left(\omega - \varepsilon_i - p\right) \left[1 - Z\left(\omega - P^{-1}\left(p\right)\right)\right]^{N-1}$$

Then, solving the symmetric equilibrium case:

$$P\left(\omega - \varepsilon_i\right) = \omega - \varepsilon_i - \int_{\varepsilon_i}^{\omega} \left[\frac{1 - Z\left(y\right)}{1 - Z\left(\varepsilon_i\right)}\right]^{N-1} dy \tag{10}$$

Then, the expected revenue is:

$$R\left(\omega, N\right) = \omega - N \int_0^{\omega} \left[1 - Z\left(\varepsilon\right)\right]^{N-1} F\left(\varepsilon\right) d\varepsilon - \int_0^{\omega} \left[1 - Z\left(\varepsilon\right)\right]^{N} d\varepsilon \tag{11}$$

Claim 1 Expected Return is increasing in N.

Claim 2 Expected payment converges to ω as $N \to \infty$.

Now consider that the winner paid a price p. How much would the market pay for this company in the next day? Remember that p is given by:

$$p = \omega - \varepsilon_w - \int_{\varepsilon_w}^{\omega} \left\{\frac{1 - Z\left(y\right)}{1 - Z\left(\varepsilon_w\right)}\right\}^{N-1} dy.$$

where ε_w is the winner's private cost of sealing the agreement with the underwriter. Considering the market agents are risk neutral, we are looking for $E\left[\omega \left| p \text{ is the winner}\right.\right]$. Then, from the expression above:

$$\omega = p + \varepsilon_w + \int_{\varepsilon_w}^{\omega} \left\{\frac{1 - Z\left(y\right)}{1 - Z\left(\varepsilon_w\right)}\right\}^{N-1} dy$$

Since the agent won the auction, ε_w is the minimum between N. Therefore, taking the expected value of the last two terms on RHS:

[8] The support being between 0 and ω is just a simplifying assumption that can be dropped without qualitative changes in the results.

$$\widehat{\omega} = p + N\int_0^{\widehat{\omega}} [1 - Z(y)]^{N-1} dy - (N-1)\int_0^{\widehat{\omega}} [1 - Z(y)]^N dy$$

It is easy to show that the RHS of the above expression is constant in $\widehat{\omega}$. Since the LHS is increasing, I can show that it crosses once. Let's show an example with a Uniform distribution

Example 2 $\varepsilon_i \sim U[0, \omega]$. Then:

$$\widehat{\omega} = p + \widehat{\omega} - \frac{N-1}{N+1}\widehat{\omega}$$

Therefore:

$$\widehat{\omega} = \frac{N+1}{N-1}p.$$

Notice that as N increases $\widehat{\omega}$ converges to p. Therefore, as N increases, p becomes a better signal of ω.

In this example, the expected first day return is given by:

$$\begin{aligned} \widehat{\omega} - p &= \frac{N+1}{N-1}p - p \\ fdr &= \frac{2}{N-1}p \end{aligned}$$

substituting p:

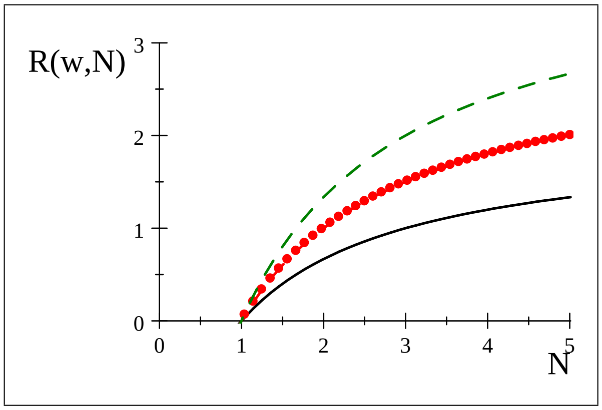
$$\begin{aligned} \widehat{fdr} &= \left(\frac{2}{N-1}\right) \times \left(\frac{N-1}{N+1}\right)\omega \\ &= \frac{2}{N+1}\omega. \end{aligned}$$

Therefore, the higher ω, the higher the expected first day return. This result is in agreement with the intuition presented by Ritter (1998) to why pre-IPO shareholders don't get upset when they see a large underprice:

"Bad news that a lot of money was left on the table arrived at the same time that the good news of high market price"

Since more valuable companies (High ω) usually have a higher first day return, a large underprice in a crowded IPO implies a high value to the shares kept by the pre-IPO shareholders. It also gives us an indication about the relationship between hot issue markets and technological shocks. Considering a technological shock as a jump in ω, this would imply an increase in the expected underprice, as advocate by some authors.

Finishing this example, it's easy to show that $R(\omega, N) = \left(\frac{N-1}{N+1}\right)\omega$ is concave in (ω, N). In this way, even if I consider that ω increases through time given VC's activity (changing managers, restructuring production, etc...), I still obtain the same results and the concave shape necessary for my previous results about optimal selling time. Graphically:



Where $\omega = 2{,}3$ and 4 in the black (solid), red (dot-dash) and green (dash) lines, respectively.$\square$

Concavity in general is not necessarily granted, especially given that there is a jump in revenue from the entrance of the second bidder in the auction[9]. However, it is easy to show that there is a cutoff number of bidders N^* such that for any $N \geq N^*$, $R(\omega, N)$ is concave in N :

$$N^* = \frac{\int_0^\omega [1 - Z(\varepsilon)]^{N^*-2} Z(\varepsilon)^2 \, d\varepsilon}{\int_0^\omega [1 - Z(\varepsilon)]^{N^*-2} Z(\varepsilon)^3 \, d\varepsilon}.$$

Up to now, I considered the number of bidders in a given IPO as constant. However, my intuition from the length of the road show and importance of good marketing skills by underwriters and VCs are related with an inflow of institutional investors. These players get to know the IPO firm and then decide to participate or not in the IPO. I will model this considering the arrival of potential buyers as a Poisson Process with average μ. Therefore, the number of investors that observed their valuations in an interval of length T is a random variable with Poisson distribution with parameter μT. The probability that an auction realized after a waiting time of length T has N bidders is $p_N(T) = \frac{e^{-\mu T}(\mu T)^N}{N!}$. I consider that all investors that where contacted and draw their ε will wait for the auction. This assumption does not affect qualitatively the results. A simple generalization would consider that investors could sample other opportunities and leave. It generates similar results since the investors that are more probable to stay waiting for the auction are the ones with low costs, and these are the agents important to my results. Therefore, the expected return of an auction realized after waiting T is given by:

[9] The introduction of reserve prices and some additional assumptions can mitigate this problem.

$$\sum_{N=2}^{\infty} p_N(T)\, R(\omega, N).$$

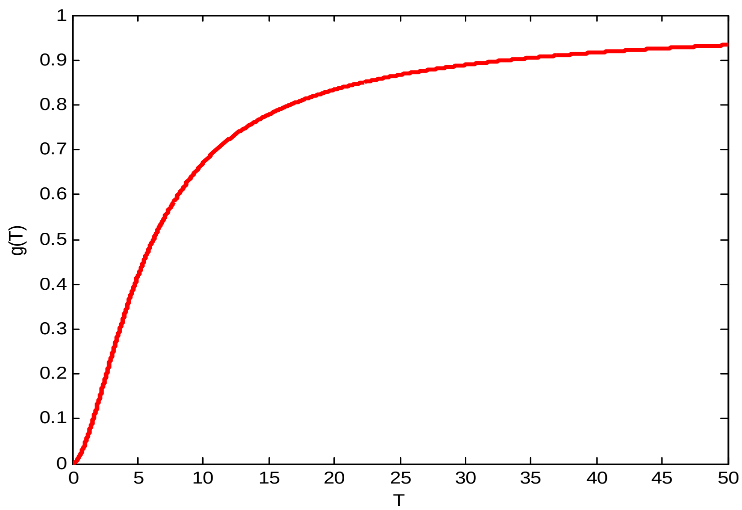
Manipulating it:

$$g(T) = \omega - \int_0^{\omega} [1 + \mu T Z(\varepsilon)]\, e^{-\mu T Z(\varepsilon)} d\varepsilon$$

It is easy to see that $g'(\cdot) > 0$ and $g''(T) < 0$, $\forall T \geq T^C$ given by:

$$T^C = \frac{\int_0^{\omega} \mu Z(\varepsilon)\, e^{-\mu T^C Z(\varepsilon)} d\varepsilon}{\int_0^{\omega} (\mu Z(\varepsilon))^2\, e^{-\mu T^C Z(\varepsilon)} d\varepsilon}$$

Plotting $g(T)$:



The initial convexity comes from the large impact on auction's expected revenue generated by the entry of a second potential buyer, since it introduces competition and raises the price from zero to a positive value. The introduction of reserve prices and assuming log concavity of the distribution $Z(\cdot)$ would help us to avoid the initial non-concavity in $g(T)$. Finally, I can see that my previous discussions on technological shocks in $g(T)$ could be modeled as jumps in ω.

