The Impact of Uncertainty Shocks on the Cross-Section of Returns

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Abstract

Uncertainty shocks can explain the value premium puzzle. Intuitively, the value of growth options increases when uncertainty is high. As a result, growth stocks provide a hedge against uncertainty risk and earn lower risk premiums than value stocks. An investment-based asset pricing model augmented with time-varying uncertainty accounts for both the value premium and the empirical failure of the capital asset pricing model (CAPM). This study also shows that uncertainty shocks influence cross-sectional investment. The investment of value firms is more severely affected by uncertainty shocks than that of growth firms.

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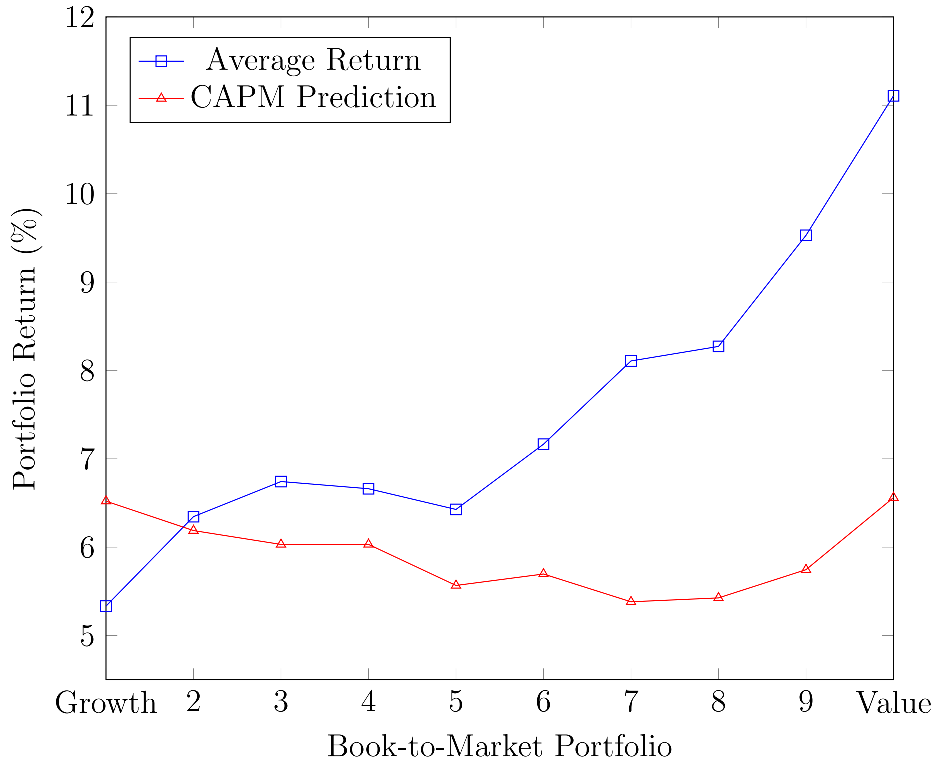
1 Introduction

Time-varying uncertainty, proxied by volatility in stock returns or macro variables, plays an important role in explaining cross-sectional returns. Ang, Hodrick, Xing, and Zhang (2006) and Adrian and Rosenberg (2008) show that market volatility is a significant cross-sectional asset pricing factor. Campbell, Giglio, Polk, and Turley (2016) extend an intertemporal capital asset pricing model (ICAPM) by allowing for stochastic volatility, and they find that volatility news explains the cross-section of returns. Bansal, Kiku, Shaliastovich, and Yaron (2014) and Segal, Shaliastovich, and Yaron (2014) explore the effects of stochastic volatility in a long-run risk model. These studies find that some assets tend to have higher risk loadings on the uncertainty factor, and consequently, show that this factor contributes to explaining the return spread across assets. However, they do not provide the theoretical mechanism for why assets tend to react differentially to the uncertainty factor.

This paper shows that, based on a structural model, time-varying uncertainty can explain the value premium puzzle. Historically, stocks with high book-to-market ratios (value stocks) earn higher average returns than those with low book-to-market ratios (growth stocks); however, this higher return of value stocks, which is called the value premium, cannot be explained by the capital asset pricing model (CAPM). Figure 1 represents the average excess returns of 10 book-to-market sorted portfolios and their expected returns as predicted by the CAPM. It shows that average excess returns rise from the growth to the value portfolio while the CAPM betas are almost the same for all portfolios. This shows that the market factor of the CAPM fails to account for the return spread between growth and value stocks. To resolve the value premium puzzle, one needs to find a new risk factor that is omitted in the asset pricing model but drives the higher risk premiums for value stocks.

I demonstrate that uncertainty shocks are the main driver of the value premium by having a differential impact on cross-sectional firms depending on their holdings of growth options. The main intuition is that the value of growth options increases when uncertainty is high, since high uncertainty expands the upside of future outcomes. As a result, when uncertainty increases, growth stocks, which have more growth options, do better than value

Figure 1: Average Returns of 10 Book-To-Market Portfolios and the CAPM Prediction
January 1963 - December 2015



stocks. Therefore, growth stocks provide a hedge against uncertainty risk and carry lower risk premiums than value stocks.

I show this in an investment-based asset pricing model augmented with time-varying uncertainty by allowing the volatilities of both aggregate and firm-specific productivity shocks to change over time. Uncertainty affects firms differentially according to the amount of their growth options. The channel is the interaction among uncertainty, investment opportunities, and adjustment costs. When firms change their level of capital, they need to pay adjustment costs, such as fixed costs and investment irreversibility. In the presence of uncertainty, these costs cause investment opportunities to behave like financial call options (See Dixit and Pindyck (1994), Pindyck (1988, 1991) and Abel, Dixit, Eberly and Pindyck (1995), among others). Investment corresponds to the exercise of the option. Once firms exercise the option, they lose opportunities that might affect their future investment decisions. This option to invest is valuable, in that it gives firms access to upside benefits. The value of options, like financial call options, increases with uncertainty. Therefore, adjustment costs generate optionality in the model, and time-varying uncertainty causes the value of options to change over time.

The model generates a sizable value premium while replicating the empirical failure of the CAPM in accounting for the value premium. This is an important finding as most prior theoretical studies that reproduce the value premium do not capture the CAPM's failure. For example, Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Cooper (2006), Ozdagli (2012), and Obreja (2013) produce the value premium, but their CAPM betas also increase with the book-to-market ratio. Therefore, in these studies, the model-generated value premium can be counterfactually explained by the CAPM. These results are common in models with productivity shocks as a single source of aggregate risk by which market returns are mainly driven. Consequently, risk premiums tend to be highly correlated with the model-generated market betas contradicting empirical evidence. To resolve this counterfactual prediction, one needs to include additional sources of aggregate risk. My model reproduces the failure of the CAPM by introducing uncertainty shocks as the second source of aggregate risk. I show that the return spreads on book-to-market portfolios are mainly driven by uncertainty shocks. The sensitivities of portfolio returns to uncertainty risk exhibit a distinct pattern that explains the return difference between the growth and the value portfolio. The pattern in the sensitivities still remains after controlling for productivity risk, which implies that the impact of uncertainty shocks on the cross-section of returns is not subsumed by productivity shocks. Therfore, the model produces both cross-sectional returns and flat CAPM betas across the book-to-market portfolios.

Empirical evidence supports the model's predictions. Using the VIX index as a proxy for uncertainty, I show that the returns of growth stocks have higher risk loadings on uncertainty shocks than those of value stocks. This evidence implies that growth stocks tend to do better than value stocks when uncertainty is high. Prior studies show that innovations to aggregate volatility or macroeconomic uncertainty are negatively priced.[1] Given this finding, if growth

[1]See, for example, Bakshi and Kapadia (2003), Ang, Hodrick, Xing, and Zhang (2006), Adrian and Rosenberg (2008), Carr and Wu (2009), Boguth and Kuehn (2013), and Bansal, Kiku, Shaliastovich, and Yaron (2014), Campbell, Giglio, Polk, and Turley (2016).

stocks have significantly higher loadings on uncertainty risk than value stocks, then growth stocks have lower risk premiums than value stocks. In addition, I find that the pattern in uncertainty risk loadings is still significant after controlling for productivity risk, which is consistent with the model predictions.

Uncertainty shocks also have a large impact on corporate investment. Bernanke (1983), Leahy and Whited (1996), Bloom (2009), and Kahle and Stulz (2013) empirically and theoretically show that higher uncertainty reduces investment. My model provides a quantitative prediction on how uncertainty affects investment in the cross-section of firms. I find that value firms more severely reduce their investment than growth firms when economic uncertainty increases. This finding indicates that uncertainty influences investment through Tobin's q. The positive relation between Tobin's q and investment offsets the negative impact of uncertainty on investment. Consequently, the investment of firms with a high Tobin's q is less affected by uncertainty, while that of firms with a low Tobin's q is more severely affected by uncertainty.

Related literature This paper is related to a recent series of studies on how uncertainty shocks affect macroeconomic variables, including Bloom, Bond, and Reenter (2007), Bloom (2009), Ferneries-Slavered and Rubin-Ramrod (2010), Appellant, Bail, and Kedge (2012), Baker and Bloom (2012), Christian, Motto, and Restaging (2012), Scala (2012), and Bloom, Falsetto, Heimlich, Sapota-Eaten, and Terry (2014), and Flagellum, Scala, and Torchere-Debouched (2014). These studies show that uncertainty negatively influences investment, consumption, output, and employment. My paper differs from the existing literature in that it focuses on asset prices. In addition, it investigates whether uncertainty shocks affect the cross-section of investment differentially depending on firm characteristics while prior studies explore the relation between uncertainty and aggregate investment.

My work is also connected to Morton (1973)'s ICAPM. With time-varying investment opportunities, factors in asset-pricing models should include state variables that predict future investment opportunity sets. If an asset has a high covariance with a state variable that improve the investment opportunity set, it has high risk premiums since the asset tends to perform poorly when investment opportunities deteriorate. In an ICAPM setting, Campbell and Vuolteenaho (2004) and Campbell, Polk, and Vuolteenaho (2010) argue that value stocks have higher average returns, as growth stocks perform better when the expected return on the stock market declines. Campbell, Giglio, Polk, and Turley (2016) point out that an increase in the volatility of stock returns also deteriorates investment opportunities. They allow for stochastic volatility and show that growth stocks do well when the volatility of stock returns increases, which is consistent with the results in this study. My paper differs from their study, in that it examines the impact of stochastic volatility on the cross-section of returns based on a structural model, and this allows for the exploration of the mechanism driving the better performance of growth stocks during times of high uncertainty.

This study is also related to the theoretical literature on the value premium. Carlson, Fisher, and Giammarino (2004) investigate the effect of operating leverage on expected returns and demonstrate that the value effect is related to fixed operating costs. Zhang (2005) shows that value stocks are riskier in the presence of investment irreversibility and the counter-cyclical price of risk, and they have higher average returns than growth stocks. Cooper (2006) incorporates the fixed adjustment costs of capital as well as investment irreversibility to explain the value effect. However, these models are based on a single source of aggregate risk, and as a result, they counterfactually generate higher market betas for value firms than for growth firms. In contrast, my model is able to replicate the failure of the CAPM by having two sources of aggregate risk, namely, productivity and uncertainty shocks.

This paper is part of the recent literature that explains both cross-sectional returns and the empirical failure of the CAPM by including additional sources of aggregate risk. Kogan and Papanikolaou (2013) include investment-specific shocks. Ai and Kiku (2013) allow for two sources of aggregate risk, namely, long-run and short-run risk in aggregate consumption growth. Bello, Lin, and Banders (2014) use stochastic adjustment costs, and Bello, Lin, and Yang (2016) use shocks to external financing costs as the second source of aggregate risk.

The organization of the paper is as follows. Section 2 presents the empirical tests. Section 3 describes the model. Section 4 calibrates the model and presents the quantitative results from simulations. Section 5 concludes the study and provides some implications.

2 Empirical Findings

In this section, I conduct some empirical analyses to examine the link between uncertainty shocks and cross-sectional returns. First, I test whether uncertainty shocks can explain the spread in returns between value and growth stocks. To this end, I regress the monthly returns of 10 book-to-market portfolios on the uncertainty factor. If the value of growth stocks increases with uncertainty, they should have higher loadings on the uncertainty factor. Prior studies show that aggregate volatility or macroeconomic uncertainty is negatively priced. This implies that if growth stocks have significantly higher loadings on uncertainty risk than value stocks, growth stocks carry lower risk premiums than value stocks. Second, I test whether the impact of the uncertainty factor is still significant after controlling for the productivity factor. If the impact of uncertainty shocks disappears after controlling for productivity shocks, it means that uncertainty shocks would not be the main driving force of the value premium. By conducting these tests, I examine whether the empirical results support for the introduction of uncertainty shocks as a significant source of aggregate risk into a model.

2.1 Data

I use the VIX index as a proxy for uncertainty. The VIX is the implied volatility of Standard & Poor's 500 index options, calculated from the prices of put and call options traded on the Chicago Board Options Exchange (CBOE). The data are taken from the CBOE, and the sample period is from January 1990 to December 2015 due to availability of data. A daily series of the VIX is aggregated to a monthly frequency by averaging the daily values within the month.

As a proxy for productivity shocks, I use Utilization-adjusted Total Factor Productivity (TFP).[2] The data are obtained from the Federal Reserve Bank of San Francisco. Since the data of Utilization-adjusted TFP are quarterly, I aggregate returns on the portfolios and the VIX by averaging their daily values within the quarter when conducting regression analyses

[2] I also use Business-sector TFP as a proxy for productivity shocks, and the results are quantitatively similar.

that involve TFP. When TFP is used as a single factor for regressions, the sample period is from 1963 to 2015. For regressions that include the VIX together with TFP, the sample period is restricted by VIX data availability and narrows from January 1990 to December 2015.

2.2 Uncertainty and cross-sectional returns

This study predicts that growth stocks tend to do better than value stocks when uncertainty is high, since the value of growth options increases with uncertainty. To test this prediction, I run a regression of the monthly returns of 10 book-to-market portfolios on innovations in the VIX index, controlling for the market factor:

$$r_{it} = \alpha_i + \beta_i^{MKT} MKT_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it}, \tag{1}$$

where r_{it} is the excess return of portfolio i at time t, MKT is the market excess return, ΔVIX_t is the proxy for innovations in the aggregate uncertainty factor, and β_i^{MKT} and β_i^{VIX} are factor loadings of portfolio i on market risk and uncertainty risk, respectively.

Panel A of Table 1 summarizes the results of regression (1). It indicates that growth stocks tend to have higher risk loadings on the uncertainty factor than value stocks. The loading of the value-minus-growth portfolio is -0.27 and is statistically significant ($t = -2.85$), while loadings on the market factor do not present any pattern across portfolios. More importantly, the uncertainty risk loading of the growth portfolio is positive while that of the value portfolio is negative. This means that uncertainty risk exhibits a positive impact on growth stocks but a negative impact on value stocks. Therefore, different from the market factor, the uncertainty factor has explanatory power for the return spread between growth and value stocks. These results suggest that uncertainty risk is potentially an important factor in explaining the value premium and consequently, in explaining why the CAPM fails to capture the value premium.

I also examine whether uncertainty shocks still affect the cross-section of returns after controlling for productivity shocks. In particular, I test whether the returns of book-to-market portfolios have any significant dispersion in their loadings on the uncertainty factor after controlling for the productivity factor. To this end, I conduct the following regression:

$$r_{it} = \alpha + \beta_i^{TFP} \Delta TFP_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it}, \tag{2}$$

where ΔTFP_t is the proxy for innovations in aggregate productivity, and β_i^{TFP} is the factor loading of portfolio i on productivity risk.