6 Empirical Evidence

I will present now some evidence about the impact of days in registration and the age of the firm on first day return and therefore, underpricing. The number of days in registration is considered here as a proxy for the length of the road show, indicating the effort of sale and/or the expected number of potential bidders.

The source of my data is SDC Platinum, from which I look at US data on IPOs from 1984-2004, focusing on Common Stocks. I include as control variables the number of days in registration, the age at which the first investment was made (difference between year of foundation and year at the first investment), number of investment rounds, book value per share, book value before offer, market indexes at the IPO date and firm's age at the IPO. I also include in my analysis dummy variables that control for: sector in which the firm is, year in which the IPO was realized, market in which the IPO was realized.

Results from regression analysis with robust errors are presented in the table below. It show that Days in Registration have an impact on First Day Return that is always significative (consider $\alpha \leq 5\%$). The size varies between $[-0.11, -0.04]$, while age at the IPO has a negative and significant impact on first day return (size varies but it is usually big: around -0.4). These results are in agreement with my theory on IPOs: The higher the time until the IPO, the lower the amount of money left on the table for investors that buy at the initial public offer[10][11].

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SEE TABLE 3

Another empirical analysis that I present here comes from a nonlinear analysis using a Multivariable fractional polynomial model. The obtained results are presented below:

SEE TABLE 4

The adjustments in the variables are obtained after 3 iterations in the fractional polynomial fitting algorithm. Dummy variables are included in the estimation, although their results are omitted here. As I can see, again there is a negative impact of days in registration and age at the IPO in the first day return, showing that my intuition that the longer the venture capitalist keeps the firm/project, the lower is the amount of money left on the table for institutional investors.

As we mentioned before, these results are only indications in favor of my theory, showing that the correlations obtained in the data are in agreement to my results. Unfortunately, a deeper empirical

[10] As additional results we have that: Dummy for 1999 is the only year dummy consistently significant. It has a positive impact on first day return. Sector dummies have the expected signals from a asymmetric information claim (positive for high tech, negative for manufacture and health) but they are not significant in many cases. All other variables are usually not statistically significant at 5%

[11] I also included dummies to take into account the impact of price revisions on underpricing, as presented by Hanley (1993). Results are robust to this exercie.

analysis, with the estimation of more structured models is not possible since most data on the IPO process is not public, not being disclosed by investment banks for further analysis.

7 Conclusion

In this paper, I present a new theory about underpricing and VC-backed companies where the key feature is VC's capacity constraints, i.e., Venture Capital firms can only handle a limited number of projects.

I show that this theory can match not only the empirical evidence that VC firms take younger companies public and that younger VC firms take even younger firms public than their more mature counterparts, but it also presents as a nice framework to address additional features in the market, as the impact of technological waves, financial shocks on time to IPO, underpricing and average time spent by a VC in a given project. These predictions seem to be aligned to what was found by the empirical literature while studying hot market issues and the grandstanding hypothesis.

Finally, this model addresses the underpricing paradox by assuming a collusion between institutional investors and underwriter. However, differently from previous models, it looks at competition between institutional investor for underpriced shares as a way to pin down underpricing's magnitude and the expected return to an IPO given the time spent marketing it. My initial empirical evidence between time to IPO -measure by firm's age at the IPO and days in registration - and underpricing corroborates the predictions of the model.

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8 Appendix A

Claim A.1 $T_{\emptyset}^{*}$ is given by:

$$\frac{g'\left(T_{\emptyset}^{*}\right)}{g\left(T_{\emptyset}^{*}\right)}=r.$$

Proof: Notice that the VC must be indifferent between keeping the project or finishing it and opening a vacancy, i.e.:

$$\beta g\left(T_{\emptyset}^{*}\right)+\frac{1}{1+rdT}\left\{\lambda dTE_{\tilde{T}}V\left(\widetilde{T}\right)+\left(1-\lambda dT\right)V^{0}\right\}=\frac{1}{1+rdT}\left\{\begin{array}{c}\lambda dT\left[\beta g\left(T_{\emptyset}^{*}+dT\right)+E_{\tilde{T}}V\left(\widetilde{T}\right)\right]\\+\left(1-\lambda dT\right)\left[\beta g\left(T_{\emptyset}^{*}+dT\right)+V^{0}\right]\end{array}\right\}$$

Simplifying:

$$\begin{aligned}g\left(T_{\emptyset}^{*}\right)&=\frac{1}{1+rdT}g\left(T_{\emptyset}^{*}+dT\right)\\\left(1+rdT\right)g\left(T_{\emptyset}^{*}\right)&=g\left(T_{\emptyset}^{*}+dT\right)\\rdTg\left(T_{\emptyset}^{*}\right)&=g\left(T_{\emptyset}^{*}+dT\right)-g\left(T_{\emptyset}^{*}\right)\end{aligned}$$

Dividing by dT and taking $dT\rightarrow0$:

$$\frac{g'\left(T_{\emptyset}^{*}\right)}{g\left(T_{\emptyset}^{*}\right)}=r$$

■

Proof of Proposition 1

To obtain the proof of this proposition, we need to show a few intermediary steps:

Claim A.2 $V\left(T\right)$ is strictly increasing.

Proof: By now, assume that V is increasing in T. Then, I know that any project with starting age $T\geq T^{*}\left(T\right)$ will induce the termination of the current project. Therefore, I have that for $T<T^{*}\left(T\right)$:

$$rV\left(T\right)=\lambda\int_{T_{c}^{*}(T)}^{T_{\emptyset}^{*}}\left[V\left(\widetilde{S}\right)+\beta g\left(T\right)-c-V\left(T\right)\right]dF\left(\widetilde{S}\right)+\frac{dV\left(T\right)}{dT}\tag{12}$$

Rearranging and using (2):

$$rV(T) = \frac{dV(T)}{dT} + \lambda \int_{T^*(T)}^{T^*_\emptyset} \left[1 - F\left(\widetilde{S}\right)\right] \frac{dV\left(\widetilde{T}\right)}{d\widetilde{T}} d\widetilde{S} \tag{13}$$

If we solve this ODE:

$$V(T) = e^{-r\left(T^*_\emptyset - T\right)} \left[\beta g\left(T^*_\emptyset\right) + V^0\right] + \lambda \int_T^{T^*_\emptyset} e^{-r(s-T)} \int_{T^*_c(s)}^{T^*_\emptyset} \frac{dV\left(\widetilde{S}\right)}{d\widetilde{T}} \left[1 - F\left(\widetilde{S}\right)\right] d\widetilde{S} ds$$

Taking the derivative with respect to T and manipulating:

$$\frac{d^2V(T)}{dT^2} = \{r + \lambda [1 - F(T^*_c(T))]\} \frac{dV(T)}{dT} - \lambda [1 - F(T^*_c(T))] \beta g'(T)$$

Note that this is a first order ODE in $\frac{dV(T)}{dT}$.