Panel B of Table 1 reports the results from regression (2). It shows that risk loadings on the productivity factor do not exhibit any significant dispersion. The loading of the value-minus-growth portfolio is not statistically significant ($t = 0.02$). In contrast, growth stocks tend to have higher loadings on the uncertainty factor, and the loading of the value-minus-growth portfolio is -0.37 and is statistically significant ($t = -2.53$). Therefore, the results indicate that uncertainty risk still accounts for the cross-section of returns after controlling for productivity risk.

Lastly, I examine the impact of uncertainty shocks on portfolio returns after controlling for both market risk and productivity shocks by running the following regression:

$$r_{it} = \alpha + \beta_i^{MKT} \Delta MKT_t + \beta_i^{TFP} \Delta TFP_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it} \tag{3}$$

Panel C of Table 1 presents the results from regression (3). It shows that both loadings on market and productivity factors do not have any patterns across portfolios. In contrast, loadings on the uncertainty factor tend to decrease with the book-to-market ratio, and the loading of the value-minus-growth portfolio is -0.54 and significant at 10% ($t = -1.86$). The pattern in uncertainty risk loadings is less distinct than that in Panel A. This seems to be because a quarterly series of the VIX data is used to match the frequency of the TFP data, and it has less accurate information on the economic uncertainty level than a monthly series of the VIX used in Panel A.

In sum, the evidence from the empirical tests supports the introduction of uncertainty shocks into a model to explain why value stocks have higher average returns than growth stocks and why the CAPM cannot account for the premium. Different from the market factor or the productivity factor, the uncertainty factor has explanatory power for the return spreads from growth to value stocks. More importantly, the uncertainty factor has a positive impact on the growth portfolio after controlling for the market factor while it has a negative impact on the value portfolio. These results suggest that the value premium would be generated by their differences in exposures to uncertainty risk between growth and value stocks; therefore, the model with the uncertainty factor would resolve the value premium puzzle. In the following sections, I examine whether predictions from the model are consistent with the empirical findings.

3 The model

In this section, I develop an investment-based asset pricing model augmented with uncertainty shocks. I explore whether the model can replicate the empirical results reported in the previous section. I also investigate what economic mechanism drives the differential impact of uncertainty shocks on the cross-section of returns.

3.1 Production and investment

In the economy, there is a large number of firms. The production function of firms is given by:

$$Y_{it} = X_t Z_{it} K_{it}^{\eta}. \tag{4}$$

At time t, a firm indexed by i produces output Y_{it}, using physical capital K_{it}. Capital differs across firms and hence generates cross-sectional heterogeneity. The productivity of firms is composed of aggregate productivity, X_t, and firm-specific productivity, Z_{it}. The aggregate productivity is common to all firms and is a source of aggregate risk while the firm-specific productivity is different across firms and is a source of heterogeneity. The capital share, $0 < \eta < 1$, implies that the production function exhibits decreasing returns to scale with capital.

Firms accumulate capital through investment. The investment of firms is as follows:

$$I_{it} = K_{it+1} - (1-\delta)K_{it}, \quad 0 < \delta < 1 \tag{5}$$

where I_{it} denotes the firm's investment, and δ represents the rate of capital depreciation.

3.2 Time varying uncertainty

I assume that both aggregate productivity and firm-specific productivity follow a first-order autoregressive process:

$$x_{t+1} = \bar{x}(1-\rho_x) + \rho_x x_t + \sigma_t^x \varepsilon_{t+1}^x, \tag{6}$$

$$z_{it+1} = \rho_z z_{it} + \sigma_t^z \varepsilon_{it+1}^z, \tag{7}$$

in which $x_t \equiv \log X_t$, $z_{it} \equiv \log Z_{it}$; ε_{t+1}^x and ε_{it+1}^z are uncorrelated for all i; ε_{it+1}^z and ε_{jt+1}^z are uncorrelated for any pair of i, j with $i \neq j$; $\bar{x}$ is the long-term mean of aggregate productivity; and ρ_x and ρ_z are the persistence of aggregate and firm-level productivity, respectively.

Following Bloom (2009), I define uncertainty as the volatility of innovations to aggregate productivity, σ_t^x, and the volatility of innovations to firm-specific productivity, σ_t^z. They are time-varying based on a two-state Markov chain:

$$\sigma_t^x \in \{\sigma_L^x, \sigma_H^x\}, \tag{8}$$

$$\sigma_t^z \in \{\sigma_L^z, \sigma_H^z\}, \tag{9}$$

$$Pr(\sigma_{t+1} = \sigma_j \quad | \quad \sigma_t = \sigma_k) = \pi_{k,j} \tag{10}$$

The time-varying volatility of aggregate productivity (σ_t^x) generates periods of low and high uncertainty in the economy, while the time-varying volatility of firm-specific productivity (σ_t^z) produces periods of low and high cross-sectional dispersion across firms. Since they both follow the same Markov process, periods of high economic uncertainty are accompanied by periods of high cross-sectional dispersion, and vice-versa.

3.3 Adjustment costs

The model incorporates both nonconvex and convex adjustment costs of physical capital.[3] The adjustment costs include installation fees for new equipment, training costs for employees

[3]Convex adjustment costs were the bedrock of investment models in the 1980s; however, they cannot account for lumpy and intermittent investment patterns at a micro-level. Nonconvex adjustment costs can capture these patterns. Cooper and Haltiwanger (2006) show that the combination of convex and nonconvex adjustment costs fits the micro-level investment data best.

on the usage of the equipment, and opportunity costs relating to installing the equipment such as temporarily shutting down factories. The adjustment cost function is given by:

$$\Phi(I_{it}, K_{it}) = \begin{cases} a^{+}K_{it} + \frac{c}{2}\left(\frac{I_{it}}{K_{it}} - \delta\right)^2 K_{it} & \text{for } I_{it} > \delta K_{it} \\ 0 & \text{for } I_{it} = \delta K_{it} \\ a^{-}K_{it} + \frac{c}{2}\left(\frac{I_{it}}{K_{it}} - \delta\right)^2 K_{it} & \text{for } I_{it} < \delta K_{it} \end{cases} \tag{11}$$

where $a^{-} > a^{+} > 0$ and $c > 0$. The adjustment costs arise only from net capital formation, and replacing depreciated capital does not incur any costs.[4]

The first terms of the function, $a^{+}K$ and $a^{-}K$, are the nonconvex part of the adjustment costs, which captures both fixed lump-sum costs and the partial irreversibility of investment. Regardless of the level of investment activity, firms need to pay these costs when they change their current level of capital. The higher coefficient for the investment case than for the disinvestment case ($a^{-} > a^{+} > 0$) represents the partial irreversibility of investment. This irreversibility means that the resale prices of capital are lower than the purchase prices due to transactions costs or information asymmetry (the "lemons" problem; see Akerlof (1970)). Both fixed costs and investment irreversibility generate an inaction region of investment. To avoid paying these costs, firms tend to be cautious when economic uncertainty is high and postpone making their investment decisions until uncertainty in the economy is resolved. Therefore, the presence of nonconvex costs creates real options for investing, which is called the "wait-and-see" effect.

The second term of the function, $\frac{c}{2}\left(\frac{I_{it}}{K_{it}} - \delta\right)^2 K_{it}$, represents convex adjustment costs. Unlike nonconvex costs, convex costs depend on investment rates. This formulation reflects the fact that more rapid changes in capital are more costly.

[4]This formulation leads firms to pay the adjustment costs only when they deviate from the non-stochastic steady state of the investment rate, δ.

3.4 Stochastic discount factor

The stochastic discount factor (SDF) is specified as:

$$\log M_{t+1} = \log \beta - \gamma_x (x_{t+1} - x_t) - \gamma_{\sigma^x} (\sigma^x_{t+1} - \sigma^x_t), \tag{12}$$

where the subjective discount factor is $\beta > 0$, the price of productivity risk is $\gamma_x > 0$, and the price of uncertainty risk is $\gamma_{\sigma^x} < 0$.