For notational reasons, let's define:

$$\Psi(S) = r + \lambda [1 - F(T^*_c(S))]$$

Then, solving the ODE:

$$\frac{dV(T)}{dT} = e^{\int_{T_0}^T \Psi(s)ds} x_0 - \lambda \int_{T_0}^T e^{\int_s^T \Psi(z)dz} \beta g'(s) [1 - F(T^*_c(s))] ds$$

where T_0 is an initial condition. Using $T^*_\emptyset$ as a terminal condition and manipulating:

$$\frac{dV(T)}{dT} = e^{-\int_T^{T^*_\emptyset} \Psi(s)ds} \beta \left\{ g'\left(T^*_\emptyset\right) + \lambda \int_T^{T^*_\emptyset} e^{\int_s^{T^*_\emptyset} \Psi(z)dz} g'(s) [1 - F(T^*_c(s))] ds \right\} > 0$$

■

Claim A.3 $\frac{dV(T)}{dT} \leq \beta g'(T)$.

Proof: From previous calculations:

$$\frac{dV(T)}{dT} = e^{-\int_T^{T^*_\emptyset} r + \lambda[1-F(T^*_c(s))]ds} \beta g'\left(T^*_\emptyset\right) + \int_T^{T^*_\emptyset} \lambda [1 - F(T^*_c(s))] e^{-\int_T^s r + \lambda[1-F(T^*_c(z))]dz} \beta g'(s) ds$$

Using integration by parts:

$$\frac{dV(T)}{dT} = \beta g'(T) + \int_T^{T^*_\emptyset} e^{-\int_T^s r + \lambda[1-F(T^*(z))]dz} \beta \left[g''(s) - rg'(s)\right] ds$$

Since $g''(T) - rg'(T) < 0$, $\forall T \in \left[0, T^*_\emptyset\right]$, I must have $\frac{dV(T)}{dT} < \beta g'(T)$

■

Now, let's derive equation (3)

Claim A.4 $T_c^*(T)$ is implicitly defined by:

$$\beta g(T_c^*(T)) - c + \int_T^{T_c^*(T)} \int_S^{T_\emptyset^*} e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega} \beta \left[g''(z) - rg'(z)\right] dzdS = 0$$

Proof:

$$\begin{aligned} \frac{dV(S)}{dS} &= e^{-\int_S^{T_\emptyset^*} r+\lambda\left[1-F\left(T_\emptyset^*(z)\right)\right]dz} \beta g'\left(T_\emptyset^*\right) \\ &\quad +\lambda e^{-\int_S^{T_\emptyset^*} r+\lambda\left[1-F\left(T_\emptyset^*(z)\right)\right]dz} \int_S^{T_\emptyset^*} e^{\int_z^{T_\emptyset^*} r+\lambda\left[1-F\left(T_\emptyset^*(\omega)\right)\right]d\omega} g'(z)\left[1-F\left(T^*(z)\right)\right]dz \end{aligned}$$

Integrating both sides:

$$\begin{aligned} \int_T^{T_\emptyset^*} \frac{dV(S)}{dS} dS &= \int_T^{T_\emptyset^*} e^{-\int_S^{T_\emptyset^*} r+\lambda\left[1-F\left(T_\emptyset^*(z)\right)\right]dz} \beta g'\left(T_\emptyset^*\right) \\ &\quad +\lambda e^{-\int_S^{T_\emptyset^*} r+\lambda\left[1-F\left(T_\emptyset^*(z)\right)\right]dz} \int_S^{T_\emptyset^*} e^{\int_z^{T_\emptyset^*} r+\lambda\left[1-F\left(T_\emptyset^*(\omega)\right)\right]d\omega} g'(z)\left[1-F\left(T^*(z)\right)\right]dzdS \end{aligned}$$

Rearranging:

$$V\left(T_\emptyset^*\right) - V(T) = \left\{ \begin{array}{c} \beta g'\left(T_\emptyset^*\right) \int_T^{T_\emptyset^*} e^{-\int_S^{T_\emptyset^*} r+\lambda[1-F(T^*(z))]dz} dS \\ +\lambda \int_T^{T_\emptyset^*} \int_S^{T_\emptyset^*} e^{\int_z^S r+\lambda[1-F(T^*(\omega))]d\omega} \beta g'(z)\left[1-F\left(T^*(z)\right)\right]dzdS \end{array} \right\}$$

Now, looking at the second term on RHS:

$$\int_T^{T_\emptyset^*} \int_S^{T_\emptyset^*} e^{\int_z^S r+\lambda[1-F(T^*(\omega))]d\omega} \beta g'(z) \lambda \left[1-F\left(T^*(z)\right)\right]dzdS =$$
$$\left\{ \begin{array}{c} -\beta g\left(T_\emptyset^*\right) \int_T^{T_\emptyset^*} e^{-\int_S^{T_\emptyset^*} r+\lambda[1-F(T^*(\omega))]d\omega} dS + \beta g\left(T_\emptyset^*\right) - \beta g(T) \\ + \int_T^{T_\emptyset^*} \int_S^{T_\emptyset^*} e^{\int_z^S r+\lambda[1-F(T^*(\omega))]d\omega} \beta \left[g''(z) - rg'(z)\right]dzdS \end{array} \right\}$$

Substituting it back:

$$V\left(T_\emptyset^*\right) - V(T) = \left\{ \begin{array}{c} \beta g\left(T_\emptyset^*\right) - \beta g(T) \\ + \int_T^{T_\emptyset^*} \int_S^{T_\emptyset^*} e^{\int_z^S r+\lambda[1-F(T^*(\omega))]d\omega} \beta \left[g''(z) - rg'(z)\right]dzdS \end{array} \right\}$$

Substituting $V\left(T_\emptyset^*\right) = \beta g\left(T_\emptyset^*\right) + V^0$ and manipulating it:

$$V^0 - V(T) = -\beta g(T) + \int_T^{T_\emptyset^*} \int_S^{T_\emptyset^*} e^{\int_z^S r+\lambda[1-F(T^*(\omega))]d\omega} \beta \left[g''(z) - rg'(z)\right] dzdS$$

$$V(T)=\beta g(T)+V^{0}-\int_{T}^{T_{\emptyset}^{*}}\int_{S}^{T_{\emptyset}^{*}}e^{\int_{z}^{S}r+\lambda[1-F(T^{*}(\omega))]d\omega}\beta\left[g''(z)-rg'(z)\right]dzdS \qquad (A.1)$$

Only a simple remark, considering $V(T_c^{*}(T))+\beta g(T)-c=V(T)$:

$$V(T_c^{*}(T))=\left\{\begin{array}{c}V^{0}+c\\ -\int_{T}^{T_{\emptyset}^{*}}\int_{S}^{T_{\emptyset}^{*}}e^{\int_{z}^{S}r+\lambda[1-F(T^{*}(\omega))]d\omega}\beta\left[g''(z)-rg'(z)\right]dzdS\end{array}\right\} \qquad (A.2)$$

From $(A.1)$, taking the Bellman at $T_c^{*}(T)$:

$$V(T_c^{*}(T))=\left\{\begin{array}{c}\beta g(T_c^{*}(T))+V^{0}\\ -\int_{T_c^{*}(T)}^{T_{\emptyset}^{*}}\int_{S}^{T_{\emptyset}^{*}}e^{\int_{z}^{S}r+\lambda[1-F(T_c^{*}(\omega))]d\omega}\beta\left[g''(z)-rg'(z)\right]dzdS\end{array}\right\} \qquad (A.1')$$

From $(A.1')$ and $(A.2)$:

$$\begin{array}{rl} & \left\{\begin{array}{c}\beta g(T_c^{*}(T))\\ -\int_{T_c^{*}(T)}^{T_{\emptyset}^{*}}\int_{S}^{T_{\emptyset}^{*}}e^{\int_{z}^{S}r+\lambda[1-F(T_c^{*}(\omega))]d\omega}\beta\left[g''(z)-rg'(z)\right]dzdS\end{array}\right\}\\ = & c-\int_{T}^{T_{\emptyset}^{*}}\int_{S}^{T_{\emptyset}^{*}}e^{\int_{z}^{S}r+\lambda[1-F(T^{*}(\omega))]d\omega}\beta\left[g''(z)-rg'(z)\right]dzdS\end{array}$$

Rearranging:

$$\beta g(T_c^{*}(T))=c-\int_{T}^{T_c^{*}(T)}\int_{S}^{T_{\emptyset}^{*}}e^{-\int_{S}^{z}r+\lambda[1-F(T_c^{*}(\omega))]d\omega}\beta\left[g''(z)-rg'(z)\right]dzdS$$