The SDF is a function of the two aggregate shocks, productivity shocks and uncertainty shocks. The positive price of productivity risk ($\gamma_x > 0$) implies that productivity risk carries positive premiums while the negative price of uncertainty risk ($\gamma_{\sigma^x} < 0$) implies that uncertainty risk carries negative premiums. The sign of the price of each risk reflects the tendency of asset returns in bad economic times. In the model, there are two types of bad times, which are times of low aggregate productivity and times of high aggregate uncertainty. If some assets do well during times of high aggregate productivity and do poorly during times of low aggregate productivity, they earn low returns when the economy is in a bad state. Investors require compensation for holding these undesirable assets, and consequently, the assets carry high risk premiums. On the other hand, if some assets perform well during times of high aggregate uncertainty and perform poorly during times of low aggregate uncertainty, they would earn high returns when the economy is in a bad state. Therefore, in equilibrium, these assets provide a hedge against uncertainty risk, and thus they carry low risk premiums. The negative price of uncertainty risk is supported by the findings of many empirical studies.[5]

3.5 Optimal Investment

The profit function for firms is as follows:

$$\Pi_{it} = Y_{it} - f, \tag{13}$$

[5]See, for example, Bakshi and Kapadia (2003), Ang, Hodrick, Xing, and Zhang (2006), Adrian and Rosenberg (2008), Carr and Wu (2009), Boguth and Kuehn (2013), and Bansal, Kiku, Shaliastovich, and Yaron (2014), Campbell, Giglio, Polk, and Turley(2016).

where Y_{it} is output and f is fixed operating costs, which must be paid by all firms participating in operational activities.

I denote by $V(K_{it}, Z_{it}; X_t, \sigma_t^x, \sigma_t^z)$ the value function of firms. There are five state variables: (i) capital stock, K_{it}; (ii) firm-specific productivity, Z_{it}; (iii) aggregate productivity, X_t; (iv) time-varying aggregate uncertainty, σ_t^x; and (v) time-varying cross-sectional uncertainty, σ_t^z.

Firms make investment decisions to maximize their market value. Optimal investment is the solution to a dynamic optimization problem defined by the Bellman equation:

$$
\begin{aligned}
V(K_{it}, Z_{it}; X_t, \sigma_t^x, \sigma_t^z) \quad &= \quad \max_{I_{it}} \{\Pi_{it} - I_{it} - \Phi(I_{it}, K_{it}) \\
&+ \quad E_t[M_{t+1} V(K_{it+1}, Z_{it+1}; X_{t+1}, \sigma_{t+1}^x, \sigma_{t+1}^z)]\} \qquad (14)
\end{aligned}
$$

$$
s.t. \quad K_{it+1} = I_{it} + (1-\delta) K_{it}, \qquad (15)
$$

where $V(\cdot)$ is the cum-dividend market value, Π_{it} is profits, I_{it} is investment, $\Phi(\cdot)$ is adjustment costs, E_t is the expectations operator, and M_{t+1} is the stochastic discount factor. The first three terms on the right hand side of (14) represent the present dividend, which is firm profits minus investment minus adjustment costs.

4 Quantitative results

This section presents the quantitative results from the model. In Section 4.1, I discuss calibration of the model and evaluate whether the model can quantitatively capture the important features in the data. In Section 4.2, the main results from the model-simulated data are presented. In Section 4.3, I investigate what features of the model drive the value premium. In Section 4.4, I compute the model-implied VIX index and test whether analyses using it generate similar results to those obtained by using the real VIX index. In Section 4.5, I conduct comparative statics and explore the mechanism in the model. Lastly, in Section 4.6, I investigate the link between uncertainty and cross-sectional investment.

4.1 Calibration

Table 2 reports the parameter values used to calibrate the model. I calibrate the model at a monthly frequency. In total, 100 artificial samples are simulated; each sample includes 5000 firms and 1000 periods. The first 364 periods are dropped to neutralize the impact of initial conditions in the simulations and also to match the length of the sample period with that in the empirical analysis.

I calibrate the model based on two approaches. I first use the parameter values reported in prior literature. Then, for the rest of the parameters, I choose values to match selected moments in the data. I mainly target the annual statistics of risk-free rates, market returns, firm-level investment rates, and firm-level market-to-book ratios. I also target the annual statistics of cross-sectional returns across book-to-market portfolios. I aggregate the simulated data on a yearly basis and compare the target moments from model simulations with those from the annual data. The sample period for target moments is 1963 through 2015. This period is chosen because the value premium puzzle has been more pronounced since 1963. Firm-level accounting data are obtained from COMPUSTAT. The returns of book-to-market sorted portfolios, risk-free rates, and market returns are taken from Kenneth French's website.

Productivity: I set the capital share, $\eta = 0.6$, which is close to the value estimated by Cooper and Ejarque (2001) and Hennessy and Whited (2007). I choose the persistence of aggregate productivity, $\rho_x = 0.983$, following Cooley and Prescott (1995). They report that the quarterly autocorrelation for aggregate output is 0.95. Since I calibrate the model at a monthly frequency, I set $\rho_x = 0.95^{\frac{1}{3}} = 0.983$. I select the persistence of firm-specific productivity, $\rho_z = 0.97$, following Zhang (2005). The long-term average of aggregate productivity is set at -3.9544 to normalize the average long-term capital stock at unity. For the parameter of fixed operating costs, I set $f = 0.0038$ to match the median of the firm-level market-to-book-ratio of 1.69.

Uncertainty process: I choose the average volatility of the aggregate productivity, $\sigma^x = 0.0041$, following Lin and Zhang (2013). There are two states of aggregate productivity volatility: high and low, given as σ^x_L and σ^x_H, respectively. Following Bloom (2009), high volatility has twice the value of low volatility, so that σ_L^x and σ_H^x are chosen to be 0.0038 and 0.0075 to match the average volatility of 0.0041. I select the average volatility of the firm-specific productivity, $\rho_z = 0.1$, following Zhang (2005). There are also two states of the volatility of firm-specific productivity. These are the high and low states, given as σ_L^z and σ_H^z, respectively. High volatility has twice the value of low volatility, so that σ_L^z and σ_H^z are set at 0.0920 and 0.1839 to match their average value of 0.1. The transition probabilities for the uncertainty process, namely $\pi_{L,L}$ and $\pi_{H,H}$, are from Bloom (2009). He calibrates the monthly transition probabilities as 0.9722 and 0.71, respectively.

Adjustment costs: The monthly rate of depreciation, δ, is set at 0.01, which is close to the empirical estimate of Cooper and Haltiwanger (2000). The parameter for convex adjustment costs, c, is chosen to be 0.03 to match the annual volatility of the firm-level investment rates of 15.87%. The parameters for nonconvex adjustment costs, a^+ and a^-, are set at 0.01 and 0.05, respectively, so that the model can match the median of the firm-level investment rates, 12.00%.

Stochastic discount factor: I select the subjective discount factor, $\beta = 0.9985$ to match the average annual risk-free rate of 0.94%. I choose the price of risk of productivity shocks, $\gamma_x = 12$, and the price of risk of uncertainty shocks, $\gamma_{\sigma^x} = -12$, to match the average and volatility of market portfolio returns, 7.50% and 17.23%, respectively.

The comparison between target moments from the data and those from the model-simulated data is summarized in Table 3.[6]

4.2 The value premium

In this subsection, I investigate whether the model generates consistent results with the empirical findings. Specifically, I compare the statistics of 10 book-to market portfolios from the empirical data with those from the model-simulated data.

[6]The Compustat accounting data and CRSP stock returns are used to estimate the moments of firm-level investment rate and market-to-book ratio. The investment rate is defined as $I_{i,t} = \frac{CAPEX_{i,t}}{K_{i,t-1}}$ where $CAPEX$ is capital expenditures, and K is gross property plant and equipment. The rate is bounded above at 1 and below at -0.1.

Table 4 reports the descriptive statistics, including the mean excess returns ($E(r)$), return volatility ($\sigma(r)$), abnormal returns (α), and market betas (β) from the CAPM regressions. Panel A in Table 4 presents the statistics in the data. It shows that portfolio returns monotonically increase from the growth to the value portfolio, and the value premium is 5.77%. Market betas are almost flat across the portfolios, and as a result, the abnormal return of the value-minus-growth portfolio is statistically significant (t-value = 2.20). These results indicate that the CAPM cannot explain why value stocks earn higher average returns than growth stocks.

Panel B in Table 4 summarizes the results from the model-generated data. The book value of a firm is defined as its capital stock, and the market value of a firm is defined as its *ex* dividend stock price.[7] Following Fama and French (1992, 1993), I form book-to-market portfolios for each simulated panel. Panel B shows that the model generates a value premium of 5.29% and also replicates the failure of the CAPM. Market betas are flat across book-to-market portfolios, and as a result, the abnormal return of the value-minus-growth portfolio is statistically significant (t-value = 8.80). Therefore, the model generates the consistent results from the empirical data.

The reproduction of the failure of the CAPM in the model is a key contribution of this paper. The models in prior studies on the value effect generate higher betas for value stocks than growth stocks. Therefore, the return difference between growth and value stocks can be counterfactually explained by the CAPM (see Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Ozdagli (2012), and Obreja (2013)). This issue is common in models with productivity shocks as a single source of aggregate risk. In the economy with one source of aggregate risk, the source affects both market returns and cross-sectional returns. As a result, market betas are highly correlated with risk premiums. By adding uncertainty shocks as a second source of risk, my model generates a multifactor structure of returns and breaks the high correlation between market betas and risk premiums.

[7] $V(K_{it}, Z_{it}; X_t, \sigma_t^x, \sigma_t^z)$ is the *cum* dividend stock price. The current dividend is defined as $D_{it} = \Pi_{it} - I_{it} - \Phi(I_{it}, K_{it})$. The *ex* dividend stock price is the firm value after the dividend is paid out.

4.3 The risk source of the value premium

To understand what drives the value premium in the model, I compute the sensitivities of portfolio returns to risk factors. Section 2 shows that in the data, the sensitivities to uncertainty risk are lower for value stocks than growth stocks, and the pattern still remains after controlling for productivity risk. This evidence implies that uncertainty shocks have explanatory power on the value premium. I explore whether the model generates the same results as the empirical findings.

In the model, uncertainty shocks are defined as the first difference between the current and lagged volatility of innovations to aggregate productivity. Panel A of Table 5 reports loadings of the portfolio returns on the market factor and the uncertainty factor. It shows that loadings on the uncertainty factor monotonically decrease from the growth to the value portfolio, while there are no significant patterns in loadings on the market factor. The loading of the value-minus-growth portfolio is -1.32 and is statistically significant ($t = -2.36$). In addition, the growth portfolio has a significantly positive loading while the value portfolio has a significantly negative loading on the uncertainty factor, which is consistent with the findings in Section 2.2. These results indicate that uncertainty shocks have a positive impact on growth stocks but a negative impact on value stocks. Therefore, growth stocks provide a hedge against uncertainty risk and carry low uncertainty risk premiums.

I investigate whether uncertainty shocks have a differential impact on cross-sectional returns after controlling for productivity shocks. Panel B shows that value stocks still exhibit lower loadings on uncertainty risk than growth stocks. The loading of the value-minus-growth portfolio is -1.35 and is statistically significant ($t = -2.39$). I also run regressions including the market factor, and the results of which are presented in Panel C of Table 5. Different from the empirical results presented in Section 2.2., loadings on the productivity factor have a increasing pattern, and the loading on the value-minus-growth portfolio is -0.19 and is statistically significant at 10% ($t = 1.85$). Given that the price of productivity risk is positive, this result indicates that productivity risk contributes to generating the value premium in the model. Panel C also shows that after controlling for both market risk and productivity risk, loadings on uncertainty risk decrease with the book-to-market ratio, and the loading on the value-minus-growth portfolio is -1.31 and is statistically significant ($t = -2.37$). In addition, the loadings of growth stocks on uncertainty risk become positive, which is consistent with the empirical findings. These results imply that the impact of uncertainty shocks is not subsumed by productivity risk and explains the return spread between growth and value stocks.

4.4 Model-implied VIX index

I compute a model-implied VIX index to ensure that the results from the model are comparable with those from the data. This index is defined as the expected conditional volatility of market returns. The model-implied VIX is calculated as follows:

$$VIX_t = 100 \times \sqrt{12 \times VAR_t(R^M_{t+1})}, \tag{16}$$

where $VAR_t(R^M_{t+1})$ is the monthly conditional variance of market returns. I convert the monthly conditional variance into annualized volatility in percentages, following the method of constructing the real VIX index.

Using the model-implied VIX, I conduct the same analysis as that conducted in the previous subsection, and then compare the results with those from the data. Table 6 reports these results. Panel A presents the sensitivities of portfolio returns to market risk and uncertainty risk, proxied by the model-implied VIX. It shows that sensitivities to market risk do not have any pattern but sensitivities to uncertainty risk are lower for value stocks than growth stocks. The uncertainty risk loading of the value-minus-growth portfolio is -0.15 and statistically significant ($t = -2.54$). In addition, the loadings of growth stocks are positive while those of value stocks are negative. Panel B shows that after controlling for productivity risk, loadings on uncertainty risk are still lower for value stocks than growth stocks. The difference in the loadings is -0.41 and is statistically significant ($t = -2.41$). Panel C shows that after controlling for both market risk and productivity risk, the decreasing pattern in uncertainty risk loadings still remains. The loading of the value-minus-growth portfolio is -0.41 and statistically significant ($t = -2.46$). The panel also shows that the loadings of portfolios on productivity risk are lower for value stocks than growth stocks. Given that the price of productivity risk is positive, this pattern in the loadings cannot explain the higher returns of value stocks.

These results are consistent with the empirical results that the productivity factor does not explain the value premium when the VIX index is used as a proxy for uncertainty shocks.

4.5 Comparative statics

In this subsection, I conduct alternative calibrations with different values for the model parameters in order to investigate the mechanism by which the model generates the value premium. These experiments shed light on which channels make significantly contribute to generating the differential effects of uncertainty shocks on cross-sectional returns. Selected moments from several alternative calibrations are reported in Table 7. Specification 0 and 1 present the results from the data and the benchmark model, respectively.

In specification 2, I shut down uncertainty shocks by setting constant volatilities for both aggregate and firm-specific productivity shocks (i.e., $\sigma_x^L = \sigma_x^H = 0.0041$ and $\sigma_z^L = \sigma_z^H = 0.1$, respectively). Without time-varying uncertainty, the model generates a value premium of 3.00%. In specification 3, I assume that there is no price of risk for uncertainty shocks by setting $\gamma_{\sigma^x} = 0$. This calibration produces a value premium of 3.10%.

Specifications 4, 5, and 6 explore the role of each component for adjustment costs. The results show that alternative calibrations for adjustment costs affect both moments of firm-level investment rates and also the magnitude of value premiums. In specification 4, I shut down the convex part of the adjustment costs by setting the coefficient of the convex term, c, to be zero. Without the convex component, the model generates a value premium of 2.59%. Without the convex part, the model generates a huge volatility of investment rate, which shows that the convex part affects the smoothness of investment activities. In specification 5, I remove investment irreversibility among the nonconvex adjustment costs by setting the coefficient of nonconvex adjustment costs of disinvestment, a^-, to be equal to that of investment, a^+. This means that disinvestment is no longer more expensive than investment. After removing the irreversibility, the model generates a value premium of 2.35%. Finally, in specification 6, I eliminate nonconvex adjustment costs by setting the coefficients of the costs, namely a^+ and a^-, at zero. By doing so, firms only need to pay convex adjustment costs to change their current level of capital. This calibration generates a value premium of 0.49%. Therefore, nonconvex adjustment costs play a key role in producing the value premium. This implies that without nonconvex adjustment costs, there is no optionality in the model, and as a result, investment opportunities do not behave like financial call options. The value of growth options no longer interacts with uncertainty; therefore, the model without nonconvex adjustment costs does not produce the value effect.