This expression implicitly defines $T_c^{*}(T)$. ■

From this expression, I obtain several of features presented in Proposition 1. First of all, together with previous claims and equation (2), I can clearly see that the optimal choice will be threshold, in which any quality above $T_c^{*}(T)$ would be accepted. To see it is decreasing, I only need to apply implicit function theorem in (3). Then:

$$\frac{dT_c^{*}(T)}{dT}=\frac{\int_{T}^{T_{\emptyset}^{*}}e^{-\int_{T}^{z}r+\lambda[1-F(T_c^{*}(\omega))]d\omega}\beta\left[g''(z)-rg'(z)\right]dzdS}{\left\{\begin{array}{c}\beta g'\left(T_{\emptyset}^{*}\right)e^{-\int_{T_c^{*}(T)}^{T_{\emptyset}^{*}}r+\lambda[1-F(T_c^{*}(\omega))]d\omega}\\ +\int_{T_c^{*}(T)}^{T_{\emptyset}^{*}}\lambda\left[1-F\left(T_c^{*}(z)\right)\right]e^{-\int_{T_c^{*}(T)}^{z}r+\lambda[1-F(T_c^{*}(\omega))]d\omega}\beta g'(z)\,dz\end{array}\right\}}<0.$$

in similar ways, I obtain results on changes in cost and market tightness:

$$\frac{dT_c^*(T)}{dc} = \frac{1}{\left\{ \begin{array}{c} \beta g'\left(T_\emptyset^*\right) e^{-\int_{T_c^*(T)}^{T_\emptyset^*} r+\lambda[1-F(T_c^*(\omega))]d\omega} \\ +\int_{T_c^*(T)}^{T_\emptyset^*} \lambda\left[1-F\left(T_c^*(z)\right)\right] e^{-\int_{T_c^*(T)}^{z} r+\lambda[1-F(T_c^*(\omega))]d\omega} \beta g'(z)\, dz \end{array} \right\}} > 0,\ \forall T$$

and

$$\begin{aligned} \frac{dT_c^*(T)}{d\lambda} &= -\frac{\int_T^{T_c^*(T)} \int_S^{T_\emptyset^*} e^{\int_z^S r+\lambda[1-F(T_c^*(\omega))]d\omega} \int_z^S \left[1-F\left(T_c^*(\omega)\right)\right] d\omega \beta\left[g''(z)-rg'(z)\right] dz dS}{\beta g'\left(T_\emptyset^*\right) e^{-\int_{T_c^*(T)}^{T_\emptyset^*} r+\lambda[1-F(T_c^*(\omega))]d\omega} + \int_{T_c^*(T)}^{T_\emptyset^*} \lambda\left[1-F\left(T_c^*(z)\right)\right] \beta g'(z)\, dz} \\ &= \frac{\int_T^{T_c^*(T)} \int_S^{T_\emptyset^*} e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega} \int_S^z \left[1-F\left(T_c^*(\omega)\right)\right] d\omega \beta\left[g''(z)-rg'(z)\right] dz dS}{\beta g'\left(T_\emptyset^*\right) e^{-\int_{T_c^*(T)}^{T_\emptyset^*} r+\lambda[1-F(T_c^*(\omega))]d\omega} + \int_{T_c^*(T)}^{T_\emptyset^*} \lambda\left[1-F\left(T_c^*(z)\right)\right] \beta g'(z)\, dz} \end{aligned}$$

Notice that, if $T < T_c^*(T)$, we have that $\frac{dT_c^*(T)}{d\lambda} < 0$, while if $T > T_c^*(T)$, I have that $\frac{dT_c^*(T)}{d\lambda} > 0$. Finally, looking at interest rates:

$$\beta g\left(T_c^*(T)\right) - c + \int_T^{T_c^*(T)} \int_S^{T_\emptyset^*} e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega} \beta\left[g''(z) - rg'(z)\right] dz dS = 0$$

$$\begin{aligned} dr \quad : \quad & -\int_T^{T_c^*(T)} \int_S^{T_\emptyset^*} e^{-\int_S^z \{r+\lambda[1-F(T_c^*(\omega))]\}d\omega} \beta g'(z)\, dz dS \\ & -\int_T^{T_c^*(T)} \int_S^{T_\emptyset^*} (z-S)\, e^{-\int_S^z \lambda[1-F(T_c^*(\omega))]d\omega} e^{-r(z-S)} \beta\left[g''(z)-rg'(z)\right] dz dS \\ & +\int_T^{T_c^*(T)} e^{-\int_S^{T_\emptyset^*} \{r+\lambda[1-F(T_c^*(\omega))]\}d\omega} \beta g'\left(T_\emptyset^*\right) \frac{dT_\emptyset^*}{dr} dS \end{aligned}$$

where $\frac{dT_\emptyset^*}{dr} = \frac{g\left(T_\emptyset^*\right)^2}{g''\left(T_\emptyset^*\right) g\left(T_\emptyset^*\right) - g'\left(T_\emptyset^*\right)^2} < 0.$

Let's take the second integral and work it out:

$$-\int_T^{T_c^*(T)}\int_S^{T_\emptyset^*} \underbrace{ze^{-\int_S^z \lambda[1-F(T_c^*(\omega))]d\omega}}\underbrace{e^{-r(z-S)}\beta\left[g''(z)-rg'(z)\right]}_{dv} dzdS =_{I.P.}$$

$$= -T_\emptyset^*\beta g'\left(T_\emptyset^*\right)\int_T^{T_c^*(T)} e^{-\int_S^{T_\emptyset^*} r+\lambda[1-F(T_c^*(\omega))]d\omega} dS + \int_T^{T_c^*(T)} S\beta g'(S)\,dS$$
$$+\int_T^{T_c^*(T)}\int_S^{T_\emptyset^*} e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega}\beta g'(z)\,dzdS$$
$$-\int_T^{T_c^*(T)}\int_S^{T_\emptyset^*} \lambda\left[1-F\left(T_c^*(z)\right)\right] ze^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega}\beta g'(z)\,dzdS$$

while

$$\int_T^{T_c^*(T)} S\int_S^{T_\emptyset^*} e^{-\int_S^z \lambda[1-F(T_c^*(\omega))]d\omega} e^{-r(z-S)}\left[\beta g''(z)-r\beta g'(z)\right]dzdS =$$
$$= \beta g'\left(T_\emptyset^*\right)\int_T^{T_c^*(T)} Se^{-\int_S^{T_\emptyset^*} r+\lambda[1-F(T_c^*(\omega))]d\omega} dS - \int_T^{T_c^*(T)} Sg'(S)\,dS$$
$$+\int_T^{T_c^*(T)} S\int_S^{T_\emptyset^*} \lambda\left[1-F\left(T_c^*(z)\right)\right] e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega}\beta g'(z)\,dzdS$$

Putting things together:

$$\begin{aligned}
& -\int_T^{T_c^*(T)}\int_S^{T_\emptyset^*} (z-S)\, e^{-\int_S^z \lambda[1-F(T_c^*(\omega))]d\omega} e^{-r(z-S)}\beta\left[g''(z)-rg'(z)\right]dzdS \\
= \quad & -\beta g'\left(T_\emptyset^*\right)\int_T^{T_c^*(T)} \left(T_\emptyset^*-S\right) e^{-\int_S^{T_\emptyset^*} r+\lambda[1-F(T_c^*(\omega))]d\omega} dS \\
& +\int_T^{T_c^*(T)}\int_S^{T_\emptyset^*} e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega}\beta g'(z)\,dzdS \\
& -\int_T^{T_c^*(T)}\int_S^{T_\emptyset^*} \lambda\left[1-F\left(T_c^*(z)\right)\right](z-S)\, e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega}\beta g'(z)\,dzdS
\end{aligned}$$