4.6 Uncertainty and cross-sectional investment

A series of empirical and theoretical studies shows that uncertainty has a significant negative impact on corporate investment.[8] However, the impact of uncertainty on cross-sectional investment is less emphasized. In this subsection, I explore whether corporate investment exhibits heterogeneous reactions to uncertainty depending on the amount of each firm's investment opportunities.

To assess the effect of uncertainty on cross-sectional investment, I run the following regression using the model-generated data:

$$\frac{I_{it}}{K_{it}} = a_i + \lambda_i \sigma_t + \epsilon_{it}, \tag{17}$$

where I_{it} is an investment flow of portfolio i over time t and K_{it} is capital stock at the beginning of time t. σ_t is uncertainty at time t, defined as the volatility of innovations to aggregate productivity.[9] I also run the same regression using the model-implied VIX index.

Table 8 reports estimates of the impact of uncertainty on the investment of 10 book-to-market portfolios. Panel A shows that uncertainty negatively affects the cross-sectional investment of firms. In addition, it shows that uncertainty has a more negative impact on the investment of firms with higher book-to-market ratios. Value firms exhibit a more severe decrease in their investment than growth firms. Panel B indicates that the results using the model-implied VIX are consistent with those in Panel A.

[8] See, for example, Bernanke (1983), Leahy and Whited (1996), Guiso and Parigi(1999), Bloom, Bond, and Reenen (2007), Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2014), Stein and Stone (2012), Kahle and Stulz (2013), and Gilchrist, Sim and Zakrajsek (2014).

[9] Capital stock, K_{it}, and uncertainty level, σ_t are known at the beginning of time t. Firms determine their optimal investment, I_{it}, based on K_{it} and σ_t.

The finding in Table 8 implies that uncertainty affects investment through Tobin's q, which is consistent with Dixit and Pindyck (1994) and Abel and Eberly (1983). They show that in the presence of investment irreversibility, uncertainty has an effect on investment only through marginal q. Since the production function in the model exhibits decreasing returns to scale, Tobin's q is not equal to marginal q; however, it can serve as a proxy for marginal q. The positive relation between Tobin's q and investment tends to offset the negative effect of uncertainty on investment. As a result, firms with a higher Tobin's q are less affected by uncertainty, while firms with a lower Tobin's q experience a more severe drop in their investment during times of high uncertainty.

5 Conclusion

This study examines the effect of uncertainty shocks on the cross-section of stock returns. Uncertainty has a heterogeneous impact across book-to-market portfolios. Growth firms tend to do better than value firms when uncertainty rises, and therefore, provide a better hedge against uncertainty shocks. The reason for this is that the value of growth options increases with uncertainty.

By introducing uncertainty shocks into an investment-based asset pricing model, my model can reproduce both the value premium and the empirical failure of the CAPM. As such, the model with uncertainty shocks generates a multifactor structure in stock returns and reduces the high correlation between CAPM betas and risk premiums in the models of prior studies on the value premium. The nonconvex adjustment costs of investment play a critical role in producing the value premium. The combination of irreversibility and fixed costs of investment generates the optionality in the model. The value of options is time-varying since uncertainty varies over time. The results from comparative statics show that the model cannot generate the value premium without nonconvex adjustment costs.

This study also finds that uncertainty affects the cross-section of investment. The investment of value firms is more negatively affected by uncertainty than that of growth. This finding suggests that uncertainty influences corporate investment through Tobin's q.

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Table 1 : Uncertainty shocks and cross-sectional returns

This table reports the risk loadings of 10 book-to-market portfolios from the following regressions:

$$\text{Panel A: } r_{it} = \alpha_i + \beta_i^{MKT} MKT_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it},$$

$$\text{Panel B: } r_{it} = \alpha_i + \beta_i^{TFP} \Delta TFP_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it},$$

$$\text{Panel C: } r_{it} = \alpha + \beta_i^{MKT} MKT_t + \beta_i^{TFP} \Delta TFP_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it},$$

where r_{it} is the excess return of portfolio i at time t, MKT is the market excess return, ΔVIX_t is the monthly (quarterly) change in the VIX index, ΔTFP is the quarterly change in Utilization-adjusted TFP, and β_i^{MKT}, β_i^{VIX}, and β_i^{TFP} are loadings of portfolio i on market risk, uncertainty risk, and productivity risk, respectively. The monthly returns on book-to-market portfolios, risk-free rates, and market portfolio returns are taken from Kenneth French's website. A daily series of the VIX index is obtained from the CBOE. A quarterly series of Utilization-adjusted TFP is obtained from the Federal Reserve Bank of San Francisco. For the analysis in Panel A, the VIX is aggregated to a monthly frequency by averaging daily values within the month. For the analyses in Panel B and Panel C, portfolio returns, market excess returns, and the VIX are averaged over every three months to form quarterly observations. Newey-West t-statistics are reported to control for heteroscedasticity and autocorrelation. The number of lags is selected following Newey and West (1994). Due to data availability for the VIX, the sample period is January 1990 through December 2015.

	Growth	2	3	4	5	6	7	8	9	Value	V-G
						Panel A					
β^{MKT}	1.08	0.94	0.93	0.91	0.83	0.90	0.82	0.81	0.89	1.00	-0.08
$t_{\beta^{MKT}}$	22.64	24.68	27.44	14.44	12.86	14.31	12.02	7.23	10.87	8.83	-0.50
β^{VIX}	0.07	-0.04	0.01	-0.08	-0.10	-0.09	-0.05	-0.10	-0.08	-0.20	-0.27
$t_{\beta^{VIX}}$	1.75	-1.17	0.40	-1.89	-1.80	-1.98	-1.18	-1.28	-1.55	-2.91	-2.85
						Panel B					
β^{TFP}	0.29	0.13	-0.10	-0.22	-0.02	-0.20	-0.07	-0.01	0.01	0.30	0.01
$t_{\beta^{TFP}}$	1.05	0.62	-0.50	-0.97	-0.08	-1.04	-0.29	-0.05	0.04	1.03	0.02
β^{VIX}	-0.93	-0.89	-0.76	-0.97	-0.85	-1.01	-0.85	-1.04	-0.95	-1.30	-0.37
$t_{\beta^{VIX}}$	-6.30	-4.93	-3.09	-5.22	-3.93	-5.99	-4.10	-7.44	-4.25	-5.52	-2.53
						Panel C					
β^{MKT}	1.05	0.94	0.99	0.86	0.79	0.87	0.88	0.75	0.89	0.87	-0.18
$t_{\beta^{MKT}}$	12.28	16.32	18.35	8.06	7.24	9.66	6.70	5.37	6.80	4.64	-0.68
β^{TFP}	0.27	0.11	-0.12	-0.24	-0.03	-0.21	-0.08	-0.03	-0.01	0.29	0.01
$t_{\beta^{TFP}}$	1.52	1.04	-1.43	-1.68	-0.22	-1.99	-0.46	-0.14	-0.05	0.86	0.03
β^{VIX}	0.04	-0.03	0.15	-0.18	-0.12	-0.22	-0.05	-0.35	-0.13	-0.51	-0.54
$t_{\beta^{VIX}}$	0.30	-0.60	1.79	-1.74	-0.90	-2.27	-0.34	-2.59	-0.91	-2.72	-1.86

Table 2 : Parameters

This table presents the parameter values used to calibrate the model. The model is calibrated at a monthly frequency.

Parameter	Notation	Value
Productivity		
Capital share	η	0.6
Persistence of aggregate productivity	ρ^x	0.983
Persistence of idiosyncratic productivity	ρ^z	0.97
Long-term average of aggregate productivity	$\bar{x}$	-3.9544
Fixed operating costs	f	0.0038
Uncertainty process		
Low volatility of aggregate productivity	σ^x_L	0.0038
High volatility of aggregate productivity	σ^x_H	0.0075
Low volatility of firm-specific productivity	σ^z_L	0.0920
High volatility of firm-specific productivity	σ^z_H	0.1839
Probability of staying in the low volatility state	$\pi_{L,L}$	0.9722
Probability of staying in the high volatility state	$\pi_{H,H}$	0.71
Adjustment costs		
Depreciation rate	δ	0.01
Convex costs	c	0.03
Nonconvex costs of positive investment	a^+	0.01
Nonconvex costs of negative investment	a^-	0.05
Stochastic discount factor		
Subjective discount factor	β	0.9985
Price of risk for productivity shocks	γ_x	12
Price of risk for uncertainty shocks	γ_{σ^x}	-12

Table 3 : Target moments

This table compares target moments from the real data with those from the model-simulated data. A total of 100 samples are simulated, and each includes 1000 months and 5000 firms. The first 364 months are dropped to neutralize the impact of initial conditions and to match the length of the sample period with that in the real data. The moments from the model-simulated data are the average values of the corresponding moments across simulations. The moments from the real data are from 1963 to 2015.

	Data	Model
Average risk-free rate (%)	0.94	0.96
Volatility of risk-free rate (%)	2.37	5.37
Average market return (%)	7.50	10.44
Volatility of market return (%)	17.23	17.21
Volatility of firm-level returns (%)	37.00	36.92
Median of firm-level investment rate (%)	12.00	12.00
Volatility of firm-level investment rate (%)	15.87	16.00
Median market-to-book ratio	1.69	1.58

Table 4 : CAPM regressions from the empirical data and the model-simulated data

This table compares summary statistics for 10 book-to-market portfolios, including mean excess returns ($E(r)$), return volatility ($\sigma(r)$), abnormal returns (α), and market betas (β) from the real data and the model-simulated data. The sample period of the real data is from 1963 to 2015. The returns on book-to-market portfolios, risk-free rates, and market returns are obtained from Kenneth French's website. In the model, 100 samples are simulated, each including 5000 firms and 1000 periods. The first 364 periods are dropped to neutralize the impact of initial conditions in the simulation and to match the length of the sample period with that in the real data. The reported values are averaged across simulations. Newey-West t-statistics are reported to control for heteroscedasticity and autocorrelation. The number of lags is selected following Newey and West (1994).

	Growth	2	3	4	5	6	7	8	9	Value	V-G
					Panel A: Data						
$E(r)$	5.33	6.35	6.74	6.66	6.43	7.17	8.11	8.27	9.53	11.11	5.77
$\sigma(r)$	17.61	16.20	15.89	16.28	15.33	15.52	15.27	15.64	16.50	20.17	15.91
α	-1.19	0.16	0.71	0.63	0.86	1.47	2.73	2.85	3.78	4.55	5.73
t_α	-1.11	0.23	0.98	0.59	0.82	1.56	2.27	2.12	3.08	2.51	2.20
β	1.07	1.01	0.99	0.99	0.91	0.93	0.88	0.89	0.94	1.07	0.01
t_β	43.78	52.85	40.49	33.88	30.14	31.96	26.26	20.36	23.79	17.85	0.08
					Panel B: Model						
$E(r)$	5.99	7.89	8.70	9.24	9.65	9.96	10.20	10.48	10.73	11.28	5.29
$\sigma(r)$	17.02	16.78	16.59	16.44	16.33	16.13	16.04	15.92	16.37	18.31	5.20
α	-3.69	-1.64	-0.71	-0.08	0.41	0.83	1.15	1.49	1.57	1.28	4.97
t_α	-15.53	-8.72	-3.94	-0.51	2.27	4.61	6.40	7.93	6.92	2.96	8.80
β	1.04	1.03	1.01	1.01	1.00	0.99	0.98	0.97	1.00	1.09	0.05
t_β	96.69	125.18	124.46	113.94	110.86	101.77	100.79	89.78	59.88	27.14	0.79

Table 5 : Results from simulations – Uncertainty shocks and cross-sectional returns

This table reports the risk loadings of 10 book-to-market portfolios from the following regressions:

$$\text{Panel A: } r_{it} = \alpha + \beta_i^{MKT} MKT_t + \beta_i^{\sigma} \Delta\sigma_t + \epsilon_{it},$$

$$\text{Panel B: } r_{it} = \alpha_i + \beta_i^{TFP} \Delta TFP_t + \beta_i^{\sigma} \Delta\sigma_t + \epsilon_{it},$$

$$\text{Panel C: } r_{it} = \alpha_i + \beta_i^{MKT} MKT_t + \beta_i^{TFP} TFP_t + \beta_i^{\sigma} \Delta\sigma_t + \epsilon_{it},$$

where r_{it} is the excess return of portfolio i at time t, MKT is market excess returns, $\Delta\sigma_t$ is uncertainty shocks defined as the monthly change in the volatility of innovations to the aggregate productivity in the model, ΔTFP is productivity shocks defined as the monthly change in aggregate productivity, and β_i^{MKT}, β_i^{σ}, and β_i^{TFP} are loadings of portfolio i on market risk, uncertainty risk, and productivity risk, respectively. A total of 100 samples are simulated, each including 5000 firms and 1000 periods. The first 364 periods are dropped to neutralize the impact of initial conditions in the simulation and to match the length of the sample period with that in the real data. The reported values are averaged across simulations. Newey-West t-statistics are reported to control for heteroscedasticity and autocorrelation. The number of lags is selected following Newey and West (1994).

	Growth	2	3	4	5	6	7	8	9	Value	V-G
					Panel A						
β^{MKT}	1.04	1.03	1.01	1.01	1.00	0.99	0.98	0.97	1.00	1.09	0.05
$t_{\beta^{MKT}}$	96.94	125.37	124.69	114.11	110.96	101.90	100.91	89.90	59.98	27.17	0.79
β^{σ}	0.52	0.31	0.24	0.19	0.08	-0.03	-0.16	-0.34	-0.47	-0.80	-1.32
$t_{\beta^{\sigma}}$	2.47	1.75	1.36	1.14	0.51	-0.11	-0.83	-1.57	-1.93	-1.85	-2.36
					Panel B						
β^{TFP}	-0.18	-0.12	-0.09	-0.09	-0.07	-0.06	-0.05	-0.03	-0.02	0.00	0.18
$t_{\beta^{TFP}}$	-0.32	-0.20	-0.13	-0.12	-0.08	-0.05	-0.03	0.02	0.05	0.11	1.76
β^{σ}	-0.11	-0.31	-0.37	-0.41	-0.51	-0.62	-0.74	-0.92	-1.07	-1.46	-1.35
$t_{\beta^{\sigma}}$	-0.00	-0.11	-0.13	-0.18	-0.24	-0.30	-0.38	-0.48	-0.55	-0.70	-2.39
					Panel C						
β^{MKT}	1.04	1.03	1.01	1.01	1.00	0.99	0.98	0.97	1.00	1.09	0.05
$t_{\beta^{MKT}}$	97.28	125.70	124.93	114.34	111.05	102.14	100.98	90.13	60.07	27.25	0.79
β^{TFP}	-0.10	-0.05	-0.02	-0.01	0.00	0.02	0.02	0.05	0.06	0.09	0.19
$t_{\beta^{TFP}}$	-2.20	-1.09	-0.50	-0.32	0.05	0.41	0.66	1.15	1.32	1.19	1.85
β^{σ}	0.52	0.31	0.24	0.19	0.08	-0.03	-0.16	-0.34	-0.47	-0.79	-1.31
$t_{\beta^{\sigma}}$	2.47	1.74	1.36	1.14	0.51	-0.12	-0.82	-1.57	-1.92	-1.86	-2.37