Now, substituting it back into dr:

$$dr:\ -\beta g'\left(T_\emptyset^*\right)\int_T^{T_c^*(T)} \left(T_\emptyset^*-S\right) e^{-\int_S^{T_\emptyset^*} r+\lambda[1-F(T_c^*(\omega))]d\omega} dS$$
$$-\int_T^{T_c^*(T)}\int_S^{T_\emptyset^*} \lambda\left[1-F\left(T_c^*(z)\right)\right](z-S)\, e^{-\int_S^z r+\lambda[1-F(T_c^*(\omega))]d\omega}\beta g'(z)\,dzdS$$
$$+\int_T^{T_c^*(T)} e^{-\int_S^{T_\emptyset^*}\{r+\lambda[1-F(T_c^*(\omega))]\}d\omega}\beta g'\left(T_\emptyset^*\right)\frac{dT_\emptyset^*}{dr}dS$$

Notice that $dr < 0$ if $T_c^*(T) > T$ and $dr > 0$ if $T > T_c^*(T)$. Since $dT_c^*(T) > 0$:

$$\frac{\partial T_c^*(T)}{\partial r} > 0 \text{ if } T < T_c^*(T)$$

and

$$\frac{\partial T_c^*(T)}{\partial r} < 0 \text{ if } T > T_c^*(T)$$

To obtain the result on $c = 0$, I just need to use the fact that $T_c^*(T)$ is decreasing, Claims A.2 and A.3 and the fact that $T_c^*(T)$ crosses the 45 degree line. Then, I can see that if $c = 0$ $T_c^*(T)$ crosses the 45 degree line at zero. Since it is decreasing, it must be zero everywhere. ■

Proof of Lemma 1:

Proof: First of all, I must have in mind that this is the optimal decision of entering or not which is taken by the entrepreneur. Therefore, he takes λ and η as given. Then, remember that the value of a partnership for the entrepreneur with a project of size T is given by

$$rP(T) = \lambda\{(1-\beta)g(T) - P(T)\} + \frac{dP(T)}{dT} \tag{14}$$

solving the ODE and remembering that:

$$P\left(T_\emptyset^*\right) = (1-\beta)g\left(T_\emptyset^*\right)$$

I have:

$$P(T) = (1-\beta)g\left(T_\emptyset^*\right)e^{-(r+\lambda)\left(T_\emptyset^*-T\right)} + \lambda\int_T^{T_\emptyset^*}(1-\beta)g(s)e^{-(r+\lambda)(s-T)}ds$$

solving the integral and rearranging,

$$P(T) = \frac{(1-\beta)}{r+\lambda}\left\{\lambda\left[g(T) + \int_T^{T_\emptyset^*}g'(s)e^{-(r+\lambda)(s-T)}ds\right] + rg\left(T_\emptyset^*\right)e^{-(r+\lambda)\left(T_\emptyset^*-T\right)}\right\}$$

Finally, let's consider the value function of a entrepreneur that is in the VC's market searching for a partner. Since the entrepreneur would accept any VC and would also be accepted by any of them, I would have:

$$S(S) = \frac{\eta}{(r+\eta)}P(S) \tag{15}$$

Then, substituting $P(S)$:

$$S(S) = \frac{\eta(1-\beta)}{(r+\lambda)(r+\eta)}\left\{\lambda\left[g(S) + \int_S^{T_\emptyset^*} g'(x)\, e^{-(r+\lambda)(x-S)}dx\right] + rg\left(T_\emptyset^*\right)e^{-(r+\lambda)\left(T_\emptyset^*-S\right)}\right\} \tag{16}$$

Then, an seller would enter the broker's market after getting a project of initial quality S, if and only if:

$$S(S) \geq A(S)$$

i.e.,

$$\frac{\eta(1-\beta)}{(r+\lambda)(r+\eta)}\left\{g\left(T_\emptyset^*\right)e^{-(r+\lambda)\left(T_\emptyset^*-S\right)} + \lambda\int_S^{T_\emptyset^*} g(x)\, e^{-(r+\lambda)(x-S)}dx\right\} \geq e^{-r\left(T_\emptyset^*-S\right)}\gamma g\left(T_\emptyset^*\right)$$

Then, rearranging the above expression:

$$e^{\lambda S}\left\{g\left(T_\emptyset^*\right)e^{-(r+\lambda)T_\emptyset^*} + \lambda\int_S^{T_\emptyset^*} g(x)\, e^{-(r+\lambda)x}dx\right\} \geq \frac{(r+\eta)\gamma}{\eta(1-\beta)}g\left(T_\emptyset^*\right)e^{-rT_\emptyset^*} \tag{$\bigstar$}$$

Take the derivative of each term:

$$\frac{d}{dS}\left\{e^{\lambda S}g\left(T_\emptyset^*\right)e^{-(r+\lambda)T_\emptyset^*}\right\} = \lambda e^{\lambda S}g\left(T_\emptyset^*\right)e^{-(r+\lambda)T_\emptyset^*} > 0.$$

$$\begin{aligned}
\frac{d}{dS}\left\{\lambda e^{\lambda S}\int_S^{T_\emptyset^*} g(x)\, e^{-(r+\lambda)x}dx\right\} &= \lambda^2 e^{\lambda S}\int_S^{T_\emptyset^*} g(x)\, e^{-(r+\lambda)x}dx - \lambda e^{-rS}g(S) \\
&= \lambda\left\{-e^{-rS}g(S) + \lambda e^{\lambda T}\int_S^{T_\emptyset^*} g(x)\, e^{-(r+\lambda)x}dx\right\}
\end{aligned}$$

Note:

$$\lambda e^{\lambda S}\int_S^{T_\emptyset^*} g(x)\, e^{-(r+\lambda)x}dx =_{I.P.} \frac{\lambda e^{\lambda S}}{r+\lambda}\left\{\begin{array}{c} -g\left(T_\emptyset^*\right)e^{-(r+\lambda)T_\emptyset^*} + \\ g(S)\, e^{-(r+\lambda)S} + \int_S^{T_\emptyset^*} g'(x)\, e^{-(r+\lambda)x}dx \end{array}\right\}$$

Now, assume that $T_\emptyset^* > 0$[12], this implies that $g'(S) - rg(S) \geq 0$ (being $= 0$ iff $S = T_\emptyset^*$). Then, $g'(S) \geq rg(S)$. Using this:

$$\lambda e^{\lambda S}\int_S^{T_\emptyset^*} g(x)\, e^{-(r+\lambda)x}dx \geq \frac{\lambda e^{\lambda S}}{r+\lambda}\left\{\begin{array}{c} -g\left(T_\emptyset^*\right)e^{-(r+\lambda)T_\emptyset^*} + \\ g(S)\, e^{-(r+\lambda)S} + r\int_S^{T_\emptyset^*} g(x)\, e^{-(r+\lambda)x}dx \end{array}\right\}$$

[12] Note that we already assumed this when we wrote $P(S)$

rearranging:

$$\lambda \int_{S}^{T_{\emptyset}^{*}} g(x)\, e^{-(r+\lambda)x} dx \geq -g\left(T_{\emptyset}^{*}\right) e^{-(r+\lambda)T_{\emptyset}^{*}} + g(S)\, e^{-(r+\lambda)S}$$

Substituting this back:

$$\frac{d}{dS}\left\{\lambda e^{S} \int_{S}^{T_{\emptyset}^{*}} g(x)\, e^{-(r+\lambda)x} dx\right\} \geq \lambda\left\{-e^{-rS} g(S) + g(S)\, e^{-rS} - g\left(T_{\emptyset}^{*}\right) e^{-(r+\lambda)T_{\emptyset}^{*}} e^{\lambda S}\right\}$$

rearranging:

$$\frac{d}{dS}\left\{\lambda e^{\lambda T} \int_{T}^{T_{\emptyset}^{*}} g(s)\, e^{-(r+\lambda)s} ds\right\} \geq -\lambda e^{\lambda T} g(T)\, e^{-(r+\lambda)T}$$

Then, putting everything together:

$$\frac{dLHS}{dS} \geq \lambda e^{\lambda S} g\left(T_{\emptyset}^{*}\right) e^{-(r+\lambda)T_{\emptyset}^{*}} - \lambda e^{\lambda S} g(S)\, e^{-(r+\lambda)S} = 0.$$

■

Proof of Proposition 2:

To prove this proposition, I will present a sequence of claims:

Claim A.5 $\frac{\partial T^{\star}}{\partial n} > 0$, i.e., the more the number of new projects in the hands of entrepreneurs, better the minimum quality project that decides to look for a VC;

Proof: From a steady state perspective:

$$n\left[1 - H\left(T^{\star}\right)\right] = \eta u^{*}$$

since $\eta = \frac{m(u^{*},v)}{u^{*}}$:

$$n\left[1 - H\left(T^{\star}\right)\right] = m\left(u^{*}, v\right)$$

Since I am assuming that v is a constant, I can define $q(u) \equiv m(u; v)$. Then:

$$u^{*} = q^{-1}\left(n\left[1 - H\left(T^{\star}\right)\right]\right)$$

I will now derive the pieces to apply implicit function theorem. From the above expression:

$$\frac{du^*}{dT^\bigstar} = \frac{-h\left(T^\bigstar\right)n}{q'\left(q^{-1}\left(n\left(1-H\left(T^\bigstar\right)\right)\right)\right)} < 0.$$

and

$$\frac{du^*}{dn} = \frac{\left(1-H\left(T^\bigstar\right)\right)}{q'\left(q^{-1}\left(n\left(1-H\left(T^\bigstar\right)\right)\right)\right)} > 0$$

Then:

$$\begin{aligned}\frac{\partial\eta}{\partial T^\bigstar} &= \frac{du^*}{dT^\bigstar}\left\{\frac{m_u\left(u^*,v\right)u^* - m\left(u^*,v\right)}{u^{*2}}\right\} \\ &= \frac{du^*}{dT^\bigstar}\left\{\frac{-m_v\left(u^*,v\right)v}{u^{*2}}\right\} > 0\end{aligned}$$

where I used the Euler Theorem from the first to the second line.

$$\frac{\partial\lambda}{\partial T^\bigstar} = -\frac{nh\left(S^\bigstar\right)}{v} < 0$$

Also applying the Euler Theorem, we obtain:

$$\frac{\partial\eta}{dn} = -\frac{du^*}{dn} * \frac{m_v\left(u^*,v\right)}{u^*}\frac{v}{u^*} < 0$$

$$\frac{\partial\lambda}{\partial n} = \frac{\left(1-H\left(T^\bigstar\right)\right)}{v} > 0$$

Then, let's obtain the sign of $\frac{dT^\bigstar}{dn}$ by IFT. Then:

$$-dn : \left\{\begin{array}{c} e^{\lambda T^\bigstar}\frac{\partial\lambda}{\partial n}\int_{T^\bigstar}^{T_\emptyset^*}\left[g'\left(x\right)-rg\left(x\right)\right]e^{-(r+\lambda)x}\left(x-T^\bigstar\right)dx \\ -\frac{\partial\eta}{\partial n}\frac{r}{\eta^2}\frac{\gamma}{1-\beta}g\left(T_\emptyset^*\right)e^{-rT_\emptyset^*}\end{array}\right\} > 0$$

$$dT^\bigstar : \left\{\begin{array}{c} e^{\lambda T^\bigstar}\int_{T^\bigstar}^{T_\emptyset^*}\left[g'\left(x\right)-rg\left(x\right)\right]e^{-(r+\lambda)x}\left(\lambda-\frac{\partial\lambda}{\partial T^\bigstar}\left(x-T^\bigstar\right)\right)dx \\ +\frac{\partial\eta}{\partial T^\bigstar}\frac{r}{\eta^2}\frac{\gamma}{1-\beta}g\left(T_\emptyset^*\right)e^{-rT_\emptyset^*}\end{array}\right\} > 0$$

Then:

$$\frac{\partial T^\bigstar}{\partial n} = \frac{-dn}{dT^\bigstar} > 0$$

■

Claim A.6 $\frac{d\lambda}{dn} > 0$, therefore the expected time a VC spends with a project $\left(\frac{1}{\lambda}\right)$ decreases as n increases

Proof: Remember that:

$$\lambda = n\left[1 - H\left(S\right)\right]$$

Then, let's look at the total differentiation of λ with respect to n. Then:

$$d\lambda = \left(1 - H\left(T^{\bigstar}\right)\right) - nh\left(T^{\bigstar}\right)\frac{dT^{\bigstar}}{dn}$$

putting $\left(1 - H\left(T^{\bigstar}\right)\right)$ in evidence and manipulating it:

$$\frac{d\lambda}{dn} = \left(1 - H\left(T^{\bigstar}\right)\right)\frac{\lambda e^{\lambda T^{\bigstar}}\int_{T^{\bigstar}}^{T_{\emptyset}^{*}}\left[g'\left(x\right) - rg\left(x\right)\right]e^{-(r+\lambda)x}dx}{\left\{\begin{array}{c} e^{\lambda T^{\bigstar}}\int_{T^{\bigstar}}^{T_{\emptyset}^{*}}\left[g'\left(x\right) - rg\left(x\right)\right]e^{-(r+\lambda)x}\left(\lambda - \frac{\partial\lambda}{\partial T^{\bigstar}}\left(x - T^{\bigstar}\right)\right)dx \\ +\frac{\partial\eta}{\partial T^{\bigstar}}\frac{r}{\eta^{2}}\frac{\gamma}{1-\beta}g\left(T_{\emptyset}^{*}\right)e^{-rT_{\emptyset}^{*}} \end{array}\right\}} > 0.$$

■

Similar calculations show that $\frac{d\eta}{dn} < 0$, which concludes my proof.■

Proof of Lemma 2:

Proof: Solving the ODE and using the terminal condition:

$$P\left(T\right) = \left(1 - \beta\right)g\left(T\right) + \int_{T}^{T_{\emptyset}^{*}}\left(1 - \beta\right)\left[g'\left(s\right) - rg\left(s\right)\right]e^{-\int_{T}^{s}\Psi(z)dz}.ds$$

Then:

$$\frac{dP\left(T\right)}{dT} = \left\{r\left(1 - \beta\right)g\left(T\right) + \Psi\left(T\right)\int_{T}^{T_{\emptyset}^{*}}\left(1 - \beta\right)\left[g'\left(s\right) - rg\left(s\right)\right]e^{-\int_{T}^{s}\Psi(z)dz}.ds\right\} > 0$$

Since $g'\left(s\right) - rg\left(s\right),\ \forall s \in \left[0, T_{\emptyset}^{*}\right]$. Then:

$$S\left(T\right) = \frac{\eta\left[1 - F\left(T^{*-1}\left(T\right)\right)\right]}{r + \eta\left[1 - F\left(T^{*-1}\left(T\right)\right)\right]}P\left(T\right)$$

As we know, a entrepreneur enters the VC market if:

$$S\left(T\right) \geq A\left(T\right)$$

which means:

$$\frac{\eta\left[1 - F\left(T^{*-1}\left(T\right)\right)\right]}{r + \eta\left[1 - F\left(T^{*-1}\left(T\right)\right)\right]}e^{-rT}P\left(T\right) \geq e^{-rT_{\emptyset}^{*}}\gamma g\left(T_{\emptyset}^{*}\right)$$

Then:

$$\frac{dLHS}{dT} = \left\{ \begin{array}{c} -\frac{r\eta f\left(T^{*-1}(T)\right)\frac{dT^{*-1}(T)}{dT}}{\{r+\eta[1-F(T^{*-1}(T))]\}^2} e^{-rT} P\left(T\right) \\ +\frac{\eta\left[1-F\left(T^{*-1}(T)\right)\right]}{r+\eta[1-F(T^{*-1}(T))]} e^{-rT}\left[-rP\left(T\right)+\frac{dP(T)}{dT}\right] \end{array} \right\}$$

notice that:

$$\frac{dP\left(T\right)}{dT} - rP\left(T\right) = \left[\lambda\left(1-F\left(T^*\left(T\right)\right)\right)\right] \int_T^{T^*_\emptyset} \left(1-\beta\right)\left[g'\left(s\right) - rg\left(s\right)\right] e^{-\int_T^s \Psi(z)dz}.ds > 0$$

Since $\frac{dT^{*-1}(T)}{dT} < 0$, we have that $\frac{dLHS}{dT} \geq 0$. Therefore, there exists a threshold $T^{\bigstar}\left(c\right)$ such that if $T \geq T^{\bigstar}\left(c\right)$ the entrepreneur enters the VC market. ■

Proof of Claim 1:

Proof: Using induction. We can easily show that it increases from $N = 1$ to $N = 2$. Now consider a given $N \geq 2$. For $N + 1$:

$$\omega - \left(N+1\right)\int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^N Z\left(\varepsilon\right) d\varepsilon - \int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^{N+1} d\varepsilon$$

while for N it was already seen:

$$\omega - N\int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^{N-1} Z\left(\varepsilon\right) d\varepsilon - \int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^N d\varepsilon$$

Therefore, the change in expected return as we increased the number of bidders in 1 is:

$$\begin{aligned} & N\int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^{N-1} Z\left(\varepsilon\right)^2 d\varepsilon + \int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^{N+1} d\varepsilon - \int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^{N+1} d\varepsilon \\ = \; & N\int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^{N-1} Z\left(\varepsilon\right)^2 d\varepsilon > 0 \end{aligned}$$

■

Proof of Claim 2:

Proof:

$$\lim_{N\to\infty} \omega - N\int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^{N-1} Z\left(\varepsilon\right) d\varepsilon - \int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^N d\varepsilon$$

notice that:

$$\lim_{N\to\infty} \int_0^\omega \left[1-Z\left(\varepsilon\right)\right]^N d\varepsilon = 0$$

Consider that there exists $z'\left(\varepsilon\right)$ and $z\left(\cdot\right) > 0$, then:

$$
\begin{aligned}
-N\int_0^\omega [1-Z(\varepsilon)]^{N-1} Z(\varepsilon)\,d\varepsilon &= \int_0^\omega N[1-Z(\varepsilon)]^{N-1}(-z(\varepsilon))\frac{Z(\varepsilon)}{z(\varepsilon)}d\varepsilon \\
&= -\int_0^\omega [1-Z(\varepsilon)]^N \left[1-\frac{Z(\varepsilon)\,z'(\varepsilon)}{z(\varepsilon)^2}\right] d\varepsilon
\end{aligned}
$$

But then, it is easy to see that this expression goes to zero as $N \to \infty$. Therefore, expected value converges to ω.■

9 Appendix B

In this appendix, I will consider the case in which the Venture Capital can handle two projects at the same time. In this case, her Bellman function is given by:

$$
\begin{aligned}
(1+rdT)\,V\left(T_1,T_2\right) &= \lambda dT E_{\widetilde{T}} \max\left\{\begin{array}{c} V\left(\widetilde{T},T_2+dT\right)+\beta g\left(T_1+dT\right)-c, \\ V\left(T_1+dT,\widetilde{T}\right)+\beta g\left(T_2+dT\right)-c, \\ V\left(T_1+dT,T_2+dT\right) \end{array}\right\} \\
&\quad +(1-\lambda dT)\max\left\{\begin{array}{c} V\left(\emptyset,T_2+dT\right)+\beta g\left(T_1+dT\right), \\ V\left(T_1+dT,\emptyset\right)+\beta g\left(T_2+dT\right), \\ V\left(\emptyset,\emptyset\right)+\beta g\left(T_1+dT\right)+\beta g\left(T_2+dT\right), \\ V\left(T_1+dT,T_2+dT\right) \end{array}\right\}
\end{aligned}
$$

Then, considering $\max\left\{T_1,T_2\right\}<T_{\emptyset}^{*}$:

$$
\begin{aligned}
(1+rdT)\,V\left(T_1,T_2\right) &= \lambda dT E_{\widetilde{T}} \max\left\{\begin{array}{c} V\left(\widetilde{T},T_2+dT\right)+\beta g\left(T_1+dT\right)-c, \\ V\left(T_1+dT,\widetilde{T}\right)+\beta g\left(T_2+dT\right)-c, \\ V\left(T_1+dT,T_2+dT\right) \end{array}\right\} \\
&\quad +(1-\lambda dT)\,V\left(T_1+dT,T_2+dT\right)
\end{aligned}
$$

Rearranging:

$$
\begin{aligned}
rdTV\left(T_1,T_2\right) &= \lambda dT E_{\widetilde{T}} \max\left\{\begin{array}{c} \left[\begin{array}{c} V\left(\widetilde{T},T_2+dT\right)+\beta g\left(T_1+dT\right) \\ -c-V\left(T_1+dT,T_2+dT\right) \end{array}\right], \\ \left[\begin{array}{c} V\left(T_1+dT,\widetilde{T}\right)+\beta g\left(T_2+dT\right) \\ -c-V\left(T_1+dT,T_2+dT\right) \end{array}\right], \\ 0 \end{array}\right\} \\
&\quad +V\left(T_1+dT,T_2+dT\right)-V\left(T_1,T_2\right)
\end{aligned}
$$

Then, dividing by dT and taking $dT\to 0$:

$$rV(T_1,T_2) = \lambda E_{\widetilde{T}} \max \left\{ \begin{array}{c} \left[\begin{array}{c} V\left(\widetilde{T},T_2\right) + \beta g(T_1) \\ -c - V(T_1,T_2) \end{array} \right], \\ \left[\begin{array}{c} V\left(T_1,\widetilde{T}\right) + \beta g(T_2) \\ -c - V(T_1,T_2) \end{array} \right], \\ 0 \end{array} \right\} + \left[\frac{\partial V}{\partial T_1}(T_1,T_2) + \frac{\partial V}{\partial T_2}(T_1,T_2) \right]$$

Then, looking at the cut off rule, let's analyze the conditions in which I have:

$$V\left(\widetilde{T},T_2\right) + \beta g(T_1) - c - V(T_1,T_2) \geq V\left(T_1,\widetilde{T}\right) + \beta g(T_2) - c - V(T_1,T_2)$$

Simplifying:

$$V\left(\widetilde{T},T_2\right) + \beta g(T_1) \geq V\left(T_1,\widetilde{T}\right) + \beta g(T_2)$$

First of all, consider a symmetry condition:

$$V\left(\widetilde{T},T_2\right) = V\left(T_2,\widetilde{T}\right)$$

Then, rearranging:

$$[\beta g(T_1) - \beta g(T_2)] - \left[V\left(T_1,\widetilde{T}\right) - V\left(T_2,\widetilde{T}\right)\right] \geq 0$$

Therefore, since both functions are increasing (as we showed before), I must have the increase in $\beta g(\cdot)$ higher than the increase in $V\left(\cdot,\widetilde{T}\right)$ as T increases. Taking $T_1 = T_2 + dT$, dividing by dT and taking $dT \to 0$:

$$\beta g'(T_2) - \frac{\partial V}{\partial T_1}\left(T_2,\widetilde{T}\right) \geq 0$$

which is exactly what I showed previously. Therefore, if the VC finishes one project to undertake a new one, she chooses the bigger one. Then, there is a decision about cutting a project or not: Imagining again that T_1 is the bigger one,

$$V\left(\widetilde{T},T_2\right)+\beta g\left(T_1\right)-c\geq V\left(T_1,T_2\right)$$

I will again have a $T_c^*\left(T_1\right)$, in which the only contribution of T_2 is that $T_2\leq T_1$.

Let's consider now calculations related to the expected time until an IPO. First of all, notice that in the usual case in which a Venture Capital firm can only handle one project, the survival rate of a project of size/quality T is given by:

$$S\left(t\left|T\right.\right)=\frac{e^{-\lambda\int_0^t[1-F(T^*(T+s))]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}[1-F(T^*(T+s))]ds}},\quad \text{for } t\in\left[0,T_\emptyset^*-T\right]$$

then the hazard rate conditional on T is:

$$\hbar\left(t\left|T\right.\right)=-\frac{S'\left(t\left|T\right.\right)}{S\left(t\left|T\right.\right)}=\lambda\left[1-F\left(T^*\left(T+t\right)\right)\right].$$

and the lifetime distribution, conditional on T is:

$$K\left(t\left|T\right.\right)=1-S\left(t\left|T\right.\right)=1-\frac{e^{-\lambda\int_0^t[1-F(T^*(T+s))]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}[1-F(T^*(T+s))]ds}}$$

which implies:

$$k\left(t\left|T\right.\right)=\lambda\left[1-F\left(T^*\left(T+t\right)\right)\right]\frac{e^{-\lambda\int_0^t[1-F(T^*(T+s))]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}[1-F(T^*(T+s))]ds}}$$