Table 6 : Results from simulations – Model-implied VIX and cross-sectional returns

This table reports the risk loadings of 10 book-to-market portfolios on the model-implied VIX index. The results are from the following regressions:

$$\text{Panel A: } r_{it} = \alpha + \beta_i^{MKT} MKT_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it},$$

$$\text{Panel B: } r_{it} = \alpha_i + \beta_i^{TFP} \Delta TFP_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it},$$

$$\text{Panel C: } r_{it} = \alpha_i + \beta_i^{MKT} MKT_t + \beta_i^{TFP} TFP_t + \beta_i^{VIX} \Delta VIX_t + \epsilon_{it},$$

where r_{it} is the excess return of portfolio i at time t, MKT is market excess returns, ΔVIX is the monthly change in the model-implied VIX index, defined as the expected conditional volatility of market returns, ΔTFP is productivity shocks defined as the monthly change in aggregate productivity, and β_i^{MKT}, β_i^{VIX}, and β_i^{TFP} are loadings of portfolio i on market risk, uncertainty risk (proxied by the model-implied VIX), and productivity risk, respectively. A total of 100 samples are simulated, each including 5000 firms and 1000 periods. The first 364 periods are dropped to neutralize the impact of initial conditions in the simulation and to match the length of the sample period with that in the empirical data. The reported values are averaged across simulations. Newey-West t-statistics are reported to control for heteroscedasticity and autocorrelation. The number of lags is selected following Newey and West (1994).

	Growth	2	3	4	5	6	7	8	9	Value	V-G
						Panel A					
β^{MKT}	1.04	1.03	1.01	1.01	1.00	0.99	0.98	0.97	1.00	1.09	0.05
$t_{\beta^{MKT}}$	97.20	125.58	124.79	114.47	111.02	102.09	100.93	90.13	60.08	27.25	0.79
β^{VIX}	0.08	0.04	0.02	0.02	0.00	-0.01	-0.02	-0.04	-0.06	-0.08	-0.15
$t_{\beta^{VIX}}$	3.06	1.87	1.16	0.81	0.17	-0.56	-1.14	-1.90	-2.08	-1.77	-2.54
						Panel B					
β^{TFP}	-0.02	-0.06	-0.10	-0.14	-0.20	-0.26	-0.35	-0.45	-0.52	-0.60	-0.59
$t_{\beta^{TFP}}$	-0.03	-0.07	-0.11	-0.16	-0.21	-0.28	-0.39	-0.47	-0.55	-0.59	-1.78
β^{VIX}	0.08	0.03	-0.01	-0.03	-0.07	-0.11	-0.16	-0.22	-0.27	-0.33	-0.41
$t_{\beta^{VIX}}$	0.12	0.01	-0.05	-0.12	-0.19	-0.29	-0.43	-0.53	-0.64	-0.69	-2.41
						Panel C					
β^{MKT}	1.04	1.03	1.01	1.01	1.00	0.99	0.98	0.97	1.00	1.09	0.05
$t_{\beta^{MKT}}$	97.35	125.97	125.06	114.85	111.10	102.26	101.24	90.31	60.21	27.30	0.79
β^{TFP}	0.24	0.19	0.14	0.11	0.04	-0.02	-0.12	-0.22	-0.28	-0.34	-0.58
$t_{\beta^{TFP}}$	2.00	1.84	1.44	1.23	0.49	-0.20	-1.10	-1.62	-1.97	-1.37	-1.80
β^{VIX}	0.18	0.12	0.08	0.07	0.02	-0.02	-0.07	-0.14	-0.18	-0.23	-0.41
$t_{\beta^{VIX}}$	3.05	2.40	1.76	1.42	0.52	-0.41	-1.40	-2.11	-2.43	-1.78	-2.46

Table 7 : Comparative statics

This table presents mean and volatility of market returns ($E(r^m)$ and $\sigma(r^m)$), median and volatility of firm-level investment rates ($E(\frac{I}{K})$ and $\sigma(\frac{I}{K})$), median of firm-level market-to-book ratio ($E(\frac{M}{B})$), and value premiums (VP) from the data, the baseline model, and alternative model calibrations. In specification 2, I shut down uncertainty shocks, by setting constant volatilities for both aggregate and firm-specific productivity shocks (i.e., $\sigma_x^L = \sigma_x^H = 0.0041$ and $\sigma_z^L = \sigma_z^H = 0.1$). In specification 3, I set zero price of uncertainty risk (i.e.,γ_{σ^x}). In specification 4, I shut down the convex part of the adjustment costs by setting its coefficient at zero (i.e., $c = 0$). In specification 5, I remove the investment irreversibility by setting the coefficient of the nonconvex adjustment costs of disinvestment to be equal to that of investment (i.e. $a^+ = a^- = 0.01$). In specification 6, I shut down the nonconvex adjustment costs by setting their coefficients at zero (i.e., $a^+ = a^- = 0$). A total of 100 samples are simulated, each including 5000 firms and 1000 periods. The first 364 periods are dropped to neutralize the impact of initial conditions in the simulation and to match the length of the sample period with that in the empirical data. The reported values are averaged across simulations.

Specification	$E(r^m)$	$\sigma(r^m)$	$E(\frac{I}{K})$	$\sigma(\frac{I}{K})$	$E(\frac{M}{B})$	VP
0-Data	7.50	17.23	12.00	15.87	1.69	5.77
1-Baseline	10.44	17.28	12.00	16.00	1.58	5.29
2-No uncertainty shocks	8.53	16.33	12.00	18.59	1.95	3.00
3-Zero price of uncertainty risk	10.66	15.01	12.00	14.50	1.40	3.10
4-No convex adjustment costs	7.09	16.22	7.22	343.44	2.95	2.59
5-No irreversibility of investment	6.23	15.28	11.85	16.63	2.75	2.35
6-No nonconvex adjustment costs	1.88	16.43	2.54	43.23	7.48	0.49

Table 8 : Results from simulations – Uncertainty shocks and cross-sectional investment

This table reports the estimated loadings on risk factors of 10 book-to-market portfolios and their t-statistics from the following regressions:

Panel A: $\frac{I_{it}}{K_{it}} = a_i + \lambda_i^{\sigma}\sigma_t + \epsilon_{it}$,

Panel B: $\frac{I_{it}}{K_{it}} = a_i + \lambda_i^{VIX} VIX_t + \epsilon_{it}$,

where I_{it} is an investment flow of portfolio i over time t and K_{it} is capital stock at the beginning of time t. σ_t is uncertainty at time t, defined as the volatility of innovations to aggregate productivity. VIX_t is the model-implied VIX index, defined as the expected conditional volatility of market returns. A total of 100 samples are simulated, with each sample containing 5000 firms and 1000 periods. The first 364 periods are dropped to neutralize the impact of initial conditions in the simulation and to match the length of the sample period with that in the empirical data. The reported values are averaged from the 100 samples. Newey-West t-statistics are reported to control for heteroscedasticity and autocorrelation. The number of lags is selected following Newey and West (1994).

	Growth	2	3	4	5	6	7	8	9	Value
					Panel A					
λ^{σ}	-1.89	-1.59	-1.70	-1.88	-2.11	-2.55	-2.74	-3.22	-4.66	-8.79
$t_{\lambda^{\sigma}}$	-6.90	-4.88	-3.84	-3.43	-3.38	-3.25	-3.37	-3.41	-3.60	-3.49
					Panel B					
λ^{VIX}	-0.06	-0.06	-0.05	-0.04	-0.04	-0.03	-0.03	-0.03	-0.04	-0.08
$t_{\lambda^{VIX}}$	-5.22	-5.57	-4.83	-4.18	-3.82	-3.35	-2.93	-2.82	-2.65	-2.86