Then the expected lifetime of a partnership given a starting size/quality T is:

$$\frac{\int_0^{T_\emptyset^*-T}e^{-\lambda\int_0^t[1-F(T^*(T+s))]ds}dt-\left(T_\emptyset^*-T\right)e^{-\lambda\int_0^{T_\emptyset^*-T}[1-F(T^*(T+s))]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}[1-F(T^*(T+s))]ds}}$$

Then the expected lifetime of a partnership is given by:

$$\int_0^{T_\emptyset^*}\frac{\int_0^{T_\emptyset^*-T}e^{-\lambda\int_0^t[1-F(T^*(T+s))]ds}dt-\left(T_\emptyset^*-T\right)e^{-\lambda\int_0^{T_\emptyset^*-T}[1-F(T^*(T+s))]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}[1-F(T^*(T+s))]ds}}dF\left(T\right).$$

Now consider the case in which a VC can handle two projects. Whenever she has a free spot, she will prefer fulfilling that spot instead of going public with the current project to accommodate the new project than cutting a current project. So she will cut projects when they are mature or if she finds a new one and have both vacancies fulfilled. Considering this second case, the Survival rate for a project with size quality T is given by:

$$S\left(t\left|T\right.\right)=\frac{e^{-\lambda\int_0^t G(T+s)[1-F(T^*(T+s))]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T} G(T+s)[1-F(T^*(T+s))]ds}},\quad \text{for } t\in\left[0,T_\emptyset^*-T\right]$$

which is bigger than the previous one, since $G\left(\cdot\right)\leq 1$. It's routine to show that in this case the expected lifetime of a partnership is given by:

$$\int_0^{T_\emptyset^*}\frac{\int_0^{T_\emptyset^*-T} e^{-\lambda\int_0^t G(T+s)[1-F(T^*(T+s))]ds}dt-\left(T_\emptyset^*-T\right)e^{-\lambda\int_0^{T_\emptyset^*-T} G(T+s)[1-F(T^*(T+s))]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T} G(T+s)[1-F(T^*(T+s))]ds}}dF\left(T\right).$$

which is bigger than the previous one.

TABLE 1: Characteristics of VC backed and non VC-backed IPOs

	VC- backed IPOs		Non VC-backed IPOs		
	Mean	*N*	*Mean*	*N*	*T-stat*
Age	7.0	1159	14.7	1446	12.13
Book Value	0.76	1961	6.63	3253	11.28
Revenue	19.9	1732	52.1	2759	3.16
EPS (% pos.)	49.7	806	76.5	1358	12.71
Total Assets	104.4	1719	543.2	2628	3.97
Net proceeds	40.5	2286	58.3	3782	5.44
First-day Return (%)	26.82	2208	19.36	4030	5.99
Underwriter Rank	7.80	2383	6.79	4030	21.02
Gross spread	7.09	2285	7.36	3781	1.76

Source: Lee and Wahal (2004)

TABLE 2: Comparison of characteristics for IPOs backed by young and old VC firms

	VC firms <6 yr. at IPO	VC firms ≥6 yr. at IPO	p-value test of no difference
Avg. time from IPO date to next follow-on fund in months	16.0	24.2	0.001
	[12.0]	[24.0]	[0.002]
Avg. size of next follow-on fund (1997 $mil)	87.9	136.6	0.018
	[68.0]	[113.4]	[0.024]
Avg. age of VC-backed company at IPO date in months	55.1	79.6	0.000
	[42.0]	[64.0]	[0.000]
Avg. duration of board representation for lead VC in months	24.5	38.8	0.001
	[20.0]	[28.0]	[0.000]
Avg. underpricing at the IPO date	0.136	0.073	0.001
	[0.067]	[0.027]	[0.036]
Avg. offer size (1997 $mil)	18.3	24.7	0.013
	[13.0]	[19.1]	[0.000]
Avg. Carter and Manaster underwriter rank	6.26	7.43	0.000
	[6.50]	[8.00]	[0.000]
Avg. number of previous IPOs	1	6	0.000
	[0]	[4]	[0.000]
Avg. fraction of equity held by all VCs prior to IPO	0.321	0.377	0.025
	[0.287]	[0.371]	[0.024]
Avg. fraction of equity held by lead VC after IPO	0.122	0.139	0.098
	[0.100]	[0.120]	[0.031]
Avg. market value of lead VC's equity after IPO (1997 $mil)	9.5	14.7	0.033
	[4.3]	[8.7]	[0.000]
Avg. aftermarket std. deviation	0.034	0.030	0.080
	[0.030]	[0.028]	[0.324]
Number of Observations	99	240	

Note: Sample is 433 VC-backed companies that went public between January 1, 1978 and December 31, 1987. Medians are in brackets. Significance tests in the third column are p-values of t-tests for differences in averages and p-values of two-sample Wilconson rank-sum tests for differences in medians in brackets.
Source: Gompers and Lerner (2001)

TABLE 3: Multivariate Regression with Robust Errors

No. obs.: 1550
F(26, 1523) = 9.64
Prob > F = 0.000
R^2 = 0.239
Root MSE = 60.18

fdr	Coef.	Std. Error	t	P>|t|	[95% Conf. Interval]	
Reg	-0.09982	0.02185	-4.57	0.000	-0.142670	-0.056962
Age	-0.48988	0.11376	-4.31	0.000	-0.713018	-0.266745
AMEXdummy	-7.01536	9.55909	-0.73	0.463	-25.76573	11.73501
NASDQdummy	0.56059	2.79067	0.20	0.841	-4.913374	6.034551
NYSEdummy	-7.36085	4.33795	-1.70	0.090	-15.86984	1.148125
Djod	-0.02486	0.01441	-1.73	0.085	-0.053122	0.003401
Dummy1999	57.81929	20.6124	2.81	0.005	17.38751	98.25106
Health	-11.10043	3.07635	-3.61	0.000	-17.13477	-5.066087
High tech	3.380294	2.82414	1.20	0.232	-2.159312	8.919901
Spod	0.196895	0.11041	1.54	0.125	-0.046873	0.386251
_cons	90.97264	44.9188	2.03	0.043	2.863437	179.0818

Where: reg ≡ days in registration; age ≡ firm age at IPO; djod ≡ dow jones index at offer date; health ≡ dummy for firm in health sector; high tech ≡ dummy for firm in tech sector; spod ≡ S & P 500 at offer date.

TABLE 4: Multivariable Fractional Polynomial Model

Source	SS	df	MS
Model	1830577.8	28	65377.8
Residual	5419252.1	1521	3562.95
Total	7249829.9	1549	4680.33

No. obs.: 1550
F(28,1521) =18.35
Prob > F = 0.000
R^2 = 0.2525
Root MSE = 59.69

fdr	Coef.	Std. Error	t	P>|t|	[95% Conf. Interval]	
lreg	-10.6595	2.183401	-4.88	0.000	-14.9423	-6.3767
Lage1	-0.50835	0.187997	-2.70	0.007	-0.87711	-0.1396
Ldjod_1	-3.47512	0.844651	-4.11	0.000	-5.13193	-1.8183
Ldjod_2	1.26426	0.313333	4.03	0.000	0.64965	1.8789
High tech	2.71745	3.927244	0.69	0.489	-4.98594	10.421
Health	-10.5799	8.051582	-1.31	0.189	-26.3733	5.2135
lspod_1	435.539	92.85873	4.69	0.000	253.394	617.68
lspod_2	-657.809	141.3594	-4.65	0.000	-935.089	-380.53
NASDQdummy	-0.15150	5.607602	-0.03	0.978	-11.1509	10.8479
NYSEdummy	-7.50743	9.710198	-0.77	0.440	-26.5542	11.5393
_cons	107.522	29.09031	3.70	0.000	50.4606	164.583

Where: lreg_1 = ln(reg/100) + 0.3457942617, lnage_1 = age – 7.612903226, Ldjod_1 = $(djod/100)^3$ – 218.8132734, Ldjod_2 = $(djod/100)^3$ln(djod/100) – 393.004593; lspod_1 = $(spod/100)^3$ – 0.4248286209; lspod_2 = $(spod/100)^3$ ln(spod/100) – 0.1212275993.